

**Theorem: An integer is a multiple of nine if and only if the sum of its digits is a multiple of nine.**

*Proof:* Express the integer in terms of its digits:  $d_0 + 10d_1 + 100d_2 + \dots + 10^n d_n$ . This may be grouped into the sum of its digits  $d_0 + d_1 + \dots + d_n$  plus the sum  $(10 - 1)d_1 + (100 - 1)d_2 + \dots + (10^n - 1)d_n$ . Since each  $10^i - 1$  is divisible by nine...

L	D	Y	T	H	H	W	I
5672	9931	2960	5553	4734	7496	9753	6065
M	K	E	E	C	R	B	X
1373	5574	3510	4324	1676	3564	9082	9875
C	M	E	K	J	S	R	N
6562	1434	1863	5192	4038	2277	9961	9470
V	N	U	A	E	L	B	D
8162	5627	6048	8878	5339	4617	4477	5388
T	A	O	T	F	O	M	H
3862	8296	3844	4473	2646	7677	7953	4650
G	L	H	N	M	L	M	F
7908	4126	2680	5776	6775	9387	5836	4936
E	T	L	B	C	O	G	U
2351	3508	7011	1526	4983	1332	4684	3659
H	C	J	W	S	D	G	U
4685	6577	9593	729	2043	7042	9267	3137