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# Steven Clontz

mathematician, professor,  
programmer, puzzler

steven.clontz@gmail.com  
<http://clontz.org>

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**Theorem:** An integer is a multiple of nine iff the sum of its digits is a multiple of nine.

*Proof:* Express the integer as the sum of its digits:  $d_0 + 10d_1 + 100d_2 + \dots + 10^n d_n$ . This may be grouped into the sum of its digits  $d_0 + d_1 + \dots + d_n$  plus the sum  $9d_1 + 99d_2 + \dots + (10^n - 1)d_n$ . Since each  $10^i - 1$  is divisible by 9, the result follows.  $\square$

63	72	36	54	54	81	72	48	85	50	27	27	90	97	36	51	54	62	63	63	18	36	45	45	36
18	26	40	24	55	11	72	56	80	81	27	88	67	45	58	72	45	14	45	55	69	37	79	62	81
36	76	45	81	54	67	63	38	88	32	40	72	57	89	36	78	18	46	72	10	90	72	72	48	72
90	82	90	54	18	29	81	35	90	45	90	45	63	45	98	18	54	44	81	12	90	27	18	60	45
18	43	81	45	54	58	45	21	16	90	24	63	27	67	54	82	71	66	54	31	18	27	36	40	45
27	74	75	85	94	66	27	11	49	18	45	72	72	82	76	37	45	70	81	12	82	50	87	86	63
90	90	18	54	63	63	27	65	36	74	63	16	63	35	36	94	45	10	18	54	18	63	63	45	36
31	84	73	74	55	38	65	83	93	54	21	75	83	46	27	81	45	92	24	22	43	22	20	68	37
63	27	88	72	36	36	90	26	14	79	26	27	36	69	54	90	72	81	49	19	59	81	52	48	43
24	73	18	45	63	72	78	72	45	77	27	43	45	63	70	89	27	16	90	91	61	18	81	45	29
98	54	25	64	65	72	54	90	49	90	42	17	18	90	45	22	91	81	77	39	20	81	53	20	27
27	87	94	97	27	36	98	84	63	81	10	60	74	87	18	84	51	30	27	79	56	68	17	89	54
37	19	24	50	10	73	36	61	65	65	27	54	30	16	71	95	53	18	55	54	87	27	18	32	10
68	63	12	92	75	18	92	98	95	45	44	63	54	14	96	72	87	24	45	36	27	36	81	25	13
58	90	84	90	27	36	36	72	73	91	61	63	55	36	18	25	70	29	67	54	12	81	57	55	36
63	63	85	63	30	95	41	27	18	81	65	36	34	45	82	81	37	36	54	82	58	63	38	46	63
72	81	72	93	72	46	27	54	17	25	54	20	27	36	54	72	72	18	45	36	90	81	59	45	81
32	64	48	74	22	47	29	37	43	81	36	45	97	36	41	11	27	52	84	28	36	92	43	64	36
27	72	45	72	90	81	90	55	22	45	36	72	81	54	76	54	54	37	72	20	81	37	89	24	94
18	62	16	62	23	41	72	29	72	48	54	18	45	54	72	96	90	60	78	66	27	54	32	27	81
27	91	90	90	81	57	36	15	72	79	48	18	88	80	90	27	36	18	81	81	90	54	72	16	39
36	55	81	63	45	75	36	71	72	97	90	85	39	67	62	38	63	84	78	97	27	63	29	72	25
54	44	63	18	27	80	45	61	39	72	45	92	49	49	40	32	63	25	63	45	31	54	63	57	13
27	56	31	30	75	91	63	32	81	72	27	71	45	18	77	84	96	21	27	45	33	27	47	21	45
81	90	54	18	81	90	18	68	81	90	27	32	21	81	54	54	81	72	54	90	48	95	27	91	18