
Proposition 1. *Any finite union of closed sets is closed, and any arbitrary intersection of closed sets is closed.*

Proof. Proposition 1

$$\begin{aligned} A, B \in \tau &\Rightarrow A \cap B \in \tau \Rightarrow \overline{A \cap B} \notin \tau \\ \overline{A \cap B} &= \overline{A} \cap \overline{B} \\ \overline{A} \cup \overline{B} &\notin \tau \end{aligned}$$

Also:

$$\begin{aligned} X &= (a, z) \in \tau \\ A &= (j, k) \in X \\ \overline{A} &= (a, j] \cup [k, z) \notin \tau \end{aligned}$$

□

Proposition 2. *A set K in a topological space X is closed if and only if K contains all its limit points.*

Proof. Proposition 2 Part 1 - Prove if K contains all its limit points then that implies $X \setminus K \in \tau$

$$\begin{aligned} \exists x, z, \alpha, \beta, \lambda, \epsilon &\in X \\ K \subseteq X, B \subseteq X, C \subseteq X \\ x, z \in K, K &= \{ y \mid x \leq y \leq z \} \\ x, \alpha, \beta \in B, B &= \{ (\alpha, \beta) \mid \alpha < x < \beta \} \\ z, \lambda, \epsilon \in C, C &= \{ (\lambda, \epsilon) \mid \lambda < z < \epsilon \} \\ K \cap B &= [x, \beta), x \neq \beta \therefore x \text{ is a limit point.} \\ K \cap C &= (\lambda, z], z \neq \lambda \therefore z \text{ is a limit point.} \\ x, z \in K, K &= \{ y \mid x \leq y \leq z \} \therefore K \text{ is closed} \Rightarrow X \setminus K \in \tau \end{aligned}$$

□

Proof. Proposition 2 Part 2 - Prove if $X \setminus K \in \tau$ then that implies that K contains all of its limit points. □