The Math of Games

at Huntingdon College

Steven Clontz
http://stevenclontz.com

Department of Mathematics and Statistics
Auburn University

June 23, 2014

Abstract

Two player games of perfect information such as chess and checkers have been played for centuries. Such games can be analyzed using tools from the field of game theory.

We will define two simple games: Takeaway and Nim. Using game theory, we will show how to create a **winning strategy** for one of the players in each game.

Also, we will show how *any* game can be analyzed in order to produce a winning strategy (given enough computational power).

About game theory

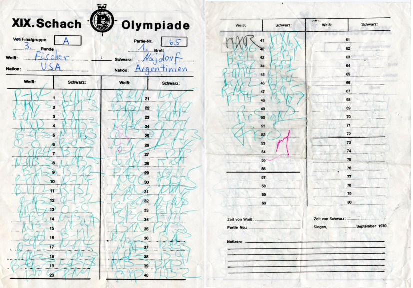
Game theory is a broad subject, but in any context, it typically involves the mathematics of decision making.

Economists use game theory to study **two-player simultaneous** games. Suppose two companies have to set a price on competiting products. The demand for each product depends on the decisions made by both companies

Another example would be **games of chance** like Solitaire or Yahtzee. One or more players use dice rolls or a shuffled deck of cards to play such games, and have to use probability to make decisions based upon future rolls of the dice or face-down cards.

Two-player Sequential Games

We'll be talking about **two-player sequential** games today. These are games of **perfect information**: the players take turns with full knowledge of the history of their opponent's moves.



Coin games

The two games we'll talk about are **coin games**, because they can be played with whatever loose change you have in your pocket.

Takeaway

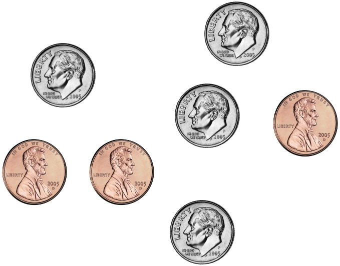
In **Takeaway**, the Players $\mathscr{A}$, $\mathscr{B}$ take turns removing 1, 2, or 3 coins from a pile (with 15 coins perhaps). The player who removes the last coin wins.

Round 1a: Player $\mathscr{A}$ takes away 3 coins, leaving 12.

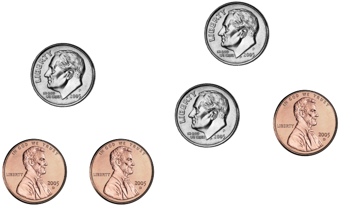
Round 1b: Player $\mathscr{B}$ takes away 2 coins, leaving 10.

Round 2a: Player $\mathscr{A}$ takes away 1 coin, leaving 9.

Round 2b: Player $\mathscr{B}$ takes away 2 coins, leaving 7.



Round 3a: Player $\mathscr{A}$ takes away 1 coin, leaving 6.



Round 3b: Player $\mathscr{B}$ takes away 2 coins, leaving 4.

Round 4a: Player $\mathscr{A}$ takes away 1 coin, leaving 3.

Round 4b: Player $\mathscr{B}$ takes away the last 3 coins, and wins!

A winning strategy

When studying sequential games, we often want to find what's called a **winning strategy**. Such a strategy should guarantee that the player following it cannot lose the game.

I claim that when Takeaway starts with 12 coins, then Player $\mathscr{B}$ has a winning strategy.

Proof: Player $\mathscr{B}$ can always end her round so that there's 8, then 4, then 0 coins. For example:

$12 - 1 = 11$

$11 - 3 = 8$

$12 - 2 = 10$

$10 - 2 = 8$

$12 - 3 = 9$

$9 - 1 = 8$

Two puzzles to try out.

Puzzle 1: Show that Player $\mathscr{A}$ had a winning strategy in Takeaway played with 15 coins (which she obviously didn't follow in the example).

Puzzle 2: Make a general rule about the number of starting coins which tells whether Player $\mathscr{A}$ or $\mathscr{B}$ has a winning strategy.

Questions? Thanks for having me!