54

$\frac{215}{224} \approx \frac{20}{20}$

Packet 2

Part 1: Sections 13.3-13.4

13.3 Arc Length and Curvature

4/4

Problem 1. Let $\vec{\mathbf{r}}(t) = \langle 6t, t^3, 3t^2\rangle$. Use the lengths of the line segments connecting $\vec{\mathbf{r}}(0)$, $\vec{\mathbf{r}}(1)$, $\vec{\mathbf{r}}(2)$, and $\vec{\mathbf{r}}(3)$ to approximate the length of the curve from $t = 0$ to $t = 3$.

Solution. $\vec{\mathbf{r}}(t) = \langle 6t, t^3, 3t^2\rangle$

$\vec{\mathbf{r}}(0) = \langle 6(0), (0)^3, 3(0)^2\rangle = \langle 0, 0, 0\rangle$
$\vec{\mathbf{r}}(1) = \langle 6(1), (1)^3, 3(1)^2\rangle = \langle 6, 1, 3\rangle$
$\vec{\mathbf{r}}(2) = \langle 6(2), (2)^3, 3(2)^2\rangle = \langle 12, 8, 12\rangle$
$\vec{\mathbf{r}}(3) = \langle 6(3), (3)^3, 3(3)^2\rangle = \langle 18, 27, 27\rangle$

Distance between these points:

Distance from $t = 0$ to $t = 1$:

$$\sqrt{(6-0)^2 + (1-0)^2 + (3-0)^2}$$
$$= \sqrt{46}$$

Distance from $t = 1$ to $t = 2$:

$$\sqrt{(12-6)^2 + (8-1)^2 + (12-3)^2}$$
$$= \sqrt{166}$$

Distance from $t = 2$ to $t = 3$:

$$\sqrt{(18-12)^2 + (27-8)^2 + (27-12)^2}$$
$$= \sqrt{622}$$

Arc Length $\approx \sqrt{46} + \sqrt{166} + \sqrt{622} \approx$? ◊

Definition 2. Let $\vec{\mathbf{r}}(t) = \langle f(t), g(t), h(t)\rangle$ be a vector function. Then the **arclength** or **length** of the curve given by $\vec{\mathbf{r}}(t)$ from $t = a$ to $t = b$ is

$$L = \int_a^b \left| \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t + \Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} \right| dt = \int_a^b |\vec{\mathbf{r}}'(t)|\, dt$$

4/4

Problem 3. Find the length of the curve given by $\vec{\mathbf{r}}(t) = \langle 6t, t^3, 3t^2\rangle$ from $t = 0$ to $t = 3$. (Hint: $9t^4 + 36t^2 + 36$ is a perfect square polynomial.)

Solution.

$$L = \int_a^b \left| \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t+\Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} \right| dt = \int_a^b |\vec{\mathbf{r}}'(t)|\, dt$$

$$\vec{\mathbf{r}}'(t) = \langle 6, 3t^2, 6t \rangle$$

$$|\vec{\mathbf{r}}'(t)| = \sqrt{6^2 + (3t^2)^2 + (6t)^2}$$

$$|\vec{\mathbf{r}}'(t)| = \sqrt{36 + 9t^4 + 36t^2}$$

$$|\vec{\mathbf{r}}'(t)| = 3\sqrt{4 + t^4 + 4t^2}$$

$$|\vec{\mathbf{r}}'(t)| = 3\sqrt{(t^2+2)^2}$$

$$|\vec{\mathbf{r}}'(t)| = 3(t^2+2)$$

$$L = \int_a^b |\vec{\mathbf{r}}'(t)|\, dt = \int_0^3 3t^2 + 6dt$$

$$L = t^3 + 6t]_0^3$$

$$L = 3^3 + 6(3) - 0 = 45$$

◊

Definition 4. Let $s(t)$ be the **arclength function/parameter** representing the length of a curve from the point given by $\vec{\mathbf{r}}(0)$ to the point given by $\vec{\mathbf{r}}(t)$. (Assume $s(t) < 0$ for $t < 0$.)

Theorem 5. The arclength function $s(t)$ is given by the definite integral

$$s(t) = \int_0^t |\vec{\mathbf{r}}'(\tau)|\, d\tau$$

Theorem 6. The derivative of the arclength function gives the lengths of the tangent vectors given by the derivative of the position function:

$$\frac{ds}{dt} = \left| \frac{d\vec{\mathbf{r}}}{dt} \right|$$

Problem 7. Compute $s(t)$ for $\vec{\mathbf{r}}(t) = \langle 6t, t^3, 3t^2 \rangle$, and use it to find the arclength parameter corresponding to $t = -2$.

4/4

Solution.

$$\vec{\mathbf{r}}(\tau) = \langle 6\tau, \tau^3, 6\tau \rangle$$

$$\vec{\mathbf{r}}'(\tau) = \langle 6, 3\tau^2, 6\tau \rangle$$

$$|\vec{\mathbf{r}}'(\tau)\,| = \sqrt{6^2 + (3\tau^2)^2) + (6t)^2)}$$

$$= \sqrt{9(4 + \tau^4 + 4\tau^2)}$$

$$= 3(\tau^2 + 2)$$

$$s(t) = \int_0^t 3\tau^2 + 6\, d\tau$$

$$s(-2) = t^3 + 6t]_{-2}^{0}$$

$$s(-2) = (0)^3 + 6(0) - ((-2)^3 + 6(-2))$$

$$s(-2) = 20$$

◊

4/4

Problem 8. Find the length of an arc of the circular helix with vector equation $\vec{\mathbf{r}}(t) = \langle \cos(t), \sin(t), t \rangle$ from $(1, 0, 0)$ to $(1, 0, 2\pi)$.

Solution.

$$L = \int_0^{2\pi} \sqrt{(-sin^2 t) + cos^2 t + 1^2} dt$$

So:

$$t = 0, t = 2\pi$$

$$L = \int_0^{2\pi} \sqrt{sin^2 t + cos^2 t + 1^2} dt$$

$$L = \int_0^{2\pi} \sqrt{2} dt$$

$$\sqrt{2}t]_0^{2\pi}$$

$$2\sqrt{2}\pi \approx 8.9$$

◊

Definition 9. The **unit tangent vector** $\vec{\mathbf{T}}$ to a curve $\vec{\mathbf{r}}$ is the direction of the derivative $\vec{\mathbf{r}}'(t) = \frac{d\vec{\mathbf{r}}}{dt}$.

Theorem 10.

$$\vec{\mathbf{T}} = \frac{d\vec{\mathbf{r}}/dt}{|d\vec{\mathbf{r}}/dt|} = \frac{d\vec{\mathbf{r}}}{ds}$$

4/4

Problem 11. Find the unit tangent vector to the curve given by $\vec{\mathbf{r}}(t) = \langle 3t^2, 2t \rangle$ at the point where $t = -3$.

Solution.

$$\vec{\mathbf{T}} = \frac{d\vec{\mathbf{r}}/dt}{|d\vec{\mathbf{r}}/dt|}$$

$$\vec{\mathbf{T}} = \frac{\langle 6t, 2\rangle}{\sqrt{36t^2+4}}$$

$$\vec{\mathbf{T}} = \frac{\langle -18, 2\rangle}{\sqrt{328}}$$

$$\vec{\mathbf{T}}(t = -3) = \langle \frac{-18}{\sqrt{328}}, \frac{2}{\sqrt{328}} \rangle$$

◊

Definition 12. The **curvature** κ of a curve C at a given point is the magnitude of the rate of change of $\vec{\mathbf{T}}$ with respect to arclength s.

Theorem 13.

$$\kappa = \left|\frac{d\vec{\mathbf{T}}}{ds}\right| = \left|\frac{1}{ds/dt}\frac{d\vec{\mathbf{T}}}{dt}\right| = \frac{1}{|d\vec{\mathbf{r}}/dt|}\left|\frac{d\vec{\mathbf{T}}}{dt}\right|$$

Theorem 14. An alternate formula for curvature is given by

$$\kappa = \frac{|\vec{\mathbf{r}}'(t) \times \vec{\mathbf{r}}''(t)|}{|\vec{\mathbf{r}}'(t)|^3}$$

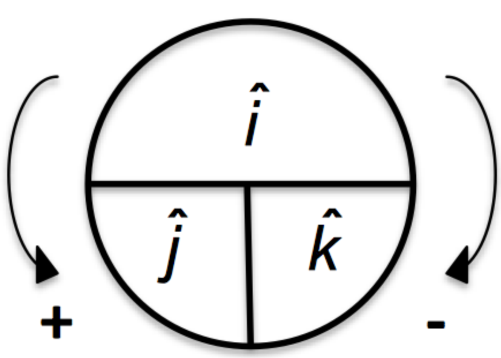
Problem 15. Prove that the helix given by the vector equation $\vec{\mathbf{r}}(t) = \langle \cos(t), \sin(t), t\rangle$ has constant curvature.

4/4

Solution.

$$\kappa = \frac{|\langle -sin(t), cos(t), 1\rangle \times \langle -cos(t), -sin(t), 0\rangle|}{|\langle -sin(t), cos(t), 1\rangle|^3}$$

$$\kappa = \frac{|\langle sin^2(t)\widehat{\mathbf{i}} \times \widehat{\mathbf{j}} - cos^2(t)\widehat{\mathbf{j}} \times \widehat{\mathbf{i}} - cos(t)\widehat{\mathbf{k}} \times \widehat{\mathbf{i}} - sin(t)\widehat{\mathbf{k}} \times \widehat{\mathbf{j}}\rangle|}{|\langle -sin(t), cos(t), 1\rangle|^3}$$



$$\kappa = \frac{|\langle sin^2(t)\widehat{\mathbf{k}} + cos^2(t)\widehat{\mathbf{k}} - cos(t)\widehat{\mathbf{j}} + sin(t)\widehat{\mathbf{i}}\rangle|}{|\langle -sin(t), cos(t), 1\rangle|^3}$$

$$\kappa = \frac{|\langle sin^2(t)\widehat{\mathbf{k}} + cos^2(t)\widehat{\mathbf{k}} - cos(t)\widehat{\mathbf{j}} + sin(t)\widehat{\mathbf{i}}\rangle|}{|\langle -sin(t), cos(t), 1\rangle|^3}$$

$$\kappa = \frac{|\langle sin(t)\widehat{\mathbf{i}} - cos(t)\widehat{\mathbf{j}} + 1\widehat{\mathbf{k}}\rangle|}{|\langle -sin(t), cos(t), 1\rangle|^3}$$

$$\kappa = \frac{|\langle \sin t\widehat{\mathbf{i}} - \cos t\widehat{\mathbf{j}} + 1\widehat{\mathbf{k}}\rangle|}{\sqrt{[sin^2(t) + cos^2(t) + (1)^2]^3}}$$

$$\kappa = \frac{|\langle \sin t\widehat{\mathbf{i}} - \cos t\widehat{\mathbf{j}} + 1\widehat{\mathbf{k}}\rangle|}{\sqrt{\sin^2 t + \cos^2 t + (1)^2}^3}$$

$$\kappa = \frac{\sqrt{\sin^2 t + (-\cos t)^2 + 1^2}}{\sqrt{\sin^2 t + \cos^2 t + (1)^2}^3}$$

$$\kappa = \frac{2^{\frac{1}{2}}}{2^{\frac{3}{2}}}$$

$$\kappa = \frac{1}{2}$$

constant

κ is a real number, therefore the helix has constant curvature.

◊

Problem 16. (OPTIONAL) Prove that the alternate formula for curvature is accurate by showing

$$\frac{1}{|d\vec{\mathbf{r}}/dt|}\left|\frac{d\vec{\mathbf{T}}}{dt}\right| = \frac{|\vec{\mathbf{r}}' \times \vec{\mathbf{r}}''|}{|\vec{\mathbf{r}}'|^3}$$

(Some of the solution has been provided.)

Solution. Begin by observing that $\vec{\mathbf{r}}' = \left|\frac{d\vec{\mathbf{r}}}{dt}\right|\vec{\mathbf{T}} = \frac{ds}{dt}\vec{\mathbf{T}}$, and by the product rule it follows that $\vec{\mathbf{r}}'' = \frac{d^2s}{dt^2}\vec{\mathbf{T}} + \frac{ds}{dt}\vec{\mathbf{T}}'$.

(...)

◊

Definition 17. The **unit normal vector** $\vec{\mathbf{N}}$ to a curve $\vec{\mathbf{r}}$ is the direction of the derivative of the unit tangent vector $\vec{\mathbf{T}}'(t) = \frac{d\mathbf{T}}{dt}$. (By definition, this vector points into the direction of the curve.)

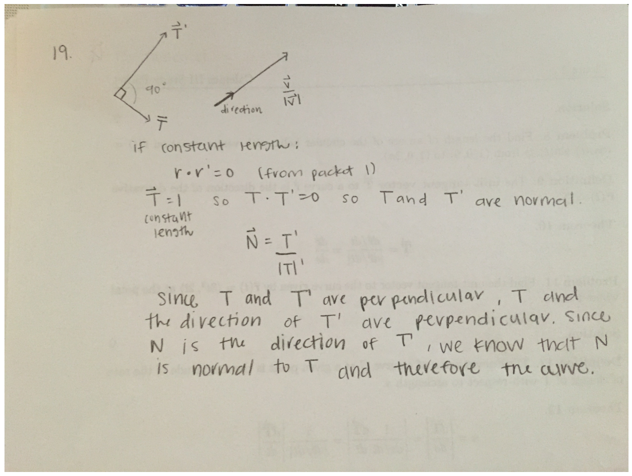
Theorem 18.

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{T}}'}{|\vec{\mathbf{T}}'|}$$

4/4

Problem 19. Prove that $\vec{\mathbf{N}}$ actually is normal to the curve by using a theorem from a previous section. (Hint: $|\vec{\mathbf{T}}| = 1$.)

Solution.


19.

$\vec{T}'$

90°

$\vec{T}$

direction

$\frac{\vec{v}}{|\vec{v}|}$

if constant length:

$r \cdot r' = 0$ (from packet 1)

$\vec{T} = 1$ so $T \cdot T' = 0$ so T and T' are normal.

constant length

$\vec{N} = \frac{T'}{|T|'}$

since T and T' are perpendicular, T and the direction of T' are perpendicular. Since N is the direction of T', we know that N is normal to T and therefore the curve.

$\diamond$

3/4

Problem 20. Plot the curve given by $\vec{\mathbf{r}}(t) = \langle \cos(2t), \sin(2t) \rangle$, along with $\vec{\mathbf{T}}, \vec{\mathbf{N}}$ at the point where $t = \frac{\pi}{2}$.

Solution.

$$\vec{\mathbf{T}} = \frac{d\vec{\mathbf{r}}/dt}{|d\vec{\mathbf{r}}/dt|} = \frac{\vec{\mathbf{r}}'}{|\vec{\mathbf{r}}'|}$$

$$\vec{\mathbf{T}} = \frac{\langle -2\sin 2t, 2\cos 2t \rangle}{\sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2}}$$

$$\vec{\mathbf{T}} = \frac{\langle -2\sin 2t, 2\cos 2t \rangle}{\sqrt{4\sin^2 2t + 4\cos^2 2t}}$$

Trig Identity: $\cos^2 t + \sin^2 t = 1$

So:

$$\vec{\mathbf{T}} = \frac{\langle -2\sin 2t, 2\cos 2t \rangle}{\sqrt{4}}$$

$$\vec{\mathbf{T}} = \langle -\sin 2t, \cos 2t \rangle$$

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{T}}'}{|\vec{\mathbf{T}}'|}$$

$$\vec{\mathbf{T}}' = \langle -2\cos 2t, -2\sin 2t \rangle$$

$$|\vec{\mathbf{T}}'| = \sqrt{(-2\cos 2t)^2 + (-2\sin 2t)^2}$$

$$|\vec{\mathbf{T}}'| = \sqrt{4\cos^2 2t + 4\sin^2 2t} = 2$$

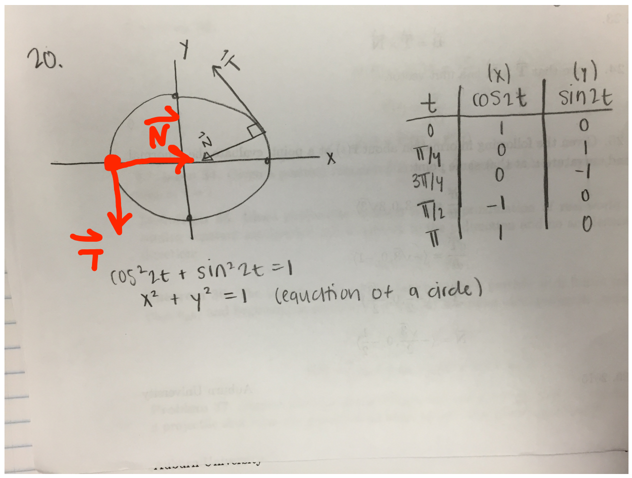
$$\vec{\mathbf{N}} = \frac{\langle -2\cos 2t, -2\sin 2t\rangle}{2}$$

$$\vec{\mathbf{N}} = \langle -\cos 2t, -\sin 2t\rangle$$

$$\vec{\mathbf{T}}(\frac{\pi}{2}) = \langle -\sin \pi, \cos \pi\rangle = \langle 0, -1\rangle$$

$$\vec{\mathbf{N}}(\frac{\pi}{2}) = \langle -\cos \pi, -\sin \pi\rangle = \langle 1, 0\rangle$$

$$\vec{\mathbf{r}}(\frac{\pi}{2}) = \langle \cos \pi, \sin \pi\rangle = \langle -1, 0\rangle$$



◊

4/4

Problem 21. Give formulas for $\vec{\mathbf{T}}, \vec{\mathbf{N}}$ in terms of t for the vector function

$$\vec{\mathbf{r}}(t) = \langle \sqrt{2}\sin t, 2\cos t, \sqrt{2}\sin t\rangle$$

Solution.

$$\vec{\mathbf{T}} = \frac{d\vec{\mathbf{r}}/dt}{|d\vec{\mathbf{r}}/dt|} = \frac{\vec{\mathbf{r}}'}{|\vec{\mathbf{r}}'|}$$

$$\vec{\mathbf{r}}' = \langle \sqrt{2}\cos t, -2\sin t, \sqrt{2}\cos t\rangle$$

$$|\vec{\mathbf{r}}'| = \sqrt{(\sqrt{2}\cos t)^2 + (-2\sin t)^2 + (\sqrt{2}\cos t)^2}$$

$$|\vec{\mathbf{r}}'| = \sqrt{2\cos^2 t + 4\sin^2 t + 2\cos^2 t}$$

$$|\vec{\mathbf{r}}'| = \sqrt{4\cos^2 t + 4\sin^2 t}$$

$$|\vec{\mathbf{r}}'| = \sqrt{4}$$

$$|\vec{\mathbf{r}}'| = 2$$

$$\frac{\vec{\mathbf{r}}'}{|\vec{\mathbf{r}}'|} = \langle \frac{\sqrt{2}\cos t, -2\sin t, \sqrt{2}\cos t\rangle}{2}$$

$$\vec{\mathbf{T}} = \langle \frac{\sqrt{2}}{2}\cos t, -\sin t, \frac{\sqrt{2}}{2}\cos t\rangle$$

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{T}}'}{|\vec{\mathbf{T}}'|}$$

$$\vec{\mathbf{T}}' = \langle -\frac{\sqrt{2}}{2}\sin t, -\cos t, -\frac{\sqrt{2}}{2}\sin t\rangle$$

$$|\vec{\mathbf{T}}'| = \sqrt{(-\frac{\sqrt{2}}{2}\sin t)^2 + (-\cos t)^2 + (-\frac{\sqrt{2}}{2}\sin t)^2}$$

$$|\vec{\mathbf{T}}'| = \sqrt{\frac{1}{2}\sin^2 t + \cos^2 t + \frac{1}{2}\sin^2 t} = \sqrt{\sin^2 t + \cos^2 t} = 1$$

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{T}}'}{|\vec{\mathbf{T}}'|} = \langle -\frac{\sqrt{2}}{2}\sin t, -\cos t, -\frac{\sqrt{2}}{2}\sin t\rangle$$

◊

Definition 22. The **binormal vector** $\vec{\mathbf{B}}$ is the direction normal to both $\vec{\mathbf{T}}$ and $\vec{\mathbf{N}}$ according to the right-hand rule.

Theorem 23.

$$\vec{\mathbf{B}} = \vec{\mathbf{T}} \times \vec{\mathbf{N}}$$

4/4

Problem 24. Prove that $\vec{\mathbf{T}} \times \vec{\mathbf{N}}$ is a unit vector.

Solution.

$$|\vec{\mathbf{u}} \times |\vec{\mathbf{v}}| = |\vec{\mathbf{u}}||\vec{\mathbf{v}}|\sin\theta$$

$$|\vec{\mathbf{T}} \times \vec{\mathbf{N}}| = |1||1|\sin 90^{\circ}$$

$$|\vec{\mathbf{T}} \times \vec{\mathbf{N}}| = 1$$

So: $\vec{\mathbf{T}} \times \vec{\mathbf{N}}$ is a unit vector. ◊

4/4

Problem 25. Given the following information about $\vec{\mathbf{r}}(t)$ at a point, evaluate the binormal vector $\vec{\mathbf{B}}$ and curvature κ at that same point:

$$\frac{d\vec{\mathbf{r}}}{dt} = \langle -3, 0, 3\sqrt{3}\rangle$$

$$\frac{d\vec{\mathbf{T}}}{dt} = \langle -\sqrt{3}, 0, -1\rangle$$

$$\vec{\mathbf{T}} = \langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\rangle$$

$$\vec{\mathbf{N}} = \langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\rangle$$

Solution.

$$\vec{\mathbf{B}} = \langle \frac{-1}{2}, 0, \frac{\sqrt{3}}{2} \rangle \times \langle \frac{-\sqrt{3}}{2}, 0, \frac{-1}{2} \rangle$$

$$(\frac{-1}{2})(\frac{-1}{2})\vec{\mathbf{i}} \times \vec{\mathbf{k}} + (\frac{\sqrt{3}}{2})(-\frac{\sqrt{3}}{2})\vec{\mathbf{k}} \times \vec{\mathbf{i}}$$

To find the cross product, use the same image as in Problem 15.

$$-\frac{1}{4}\widehat{\mathbf{j}} - \frac{3}{4}\widehat{\mathbf{j}} = -\widehat{\mathbf{j}}$$
$$\vec{\mathbf{B}} = -\widehat{\mathbf{j}}$$

$$\kappa = \frac{1}{|d\vec{\mathbf{r}}/dt|}\left|\frac{d\vec{\mathbf{T}}}{dt}\right|$$
$$= \frac{1}{|\langle -3, 0, 3\sqrt{3}\rangle|}|\langle -\sqrt{3}, 0, -1\rangle|$$
$$= \frac{4}{\sqrt{36}}$$
$$\kappa = \frac{2}{3}$$

◊

Definition 26. A **right-handed frame** is a group of three unit vectors which are all normal to one another and satisfy the right hand rule.

Example 27. $\widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}}$ and $\vec{\mathbf{T}}, \vec{\mathbf{N}}, \vec{\mathbf{B}}$ are examples of right-handed frames.

Theorem 28. Any vector is a linear combination of the vectors in a right-handed frame.

13.4 Motion in Space, Velocity, and Acceleration

Definition 29. The **velocity** $\vec{\mathbf{v}}(t)$ of a particle at time t on a position function $\vec{\mathbf{r}}(t)$ is its rate of change with respect to t.

Definition 30. The **speed** $|\vec{\mathbf{v}}(t)|$ of a particle at time t on a position function $\vec{\mathbf{r}}(t)$ is the magnitude of its velocity.

Definition 31. The **direction** $\vec{\mathbf{T}}(t)$ of a particle at time t on a position function $\vec{\mathbf{r}}(t)$ is the direction of its velocity.

Definition 32. The **acceleration** $\vec{\mathbf{a}}(t)$ of a particle at time t on a position function $\vec{\mathbf{r}}(t)$ is the rate of change of its velocity with respect to t.

Theorem 33.

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{r}}'(t)$$

$$|\vec{\mathbf{v}}(t)| = |\vec{\mathbf{r}}'(t)| = \frac{ds}{dt}$$

$$\vec{\mathbf{T}}(t) = \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$$

$$\vec{\mathbf{a}}(t) = \vec{\mathbf{v}}'(t) = \vec{\mathbf{r}}''(t)$$

4/4

Problem 34. Given a position function $\vec{\mathbf{r}}(t) = \langle t^3, t^2 \rangle$ find its velocity, speed, and acceleration at $t = 1$.

Solution.

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{r}}'(t)$$

$$\vec{\mathbf{v}}(t) = \langle 3t^2, 2t \rangle$$

$$|\vec{\mathbf{v}}(t)| = \sqrt{(3t^2)^2 + (2t)^2}$$

$$= \sqrt{9t^4 + 4t^2}$$

$$\vec{\mathbf{a}}(t) = \vec{\mathbf{v}}(t)' = \langle 6t, 2 \rangle$$

velocity

~~speed~~ =

$$\vec{\mathbf{v}}(1) = \langle 3, 2 \rangle$$

speed

~~velocity~~ =

$$|\vec{\mathbf{v}}(1)| = \sqrt{13}$$

acceleration =

$$\vec{\mathbf{a}}(1) = \langle 6, 2 \rangle$$

$\Diamond$

Definition 35. Ideal projectile motion is an approximation of real-world motion assuming constant acceleration due to gravity in the y direction and no acceleration in the x direction:

$$\vec{\mathbf{a}}(t) = \langle 0, -g \rangle$$

Theorem 36. The velocity and position functions for a particle with initial velocity $\vec{\mathbf{v}}_0 = \langle v_{x,0}, v_{y,0} \rangle$ and beginning at position $P_0 = \langle x_0, y_0 \rangle$ assuming ideal projectile motion are:

$$\vec{\mathbf{v}}(t) = \langle v_{x,0}, -gt + v_{y,0} \rangle$$

$$\vec{\mathbf{r}}(t) = \left\langle v_{x,0}t + x_0, -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \right\rangle$$

4/4

Problem 37. Assume ideal projectile motion and and $g = 10\frac{m}{s^2}$. What is the flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100\frac{m}{s}$?

Solution. Finding the component vectors given speed and θ using Trig:

$$v_{x0} = |\vec{\mathbf{v}}| \cos\theta$$

$$v_{y0} = |\vec{\mathbf{v}}| \sin\theta$$

$$|\vec{\mathbf{v}}_0| = 100m/s$$

Being only concerned with flight time, we will only use v_{y0} becasue it is affected by gravity and the v_x component doesn't change. ??

$$v_{y0} = 100 \sin\frac{\pi}{6} = 50m/s$$

$$\vec{\mathbf{r}}(t) = \left\langle v_{x,0}t + x_0, -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \right\rangle$$

We want to find ~~when~~ time when the position of the y component of $\vec{\mathbf{r}}(t) = 0$ so $\vec{\mathbf{r}}_y(t) = 0$

$$-\frac{1}{2}gt^2 + v_{y,0}t + y_0 = 0$$

$$-\frac{1}{2}(10)t^2 + (50)t + 0 = 0$$

$$-5t^2 + 50t = 0$$

$$-5t(t - 10) = 0$$

$$t = 0s, t = 10s$$

The flight time of the projectile is 10 seconds. ✓

◊

3/4

Problem 38. Assume ideal projectile motion and and $g = 10\frac{m}{s^2}$. What must have been the initial speed of a projectile shot from the ground at an angle of $\pi/3$ if it traveled 60 meters horizontally after 4 seconds?

Solution.

$$\vec{\mathbf{r}}(t) = \langle V_{ix}t + x_i, -\frac{1}{2}gt^2 + V_{iy}t + y_i\rangle$$

?

$$60 = 4V_i cos(\frac{\pi}{3})$$

$$V_i = 30m/s$$

◊