Proposition 1. *Any finite union of closed sets is closed, and any arbitrary intersection of closed sets is closed.*

Proof. Proposition 1 - Any finite union of closed sets is closed.
Let C, D be closed sets.
Let A, B be compliments of C, D so A, B are open.
Then, A $\cap$ B is also open.
Thus, X$\backslash$(A $\cap$ B) is closed.
X$\backslash$(A $\cap$ B) = X$\backslash$A $\cap$ X$\backslash$B = C $\cup$ D

Inductive Proof:
$\mathcal{C}_1$ = C $\cup$ D, $\mathcal{C}_1$ is closed.
$\mathcal{C}_2$ = $\mathcal{C}_1 \cup$ E, $\mathcal{C}_2$ is closed.
$\mathcal{C}_n = \mathcal{C}_1 \cup \mathcal{C}_2 \cup ... \cup \mathcal{C}(n-1)$, $\mathcal{C}_n$ is closed.
$\mathcal{C}(n+1) = \mathcal{C}_n \cup$ Z, $\mathcal{C}(n+1)$ is closed.

Proposition 1 - Any arbitrary intersection of closed sets is closed.
Let $\mathcal{C}$ be a collection of closed sets.
Let $\mathcal{U}$ be the compliment of $\mathcal{C}$, so $\mathcal{U}$ is a collection of open sets.
Then, $\bigcup \mathcal{U}$ is also open.
Thus, X$\backslash \bigcup \mathcal{U}$ is closed.
X$\backslash \bigcup \mathcal{U} = \bigcap \mathcal{C}$
Therefore, $\bigcap \mathcal{C}$ is closed. □

Proposition 2. *A set K in a topological space X is closed if and only if K contains all its limit points.*

Proof. Proposition 2
Let K = X, then K is closed because $\emptyset$ is open.
Let x $\in$ X$\backslash$K, so x is not a limit point of K.
$\exists \ell \in \mathcal{U}_\ell \in \tau$ such that $\forall$ k $\in$ K besides ℓ, k $\notin \mathcal{U}_\ell$
Therefore, $\mathcal{U}_\ell$ is the compliment of K, so $\mathcal{U}_\ell$ is open.
$\mathcal{U}_\ell$ = X$\backslash$K
Thus, X$\backslash$K = $\bigcup$ { $\ell \in \mathcal{U}_\ell$: $\ell \notin$ K } is open. □