

Suppose T be a formula representing a gofirst-fair set of $n+1$ dice extending a formula S for n dice such that for each symbol in T representing a face of the extra die, S restricted below that symbol is permutation-fair. Then T is permutation-fair.

Proof. Since gofirst-fair implies permutation-fair in the base case of one (or two, or zero, whatever) dice, assume the theorem holds for formulas T of length $n+1$, and let T' be length $n+2$ extending S' . \square