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Theorem: An integer is a multiple of nine iff the sum of its digits is a multiple of nine.

Proof: Express the integer in terms of its digits: $d_0 + 10d_1 + 100d_2 + \ldots + 10^n d_n$. This may be grouped into the sum of its digits $d_0 + d_1 + \ldots + d_n$ plus the sum $9d_1 + 99d_2 + \ldots + (10^n - 1)d_n$. Since each $10^i - 1$ is divisible by 9, the result follows. □

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18	90	45	63	90	72	81	59	14	90	80	36	72	84	90	79	54	19	45	18	90	72	90	72	36
72	46	25	66	23	19	36	42	11	51	45	36	63	12	95	63	38	48	81	22	41	49	15	29	81
54	15	54	54	81	34	72	31	72	72	39	54	45	91	49	36	90	71	36	43	18	18	45	44	27
45	42	45	90	27	39	18	33	96	30	80	49	59	89	90	73	72	44	27	78	81	27	18	73	81
27	62	54	63	27	60	81	85	36	61	27	72	46	18	54	66	36	53	90	56	90	27	18	82	27
54	98	74	24	21	23	63	24	27	34	27	72	90	54	18	63	81	62	45	68	93	91	48	23	54
72	63	54	54	36	27	72	96	18	56	81	75	18	68	36	71	81	20	54	90	63	54	90	36	63
33	98	75	15	23	57	28	47	21	38	36	79	54	54	81	63	90	59	62	47	97	32	94	25	74
90	74	22	27	48	14	81	62	18	72	29	35	63	26	34	72	18	18	76	45	45	70	72	61	22
54	57	22	72	80	82	37	29	51	72	31	49	80	35	81	64	88	58	45	42	27	72	81	64	51
45	18	34	45	26	39	27	29	27	63	18	36	54	97	73	18	34	36	45	66	27	54	36	27	50
75	47	36	45	71	88	58	72	21	50	30	73	75	10	40	54	36	98	27	86	63	26	63	18	17
79	34	48	13	72	63	72	27	41	63	59	66	82	57	63	62	68	81	36	18	90	52	90	23	63
18	91	90	69	63	81	38	31	27	81	13	81	54	25	36	63	39	83	27	60	14	37	27	37	18
11	54	36	35	29	26	90	80	13	45	45	85	27	77	27	81	36	90	89	14	15	51	54	45	45
21	81	70	12	33	63	76	31	35	54	27	72	45	63	38	83	18	63	81	47	58	64	54	36	81
63	45	81	29	36	36	72	18	36	13	45	63	81	24	98	27	90	45	63	36	36	36	69	72	90
46	79	94	76	62	98	56	75	27	81	36	33	93	18	72	97	45	75	91	64	54	30	28	62	52
81	90	27	81	18	90	81	76	88	72	72	18	72	27	55	72	18	65	81	79	45	40	85	34	30
54	70	78	91	47	89	18	67	25	11	80	54	72	49	72	62	63	89	19	76	54	72	81	27	36
90	41	45	54	81	83	81	92	17	43	44	20	61	35	89	45	36	72	18	72	27	72	54	81	50
45	35	90	27	36	64	81	50	45	84	63	17	16	17	62	31	81	82	16	98	72	45	14	81	48
90	14	63	36	90	62	27	68	77	54	37	73	19	72	31	35	39	42	90	41	92	81	22	55	70
72	37	24	35	83	53	81	95	57	54	45	72	36	63	54	82	96	36	27	27	45	72	89	27	45
45	90	45	18	27	45	18	98	63	45	45	85	60	54	81	63	45	72	45	18	47	88	90	83	72