Project Summary

Intellectual Merit

Broader Impacts

Project Description

1 Overview

In the summers of 1967 and 1968, Lynn Arthur Steen and J. Arthur Seebach, Jr. coordinated two NSF-funded undergraduate research experiences at Saint Olaf College. Each summer, five undergraduate students assisted them in investigating and cataloging known and novel results in general topology. Specifically, their work was a treatment of topological spaces, properties preserved by homeomorphism, and theorems connecting these properties (e.g. all compact Hausdorff spaces are normal). In 1970, Steen and Seebach published a handbook covering this work, titled *Counterexamples in Topology* [4].

Mary Ellen Rudin wrote the following in her review [3] of the original text. "*Counterexamples in Topology* is a valuable addition to the small collection of books I keep on the shelf in my office." "The book is completely unique; no other book now in print serves its purpose." Recognizing the maze of counterexamples littered throughout the field of set-theoretic topology, Rudin suggested that students could benefit from the text as a guidebook. "Even those of us who work exactly in the area will profit from its organization." Several other "*Counterexamples*" texts in the tradition of Stein and Seebach have been published in several fields, including (but not limited to) real analysis [7], differential equations [2], probability [5], and graph theory [1].

Of the students involved in these topology REUs, several moved on to careers in mathematical research. These include John Feroe, Professor of Mathematics and Statistics at Vassar College; Gary Gruenhage, Professor Emeritus of Mathematics at Auburn University, specializing in set-theoretic topology; and Linda Ness, Chief Research Scientist: Applied Research, Vencore Labs. Stein and Seebach write the following in their introduction to *Counterexamples*, "We acknowledge that theirs was a twofold contribution: not only did they explore and develop many examples, but they proved by their own example the efficacy of examples for the undergraduate study of topology."

The handbook stands as a useful resource, even today. However, the examples in this original work are relatively elementary, and the research community has shifted focus away from some topics and onto others over the last four decades. To this end, the proposed REU program is a next-generation approach to extending *Counterexamples*, keeping in mind the needs of modern students and researchers in topology, and taking advantage of modern technology.

{TODO: give overview of piBase}

Participants in the propsoed REU will contribute to the piBase database, auditing its current entries as well as adding new content from more recent publications such as Watson's survey of topological planks and resolutions [6]. Participants will also be given mentorship as they choose a problem based on this work for the purpose of original research.

As a result of the proposed program, all students of set-theoretic topology will benefit, especially the program's participants. General topology is the backbone of many mathematical fields, and participants will be given the opportunity to develop their knowledge of this core. Generally, it is difficult to develop novel open questions in set-theoretic topology which are accessible by an undergraduate researcher. However, the piBase application automatically detects unknown

properties of the spaces within its database, providing a plethora of material on which the REU participants, as well as any undergraduate student, may work as original research. By uncovering the questions which researchers have not yet thought to ask or rigorously pursue, a robust piBase will allow students to obtain valuable experience working on truely open problems, while contributing to the collective knowledge of the set-theoretic topology research community.

The proposers' aim is to simultaneously provide an authentic research experience for the undergraduate participants, adjusting for the length of the program, the inexperience of the participants, and the goal of developing the piBase database. Students are not all expected to pursue mathematical research as a career; however, the problem-solving skills developed during the program, and the exposure to software development and research cyberinfrastructure, will certainly benefit all participants regardless of their eventual careers. However, it is the investigators' hope that many participants will have their interest in mathematical research solidified by this experience, or even have it germinated for the first time.

Researchers of set-theoretic topology will also appreciate the product of this REU. Several major open questions in topology simply ask for the existence of, or counterexample to, a topological space satisfying certain properties (perhaps under various set-theoretic axioms). Additionally, it is not uncommon for seminar talks to be derailed by pondering the existence of one counterexample or another. So much of the community's knowledge is scattered across a diaspora of peer-reviwed papers in numerous journals, meaning many "open" questions may actually be a simple corollary of results from two or more heretofore unconnected articles. Likewise, several properties have been studied under various names, whether for historical reasons, or because these properties were later shown to be equivalent; other properties share the same names, while actually being distinct (at least in a sufficiently general setting). Spaces and properties in the piBase are tagged with unique IDs, preventing any ambiguity, and providing researchers a common language when referencing existing spaces and properties from the literature.

The benefits of this program will not be restricted to only students of topology, or even researchers in set-theoretic topology. After the piBase database has been updated to reflect the modern status of topological research, data on its utility as a tool for students and researchers may be collected. At its core, piBase is a tool which may be generalized to relate the objects, categorical invariants, and theorems relating those invariants within any given mathematical category. Once the piBase is battled-tested within one field, it will serve as a proof of concept for researchers of different categories, and can be adapted to serve those communities as well.

{TODO: Provide a brief description of the targeted student participants, organizational structure, timetable, and participating organizations' commitment to the REU activity.}

2 Nature of Student Activities

3 The Research Environment

4 Student Recruitment and Selection

5 Project Evaluation and Reporting

6 Broader Impacts

7 Results From Prior NSF Support

No prior NSF support has been given for this project.

References Cited

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[3] Mary Ellen Rudin. Reviews: Counterexamples in Topology. *Amer. Math. Monthly*, 78(7):803–804, 1971.

[4] Lynn Arthur Steen and J. Arthur Seebach, Jr. *Counterexamples in topology.* Dover Publications, Inc., Mineola, NY, 1995. Reprint of the second (1978) edition.

[5] Jordan M. Stoyanov. *Counterexamples in probability.* Wiley Series in Probability and Mathematical Statistics: Probability and Mathematical Statistics. John Wiley & Sons, Ltd., Chichester, 1987.

[6] Stephen Watson. The construction of topological spaces: planks and resolutions. In *Recent progress in general topology (Prague, 1991)*, pages 673–757. North-Holland, Amsterdam, 1992.

[7] Gary L. Wise and Eric B. Hall. *Counterexamples in probability and real analysis.* The Clarendon Press, Oxford University Press, New York, 1993.

Biographical Sketch: Your Name