Calculus III - Spring 2015 - Mr. Clontz - Test 1

Name: Solutions Class: ______

- Write complete solutions for each of the given problems. You may cite definitions or theorems from the notes rather than rewrite them.
- This exam is open-"everything", provided you do not plagiarize.
- Write your solutions so that an A-student in Cal 1 and Cal 2 who has never seen that type of problem before could follow your work.
- Individual Test: You will have 40 minutes to complete this test on your own. You may not communicate with anyone during this period.
- Group Test: You will have 40 minutes to complete an identical test. You may collaborate with your group members during this time. All solutions must still be written by yourself, and may not be directly copied from another student.

1. (5 points) Compute the angle between the vectors $\vec{\mathbf{u}} = \langle 2, 2\sqrt{3}\rangle$ and $\vec{\mathbf{v}} = \langle -5, 0\rangle$.

By defn. 62:

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$|\vec{u}| = \sqrt{(2)^2+(2\sqrt{3})^2} = \sqrt{4+4(3)} = \sqrt{16} = 4$$

$$|\vec{v}| = \sqrt{(-5)^2+0^2} = 5$$

$$\vec{u}\cdot\vec{v} = 4(5)\cos\theta = 20\cos\theta$$

By theorem 64:

$$\vec{u}\cdot\vec{v} = 2(-5) + 2\sqrt{3}(0)$$

$$= -10$$

Therefore:

$$20\cos\theta = -10$$

$$\cos\theta = -1/2$$

$$\theta = \operatorname{Arccos}(-1/2) = 2\pi/3 = 120°$$

2. (5 points) Find an equation for the plane passing through $\overset{P}{(0,0,0)}$, $\overset{Q}{(1,0,-3)}$, and $\overset{R}{(-2,3,0)}$.

For a plane EQ, we need a point $(0,0,0)$ and a normal vector.

To get the normal vector, we can use

$$\vec{PQ} \times \vec{PR} = \langle 1,0,-3 \rangle \times \langle -2,3,0 \rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} = \left\langle \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix}, -\begin{vmatrix} 1 & -3 \\ -2 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ -2 & 3 \end{vmatrix} \right\rangle$$

$$= \langle 0 - -9, -(0-6), 3-0 \rangle$$

$$= \langle 9,6,3 \rangle$$

So by the plane EQ (Thm 86):

$$9(x-0) + 6(y-0) + 3(z-0) = 0$$

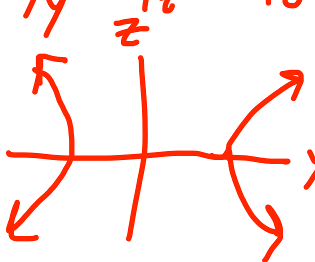
$$9x + 6y + 3z = 0 \quad \text{OR} \quad 3x + 2y + z = 0$$

3. (5 points) Sketch $x^2 + 9y^2 - 4z^2 = 16$ and its traces in the planes $x = 0$, $y = 0$, and $z = 0$. Then use these traces to name the quadric surface.

Its traces are:

$\underline{x=0}$

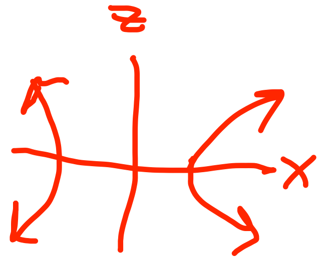
$9y^2 - 4z^2 = 16$



(hyperbola)

$\underline{y=0}$

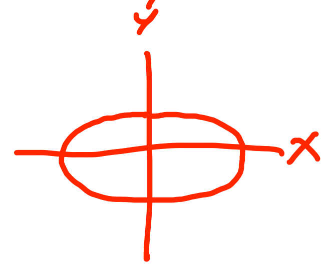
$x^2 - 4z^2 = 16$



(hyperbola)

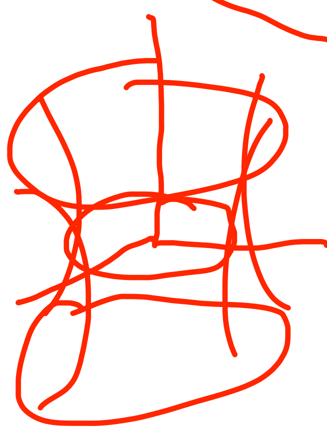
$\underline{z=0}$

$x^2 + 9y^2 = 16$



(ellipse)

3D Sketch:



By its traces this is a hyperboloid of one sheet (Def'n 110)

4. (5 points) Explain why the parametric equations $x = 4\sin t$ and $y = \cos t$ yield points on the ellipse $x^2 + 16y^2 = 16$.

For any value of t, we may plug in the parametric equations to see that:

$$x^2+16y^2 = (4\sin t)^2 + 16(\cos t)^2$$

$$= 16\sin^2 t + 16\cos^2 t$$

$$= 16(\sin^2 t + \cos^2 t)$$

$$= 16(1)$$

$$= 16$$

Therefore all points yielded by the EQs satisfy $x^2+16y^2=16$.

5. (5 points) Find $\vec{\mathbf{r}}(t)$ given $\vec{\mathbf{r}}'(t) = \langle 4t^3, -\sin t, e^t \rangle$ and $\vec{\mathbf{r}}(0) = \langle 1, 2, 3 \rangle$.

$\vec{r}(t)$ is an antiderivative of $\vec{r}'(t)$.

By Defn 138:

$$\vec{r}(t) = \left\langle \int 4t^3\,dt, \int -\sin t\,dt, \int e^t\,dt \right\rangle$$

$$= \langle t^4, \cos t, e^t \rangle + \vec{C}$$

Substituting in $\vec{r}(0) = \langle 1, 2, 3 \rangle$ allows us to solve for $\vec{C}$:

$$\vec{r}(0) = \langle 0, \cos 0, e^0 \rangle + \vec{C} = \langle 1, 2, 3 \rangle$$

$$\langle 0, 1, 1 \rangle + \vec{C} = \langle 1, 2, 3 \rangle$$

$$\vec{C} = \langle 1, 1, 2 \rangle$$

Therefore:

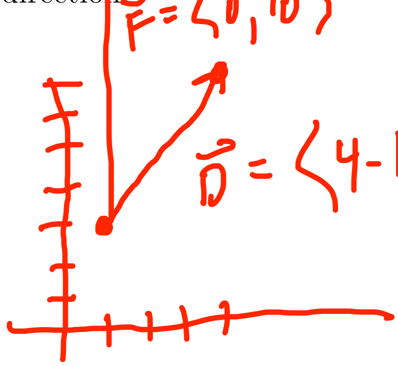
$$\vec{r}(t) = \langle t^4 + 1, \cos t + 1, e^t + 2 \rangle.$$

6. (5 points) Recall from the notes that the work done by moving an object along the displacement vector $\vec{\mathbf{D}}$ using a force vector $\vec{\mathbf{F}}$ is given by

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}$$

Use this formula to show how much work is done in moving an object from the point $(1, 3)$ to the point $(4, 7)$ using a force of magnitude 10 units oriented in the positive y-direction.

$\vec{F} = \langle 0, 10 \rangle$



$\vec{D} = \langle 4-1, 7-3 \rangle = \langle 3, 4 \rangle$

Therefore:

$$W = \vec{F} \cdot \vec{D} = \langle 0, 10 \rangle \cdot \langle 3, 4 \rangle$$
$$= 0 + 40$$
$$= 40$$