Packet 2

Part 2: Sections 14.4-14.6

14.4 Tangent Planes and Linear Approximations

Definition 1. A **normal vector** to a surface is a vector normal to any vector tangent to a curve on the surface.

Theorem 2. Let $f(x, y)$ be a function of two variables with continuous partial derivatives, and let (a, b) be a point in the interior of f's domain. Then $\langle f_x(a, b), f_y(a, b), -1\rangle$ is normal to the surface at the point $(a, b, f(a, b))$.

Problem 3. OPTIONAL. Prove the previous theorem by using the curves $\vec{\mathbf{r}}(t) = \langle t, b, f(t, b)\rangle$ and $\vec{\mathbf{q}}(t) = \langle a, t, f(a, t)\rangle$ to yield the tangent vectors $\langle 1, 0, f_x(a, b)\rangle$ and $\langle 0, 1, f_y(a, b)\rangle$.

Solution. ◊

Definition 4. The **tangent plane** to a surface at a point is the plane passing through that point sharing the same normal vectors as the surface.

Theorem 5. The tangent plane to the surface $z = f(x, y)$ above the point (a, b) is given by the equation

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

Problem 6. Prove the previous theorem.

Solution. ◊

Problem 7. Find an equation for the plane tangent to the surface $z = 4x^2 + y^2$ above the point $(1, -1)$.

Solution. ◊

Definition 8. The **linearization** $L(x, y)$ of a function $f(x, y)$ at the point (a, b) is given by the formula:

$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

Definition 9. A function f is **differentiable** at a point if its linearization at that point approximates the value of the function nearby.

Remark 10. Basically, a differentiable function is one which looks similar to its tangent planes when zoomed in sufficiently far.

Problem 11. Approximate the value of the differentiable function $f(x, y) = 4xy + 3y^2$ at $(1.1, -2.05)$ by using its linearization at the point $(1, -2)$. Then use a calculator to approximate $f(1.1, -2.05)$.

Solution. $\Diamond$