Example 1. Let $L \subseteq \prod_{i=1}^{\infty}[0,1]$ be the inverse limit space with bonding functions all equal to

$$f(x) = \begin{cases} 2x & : x \leq 0.5 \\ 2-2x & : x \geq 0.5 \end{cases}$$

Then the following hold:

1. The subspaces $C_t = \{\alpha \in L : \alpha(1) = t\}$ are each homeomorphic to the Cantor Set.
2. All proper subcontinuua are homeomorphic to the unit interval $[0,1]$
3. All proper subcontinuua are nowhere dense in the space.

Proof. For (1), consider that for each number $t \in (0,1)$, there are exactly two preimages under f inside $(0,1)$, giving a natural corespondance with the Cantor tree with branches given by points in the Cantor set 2^{ω}. The arguments for $t = 0,1$ are similar.

For (2), we first claim that all basic open sets are of the form $L \cap \prod_{n=1}^{\infty} B_n$ where $B_n = [0,1]$ for all $n \neq N$. By definition all basic open sets are of the form $L \cap \prod_{n=1}^{\infty} A_n$ where $A_n = [0,1]$ for all $n \neq N_1, \ldots, N_m$. It's easily seen that if $N = N_m$ and $B_N = \bigcap_{i=1}^{m}(f^{-1})^{(N_m - N_i)}(A_{N_i})$ and $B_n = [0,1]$ otherwise, then $L \cap \prod_{n=1}^{\infty} A_n = L \cap \prod_{n=1}^{\infty} B_n$.

Using this fact, we can argue that each proper subcontinuum K is of the form $L \cap \prod_{n=1}^{\infty}[a_n, b_n]$ where $0 \leq a_N < b_N < 1$ for some N.

First note that every maximal basic open set missed by K must be of the form $L \cap \prod_{n=1}^{\infty} B_n$ here $B_n = [0,1]$ for all $n \neq N$ and $[0,1] \setminus B_N$ is connected, that is, $B_N = [0,a) \cup (b,1]$. (If not, then the disconnection of $[0,1] \setminus B_N$ yields a disconnection for K.) Thus $K = \prod_{n=1}^{\infty}[0,1] \setminus ([0,a_n) \cup (b_n,1]) = \prod_{n=1}^{\infty}[a_n,b_n]$.

We can see that $a_n < b_n$ always; otherwise the continuum is a single point. Suppose $b_n = 1$ for all n, then either $a_n = 0$ for all n (contradiction since K is a proper subcontinuum), or $a_N > 0$, and $b_{N+1} \leq 1 - \frac{a_N}{2}$ (contradition).

Finally, considering $[a_N, b_N]$ where $0 \leq a_N < b_n < 1$, we note that as only a single sequence α in K may satisfy $\alpha(N) = t$ for each $t \in [a_N, b_N]$, the projection from K onto the N^{th} coordinate is a homeomorphism onto $[a_N, b_N]$.

For (3), we need only observe that for any sequence $\alpha \in K$, an open neighborhood of α would contain infinte sequences β such that $\alpha(N) = \beta(N)$.

$\square$