MA437 - 2019 Spring - Assessment 03/14 Mult. Choice - Dr. Clontz

Name: ______________________________

Instructions

Use the provided answer sheet to select the most appropriate response for each multiple choice Computation/Knowledge question, skipping any sections already checked-off as mastered on your progress report.

Chapter 2 Computation

1. Let $f(z) = ye^{ix}$ whenever $z = x + iy$. Find $f(4i)$.
   - A. 4
   - B. -4
   - C. ie^4
   - D. $-ie^4$
   - E. None of these.
2. Compute the domain of $f(z) = \dfrac{1}{z\overline{z}}$.
   - A. $\{x + iy \in \mathbb{C} : x + y \neq 0\}$
   - B. $\{x + iy \in \mathbb{C} : x + y = 0\}$
   - C. $\{z \in \mathbb{C} : z \neq 0\}$
   - D. $\{z \in \mathbb{C} : z = 0\}$
   - E. None of these.
3. Find $\displaystyle\lim_{z \to 2i} \frac{z - 2i}{z^2 + 4}$.
   - A. $\frac{1}{2}i$
   - B. $\frac{1}{4}$
   - C. $-\frac{1}{2}$
   - D. $-\frac{1}{4}i$
   - E. None of these.
4. Find the value of $\frac{d}{dz}[f(z)g(z)]$ at $z = 1 + i$ given $f(1 + i) = 3$, $f'(1 + i) = 2i$, $g(1 + i) = 1 - i$, and $g'(1 + i) = \sqrt{2}$.
   - A. $2 - 3\sqrt{2}i + 2i$
   - B. $3\sqrt{2} + 2 + 2i$
   - C. $-\sqrt{2} + 5i$
   - D. $-2 + 2\sqrt{2}i + 3i$
   - E. None of these.
5. The function $f(z) = x^2 + y^2 + i(\frac{1}{2}y^2 - 4x)$ is differentiable at $(x, y) = (1, 2)$. Find $f'(1 + 2i)$.
   - A. $2 - 4i$
   - B. $4 + 2i$
   - C. $-2 + 4i$
   - D. $-4 - 2i$
   - E. None of these.

Chapter 3 Computation

6. Simplify $e^{1+i}e^{1-i}$.
   A. $\cos(2) + i\sin(2)$
   B. $\cos(1) - i\sin(1)$
   C. e^2
   D. $e^2(\cos(1) + i\sin(1))$
   E. None of these.

7. Simplify $e^{\frac{3-i\pi}{3}}$.
   A. $e(1+\sqrt{3})$
   B. $\frac{e}{2}(1-\sqrt{3})$
   C. $\frac{e}{2}(-1+\sqrt{3})$
   D. $e(-1-\sqrt{3})$
   E. None of these.

8. Simplify $\mathrm{Log}(e+ei)$.
   A. $1 + \frac{1}{2}\ln 2 + i\frac{\pi}{4}$
   B. $\sqrt{2}e - i\frac{\pi}{8}$
   C. $e - \frac{1}{2}\ln 2 - i\frac{2\pi}{3}$
   D. $1 + i$
   E. None of these.

9. Simplify $\sqrt{2i}$.
   A. $\pm(\sqrt{2} - \sqrt{2}i)$
   B. $\pm(1+i)$
   C. $\pm(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i)$
   D. $\pm(-2-2i)$
   E. None of these.

10. Let $z = 8e^{3i\pi/2}$. Which of these is the principle value of $z^{1/3}$?
    A. $2e^{-i\pi/6}$
    B. $2e^{i\pi/2}$
    C. $2e^{7i\pi/6}$

Chapter 4a Computation

11. Find $\int_0^1 (3t^2 - 4it^3)dt$.

    A. $3 + 4i$

    B. $-1 + i$

    C. $-3 - 4i$

    D. $1 - i$

    E. None of these.

12. Find $\int_\pi^{3\pi} ie^{i\theta} d\theta$.

    A. 0

    B. 2π

    C. $-2\pi i$

    D. $-2i$

    E. None of these.

13. Which of these is a parametrization of a parabola in the complex plane?

    A. $z(t) = t^2 - 2it^2$

    B. $z(t) = -t + 2it$

    C. $z(t) = it^2$

    D. $z(t) = 2t - it^2$

    E. None of these.

14. Which of these is a parametrization of the unit circle in the complex plane starting at i and rotating exactly once clockwise for $0 \leq t \leq 1$?

    A. $z(t) = e^{it}$

    B. $z(t) = e^{2\pi - it}$

    C. $z(t) = e^{\pi i(1/2 - 2t)}$

    D. $z(t) = e^{\pi(t-i)}$

    E. None of these.

15. Let $f(z) = 3e^z$ and C be the line segment joining 0 to $1 + \pi i$. Find $\int_C f(z)dz$.

    A. $-3e$

    B. $e + 3i$

    C. $3 - i$

    D. $3ie$

    E. None of these.

Chapter 2 Knowledge

16. If $\lim_{h\to 0} \frac{f(z+h)-f(z)}{z}$ exists, then f is differentiable at z.
    A. True
    B. False
17. If $\lim_{z\to w} f(z)$ exists and $\lim_{z\to w} g(z)$ exists, then $\lim_{z\to w} \frac{f(z)}{g(z)}$ always exists.
    A. True
    B. False
18. If $u_x \neq v_y$ at a point, then $f(z) = u(z) + iv(z)$ is not differentiable at that point.
    A. True
    B. False
19. If $rv_r \neq -u_\theta$ at a point, then $f(z) = u(z) + iv(z)$ is not differentiable at that point.
    A. True
    B. False
20. If f is differentiable for all complex numbers, then f is entire.
    A. True
    B. False

Chapter 3 Knowledge

21. e^z is a multi-valued expression.
    A. True
    B. False
22. $|e^{2z+1+i}| > 0$ for all complex z.
    A. True
    B. False
23. $\log(e^z)$ is a multi-valued expression.
    A. True
    B. False
24. $\mathrm{Log}(z)$ is well-defined for all complex numbers z.
    A. True
    B. False
25. The principle value of $z^{1/4}$ has a principle argument greater than $-\pi/4$ and less than or equal to $\pi/4$.
    A. True
    B. False

Chapter 4a Knowledge

26. $Re(\int_a^b w(t)dt) = \int_a^b Im(w(t))dt$.
    A. True
    B. False
27. The Mean Value Theorem holds for all complex functions.
    A. True
    B. False
28. Joining two contours end-to-end results in a contour.
    A. True
    B. False
29. Let $-C$ be the reversal of the contour C. Then $\int_C f(z)dz = \int_{-C} f(z)dz$.
    A. True
    B. False
30. The value of $\int_C f(z)dz$ depends only on the starting and ending points of C.
    A. True
    B. False

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Proofs

Solve at most one of the exercises from each chapter, skipping any chapters already checked-off as mastered on your progress report.

1. **Ch1** Prove or disprove that $|\overline{zw}| = |z||w|$.
2. **Ch2** Show that $\lim\limits_{z\to i}(z-i)(\overline{z-i})^{-1}$ does not exist.
3. **Ch2** Give an example of a complex function that is continuous but not differentiable at 0, and explain why.
4. **Ch3** Prove that $z^{1/3}$ takes on exactly three values for each non-zero z.
5. **Ch3** Prove that $\frac{d}{dz}[\operatorname{Log} z]$ is $-i$ at $z = i$ by using the derivative definition $\lim_{z\to i} \frac{\operatorname{Log} z - \operatorname{Log} i}{z-i}$.
6. **Ch4a** Prove that $\int_0^{\pi/2} e^{(2+i)\theta} d\theta = \frac{e^\pi - 2}{5} + i(\frac{1+2e^\pi}{5})$.
7. **Ch4a** Use the fact that $\int_0^{\pi/2} e^{(2+i)\theta} d\theta = \frac{e^\pi - 2}{5} + i(\frac{1+2e^\pi}{5})$ to compute $\int_0^{\pi/2} e^{2x} \sin(x) dx$ without using integration by parts.