MA 126 — Spring 2017 — Prof. Clontz — Readiness Quizzes

- Which of these is a definition of a^x for all positive numbers a and all real numbers x? (2017-01-11, 1.1, practice)
  - A. $\ln(x \cdot e^a)$
  - B. a multiplied by itself x times
  - C. the unique function for which $\frac{d}{dx}[a^x] = a^x$
  - D. $\exp(x \ln a)$
- Which of these statements is false? (2017-01-11, 1.1, practice)
  - A. $\ln(abc) = \ln(a) + \ln(b) + \ln(c)$
  - B. $\frac{d}{dx}[\ln x] = \frac{1}{|x|}$ for all nonzero numbers x
  - C. $y = \exp(x)$ if and only if $x = \ln(y)$
  - D. $e^x = \exp(x)$

1. The Substitution Rule is the opposite of which derivative rule? (2017-01-19, 2.1)

   A. Chain Rule

   B. Product Rule

   C. Quotient Rule

   D. Power Rule

2. What is incorrect about the following attempt at using the Substitution Rule?

$$\int_0^1 (3-2x)^5\,dx = \int_0^1 u^5 \left(-\frac{1}{2}du\right)$$

   (2017-01-19, 2.1)

   A. dx should have been replaced with $+\frac{1}{2}\,du$.

   B. u shouldn't be raised to the 5th power.

   C. dx should have been replaced with $-2\,du$.

   D. The bounds are incorrect.

3. Which of these formulas would be most useful in finding $\int \sin^4\theta\cos^2\theta\,d\theta$? (2017-01-25, 2.2)

    A. $\sin^2(\theta) = \frac{1}{2} + \frac{1}{2}\sin(2\theta)$

    B. $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

    C. $\cos^2(\theta) = 1 - \sin^2(\theta)$

    D. $\sin^2(\theta) = 1 - \cos^2(\theta)$

4. Which of these formulas would be most useful in finding $\int \sec^4(\theta)\,d\theta$? (2017-01-25, 2.2)

    A. $\sec^2(\theta) = 1 + \tan^2(\theta)$

    B. $\sec^2(\theta) = 1 - \tan^2(\theta)$

    C. $\tan^2(\theta) = 1 + \sec^2(\theta)$

    D. $\tan^2(\theta) = 1 - \sec^2(\theta)$

5. Which of these substitutions would be most useful in finding $\int \frac{1}{25x^2+9}\,dx$? (2017-01-27, 2.3)

   A. Let $25x^2 + 9 = 25\sec^2\theta + 25$.

   B. Let $25x^2 + 9 = 9\sin^2\theta + 9$.

   C. Let $25x^2 + 9 = 9\tan^2\theta + 9$.

   D. Let $25x^2 + 9 = 25\cos^2\theta + 25$.

6. Which of these substitutions would be most useful in finding $\int \frac{1}{x\sqrt{4-16x^2}}\,dx$? (2017-01-27, 2.3)

   A. Let $4 - 16x^2 = 16 - 16\cos^2\theta$.

   B. Let $4 - 16x^2 = 4 - 4\sin^2\theta$.

   C. Let $4 - 16x^2 = 4 + 4\tan^2\theta$.

   D. Let $4 - 16x^2 = 16 + 16\sec^2\theta$.

7. Which of these substitutions would be most useful in finding $\int_3^5 \frac{1}{\sqrt{x^2-9}}\,dx$? (2017-01-27, 2.3)

   A. Let $x^2 - 9 = 9\sin^2\theta + 9$.

   B. Let $x^2 - 9 = \tan^2\theta - 1$.

   C. Let $x^2 - 9 = \cos^2\theta + 1$.

   D. Let $x^2 - 9 = 9\sec^2\theta - 9$.

8. Which of these sums is the first step in expanding $\frac{4x^2+16x+17}{(x+2)^2(x^2+1)^2}$ into partial fractions? (2017-02-01, 2.4)

   A. $\frac{A}{x+2} + \frac{Bx}{x+2} + \frac{C}{(x^2+1)^2}$

   B. $\frac{A}{x+2} + \frac{Bx}{(x+2)^2} + \frac{Cx+D}{(x^2+1)^2}$

   C. $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$

   D. $\frac{Ax+B}{(x+2)^2} + \frac{Cx+D}{(x^2+1)^2}$

9. Why must $\frac{3+5x^5}{(x+1)(x+3)^2}$ first be simplifed using long polynomial division before using the method of partial fractions? (2017-02-01, 2.4)

   A. It is a rational function of x.

   B. The degree of its numerator is odd, while the degree of its demoninator is even.

   C. The degree of its numerator is greater than or equal to the degree of its denominator.

   D. It is an irrational function of x.

10. Which of these techniques is most appropriate as the first step to find $\int z^2 \sin(z^3)\, dz$? (2017-02-09, 2.6)

   A. Integration by Substitution

   B. Integration by Parts

   C. Method of Partial Fractions

   D. Trigonometric Identities

11. Which of these techniques is most appropriate as the first step to find $\int \frac{t^2+3t+1}{t^3+t}\, dt$? (2017-02-09, 2.6)

   A. Method of Partial Fractions

   B. Trigonometric Substitution

   C. Trigonometric Identities

   D. Integration by Substitution

12. Which of these integrals represents the area bounded by the curves $x = y^2$ and $x = 4$? (2017-02-13, 3.1)

A. $\int_0^4 ((\sqrt{x}) - (-\sqrt{x}))\,dx$

B. $\int_0^2 ((x^2) - (4))\,dx$

C. $\int_2^4 ((x^2) - (-x^2))\,dx$

D. $\int_2^0 ((x^2) - (\sqrt{x}))\,dx$

Answer the following questions about the solid of revolution obtained by rotating the triangle with vertices $(1, 1), (2, 2), (2, 3)$ around the line $y = -1$.

13. Which of these curves should be used to find the outer radius $R(x)$? (2017-02-16, 3.3)

    A. $y = \frac{1}{2}x$

    B. $y = 2x - 1$

    C. $y = -2x + 2$

    D. $y = -\frac{1}{2}x + 1$

14. What formula should be used for $R(x)$? (2017-02-16, 3.3)

    A. $R(x) = 2x$

    B. $R(x) = 1 - 2x$

    C. $R(x) = 2 - \frac{1}{2}x$

    D. $R(x) = \frac{1}{2}x - 1$

15. What are the correct bounds for the washer method integral? (2017-02-16, 3.3)

    A. $\pi \int_{-1}^{3}([R(x)]^2 - [r(x)]^2)\, dx$

    B. $\pi \int_{2}^{3}([R(x)]^2 - [r(x)]^2)\, dx$

    C. $\pi \int_{1}^{2}([R(x)]^2 - [r(x)]^2)\, dx$

    D. $\pi \int_{0}^{2}([R(x)]^2 - [r(x)]^2)\, dx$

16. In the work integral $\int_a^b F(x)\,dx$, the function $F(x)$ represents... (2017-02-24, 3.5)

    A. Friction

    B. Speed

    C. Force

    D. Mass

17. When computing the work done in pumping water out of a container, the video suggests using which formula? (2017-02-24, 3.5)

    A. $\int_{y=c}^{y=d} dW$

    B. $\int_a^b F(x)\,dx$

    C. $\int_a^b \frac{W(x)}{x}\,dx$

    D. $\int_{y=c}^{y=d} yF(y)\,dy$

18. The equations $x = 3 + 4\cos(t), y = -2 + 4\sin(t)$ for $0 \leq t \leq 2\pi$ parametrize which kind of curve? (2017-03-01, 4.1)

    A. A parabola

    B. A line segment

    C. A circle oriented counter-clockwise

    D. A circle oriented clockwise

19. The equations $x = 3 + 4t, y = -2 + 4t$ for $0 \leq t \leq 2\pi$ parametrize which kind of curve? (2017-03-01, 4.1)

    A. A parabola

    B. A line segment

    C. A circle oriented counter-clockwise

    D. A circle oriented clockwise

20. The formula $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$ for computing the slope of a tangent line to a curve defined by parametric equations is a result of the... (2017-03-02, 4.2)

    A. Product Rule

    B. Pythagorean Theorem

    C. Method of Partial Fractions

    D. Chain Rule

21. The formula $L = \int_a^b \sqrt{[dx/dt]^2 + [dy/dt]^2}\, dt$ for computing arclength is a result of the... (2017-03-02, 4.2)

    A. Product Rule

    B. Pythagorean Theorem

    C. Method of Partial Fractions

    D. Chain Rule

22. Find the first few terms of the sequence defined recursively by $a_0 = 1$, $a_1 = 2$, $a_{n+2} = 2a_n + a_{n+1}$. (2017-03-22, 5.1)

   A. $\langle 1, 2, 3, 4, 5, \ldots \rangle$

   B. $\langle 1, 2, 3, 5, 8, \ldots \rangle$

   C. $\langle 1, 2, 4, 8, 16, \ldots \rangle$

   D. $\langle 1, 2, 4, 7, 15, \ldots \rangle$

23. The limit $\lim\limits_{n\to\infty} \dfrac{n}{1+n^2}$ is equal to which of the following limits? (2017-03-22, 5.2)

   A. $\lim\limits_{n\to 0} \dfrac{1+n^2}{n}$

   B. $\lim\limits_{x\to\infty} \dfrac{x}{1+x^2}$

   C. $\lim\limits_{x\to 0} \left(x + \dfrac{1}{x}\right)$

   D. $\lim\limits_{n\to\infty} \left(n + \dfrac{1}{n}\right)$

24. Which of the following describes the sequence $\langle (-\frac{2}{3})^n \rangle_{n=0}^{\infty} = \langle 1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots \rangle$? (2017-03-22, 5.2)

   A. It is bounded and monotonic, and therefore convergent by the Monotonic Sequence Theorem.

   B. It is bounded and convergent, but not monotonic.

   C. It is monotonic, but not bounded nor convergent.

   D. It is convergent and monotonic, but not bounded.

25. Which of the following statements about the sequence $\langle(\frac{1}{2})^n\rangle_{n=0}^{\infty} = \langle 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\rangle$ is false? (2017-03-27, 5.3)

A. The sequence is bounded and monotonic, and therefore convergent by the Monotonic Sequence Theorem.

B. Its partial sum sequence $\langle 1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{4}, \ldots\rangle = \langle 1, \frac{3}{2}, \frac{7}{4}, \ldots\rangle$ is bounded and monotonic, and therefore convergent by the Monotonic Sequence Theorem.

C. Its corresponding series $\sum_{n=0}^{\infty}(\frac{1}{2})^n = 1 + \frac{1}{2} + \frac{1}{4} + \ldots$ converges to $\frac{1}{1-\frac{1}{2}} = 2$.

D. Its corresponding series $\sum_{n=0}^{\infty}(\frac{1}{2})^n = 1 + \frac{1}{2} + \frac{1}{4} + \ldots$ is an infinite sum and therefore does not exist.

26. Which of these techniques is NOT valid for determining the convergence of $\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$? (2017-03-31, 5.5)

A. Geometric Series: converges to $\frac{1}{1+\frac{1}{e}}$ because $|\frac{1}{e}| < 1$

B. Series Convergence Test: converges because $\lim_{n\to\infty} \frac{1}{e^n} = 0$.

C. Integral Test: converges because $\int_0^\infty e^{-x}\,dx$ converges

27. Which of these series converges? (2017-03-31, 5.5)

A. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

B. $\sum_{n=3}^{\infty} \frac{1}{n}$

C. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}}$

D. $\sum_{n=4}^{\infty} \frac{1}{n^{1/3}}$

28. Describe $\sum_{j=0}^{\infty} \frac{4^j+1}{3^{j+1}}$ and $\sum_{k=4}^{\infty} \frac{k}{k^2-4}$. (2017-04-10, 5.7)

A. Both converge.

B. The first converges, and the second diverges.

C. The first diverges, and the second converges.

D. Both diverge.

29. Describe $\sum_{m=8}^{\infty} \frac{m+3^m}{m!}$ and $\sum_{n=5}^{\infty} \frac{2}{n+\ln(n)}$. (2017-04-10, 5.7)

A. Both converge.

B. The first converges, and the second diverges.

C. The first diverges, and the second converges.

D. Both diverge.

30. Which of the following power series is equal to $1-x+\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}-\frac{x^5}{120}+\ldots$? (2017-04-14, 6.1)

    A. $\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}$

    B. $\sum_{n=1}^{\infty}\frac{x^n}{(2n)!}$

    C. $\sum_{n=1}^{\infty}\frac{(-x)^n}{3^n}$

    D. $\sum_{n=0}^{\infty}2(\frac{x}{3})^n$

31. The function $f(x)=\sum_{n=0}^{\infty}2(\frac{x}{3})^n$ with domain $|x|<3$ may be simplified to $f(x)=\frac{2}{1-x/3}$ because it is... (2017-04-14, 6.1)

    A. a telescoping series.

    B. a geometric series.

    C. a ratio series.

    D. an alternating series.

32. Let $f(x) = e^x$. It follows that its k^{th} derivative $f^{(k)}(x)$ equals which of the following? (2017-04-18, 6.2)

   A. ke^x.

   B. e^{kx}.

   C. e^x.

   D. $k!e^{x-k}$.

33. Recall that $f(x) = e^x$ converges to its Maclaurin series $\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$ for all x. Therefore, which of the following must hold? (2017-04-18, 6.2)

   A. $e^x = 1 + x + x^2 + x^3 + \ldots$

   B. $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$

   C. $e^x = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$

   D. $e^x = 1 + \frac{x^2}{2} + \frac{x^4}{16} + \frac{x^6}{64} + \ldots$

34. Recall that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$. Which of these is the Maclaurin series converging to xe^{x^3} for all real numbers x? (2017-04-24, 6.3)

    A. $\sum_{k=1}^{\infty} \frac{x^{k+3}}{(k+1)!}$

    B. $\sum_{k=0}^{\infty} \frac{x^{k+1}}{(3k)!}$

    C. $\sum_{k=0}^{\infty} \frac{x^{3k+1}}{k!}$

    D. $\sum_{k=3}^{\infty} \frac{x^{k}}{(k+1)!}$

35. Suppose that $f(x) = 3 + 2x - 4x^2 + 7x^3 - 5x^4 + \ldots$ for all real numbers x. Which of these is the first few terms of a power series converging to $f'(x)$ for all real numbers x? (2017-04-24, 6.3)

    A. $f(x) = 6 - 6x + 14x^2 - 35x^3 + \ldots$

    B. $f(x) = 2 - 8x + 21x^2 - 20x^3 + \ldots$

    C. $f(x) = 3 + 5x + x^2 + 8x^3 + \ldots$

    D. $f(x) = 1 - 2x + 4x^2 - 8x^3 + \ldots$

36. The term $R_n(x) = \frac{f^{(n+1)}(x_n)}{(n+1)!}(x-a)^{n+1}$ in Taylor's formula represents which of the following? (2017-04-27, 6.4)

   A. The ratio between terms of a Taylor series.

   B. The error or remainder between a Taylor polynomial and the generating function.

   C. The randomness of the n^{th} term of the Taylor polynomial.

   D. The Riemann sum used to obtain the integral of the generating function.

37. The number x_n in the $R_n(x)$ term of Taylor's formula satisfies which of the following? (2017-04-27, 6.4)

   A. $|x_n| \leq nx$ for all $n > 1$.

   B. Its only non-negative factors are 1 and itself.

   C. $f(x_n)$ is infinitely differentiable.

   D. x_n must be between x and a.