MA 227-103 — Summer 2017 — Dr. Clontz — MIDTERM

Name:	
J#:	**Midterm**
Date: 2017 June 30	

Instructions:

- **Your student ID is required to take this exam.**
- Do **not** separate these pages.
- All items other than writing utensils must be put away for the duration of the exam. You will be provided with an updated progress report.
- You have **75 minutes** to complete any of the provided exercises: two for each Core Standard C01-C06 and one for each Supporting Standard S01-S07. *You are not expected to answer every exercise.* Instead, only answer the exercises for standards where you have not earned all possible ✓s and you are confident of the correct solution.
- Use the space provided in the back of the packet if you run out of room for an exercise.
- Each worked exercise will be marked with $\times$, $\star$, or ✓ and treated similarly to quiz exercises. Details on improving $\star$ marks will be provided at a later date.
- All the necessary information to answer each question is provided on the exam. The proctor will not answer questions or make clarifications. In the unlikely event of an error, a make-up exercise will be offered at a later date.
- When you are satisfied with your solutions, submit this packet to the proctor. Then collect your belongings and exit the classroom.
- **Exams not submitted to the proctor in time will not be graded.**

Standard: This student is able to... **C01: SurfaceEQ.** Identify and sketch surfaces in three-dimensional Euclidean space. a	Mark:

Sketch the surface $(x+1)^2 + (y-3)^2 + z^2 = 1$ in xyz space.

Standard: This student is able to... **C01: SurfaceEQ.** Identify and sketch surfaces in three-dimensional Euclidean space. b	Mark:

Sketch the equation $yz = 9$ first as a curve in the yz plane, then as a surface in xyz space.

Standard: This student is able to... **C02: VectFunc.** Model curves in Euclidean space with vector functions. a	Mark:

Give a vector function modeling the line passing through $\langle 3, 3, 2\rangle$ and parallel to the vector $\langle 3, -1, 4\rangle$.

Standard: This student is able to... **C02: VectFunc.** Model curves in Euclidean space with vector functions. b	Mark:

Give a vector function $\mathbf{r}(t)$ parameterizing one counter-clockwise motion around the circle with center $\langle 5, 0\rangle$ from the point $\langle 5, 3\rangle$ back to itself.

Standard: This student is able to... **C03: VectCalc.** Compute and apply vector function limits, derivatives, and integrals. a	Mark:

Find a vector tangent to the curve parameterized by $\mathbf{r}(t) = \langle 3t, 2t^2 + t - 2 \rangle$ at the point $\langle 0, -2 \rangle$.

Standard: This student is able to... **C03: VectCalc.** Compute and apply vector function limits, derivatives, and integrals. b	Mark:

Find $\mathbf{r}(t)$ given $\mathbf{r}'(t) = \langle e^t, 3t^2 \rangle$ and $\mathbf{r}(1) = \langle 2e, 3 \rangle$.

Standard: This student is able to... **C04: VectFuncSTNB.** Compute and apply the arclength parameter and TNB frame for a vector function. a	Mark:

Find the arclength parameter $s(t)$ for the curve given by $\mathbf{r}(t) = \langle 4\cos t, 3t, -4\sin t\rangle$. Then give the arclength from $t = 0$ to $t = \pi$.

Standard: This student is able to... **C04: VectFuncSTNB.** Compute and apply the arclength parameter and TNB frame for a vector function. b	Mark:

Suppose the unit tangent and normal vectors at a point on a parametrized curve are given by $\mathbf{T} = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle$ and $\mathbf{N} = \left\langle \frac{2}{5}, \frac{\sqrt{3}}{2}, \frac{3}{10} \right\rangle$. Find the binormal vector $\mathbf{B}$ at that same point.

Standard: This student is able to... **C05: MulivarCalc.** Compute and apply the partial derivatives, gradient, and directional derivatives of a multivariable real-valued function. a	Mark:

Find the minimal value of the directional derivative for the function $f(x, y) = 4xy+2x^2+y-1$ at the point $\langle -1, 2\rangle$.

Standard: This student is able to... **C05: MulivarCalc.** Compute and apply the partial derivatives, gradient, and directional derivatives of a multivariable real-valued function. b	Mark:

Verify the mixed derivative theorem $f_{yz} = f_{zy}$ for $f(x, y, z) = xy^3 - 5xyz^2$ by computing the second partial derivative both ways.

Standard: This student is able to... **C06: ChainRule.** Apply the multivariable Chain Rule to compute derivatives and find normal vectors. a	Mark:

Use the multivariable Chain Rule to find $\frac{df}{dt}$ at $t = 1$ given $f(x, y) = 3xy^2$ and $\mathbf{r}(t) = \langle t^2 + 2, 1 - 2t\rangle$.

Standard: This student is able to... **C06: ChainRule.** Apply the multivariable Chain Rule to compute derivatives and find normal vectors. b	Mark:

Find an equation for the plane tangent to the surface $xy = z^2 + 3$ at the point $\langle 2, 2, -1\rangle$.

Standard: This student is able to... **S01: 3DSpace.** Plot and analyze points and vectors in three-dimensional Euclidean space.	Mark:

In xy plane, sketch the vector $\mathbf{v} = \langle 3, 4\rangle$, the vector $\mathbf{w}$ pointing from $\langle 3, 4\rangle$ to $\langle -1, 2\rangle$, and the vector $\mathbf{v} + \mathbf{w}$.

Standard: This student is able to... **S02: DotProd.** Compute and apply the dot product of two vectors.	Mark:

Find the work done by a force vector $\langle 5, -3\rangle$ over the displacement vector $\langle 3, -4\rangle$.

Standard: This student is able to... **S03: CrossProd.** Compute and apply the cross product of two vectors.	Mark:

Use the cross product to prove that $\langle 3,1,-4\rangle$ and $\langle 6,2,-8\rangle$ are parallel vectors.

Standard: This student is able to... **S04: Kinematics.** Compute and apply position, velocity, and acceleration vector functions.	Mark:

Recall that position in ideal projectile motion is given by $\mathbf{r}(t) = P_0 + \mathbf{v}_0 t - \frac{1}{2} g \hat{\jmath} t^2$ where P_0 is the initial position, $\mathbf{v}_0$ is initial velocity, and g is acceleration due to gravity.
Assume $g = 10$ meters per second squared. Find the height of a projectile after 3 seconds if it is launched from the ground with initial velocity $\langle 8, 20\rangle$ meters per second.

Standard: This student is able to... **S05: MulivarFunc.** Sketch and analyze the domain, level curves, and graph of a two-variable real-valued function.	Mark:

Graph $f(x, y) = \sqrt{25 - x^2 - y^2}$.

Standard: This student is able to... **S06: Lineariz.** Compute the linearization of a two-variable real-valued function at a point and use it for approximation.	Mark:

Find the linearization $L(x, y)$ for $f(x, y) = (6x + 3y)^{1/2}$ at the point $\langle 1, 1 \rangle$. Then use it to show that $f(1.1, 0.98) \approx 3.09$.

Standard: This student is able to... **S07: Optimiz.** Use the first-derivative test and Lagrange multipliers to optimize a real-valued multivariable function.	Mark:

Find the maximum value of the function $f(x, y) = 2x^2 + y^2 + 1$ on the closed and bounded disk $x^2 + y^2 \leq 4$. (Hint: You can check the critical points on the boundary by using $\mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle$ with $2\sin t\cos t = \sin(2t)$, or alternatively by using Lagrange multipliers with $g(x, y) = x^2 + y^2 = 4$.)

Use this page if additional space is required.

Use this page if additional space is required.