

Draw a box around your answer. Show your work. Pens/calculators not allowed.

1. Give the domain and range of the function $f(x, y) = \frac{3}{x-y}$ in set notation or interval notation.
- Write the domain in set notation (3 pts)
 - Write the range in set notation or interval notation (1 pt)

Now, sketch the function's domain, tell if it is open/closed/both/neither, and tell if it is bounded/unbounded.

- Sketch the domain correctly (2 pts)
 - Identify the domain as open/closed/both/neither (2 pts)
 - Identify the domain as bounded/unbounded (2 pts)
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2. Sketch three typical level curves for the function $f(x, y) = \frac{3}{x-y}$.

- Use the format $f(x, y) = c$ for level curves (2 pts)
 - Write $f(x, y) = c$ for three values of c (2 pts)
 - Simplify each $f(x, y) = c$ into an identifiable curve equation (3 pts)
 - Sketch each $f(x, y) = c$ in the xy -plane accurately (3 pts)
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3. Compute $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin 2y - \sin 2y}{y - xy}$ without restricting to a path of approach.

- Factor the function appropriately, including using a conjugate if necessary (4 pts)
 - Cancel fractions and apply L'Hopital's Rule as appropriate (3 pts)
 - Compute the correct value of the limit (3 pts)
- (Note: if the student uses a substitution to restrict the path of approach and reduce the number of variables in the limit, these 3 points are the only credit that can be awarded)
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4. Prove that $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{y^2 + x^4}$ doesn't exist.

- Restrict the limit to two paths of approach, or a path of approach depending on k (3 pts)
 - Conclude the limit DNE since the limit equals different values along two different paths of approach (3 pts)
 - Correctly compute the limits along two paths of approach (4 pts)
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5. Find all of the first-order and second-order partial derivatives for $f(x, y) = x^3 + 3xy^2 + e^y$.

- Compute each of f_x , f_y , f_{xx} , $f_{xy} = f_{yx}$, and f_{yy} correctly (2 pts each)

6. Find $\frac{df}{dt}$ at $t = \frac{\pi}{4}$ given $f(x, y, z) = \ln(xyz)$, $x = \cos t$, $y = \sec t$, and $z = 4t^2$.

- If using Chain Rule:
 - Use a correct Chain Rule formula $\frac{df}{dt} = \nabla f \cdot \frac{d\vec{r}}{dt}$ (2 pts)
 - Compute each of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$, $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dz}{dt}$ correctly (1 pt each)
 - Give the correct derivative (2 pts)
 - If using substitution:
 - Plug in correctly (2 pts)
 - Simplify if possible (2 pts)
 - Compute the correct derivative (6 pts)
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7. Find the directional derivative of $f(x, y, z) = 3xy^2 + 6yz^2$ at the point $(1, 2, -1)$ in the direction of $\langle 2, -1, 2 \rangle$.

- Use the formula $\frac{df}{ds_{\vec{u}}} = \nabla f \cdot \vec{u}$ (3 pts)
 - Compute the direction \vec{u} (2 pts)
 - Compute the gradient ∇f (2 pts)
 - Compute the directional derivative (3 pts)
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8. Find the plane tangent to the surface $xy + 2yz - xz = 24$ at the point $(1, 2, -1)$.

- Compute the gradient ∇f (2 pts)
 - Use the plane equation $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ (2 pts)
 - Use the gradient as A, B, C (2 pts)
 - Use the point as x_0, y_0, z_0 (2 pts)
 - Simplify the plane equation correctly (2 pts)
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9. Find and label all the points yielding local maximum values, local minimum values, and saddle points for $f(x, y) = x^3 + 3xy + y^3 + 2$.

- Compute the gradient ∇f (2 pts)
 - Set $\nabla f = 0$ to find its critical points (1 pt)
 - Find all the critical points (2 pts)
 - Compute each of f_{xx}, f_D correctly (1 pt each)
 - Use the 2nd derivative test to correctly label each critical point (0, 1, or 3 pts)
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10. Find the radius and height of a cylinder with volume 2π such that the surface area is as small as possible. (HINT: Use the Method of Lagrange Multipliers along with the formulas $V = \pi r^2 h$ and $SA = 2\pi r h + 2\pi r^2$.)
- Identify and label the function f to optimize (2 pt)
 - Identify and label the restriction function $g = c$ (2 pt)
 - Use the method of Lagrange Multipliers $\nabla f = \lambda \nabla g$ (3 pts)
 - Find the optimizing input (3 pts)
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Include extra scratch work below:

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