CODING STRATEGIES IN BAKER'S GAME

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Abstract. TODO

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Let $f^{\leftarrow}(y) = \{x \in A : f(x) = y\}$.

Proposition 1. *There exists a function $f : \mathbb{R} \to \mathbb{R}^{<\omega}$ such that for each $s \in \mathbb{R}^{<\omega}$, $f^{\leftarrow}(s)$ is dense in $\mathbb{R}$.*

Proof. Noting that $|\mathbb{R}^{<\omega}| = |\mathbb{R}| = \mathfrak{c}$, we recall that $\mathbb{R}$ may be partitioned into $\mathfrak{c}$-many parts, each dense in $\mathbb{R}$ (e.g., the equivalence classes of $x \sim y$ iff $x - y \in \mathbb{Q}$). We may then let f assign each equivalence class to a distinct sequence in $\mathbb{R}^{<\omega}$. $\square$

(We thank Lynne Yengulalp for suggesting the partition result that greatly simplified the preceding proof.)

Theorem 2. $\mathrm{II} \uparrow G(W)$ *if and only if* $\mathrm{II} \underset{code}{\uparrow} G(W)$.

Proof. We need only show $\mathrm{II} \underset{\text{code}}{\not\uparrow} G(W)$ implies $\mathrm{II} \not\uparrow G(W)$. Let σ be a perfect-information strategy for II, and let f be given by Prop 1.

First, we choose

$$\tau(\langle a_0 \rangle) \in \big(a_0, \sigma(\langle a_0 \rangle)\big) \cap f^{\leftarrow}\big(\, \langle a_0 \rangle \,\big)$$

that is, we guarantee $a_0 < \tau(\langle a_0 \rangle) < \sigma(\langle a_0 \rangle)$ and $f\big(\tau(\langle a_0 \rangle)\big) = \langle a_0 \rangle$.

Given b_n, a_{n+1}, define $b'_n = \min\big(b_n, \sigma(f(b_n)^\frown \langle a_{n+1} \rangle)\big)$, noting $a_{n+1} < b'_n$. Now choose

$$\tau(\langle b_n, a_{n+1} \rangle) \in \big(a_{n+1}, b'\big) \cap f^{\leftarrow}\big(f(b_n)^\frown \langle a_{n+1} \rangle\,\big)$$

that is, we guarantee $a_{n+1} < \tau(\langle b_n, a_{n+1} \rangle) < b'_n \leq b_n$ and $f\big(\tau(\langle b_n, a_{n+1} \rangle)\big) = f(b_n)^\frown \langle a_{n+1} \rangle$.

Then τ defines a coding strategy for II; suppose it is defeated by I choosing a_n during round n.

Then if b_n is the move designated by τ for player II during round n, that is, $b_0 = \tau(\langle a_0 \rangle)$ and $b_{n+1} = \tau(\langle b_n, a_{n+1} \rangle)$, we have $f(b_n) = \langle a_0, \ldots, a_n \rangle$.

First, we see that

$$a_0 < a_1 < \tau(\langle a_0 \rangle) < \sigma(\langle a_0 \rangle)$$

And finally, we see that

$$a_{n+1} < a_{n+2} < \tau(\langle b_n, a_{n+1} \rangle) < b'_n \leq \sigma(f(b_n)^\frown \langle a_{n+1} \rangle) = \sigma(\langle a_0, \ldots, a_{n+1} \rangle)$$

Therefore we have shown that a_n is also a legal move against the perfect information strategy σ each round, and therefore σ is also not a winning strategy. $\square$

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