Proximal compact spaces are Corson compact

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A *topological game* is a two-player game $G(X)$ of length $\omega = \{0, 1, 2, \dots\}$ defined for certain topological spaces X.

During each round n, the first and second player take turns choosing certain topological objects from X (e.g. point, open set, open cover, etc.).

At the "end" of the game, a winner is declared by inspecting the sequences of choices made throughout the game.

The study of such games involves finding when a player has a *winning strategy* which defeats every possible counterattack by the opponent.

Canonical example: *Banach-Mazur Game* (1935) [5]

(insert image here)

The first player wins the game if the intersection of all the chosen open sets is empty.

Theorem

X is Baire if and only if the first player lacks a winning strategy in the Banach Mazur game.

See Telgarsky's excellent survey on topological games for more details: [8]

Proximal Game for compact T_1 zero-dimensional spaces (2011) [1]

(insert image here)

The first player $\mathscr{D}$ wins the game if the points chosen by the second player $\mathscr{P}$ converge. If the first player has a winning strategy for this game, call *X proximal*.

Some results related to the Proximal Game due to Bell:

Proposition

If X is metrizable, then X is proximal.

Theorem

If X is proximal, then X is collectionwise normal.

Theorem

Σ-product and closed subspaces of proximal spaces are proximal.

Corollary

The Σ-product of metrizable spaces is collectionwise normal. [4] [7]

A *Corson compact* space is a space homeomorphic to a compact subset of the Σ-product of real lines.

Peter Nyikos observed:

Proposition

Every Corson compact space is proximal compact. [6]

C. and Gruenhage showed in [2] that any compact proximal space must be Corson compact, using another game-theoretic characterization:

Grunehage's Diagonal Game (1984) [3]:

(show picture here)

The first player $\mathscr{O}$ wins the game if the points chosen by the second player $\mathscr{P}$ converge.

Theorem

If the first player has a winning strategy for Gruenhage's Diagonal Game and X is compact, then X is Corson compact.


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