

STEVEN CLONTZ

Research Statement

My scholarship is primarily focused on a strong research program studying general topology, particularly selection games. For context, a selection game is a two-player game that characterizes the structure of a topological space based upon which player has a winning strategy, a rule that a player can use to guarantee victory in the game no matter how their opponent plays. Key results published during my time at South include the following.

Theorem: A family of almost-compatible finite-to-one functions mapping countable subsets of a cardinal to the natural numbers cannot be used to construct a family of one-to-one functions with the same property. [0]

This result has important ramifications in set theory and topology. In particular, such a family of finite-to-one functions for the second uncountable cardinal may be constructed using the usual ZFC axioms of set theory, but if the Continuum Hypothesis holds, then it is impossible to construct such one-to-one functions. Considering the Menger selection game characterizing topological properties of a topological space, finite-to-one functions are sufficient to design “Markov” strategies that are able to track the number of moves made in a topological game, but one-to-one functions seem to be required to design “tactical” strategies that do not need to track this additional information.

Theorem: A Markov strategy for the Menger selection game using finite information may always be improved to only require memory of the two most recent moves of the opponent. [1]

This result is striking as it does not hold for another well-studied game attributed to Banach and Mazur which characterizes the Baire topological property. Telgarsky conjectures that there exists a topological space for which a winning strategy using three moves of the opponent exists for the Banach-Mazur game, but it cannot be improved to a strategy using two moves of the opponent. My result demonstrates why this conjecture is not a trivial assumption to make.

Theorem: Any strategy for a selection game satisfying a certain countability requirement may be improved to a Markov strategy requiring memory of only the most recent move of the opponent. [2]

This result improves the previous theorem whenever the countability requirement is satisfied, as is the case in many applications. The importance of this paper is witnessed by its inclusion in an upcoming special issue of Topology and its Applications dedicated to selection principles and their applications.

In addition to the papers cited above, I have two co-authored papers currently under review, making my research productivity around two peer-reviewed papers a year. To date every paper I have submitted for publication has been accepted pending minor revisions, and the journals I publish in are the standard for publications in my field. For comparison, my PhD advisor (Gary

Gruenhagen, Auburn University) has published fifteen out of his eighteen research papers over the past decade from these same journals.

[0]: Clontz, Steven; Dow, Alan; Almost compatible functions and infinite length games. Rocky Mountain J. Math. 48 (2018), no. 2, 463483.

[1]: Clontz, Steven; Applications of limited information strategies in Menger's game. Comment. Math. Univ. Carolin. 58 (2017), no. 2, 225239.

[2]: Clontz, Steven; Relating games of Menger, countable fan tightness, and selective separability. Topology Appl., to appear.