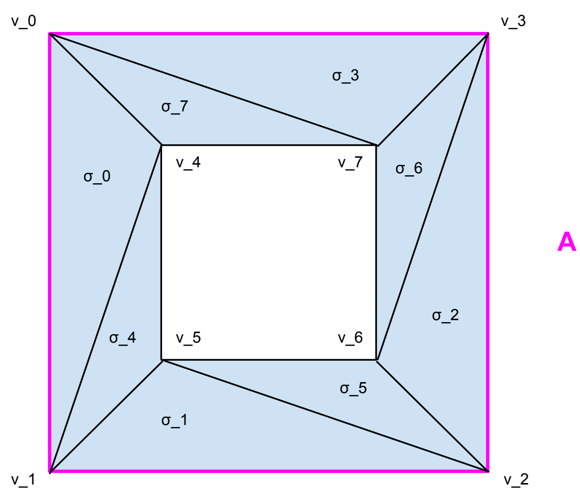
Question 1. *Week 3 (1b)*

Show that there is no no-trivial chain $\gamma \in C_2(\mathcal{S}(E), \mathbb{Z})$ whose boundary is in A.



Proof. Assume γ has boundary completely in A. Then $\partial(\gamma) = \sum_{i<4} k_i [v_i, v_{i+1(\text{mod}4)}]$.

We claim $\gamma(\sigma_i) = 0$ for $4 \leq i < 8$. This can be verified by observing that giving the faces $\sigma_4, \ldots, \sigma_7$ nonzero value in γ results in non-zero boundary values for the edges on the inner square.

Next, we observe that since the values for the 45° degree diagonals $[v_i, v_{i+4}]$ $(i < 4)$ in the boundary of γ are each 0, that the two faces at each edge, σ_0:σ_7, σ_1:σ_3, etc., must be inversely included in γ. Thus $\gamma(\sigma_0) = -\gamma(\sigma_7) = 0$, $\gamma(\sigma_1) = -\gamma(\sigma_4) = 0$, etc.

Therefore $\gamma(\sigma_i) = 0$ for $0 \leq i < 8$. □