Proposition 1. *Any finite union of closed sets is closed, and any arbitrary intersection of closed sets is closed.*

Proof. Proposition 1
A, B $\in \tau \Rightarrow$ A $\cap$ B $\in \tau \Rightarrow \overline{A \cap B} \notin \tau$
$\overline{A \cap B} = \overline{A} \cup \overline{B}$
$\overline{A} \cup \overline{B} \notin \tau$

Also:
X = (a, z) $\in \tau$
A = (j, k) $\in$ X
$\overline{A}$ = (a, j] $\cup$ [k, z) $\notin \tau$

If $A \subseteq \tau$ then $\bigcup A \in \tau$
If A contains at least one limit point k, we call this $\overline{A}$ if $\overline{A} \notin A$, then $\overline{A} \notin \tau$
If $\overline{A} \notin \tau$, then $\overline{\bigcup A} \notin \tau$
$\overline{\bigcup A} = \bigcap \overline{A} \notin \tau$ □

Proposition 2. *A set K in a topological space X is closed if and only if K contains all its limit points.*

Proof. Proposition 2 Part 1 - Prove if K contains all its limit points then that implies X\K $\in \tau$

□

Proof. Proposition 2 Part 2 - Prove if X\K $\in \tau$ then that implies that K contains all of its limit points.
Suppose $\exists K \in X$ and that K contains all its limit points.
$\exists x$ such that $x \in X \setminus K$. It follows that if $x \in X \setminus K$ then $x \notin K \Rightarrow x \neq$ limit point $\in K$. $\therefore \exists \bigcup x'$ limit points $\in K$ □