In this paper we investigate an open question posed to us by Gruenhage:

Question 1. *Let P be the subspace of the Sorgenfrey line containing only irrational numbers. Does there exist a base $\mathcal{B}$ for P such that for every $\mathcal{C} \subseteq \mathcal{B}$ which is also a base for X, there exists a locally finite subcover $\mathcal{C}' \subseteq \mathcal{C}$?*

We begin by tackling a simpler (solved) problem:

Proposition 2. *Let R be the Sorgenfrey line, the set of real numbers with the topology generated by the base $\mathcal{B} = \{[a, b) : a < b \in R\}$ (where $[a, b) = \{x : a \leq x < b\}$).*

For every $\mathcal{C} \subseteq \mathcal{B}$ which is also a base for R, there exists a pairwise disjoint subcover $\mathcal{C}' \subseteq \mathcal{C}$ (and thus a locally finite subcover).

Proof. We begin by letting $b_{n,0} = n$ for each $n < \omega$, and if $b_{n,\alpha}$ is defined for some ordinal $\alpha < \omega_1$ and $b_{n,\alpha} < n + 1$, we define its successor $b_{n,\alpha+1}$ as follows:

- $b_{n,\alpha} < b_{n,\alpha+1} \leq n + 1$
- $[b_{n,\alpha}, b_{n,\alpha+1}) \in \mathcal{C}$

  (This is possible as $\mathcal{C}$ is a base, and there must be some element of $\mathcal{C}$ which contains $b_{n,\alpha}$ and is a subset of $[b_{n,\alpha}, n + 1)$.)

(If $b_{n,\alpha} = n + 1$, then let $b_{n,\alpha+1} = n + 1$ as well.) Finally, if $\alpha < \omega_1$ is a limit ordinal, let $b_{n,\alpha} = \lim_{\beta \to \alpha} b_{n,\beta}$.

Let $C_{n,\alpha} = [b_{n,\alpha}, b_{n,\alpha+1})$. We claim $\mathcal{C}' = \{C_{n,\alpha} : n < \omega, \alpha < \omega_1\}$ is a pairwise disjoint cover of R. Pairwise disjoint is evident by definition. To see that it is a cover, suppose it wasn't and missed some $x \in [n, n + 1)$. Then we have an uncountable increasing sequence of numbers $\{b_{n,\alpha} : \alpha < \omega_1\}$, which contradicts the countable chain condition on the real line. $\square$