MA 227-103 — Summer 2017 — Dr. Clontz — FINAL EXAM

Name:	
J#:	
Date: 2017 July 26	

(example cover page)

Final Exam

Instructions (tentative):

- **Your student ID is required to take this exam.**
- Do **not** separate these pages.
- All items other than writing utensils must be put away for the duration of the exam. You will be provided with an updated progress report.
- You have **120 minutes** to complete up to **18 exercises** of the 36 exercises provided in a separate packet: two for each Core Standard C01-C12 and one for each Supporting Standard S01-S12. On each page, clearly mark the Standard Code and, for Core Standards, the exercise letter (for example: C07b or S11).
- Each worked exercise will be marked with $\times$, $\star$, or $\checkmark$.
- Three $\star$ marks will be converted to $\checkmark$ marks. Students with few $\times$ marks on quizzes since July 06 will have one or two additional $\star$ marks converted to $\checkmark$ marks.
- All the necessary information to answer each question is provided on the exam. The proctor will not answer questions or make clarifications.
- When you are satisfied with your solutions, submit this packet and the separate exercise book to the proctor. Then collect your belongings and exit the classroom.
- **Exams not submitted to the proctor in time will not be graded.**

Write the Standard code (C##a or C##b or S##) for the exercise you are attempting:	Mark:

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S01: 3DSpace.

Find the magnitude $\|\mathbf{v}\|$ and direction $\frac{1}{\|\mathbf{v}\|}\mathbf{v}$ of the vector $\mathbf{v} = 6\hat{\imath} - 8\hat{k}$.

S02: DotProd.

Find $\cos\theta$, where θ is the angle between the vectors $\langle 1, -2, 2\rangle$ and $\langle 3, -4, 0\rangle$.

S03: CrossProd.

A force of 6 units is applied to a wrench at an angle of $\pi/6$ radians to a point 4 units away from a bolt. What is the mangitude of the resulting torque?

C01a: SurfaceEQ.

Sketch the surface $2x + y + 4z = 8$.

C01b: SurfaceEQ.

Sketch the equation $x = z^2$ first as a curve in the xz plane, then as a surface in xyz space.

C02a: VectFunc.

Give a vector function parametrizing the line passing through $\langle 0, -2, 1\rangle$ and parallel to the line with vector function $\mathbf{r}(t) = \langle 3 - 2t, 5 + 3t, -2 + 4t\rangle$.

C02b: VectFunc.

Give a vector function parameterizing the portion of the parabola $y = x^2 + 2x + 1$ beginning at $\langle -1, 0\rangle$ and ending at $\langle 3, 16\rangle$.

C03a: VectCalc.

Find a vector tangent to the curve parameterized by $\mathbf{r}(t) = \langle \sin(t), t, \cos(t)\rangle$ at the point $\langle 0, \pi, -1\rangle$.

C03b: VectCalc.

Find $\mathbf{r}(t)$ given $\mathbf{r}'(t) = \langle \sin t, 3t^2\rangle$ and $\mathbf{r}(0) = \langle -2, 3\rangle$.

S04: Kinematics.

Recall that position in ideal projectile motion is given by $\mathbf{r}(t) = P_0 + \mathbf{v}_0 t - \frac{1}{2}g\hat{\jmath}t^2$ where P_0 is the initial position, $\mathbf{v}_0$ is initial velocity, and g is acceleration due to gravity.

Assume $g = 10$ meters per second squared. Find the speed of a projectile after 0.5 seconds if it is launched from the ground with initial speed $20\sqrt{2}$ meters per second at an angle of $\pi/4$ radians.

C04a: VectFuncSTNB.

Find the arclength parameter $s(t)$ for the curve given by $\mathbf{r}(t) = \left\langle 2t, \frac{1}{3}t^3, t^2\right\rangle$. (Hint: $z^4 + 4z^2 + 4 = (z^2 + 1)^2$.)

C04b: VectFuncSTNB.

Sketch the curve $x^2 + y^2 = 1$. Find $\mathbf{T}$ and $\mathbf{N}$ at the point $\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$ and add them to your sketch.

S05: MulivarFunc.

Sketch the level curves for the function $f(x, y) = \sqrt{x^2 + y^2}$ where $k = 0, 1, 2, 3$. Then sketch a graph of the function in xyz space.

C05a: MulivarCalc.

Find ∇g for $g(x, y, z) = \ln(x + z) + 3xy^2$.

C05b: MulivarCalc.

Find rate of change of $f(x, y, z) = xyz + 4y^2z$ at the point $\langle -2, 1, 0\rangle$ as the variables change in the direction of $\mathbf{u} = \left\langle \frac{3}{5}, 0, -\frac{4}{5}\right\rangle$.

C06a: ChainRule.

Let $f(x, y, z) = x^2y - yz + 2xz^2$ and $\mathbf{r}(t) = \langle 2t, e^t, t + 3\rangle$. Use the multivariable Chain Rule by to find $\frac{df}{dt}$ when $t = 0$.

C06b: ChainRule.

Let the equation $3xy^2 - 2x^2 = 4y - 3$ define y as a differentiable function of x near the point $\langle 1, 1\rangle$. Use partial derivatives to find the slope of the line tangent to this curve at the point $\langle 1, 1\rangle$.

C07a: DoubleInt.

Change the order of integration for the integral $\int_0^9 \int_{\sqrt{y}}^3 \cos(x^3)\,dx\,dy$. (Do not solve this integral.)

C07b: DoubleInt.

Give an expression involving an iterated integral that equals the average value of the function $f(x, y) = xy^2$ over the rectangle where $0 \leq x \leq 2$ and $1 \leq y \leq 4$. (Do not solve this integral.)

C08a: TripleInt.

Express the volume of the solid D in the first octant (where x, y, z are all non-negative) bounded by the plane $x+y+z = 2$ as a triple iterated integral. (Do not solve this integral.)

C08b: TripleInt.

Let D be the solid where $0 \leq z \leq \sqrt{4 - x^2 - y^2}$. Express $\iiint_D xy\,dV$ as a triple iterated integral of the variables x, y, z. (Do not solve this integral.)

S08: TransVar.

Find an affine transformation from the unit square with vertices $\langle 0, 0\rangle, \langle 1, 0\rangle, \langle 1, 1\rangle, \langle 0, 1\rangle$ in the uv plane to the rectangle with vertices $\langle 1, 1\rangle, \langle 3, 0\rangle, \langle 5, 4\rangle\ \langle 3, 5\rangle$ in the xy plane.

C09a: PolCylSph

Let D be the solid where $0 \leq z \leq \sqrt{4 - x^2 - y^2}$. Express $\iiint_D xy\,dV$ as a triple iterated integral of either spherical or cylindrical coordinates. (Do not solve this integral.)

C09b: PolCylSph

Find $\iint_R \sqrt{x^2 + y^2}\,dA$ where R is the circle bounded by $x^2 + y^2 = 4$.

C10a: VectField.

Find the curl and divergence of the vector field $\mathbf{F}(x, y) = \langle xyz, 4xz, 2xy\rangle$. Then compute the curl and divergence of the vector field at the point $\langle 1, 1, 1\rangle$.

C10b: VectField.

Find the curl and divergence of the vector field $\mathbf{F}(x, y) = \hat{\imath} + x^2\hat{\jmath} - y\hat{k}$. Then compute the curl and divergence of the vector field at the point $3\hat{k}$.

C11a: LineInt.

Find the circulation of the vector field $\mathbf{F} = \langle -y, x + 1\rangle$ counter-clockwise around the circle $x^2 + y^2 = 4$.

C11b: LineInt.

Rewrite $\int_C xy\,ds$ as a definite integral with respect to t, where C is the portion of the parabola $y = x^2$ starting at $\langle 3, 9\rangle$ and ending at $\langle -2, 4\rangle$. (Do not solve this integral.)

C12a: FundThmLine.

Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y^2 z, 2xzy, xy^2\rangle$ and C is an unknown curve that begins at $\langle 2, 2, 1\rangle$ and ends at $\langle -1, 0, 4\rangle$.

C12b: FundThmLine.

Compute the work done by the force vector field $\langle \cos(x + 2z) + e^y, xe^y, 2\cos(x + 2z)\rangle$ along any path that begins and ends at the same point.

S09: ParamSurf.

Parameterize the portion of the surface $z = y^2 - x^2$ above the square $0 \leq x \leq 3$, $1 \leq y \leq 4$.

S10: SurfInt.

The function $\mathbf{r}(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z\rangle$ parametrizes the cylinder $x^2 + y^2 = 4$. Let S be the portion of the cylinder $x^2 + y^2 = 4$ where $1 \leq z \leq 4$ and $x \geq 0$. Express the surface integral $\iint_S (x^2 + y^2)\, d\sigma$ as a double iterated integral of θ and z. (Do not solve this integral.)

S11: GreenStokes.

Green's Theorem states that if the boundary ∂R of a 2D region R is oriented counter-clockwise, then circulation may be computed as $\int_{\partial R} \mathbf{F} \cdot d\mathbf{r} = \iint_R \operatorname{curl} \mathbf{F} \cdot \mathbf{k}\, dA$.
Let C be the boundary of the triangle bounded by $y = x, y = 2x, y = 4$ oriented counter-clockwise. Express the circulation of the vector field $\langle x^2 y, x + y\rangle$ around C as a double iterated integral. (Do not solve this integral.)

S12: DivThm.

The Divergence Theorem states that if ∂D is the outward-oriented boundary of a 3D solid D, then flux may be computed as $\iint_{\partial D} \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_D \operatorname{div} \mathbf{F}\, dV$.
Let D be the cube where $1 \leq x \leq 2$, $0 \leq y \leq 1$, and $3 \leq z \leq 4$. Express the flux $\iint_{\partial D} \langle x^2, 4yz, 3xz\rangle \cdot \mathbf{n}\, d\sigma$ as a triple iterated integral. (Do not solve this integral.)