Project Summary

Intellectual Merit

Broader Impacts

Project Description

1 Overview

In the summers of 1967 and 1968, Lynn Arthur Steen and J. Arthur Seebach, Jr. coordinated two NSF-funded undergraduate research experiences at Saint Olaf College. Each summer, five undergraduate students assisted them in investigating and cataloging known and novel results in general topology. Specifically, their work was a treatment of topological spaces, properties preserved by homeomorphism, and theorems connecting these properties (e.g. all compact Hausdorff spaces are normal). In 1970, Steen and Seebach published a handbook covering this work, titled *Counterexamples in Topology* [4].

Mary Ellen Rudin wrote the following in her review [3] of the original text. "*Counterexamples in Topology* is a valuable addition to the small collection of books I keep on the shelf in my office." "The book is completely unique; no other book now in print serves its purpose." Recognizing the maze of counterexamples littered throughout the field of set-theoretic topology, Rudin suggested that students could benefit from the text as a guidebook. "Even those of us who work exactly in the area will profit from its organization." Several other "*Counterexamples*" texts in the tradition of Stein and Seebach have been published in several fields, including (but not limited to) real analysis [7], differential equations [2], probability [5], and graph theory [1].

Of the students involved in these topology REUs, several moved on to careers in mathematical research. These include John Feroe, Professor of Mathematics and Statistics at Vassar College; Gary Gruenhage, Professor Emeritus of Mathematics at Auburn University, specializing in set-theoretic topology; and Linda Ness, Chief Research Scientist: Applied Research, Vencore Labs. Stein and Seebach write the following in their introduction to *Counterexamples*, "We acknowledge that theirs was a twofold contribution: not only did they explore and develop many examples, but they proved by their own example the efficacy of examples for the undergraduate study of topology."

The handbook stands as a useful resource, even today. However, the examples in this original work are relatively elementary, and the research community has shifted focus away from some topics and onto others over the last four decades. To this end, the proposed REU program is a next-generation approach to extending *Counterexamples*, keeping in mind the needs of modern students and researchers in topology, and taking advantage of modern technology.

{TODO: give overview of PiBase}

Participants in the propsoed REU will contribute to the PiBase database, auditing its current entries as well as adding new content from more recent publications such as Watson's survey of topological planks and resolutions [6]. Participants will also be given mentorship as they choose a problem based on this work for the purpose of original research.

As a result of the proposed program, all students of set-theoretic topology will benefit, especially the program's participants. General topology is the backbone of many mathematical fields, and participants will be given the opportunity to develop their knowledge of this core. Generally, it is difficult to develop novel open questions in set-theoretic topology which are accessible by an undergraduate researcher. However, the PiBase application automatically detects unknown properties of the spaces within its database, providing a plethora of material on which the REU participants, as well as any undergraduate student, may work as original research. By uncovering the questions which researchers have not yet thought to ask or rigorously pursue, a robust PiBase will allow students to obtain valuable experience working on truely open problems, while contributing to the collective knowledge of the set-theoretic topology research community.

The proposers' aim is to simultaneously provide an authentic research experience for the undergraduate participants, adjusting for the length of the program, the inexperience of the participants, and the goal of developing the PiBase database. Students are not all expected to pursue mathematical research as a career; however, the problem-solving skills developed during the program, and the exposure to software development and research cyberinfrastructure, will certainly benefit all participants regardless of their eventual careers. However, it is the investigators' hope that many participants will have their interest in mathematical research solidified by this experience, or even have it germinated for the first time.

Researchers of set-theoretic topology will also appreciate the product of this REU. Several major open questions in topology simply ask for the existence of, or counterexample to, a topological space satisfying certain properties (perhaps under various set-theoretic axioms). Additionally, it is not uncommon for seminar talks to be derailed by pondering the existence of one counterexample or another. So much of the community's knowledge is scattered across a diaspora of peer-reviwed papers in numerous journals, meaning many "open" questions may actually be a simple corollary of results from two or more heretofore unconnected articles. Likewise, several properties have been studied under various names, whether for historical reasons, or because these properties were later shown to be equivalent; other properties share the same names, while actually being distinct (at least in a sufficiently general setting). Spaces and properties in the PiBase are tagged with unique IDs, preventing any ambiguity, and providing researchers a common language when referencing existing spaces and properties from the literature.

The benefits of this program will not be restricted to only students of topology, or even researchers in set-theoretic topology. After the PiBase database has been updated to reflect the modern status of topological research, data on its utility as a tool for students and researchers may be collected. At its core, PiBase is a tool which may be generalized to relate the objects, categorical invariants, and theorems relating those invariants within any given mathematical category. Once the PiBase is battled-tested within one field, it will serve as a proof of concept for researchers of different categories, and can be adapted to serve those communities as well.

As described in more detail below, the proposed program will take place over two summers, each summer lasting eight weeks with a team of six talented undergraduate students. The first summer will take place at the University of South Alabama, located in Mobile, Alabama, and targeting participants from the southeastern United States. The second summer will be held virtually, without a bias towards any students in a particular geographical region of the United States.

Leadership of the proposed program is divided between Dr. Steven Clontz, handling the logistics of running the program and the combined research and cyberinfrastructure expertise required for nurturing a next-generation *Counterexamples*, and Dr. Ziqin Feng, bringing his research expertise in the fields of set-theroetic topology and set theory. They will be assisted by James Dabbs, the developer and maintainer of the PiBase project, who received his Masters in mathematics studing set-theoretic topology before entering industry as a software engineer. The first summer will bring these players together with the first batch of student participants onsite, to build the foundation upon which future summers will be run virtually. By running the second summer virtually, a more diverse audience of potential participants may be reached, as well as opening up participation to potential speakers and mentors across the country. In addition, participants will be better equipped to continue contributions to the PiBase database after the end of the program as their environment will not change, and the costs of running the program will be reduced.

{TODO Add participating organizations' commitment to the REU activity.}

2 Nature of Student Activities

Despite the differing nature of onsite and virtual undergraduate research experiences, student activities will remain the same between the first summer and the second. In the second summer, all meetings will take place between organizers and participants using free videoconferencing services such as Google Hangouts, utilizing screensharing alongside virtual whiteboarding and mathematical typesetting programs in order to share sketched and written mathematical concepts. All virtual meetings will be scheduled between 9am Pacific and 6pm Eastern to accommodate varying timezones of the participants.

The first two weeks of the program will serve to orient participants to the field, building upon their existing knowledge of the topology of Euclidean space $\mathbb{R}^n$ and any previous topology courses they have taken. This orientation will begin by covering selected spaces and their properites from the PiBase database, and later expanding to theorems in the database. Group investigations will take place during morning meetings, led by one of the organizers, followed by assigned work to be done in pairs by the participants in the afternoons. The length of this orientation period may be adjusted based upon the competency of the participants, but particularly in the first summer this will also serve to audit the existing contents of the PiBase database. Organizers will serve as mentors during this period, and will check in with the participants during the afternoon to provide advice or assistance as required.

Following this introductory period, the remainder of the summer will consist of each student being assigned several spaces and properties from the literature, to be studied and catalogued in the PiBase database. These assignments will be chosen by the organizers based upon the competencies and interests of the participants, and some will be assigned to multiple participants for the purposes of peer review and collaboration. Once or twice each day, a student will lead a presentation with an organizer and the other participants based upon the material they have investigated since their last presentation. Occassionally, these presentations will be substituted with talks from active researchers in set-theoretic topology or other fields of interest to the participants. These speakers may be in person during the onsite summer, or be brought in virtually through free videoconferencing solutions from anywhere in the world.

As participants work through their assigned material, several questions will arise from blanks in the database which cannot be directly answered from the literature. Under the organizers' mentorship, participants will choose several questions for their original research. Generally, these questions will be of one of the following forms:

- “Does space S have property P?”
- “Does property P imply Q, or does there exist a counterexample?”

Such questions can be automatically generated by PiBase; however, it is up to the participants and their mentors to choose questions appropriate for their competencies and interests. The expectation for each participant is that his or her work will result in one or more publications within journals appropriate for the significance of the obtained results. Each participant is also expected to give a talk on their research at either a meeting of the AMS/MAA, a conference on undergraduate research, or a topology conference, following the conclusion of the program.

{TODO add note on the significance of the research area}

In addition to the mathematical training and experience the participants will receive, they will also be exposed to several technologies and topics outside set-theoretic topology. These topics include, but are not limited to, the cyberinfrastructure of mathematical research, mathematical typesetting, web application development, teamwork/leadership skills, pedagogy, and presentation skills. These skills will benefit the student participants in nearly any chosen career.

While the development of PiBase as a tool for modern students and researchers of topology is a major goal, the primary focus of the proposed program is the academic and professional development of the participating students. These participants will develop relationships not only with their peers and the program organizers, but also with various invited speakers and the greater community developing the PiBase source and contributing to its database. During both the onsite and virtual summers, participants will be required to show evidence of collaboration with each other and their mentors on their assigned projects in order to develop these important collegial relationships. Participants will be encouraged to stay involved in the PiBase community after the completion of the REU, perhaps even returning as a virtual speaker during a future year, as long as their research interests overlap with the content of the program.

In order to foster commraderie amongst the participants, games and other social events will be organized or offered, particularly during the onsite year. Some, especially those mathematical in nature, will be required and take place during the program itself. Examples of such activities include working on puzzles provided by Mathematical Puzzle Programs and other online resources, and playing and studying games with connections to mathematics such as Nim. During the onsite year, participants will have the opportunity to visit the beach at Gulf Shores, {TODO add other fun social stuff here, or remove paragraph if inappropriate?}

{TODO Ziqin Add more details specific to the research}

3 The Research Environment

{TODO Ziqin}

{TODO Steven add research bio}

4 Student Recruitment and Selection

Recruitment for this program will be centralized on a webpage hosted by the University of South Alabama. Participants will be able to apply through a responsive online form a desktop computer, tablet, or mobile device. The site will include a brief description of the program, describing the PiBase application and the matheatical concepts involved. Emails advertising this site will be disseminated to every mathematics and/or computer science department of every four-year college and university in Alabama. Other major schools will also be notified within adjoining states, expanding outward geographically from South Alabama. In addition, colleagues of the organizers in the area of set-theoretic topology will be encouraged to promote the opportunity to their departments and students as well. In particular, the organizers will rely on this network of fellow researchers to attract promising candidates from across the country with an interest in topology and the cyber-infrastructre of mathematical research. Demographics underrepresented in mathematics will be especially encouraged to apply.

{The overall quality of the student recruitment and selection processes and criteria will be an important element in the evaluation of the proposal. The recruitment plan should be described with as much specificity as possible, including the types and/or names of academic institutions where students will be recruited and the efforts that will be made to attract members of underrepresented groups (women, minorities, and persons with disabilities). A significant fraction of the student participants at an REU Site must come from outside the host institution or organization, and at least half of the student participants must be recruited from academic institutions where research opportunities in STEM are limited (including two-year colleges). The number of students per project should be appropriate to the institutional or organizational setting and to the manner in which research is conducted in the discipline. (The typical REU Site hosts 8-10 students per year.) Proposals involving fewer than six students per year are discouraged.

Undergraduate student participants supported with NSF funds in either REU Sites or REU Supplements must be U.S. citizens, U.S. nationals, or permanent residents of the United States.}

5 Project Evaluation and Reporting

6 Broader Impacts

7 Results From Prior NSF Support

No prior NSF support has been given for this project.

References Cited

[1] Michael Capobianco and John C. Molluzzo. *Examples and counterexamples in graph theory*. North-Holland, New York-Amsterdam-Oxford, 1978. Foreword by Gary Chartrand.

[2] John M. Rassias. *Counterexamples in differential equations and related topics*. World Scientific Publishing Co., Inc., Teaneck, NJ, 1991.

[3] Mary Ellen Rudin. Reviews: Counterexamples in Topology. *Amer. Math. Monthly*, 78(7):803–804, 1971.

[4] Lynn Arthur Steen and J. Arthur Seebach, Jr. *Counterexamples in topology*. Dover Publications, Inc., Mineola, NY, 1995. Reprint of the second (1978) edition.

[5] Jordan M. Stoyanov. *Counterexamples in probability*. Wiley Series in Probability and Mathematical Statistics: Probability and Mathematical Statistics. John Wiley & Sons, Ltd., Chichester, 1987.

[6] Stephen Watson. The construction of topological spaces: planks and resolutions. In *Recent progress in general topology (Prague, 1991)*, pages 673–757. North-Holland, Amsterdam, 1992.

[7] Gary L. Wise and Eric B. Hall. *Counterexamples in probability and real analysis*. The Clarendon Press, Oxford University Press, New York, 1993.

Biographical Sketch: Your Name