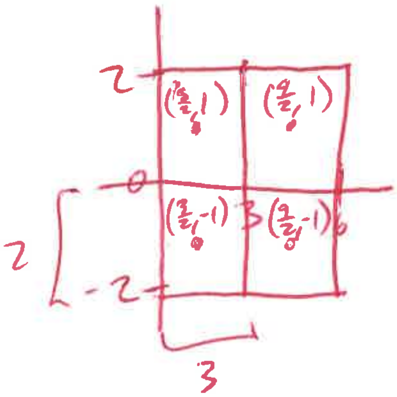
Calculus III - Spring 2015 - Mr. Clontz - Test 3

Name: Solutions Class: ________ Type: Group

- This exam is open-"everything", provided you do not plagiarize. *Do not leave any unsupported answers!*
- Write your solutions so that a fellow student who has a perfect understanding of previous math classes and packets but has never seen that type of problem before could follow your work.
- Individual Test: You will have 40 minutes to complete this test on your own. You may not communicate with anyone during this period.
- Group Test: You will have 40 minutes to complete an identical test. You may collaborate with your group members during this time. All solutions must still be written by yourself, and may not be directly copied from another student.

1. (5 points) Divide $R : 0 \leq x \leq 6, -2 \leq y \leq 2$ into four congruent pieces arranged two-by-two, and then use the midpoint rule to give an arithmetic expression approximating the double integral $\iint_R \underbrace{2xy^2}_{f(x,y)}\, dA$. (You do not need to simplify the arithmetic.)



$$\iint_R 2xy^2\, dA \approx \sum_{\text{midpoints}} f(x,y)\,\Delta A$$

$\Delta A = 6$

$$= \boxed{2\left(\tfrac{3}{2}\right)(1)^2(6) + 2\left(\tfrac{9}{2}\right)(1)^2(6) + 2\left(\tfrac{3}{2}\right)(-1)^2(6) + 2\left(\tfrac{9}{2}\right)(-1)^2(6)}$$

$$\left(\begin{aligned} &= 18 + 54 + 18 + 54 \\ &= 72 + 72 \\ &= 144 \end{aligned} \right)$$

2. (5 points) Evaluate $\iint_R 2xy^2\, dA$ for $R : 0 \leq x \leq 6, -2 \leq y \leq 2$.

$$= \int_{-2}^{2}\int_{0}^{6} 2xy^2\, dx\, dy$$

$$= \int_{-2}^{2} \left[x^2y^2\right]_0^6 dy$$

$$= \int_{-2}^{2} 36y^2\, dy$$

$$= \left[12y^3\right]_{-2}^{2}$$

$$= 12(8) - 12(-8)$$

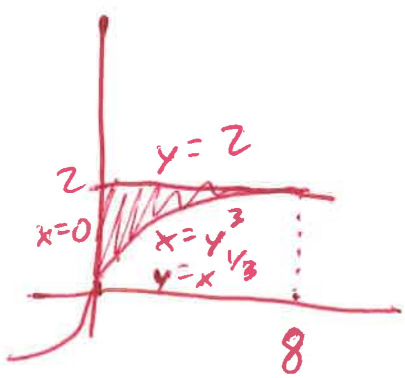
$$= 96 + 96$$

$$= 192$$

3. (5 points) Rewrite the Type II integral

$$\int_{x=0}^{x=8}\int_{y=x^{1/3}}^{y=2} \frac{1}{1+y^2}\, dy\, dx$$

as a Type I integral by switching the order of integration. (You do not need to evaluate this integral.)



$$= \int_0^2 \int_0^{y^3} \frac{1}{1+y^2}\, dx\, dy$$

$$= \int_0^2 \frac{y^3}{1+y^2}\, dy$$

Let $u = 1+y^2$, $\frac{1}{2}du = y\,dy$

$$= \int_1^5 \frac{\frac{1}{2}(u-1)}{u}\, du$$

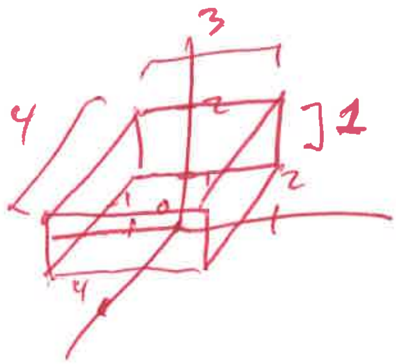
$$= \int_1^5 \frac{1}{2} - \frac{1}{u}\, du$$

$$= \left[\frac{1}{2}u - \ln u\right]_1^5$$

$$= \left(\frac{5}{2} - \ln 5\right) - \left(\frac{1}{2} - \cancel{\ln 1}\right)$$

$$= 2 - \ln 5$$

4. (5 points) Express the average value of the function $f(x, y, z) = xy^2 - z$ over the solid cube given by $0 \le x \le 4$, $-1 \le y \le 2$, and $1 \le z \le 2$ as a quotient involving one or more triple integrals. (You do not need to evaluate any integrals.)



Volume $= 12$

$$\text{Avg val} = \frac{1}{\text{Volume}} \iiint_D xy^2 - z \, dV$$

$$= \frac{1}{12} \int_0^4 \int_{-1}^2 \int_1^2 xy^2 - z \, dz\,dy\,dx$$

5. (5 points) Use the transformation $\vec{r}(u,v) = \langle u+v, 1-u+3v\rangle$ from the unit square in the uv plane to the parallelogram R with vertices $(0,1),(1,0),(2,3),(1,4)$ in the xy plane to change

$$\iint_R x + y \, dA$$

to an integral of the varibles u and v. (You do not need to evaluate this integral.)

$$\vec{r}_J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 3-(-1) = 4$$

$$\iint_R x+y \, dA = \iint_G \big(x(u,v) + y(u,v)\big)\left|\vec{r}_J(u,v)\right| dA$$

$$= \int_0^1\int_0^1 \big((\cancel{u}+v) + (1-\cancel{u}+3v)\big)(4)\,dv\,du$$

$$= \int_0^1\int_0^1 4+16v\,dv\,du$$

$$\left(\begin{aligned} &= \int_0^1 12\,du \\ &= 12 \end{aligned} \right)$$

6. (5 points) Describe the geometric property that

Cylindrical coordinates

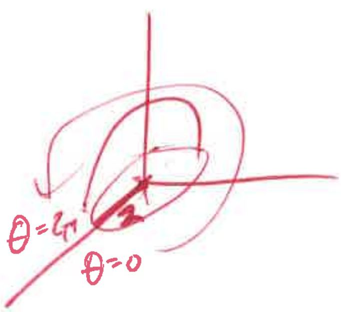
$$\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

represents. Be as specific as possible. (You shouldn't have to evaluate the integral to answer this question.)

$z=\sqrt{4-r^2}$

$z=\sqrt{4-x^2-y^2}$

$x^2+y^2+z^2=4 \ (z\geq 0)$



Since volume $=\iiint_D 1\,dV$

$=\iiint_G r\,dV$ in cylindrical coordinates, this is the volume of a hemisphere of radius 2.