C01a: LogExpDerInt.

Find $\frac{d}{dx}[e^{2x+\ln(x)}]$.

C01b: LogExpDerInt.

Find $\int \frac{1+xe^x}{x}\,dx$.

S01: LogExpPrf.

Use the definitions $\log_b x = \frac{\ln x}{\ln b}$ and $b^x = \exp(x \ln b)$ to prove that $x = b^{\log_b(x)}$.

C02a: HypDerInt.

Find $\frac{d}{dx}[\sinh(x^3 + x)]$.

C02b: HypDerInt.

Find $\int (4\cosh(x) - 5\sinh(x))\,dx$.

S02: HypPrf.

Use the definitions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

to prove that $\sinh^2(x) + \cosh^2(x) = \cosh(2x)$.

C03a: IntSub.

Find $\int y(y+3)^9\,dy$.

C03b: IntSub.

Find $\int z\cos(z^2+1)\,dz$.

S03: TrigId.

Find $\int \cos^5(\theta)\sin^5(\theta)\,d\theta$.

S04: TrigSub.

Find $\displaystyle\int \frac{4}{9+4x^2}\,dx$.

S05: PartFrac.

Expand $\displaystyle\frac{2x^2+8x+7}{(x+2)^3}$ using partial fractions. Do not integrate.

C04: IntParts.

Find $\int 4x^3 \ln(x)\,dx$.

C04: IntParts.

Find $\int xe^x\,dx$.

C05a: IntTech.

Which integration technique is most appropriate for each integral?
Choices: *Integration by Substiution — Method of Partial Fractions — Trigonometric Identities — Trigonometric Substitution — Integration by Parts*

1. $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$
2. $\int x^2\sin(x)\,dx$
3. $\int \cos^2(x)\sin^2(x)\,dx$
4. $\int \frac{x+4}{x^2+3x+2}\,dx$
5. $\int x\sinh(x^2)\,dx$

C05b: IntTech.

Which integration technique is most appropriate for each integral?
Choices: *Integration by Substiution — Method of Partial Fractions — Trigonometric Identities — Trigonometric Substitution — Integration by Parts*

1. $\displaystyle\int x\cos(x)\,dx$
2. $\displaystyle\int \sin^4(x)\,dx$
3. $\displaystyle\int \frac{9}{x\sqrt{x^2+4}}\,dx$
4. $\displaystyle\int \frac{6x}{3x^2-1}\,dx$
5. $\displaystyle\int \frac{x^2+x+12}{x^3+3x}\,dx$

C06a: AreaBtCurv.

Find a definite integral equal to the area bounded by $y = x$, $y = 3x$ and $x = 2$.

C06b: AreaBtCurv.

Find a definite integral equal to the area bounded by $y = x^2 - 4x + 4$ and $x + y = 4$.

S06: CrossSect.

Find a definite integral equal to the volume of a cone of height 2 that has a circular base of radius length 6.

C07a: WashShell.

Find a definite integral equal to the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 3x$ and $x = 2$ around the axis $x = 0$.

C07b: WashShell.

Find a definite integral equal to the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 4$ and $x + y = 4$ around the axis $y = -1$.

C08a: Work.

Find a definite integral equal to the work required to pull up 20 meters of cable if it weighs 120 newtons and is fully extended downward into a hole. (Do not solve your integral.)

C08b: Work.

Hooke's Law states that the force required to stretch or compress a spring x units from its natural length requires $F(x) = kx$ units of force for some constant k (depending on the spring). Suppose a spring satisfies $k = 4$ and is naturally length 7. Find a definite integral equal to the work required to compress this spring from length 11 to length 13. (Do not solve your integral.)

S07: WorkDiff.

Assume salt water weighs $10kN/m^3$. Find an expression in terms of y for the work differential dW required to pump a cross-section of water at height y from a conical tank pointed downwards that stands 10 meters tall, with a circular lid with radius 5 meters. Then give a definite integral equal to the work required to pump this tank if it is filled 7 meters deep with salt water.

C09a: Param.

Parametrize the line segment starting at $(-1, 0)$ and ending at $(2, 4)$.

C09b: Param.

Parametrize the portion of the circle $x^2 + (y - 2)^2 = 4$ from $(2, 2)$ to $(0, 4)$.

S08: ParamAppl.

The arclength of a curve parametrized by x, y in terms of t from $a \leq t \leq b$ is given by $\int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$. Give a definite integral equal to the length of the parabolic arc along the curve $y = x^2$ from $(-1, 1)$ to $(3, 9)$.

C10a: Polar.

Convert the polar coordinates $p(6, \frac{5\pi}{6})$ to Cartesian.

C10b: Polar.

Convert the Cartesian equation $(x + 3)^2 + y^2 = 9$ to a polar equation.

S09: PolarAppl.

The area bounded by an outside curve with polar equation $r = R(\theta)$ and inside curve with polar equation $r = r(\theta)$ where $\alpha \leq \theta \leq \beta$ is given by $\frac{1}{2}\int_\alpha^\beta ((R(\theta))^2 - (r(\theta))^2)\, d\theta$. Give a definite integral equal to the area inside the circle $r = 8 \sin\theta$.

S10: SeqForm.

Find a formula for the sequence $\langle 2, 5, 10, 17, 26, 37, 50, \ldots\rangle$. (You may choose whatever starting index you like.)

C11a: SeqLim.

Find $\displaystyle\lim_{m\to\infty} \frac{m + 4m^2 - 3}{m^2 + 7}$.

C11b: SeqLim.

Recall that $e^x = \lim_{n\to\infty}(1 + \frac{x}{n})^n$. Find $\displaystyle\lim_{k\to\infty} \left(\frac{k}{3 + k}\right)^{-k}$.

C12a: PartSum.

Find a formula for the partial sum $s_n = a_0 + a_1 + \cdots + a_n$ where $a_n = (\frac{3}{2n+1} - \frac{3}{2n+3})$. Then use this formula to find the value of $\sum_{n=0}^{\infty}(\frac{3}{2n+1} - \frac{3}{2n+3})$.

C12b: PartSum.

Find a formula for the partial sum $s_n = a_1 + a_2 + \cdots + a_n$ where $a_n = (\frac{3}{4})^n$. Then use this formula to find the value of $\sum_{n=1}^{\infty}(\frac{3}{4})^n$.

S11: GeoAlt.

Recall that the geometric series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ when $|r| < 1$ and diverges otherwise. Use this to find $\sum_{n=1}^{\infty}(\frac{3}{4})^n$.

S12: IntTest.

Does $\int_0^{\infty} \frac{3}{x+1}\,dx$ converge or diverge?
Does $\sum_{k=2}^{\infty} \frac{3}{k+1}$ converge or diverge?

S13: RatioRoot.

Does $\displaystyle\sum_{n=0}^{\infty} \frac{(1/2)^n}{n!}$ converge or diverge?

S14: CompTests.

Does $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{n^2}{n^3+4}}$ converge or diverge?

C13: SerTech. (worth double)

For each series, choose **one** technique that would be appropriate to determine convergence/divergence. (There may be multiple correct responses.) Then choose whether the series is convergent or divergent. You do not need to show your work.
Choices: *Partial Sum Sequence — Divergence Test — Geometric Series Test — Alternating Series Test — Integral Test — p-Series Test — Ratio Test — Root Test — Direct/Limit Comparison Test*

1. $\sum_{k=0}^{\infty} \frac{3^k}{5^{2k}}$
2. $\sum_{m=3}^{\infty} \frac{m^2}{m^3+1}$
3. $\sum_{n=2}^{\infty} 5n^{-1/2}$

C14a: PowSer.

Find the domain of $f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{4^k}$. For each endpoint, if they exist, write the appropriate series and label it as converges/diverges, but you do not need to show your work in determining if the series converges or diverges.

C14b: PowSer.

Find the domain of $f(x) = \sum_{k=2}^{\infty} \frac{(x-3)^k}{k!}$. For each endpoint, if they exist, write the appropriate series and label it as converges/diverges, but you do not need to show your work in determining if the series converges or diverges.

C15a: TaySer.

Generate the MacLaurin series (Taylor series where $a = 0$) for $f(x) = e^x$.

C15b: TaySer.

Generate the MacLaurin series (Taylor series where $a = 0$) for $f(x) = \cos(x)$.

S15: PowSerConv.

Use the fact that $\sinh(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$ for all real numbers x to find a power series converging to $f(x) = \sinh(x^2)$ for all real numbers x.

C16: Approx. (worth double)

The Maclaurin Series for $\cosh(x)$ is given by $\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$, and the Taylor polynomial error at x for $a = 0$ is given by $|R_n(x)| = \frac{|f^{(n+1)}(x_n)|}{(n+1)!}|x|^{n+1}$ for some value of x_n between 0 and x. First, find a sufficiently large value of n such that $|R_n(1)| < 0.01$, given that $\sinh(1) \le \cosh(1) \le e \le 3$. Then, approximate the value of $\cosh(1)$ with an error no larger than 0.01.

MA 126-103 — Summer 2017 — Dr. Clontz — Final Exam

Name:	
J#:	
Date: **2017 July 26**	

Exercise Type (Cost):

Final Exam (0AP each)

Write the Standard code (C##a or C##b or S##) for the exercise you are attempting:	Mark: