

Definition 1. A *flexibly Markov* strategy for a game $\langle G, M \rangle$ is a pair of functions $\langle \sigma, \rho \rangle \in M^{M \times \omega} \times M^{M^{<\omega}}$. Intuitively, σ is a Markov strategy using a single move of the opponent and the round number, and τ is a strategy which applies the Markov strategy to each of the opponent's previous moves individually each round.

Let $\sigma' : M^{<\omega} \rightarrow M^{<\omega}$ be defined by $|\sigma'(s)| = |s|$ and $\sigma'(s)(m) = \sigma(s(m), |s|)$. Then more rigorously, $\langle \sigma, \rho \rangle$ is a winning flexibly Markov strategy if and only if $\tau : M^{<\omega} \rightarrow M$ defined by $\tau(s) = \rho \circ \sigma'(s)$ is a winning strategy.

Definition 2. A *semi-flexibly Markov* strategy for a game $\langle G, M \rangle$ is a pair of functions $\langle \sigma, \phi \rangle \in M^{M \times \omega} \times \omega^\omega$. Intuitively, σ is a Markov strategy using a single move of the opponent and the round number, and ϕ is a strategy for choosing how far in the past the move of the opponent should be chosen.

More rigorously, $\langle \sigma, \phi \rangle$ is a winning semi-flexibly Markov strategy if and only if $\tau : M^{<\omega} \rightarrow M$ defined by $\tau(s) = \sigma(s(|s| - 1 - \phi(|s|)), |s|)$ is a winning strategy.

Proposition 3. $\mathcal{F} \underset{\text{semiflexmark}}{\uparrow} Sch_{C,F}^\cup(\kappa)$

Proof. Let ϕ satisfy $|\phi^{-1}(n)| = \omega$ and $\sigma(C, n) = C \upharpoonright n$. Then since σ sees every move of \mathcal{C} infinitely often, it follows that it covers every move of \mathcal{C} . \square

Proposition 4. For compact X , $\mathcal{D} \underset{\text{flexmark}}{\uparrow} Bell_{D,P}^\rightarrow(X)$ if and only if $\mathcal{D} \underset{\text{flexmark}}{\uparrow} Bell_{D,P}^\rightarrow(X)$

Proof. The forward direction is the only interesting direction. In this case, X is Corson compact and embeddable in the Σ -product of reals. Let ϕ satisfy $|\phi^{-1}(n)| = \omega$, and let $\text{supp}(x)$ be the countable support of x , with $\text{supp}_{n+1}(x) \supseteq \text{supp}_n(x)$ finite and $\bigcup_{n < \omega} \text{supp}_n(x) = \text{supp}(x)$.

Let $\sigma(x, n) = D(2^{-n}, \text{supp}_n(x))$. Then since σ sees every move infinitely often, the non-zero coordinates are all Cauchy and converge. \square