

Remark 1. Scheeper's $S(\kappa)$ requiring injections is stronger than my $S'(\kappa)$ requiring finite-to-one maps. Dow suggests that $S'(\omega_2)$ holds in ZFC. Here's my recreation of his argument.

Claim. *There exists a topology on ω_2 which is locally countable, locally compact, and \aleph_0 -bounded.*

Theorem. $S'(\omega_2)$.

Proof. For $\alpha < \omega_2$, let W_α be a compact neighborhood for which $\beta \in W_\alpha$ implies $\text{rank}\beta \leq \text{rank}\alpha$. \square