

Example 1. $\mathcal{D} \uparrow_{\text{tact}} Bell_{D,P}^{\rightarrow}(\kappa^*)$, so $\mathcal{D} \uparrow_{\text{mark}} Bell_{D,P}^{\rightarrow}((\kappa^*)^\omega)$.

Definition 2. A space X is *strong Eberlein compact* if it embeds in $\sigma 2^\kappa = \{x \in 2^\kappa : |\{\alpha : x(\alpha) = 1\}| < \omega\}$.

Theorem 3. If X is strong Eberlein compact, then $\mathcal{D} \uparrow_{\text{tact}} Bell_{D,P}^{\rightarrow}(X)$.

Proof. Consider $Bell_{D,P}^{\rightarrow}(\sigma 2^\kappa)$. Let $\text{supp}(x) = \{\alpha : x(\alpha) = 1\} \in [\kappa]^{<\kappa}$.

Define the tactic σ for \mathcal{D} such that

$$\sigma(\langle x \rangle) = \bigcap \{P_\alpha(\Delta) : \alpha \in \text{supp}(x)\}$$

Fix a legal attack $p : \omega \rightarrow \sigma 2^\kappa$, and let $\alpha < \kappa$. If $p_\alpha : \omega \rightarrow \sigma 2^\kappa$ defined by $p_\alpha(n) = p(n)(\alpha)$ converges, then σ is a winning tactic. So assume $p_\alpha(n) \neq 0$ for all $n < \omega$. Then $p_\alpha(n) = 1$ for some n , and as $\alpha \in \text{supp}(p(n))$, $\sigma(p(n)) \subseteq P_\alpha(\Delta)$. As p is a legal attack, it follows that $p_\alpha(m) = p_\alpha(m+1)$ for all $m > n$, so p_α converges.

Since every strong Eberlein compact X embeds as a closed subset of $\sigma 2^\kappa$, it follows that $\mathcal{D} \uparrow_{\text{tact}} Bell_{D,P}^{\rightarrow}(X)$. □