
Proposition 1. *Any finite union of closed sets is closed, and any arbitrary intersection of closed sets is closed.*

Proof. Proposition 1

$X \in \tau$

If $U \subseteq X \wedge X \setminus U \in \tau$ then $X \setminus \bigcap U \in \tau$

If $U, V \subseteq X \wedge X \setminus U \in \tau \wedge X \setminus V \in \tau$ then $X \setminus (U \cup V) \in \tau$ □

Proposition 2. *A set K in a topological space X is closed if and only if K contains all its limit points.*

Proof. Proposition 2 Part 1 - Prove if K contains all its limit points then that implies $X \setminus K \in \tau$

$\exists x, z, \alpha, \beta, \lambda, \epsilon \in X$

$K \subseteq X, B \subseteq X, C \subseteq X$

$x, z \in K, K = \{ y \mid x \leq y \leq z \}$

$x, \alpha, \beta \in B, B = \{ (\alpha, \beta) \mid \alpha < x < \beta \}$

$z, \lambda, \epsilon \in C, C = \{ (\lambda, \epsilon) \mid \lambda < z < \epsilon \}$

$K \cap B = [x, \beta), x \neq \beta \therefore x$ is a limit point.

$K \cap C = (\lambda, z], z \neq \lambda \therefore z$ is a limit point.

$x, z \in K, K = \{ y \mid x \leq y \leq z \} \therefore K$ is closed $\Rightarrow X \setminus K \in \tau$ □

Proof. Proposition 2 Part 2 - Prove if $X \setminus K \in \tau$ then that implies that K contains all of its limit points. □