

**Example 1.** If  $\mathcal{F}$  is a free ultrafilter on  $\omega$ , let  $L(\mathcal{F}) = \omega \cup \{\mathcal{F}\}$  as a subspace of the Stone-Cech compactification  $\beta\omega$  be the **single ultrafilter line**. There is some ultrafilter  $\mathcal{F}$  such that  $K \uparrow_{\text{pre}} LF_{K,P}(L(\mathcal{F}))$  and  $K \uparrow_{\text{tact}} LF_{K,P}(L(\mathcal{F}))$ .

( $L(\mathcal{F})$  is not compactly generated, and thus not locally compact.)

*Proof.* Let  $a_n$  be a sequence such that the sequence  $\frac{a_n}{n}$  is unbounded above. Then there is an ultrafilter  $\mathcal{F}$  such that  $\sigma(n) = (\sum_{m \leq n} a_m) \cup \{\mathcal{F}\}$  is a winning predetermined strategy for  $K$  in  $LF_{K,P}(L(\mathcal{F}))$ .

Let  $\mathcal{P}$  be the collection of all legal plays by  $P$  against the strategy  $\sigma$ . Consider a finite collection of plays  $P_0, \dots, P_{n-1} \in \mathcal{P}$ . As  $\frac{a_m}{m}$  is unbounded above, we may find infinitely many  $m$  such that  $\frac{a_m}{m} > n \Rightarrow mn < a_m$ . As the  $a_m$  partition  $\omega$  such that  $P$  may only play at most  $m$  points in each part, there are infinitely many parts which are not filled, and thus  $\bigcup_{m < n} P_m$  is not cofinite.

It then follows that the closure of  $\mathcal{P}$  under finite unions and subsets, along with all finite sets, is an ideal. Its dual filter may then be extended to an ultrafilter  $\mathcal{F}$  such that every possible play by  $P$  is the complement of some member of  $\mathcal{F}$ , making  $\sigma$  a winning predetermined strategy.

A winning tactic can then be easily constructed by using the moves by  $P$  as the round number in the predetermined strategy.  $\square$

**Example 2.** Let  $T(\mathcal{F}) = 2^{<\omega}$  where  $2^{<\omega}$  is discrete and for each  $c \in 2^\omega$ ,  $\{c \upharpoonright \alpha : \alpha \leq \omega\}$  is homeomorphic to  $L(\mathcal{F})$ . This is called the **single ultrafilter tree**. There is some ultrafilter  $\mathcal{F}$  such that  $K \uparrow_{\text{pre}} LF_{K,P}(L(\mathcal{F}))$  and  $K \uparrow_{\text{tact}} LF_{K,P}(L(\mathcal{F}))$ .

( $T(\mathcal{F})$  is not compactly generated, and thus not locally compact.)

*Proof.* Assume without loss of generality that  $P$  does not play points in  $2^\omega$ .

We use a winning predetermined strategy  $\sigma^*(n)$  for  $L(\mathcal{F})$  and let  $\sigma(n) = \bigcup_{m \in \sigma^*(n)} 2^m$ . Note that if  $P$  has a counter which converges to some  $c \in {}^\omega 2$ , then  $P$  would have a counter within a single branch. Since each branch is homeomorphic to  $L(\mathcal{F})$ ; this is impossible.

A winning tactic can then be easily constructed by using the moves by  $P$ , taking the level of the tree played upon as the round number in the predetermined strategy.  $\square$

**Example 3.** Let  $M = \omega^2 \cup \{\infty\}$  be the **metric fan** where  $\omega^2$  is discrete and  $\infty$  has neighborhoods of the form  $M \setminus (n \times \omega)$  for any  $n < \omega$ . Then  $K \nmid LF_{K,P}(M)$ . (In fact,  $P \uparrow_{\text{mark}} LF_{K,P}(M)$ .)

( $M$  is not locally compact, but is compactly generated.)

*Proof.* For each compact set  $C$  in  $M$ , there exists a minimal dominating function  $f_C$  such that for each  $(x, y) \in C \setminus \{\infty\}$ ,  $f(x) > y$ .

So let  $P$  respond to the move  $C \in K[X]$  by  $K$  on round  $n$  with the point  $p = (n, s_C)$  such that  $s_C = \min(\{y < \omega : f_C(n) < y\})$ . It is easy to see that  $p_n \rightarrow \infty$ , so  $P$  has a winning Markov strategy.  $\square$

**Example 4.** Let  $S = \omega^2 \cup \{\infty\}$  be the **sequential fan** where  $\omega^2$  is discrete and  $\infty$  has neighborhoods of the form  $M \setminus \{(x, y) : x < f(y)\}$  for any  $f : \omega \rightarrow \omega$ . Then  $K \uparrow_{pre} LF_{K,P}(S)$  and  $K \uparrow_{tact} LF_{K,P}(S)$ .

*( $S$  is not locally compact, but is compactly generated.)*

*Proof.* Let  $\sigma(n) = \omega \times (n+1) \cup \{\infty\}$ . By defining  $f(y)$  to be greater than the  $x$ -coordinate of all  $P$ 's plays through round  $y$ , we see that  $M \setminus \{(x, y) : x < f(y)\}$  misses every move by  $P$ , so  $P$  cannot converge to  $\infty$ .

A winning tactic can be easily constructed by using the  $y$ -coordinate of  $P$ 's moves as the round number in the predetermined strategy.  $\square$