Chapter 12

Vectors and the Geometry of Space

12.1 Two and Three Dimensional Space

Definition 1. Let $\mathbb{R}$ be the collection of real numbers, let $\mathbb{R}^2$ be the collection of all **ordered pairs** of real numbers, and let $\mathbb{R}^3$ be the collection of all **ordered triples** of real numbers.

$\mathbb{R}$ is known as the **real line**, $\mathbb{R}^2$ is known as the **real plane** or the xy**-plane**, and $\mathbb{R}^3$ is known as **real (3D) space** or xyz**-space**.

Definition 2. The **distance** between two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ in $\mathbb{R}^2$ is given by the formula

$$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **distance** between two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ in $\mathbb{R}^3$ is given by the formula

$$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Problem 3. Plot and find the distance between the following pairs of points:

- $(-2, 6)$ and $(3, -6)$
- $(0, 0, 0)$ and $(4, 2, 4)$
- $(3, 7, -2)$ and $(-1, 7, 1)$
- $(8, 2, 1)$ and $(4, -2, 7)$

Definition 4. **Simple lines** in $\mathbb{R}^2$ are given by the relations $x = a$, and $y = b$ for real numbers a, b.

Simple planes in $\mathbb{R}^3$ are given by the relations $x = a$, $y = b$, $z = c$ for real numbers a, b, c.

Definition 5. A **circle** in $\mathbb{R}^2$ is the set of all points a fixed distance (called its **radius**) from a fixed point (called its **center**). For a center (a, b) and radius r, the equation for a circle is

$$(x - a)^2 + (y - b)^2 = r^2$$

A **sphere** in $\mathbb{R}^3$ is the set of all points a fixed distance (called its **radius**) from a fixed point (called its **center**). For a center (a, b, c) and radius r, the equation for a sphere is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

Question 6. Sketch the following curves and surfaces.

- $x = 3$ in the xy-plane and xyz-space.
- $y = -1$ in the xy-plane and xyz-space.
- $z = 0$ in xyz-space.
- $(x - 2)^2 + (y + 1)^2 = 9$ in the xy-plane.
- $x^2 + y^2 + z^2 = 4$ in xyz-space.
- $x^2 + (y - 1)^2 + z^2 = 1$ in xyz-space.

Suggested Homework: Section 12.1 numbers 4, 6, 7, 8, 10, 11, 12, 14, 15, 16

12.2 Vectors

Definition 7 (Vector)**.** A **vector** $\vec{\mathbf{v}}$ is a mathematical object that stores a **magnitude** (a nonnegative real number often thought of as length) and **direction**. Two vectors are **equal** if and only if they have the same magnitude and direction.

Definition 8. The **zero vector** $\vec{\mathbf{0}}$ has zero magnitude and no direction. (This is the only vector without a direction.)

Definition 9. For a given point $P = (a, b)$ in $\mathbb{R}^2$, its **position vector** is given by $\vec{\mathbf{P}} = \langle a, b\rangle$: the vector from the origin $(0, 0)$ to the point $P = (a, b)$.

For a given point $P = (a, b, c)$ in $\mathbb{R}^3$, its **position vector** is given by $\vec{\mathbf{P}} = \langle a, b, c\rangle$: the vector from the origin $(0, 0, 0)$ to the point $P = (a, b, c)$.

Theorem 10. Two vectors are equal if and only if they share the same magnitude and direction as a common position vector.

Definition 11. Since all vectors are equal to some position vector $\langle a, b\rangle$ or $\langle a, b, c\rangle$, we usually define vectors by a position vector written in this **component form**. Since the component form of a vector stores the same information as a point, we will use both interchangeably, that is, $\langle a, b\rangle = (a, b) \in \mathbb{R}^2$ and $\langle a, b, c\rangle = (a, b, c) \in \mathbb{R}^3$ (although we usually sketch them differently).

Problem 12. Plot the following points and position vectors.

- $(1, 3)$ and $\langle 1, 3\rangle$ in the xy-plane.
- $(-2, 5)$ and $\langle -2, 5\rangle$ in the xy-plane.
- $(1, 1, -3)$ and $\langle 1, 1, -3\rangle$ in xyz-space.
- $(0, 5, 0)$ and $\langle 0, 5, 0\rangle$ in xyz-space.

Definition 13. Let $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$. Then the vector with initial point P and terminal point Q is defined as

$$\overrightarrow{\mathbf{PQ}} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Problem 14. Plot and sketch the points P, Q and the vector $\overrightarrow{\mathbf{PQ}}$ for each.

- $P = (1, 3)$, $Q = (-3, 6)$ in the xy-plane
- $P = (3, 1)$, $Q = (0, -2)$ in the xy-plane
- $P = (1, 1, 1)$, $Q = (-3, -1, 3)$ in xyz-space
- $P = (-2, 0, 3)$, $Q = (1, 3, -3)$ in xyz-space

Definition 15. The magnitude $|\vec{\mathbf{v}}|$ of a vector $\vec{\mathbf{v}}$ in $\mathbb{R}^2$ or $\mathbb{R}^3$ is the distance between its initial and terminal points.

Theorem 16. The magnitude of $\vec{\mathbf{v}} = \langle a, b \rangle$ is given by

$$|\vec{\mathbf{v}}| = \sqrt{a^2 + b^2}$$

The magnitude of $\vec{\mathbf{v}} = \langle a, b, c \rangle$ is given by

$$|\vec{\mathbf{v}}| = \sqrt{a^2 + b^2 + c^2}$$

Problem 17. Evaluate the magnitude of the following vectors:

- $\langle 5, 5\rangle$
- $\langle -4, 3\rangle$
- $\langle 12, -5\rangle$
- $\langle 3, 1, -2\rangle$
- $\langle 4, -2, -4\rangle$
- $\langle 8, 0, -6\rangle$

12.2.1 Basic Vector Operations

Definition 18. **Vector addition** is defined component-wise as follows for $\mathbb{R}^2$ and $\mathbb{R}^3$

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1, u_2\rangle + \langle v_1, v_2\rangle = \langle u_1 + v_1, u_2 + v_2\rangle$$

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle u_1, u_2, u_3\rangle + \langle v_1, v_2, v_3\rangle = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3\rangle$$

Definition 19. A **scalar** is simply a real number by itself (as opposed to a vector of real numbers).

Definition 20. **Scalar multiplication of a vector** is defined component-wise as follows for $\mathbb{R}^2$ and $\mathbb{R}^3$:

$$k\vec{\mathbf{u}} = k\langle u_1, u_2\rangle = \langle ku_1, ku_2\rangle$$

$$k\vec{\mathbf{u}} = k\langle u_1, u_2, u_3\rangle = \langle ku_1, ku_2, ku_3\rangle$$

Problem 21. Sketch the following vectors.

- $\vec{\mathbf{u}} = \langle 1, -3 \rangle$, $\vec{\mathbf{v}} = \langle 3, 1 \rangle$ and $\vec{\mathbf{u}} + \vec{\mathbf{v}}$ in the xy-plane.
- $\vec{\mathbf{u}} = \langle 2, 0, 1 \rangle$, $\vec{\mathbf{v}} = \langle -2, 4, 2 \rangle$ and $\vec{\mathbf{u}} + \vec{\mathbf{v}}$ in xyz-space.
- $\vec{\mathbf{u}} = \langle 8, -2 \rangle$ and $\frac{1}{2}\vec{\mathbf{u}}$ in the xy-plane.
- $\vec{\mathbf{u}} = \langle 5, 3, -1 \rangle$ and $3\vec{\mathbf{u}}$ in xyz-space.

Definition 22. A vector $\vec{\mathbf{v}}$ is a **unit vector** if $|\vec{\mathbf{v}}| = 1$.

Theorem 23. For any non-zero vector $\vec{\mathbf{v}}$, the vector

$$\frac{1}{|\vec{\mathbf{v}}|}\vec{\mathbf{v}} = \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$$

is a unit vector.

Definition 24. The **direction** of a vector $\vec{\mathbf{v}}$ is the unit vector $\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$.

Theorem 25. Any vector $\vec{\mathbf{v}}$ is the scalar product of its magnitude and direction:

$$\vec{\mathbf{v}} = |\vec{\mathbf{v}}|\frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$$

Problem 26. Write the following vectors as the scalar product of their magnitude and direction:

- $\langle 5, 5\rangle$
- $\langle -4, 3\rangle$
- $\langle 12, -5\rangle$
- $\langle 3, 1, -2\rangle$
- $\langle 4, -2, -4\rangle$
- $\langle 8, 0, -6\rangle$

Definition 27. The **standard unit vectors** in $\mathbb{R}^2$ are $\widehat{\mathbf{i}} = \langle 1, 0\rangle$ and $\widehat{\mathbf{j}} = \langle 0, 1\rangle$, and any vector in $\mathbb{R}^2$ can be expressed in **standard unit vector form**:

$$\langle a, b\rangle = a\widehat{\mathbf{i}} + b\widehat{\mathbf{j}}$$

The **standard unit vectors** in $\mathbb{R}^3$ are $\widehat{\mathbf{i}} = \langle 1, 0, 0\rangle$, $\widehat{\mathbf{j}} = \langle 0, 1, 0\rangle$, and $\widehat{\mathbf{k}} = \langle 0, 0, 1\rangle$, and any vector in $\mathbb{R}^3$ can be expressed in **standard unit vector form**:

$$\langle a, b, c\rangle = a\widehat{\mathbf{i}} + b\widehat{\mathbf{j}} + c\widehat{\mathbf{k}}$$

Note 28. Since the xy-plane is the the plane $z = 0$ in xyz-space, we say the points $(a, b) = (a, b, 0)$ and vectors $\langle a, b\rangle = \langle a, b, 0\rangle = a\widehat{\mathbf{i}} + b\widehat{\mathbf{j}} + 0\widehat{\mathbf{k}}$ are equal.

Problem 29. Write the following vectors in standard unit vector form.

- $\langle 5, 5\rangle$
- $\langle -4, 3\rangle$
- $\langle 12, -5\rangle$
- $\langle 3, 1, -2\rangle$
- $\langle 4, -2, -4\rangle$
- $\langle 8, 0, -6\rangle$

Theorem 30. The following properties hold for any two vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$ and scalars a, b.

- $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$
- $(\vec{\mathbf{u}} + \vec{\mathbf{v}}) + \vec{\mathbf{w}} = \vec{\mathbf{u}} + (\vec{\mathbf{v}} + \vec{\mathbf{w}})$
- $\vec{\mathbf{u}} + \vec{\mathbf{0}} = \vec{\mathbf{u}}$
- $\vec{\mathbf{u}} + (-\vec{\mathbf{u}}) = \vec{\mathbf{0}}$
- $0\vec{\mathbf{u}} = \vec{\mathbf{0}}$
- $1\vec{\mathbf{u}} = \vec{\mathbf{u}}$
- $a(b\vec{\mathbf{u}}) = (ab)\vec{\mathbf{u}}$
- $a(\vec{\mathbf{u}} + \vec{\mathbf{v}}) = a\vec{\mathbf{u}} + a\vec{\mathbf{v}}$
- $(a + b)\vec{\mathbf{u}} = a\vec{\mathbf{u}} + b\vec{\mathbf{u}}$

Definition 31. **Vector subtraction** is defined as the addition of a negative:

$$\vec{\mathbf{u}} - \vec{\mathbf{v}} = \vec{\mathbf{u}} + (-\vec{\mathbf{v}}) = \langle u_1 - v_1, u_2 - v_2\rangle$$

$$\vec{\mathbf{u}} - \vec{\mathbf{v}} = \vec{\mathbf{u}} + (-\vec{\mathbf{v}}) = \langle u_1 - v_1, u_2 - v_2, u_3 - v_3\rangle$$

Suggested Homework: Section 12.2 numbers 3, 5, 13, 14, 15, 19, 21, 24, 26

12.3 The Dot Product

Definition 32. Let θ be the angle between two non-zero vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$. The **dot product** $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$ is the product of their lengths when projected into the same direction, obtained by this formula:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \cos\theta$$

Definition 33. The dot product with a zero vector is always zero:

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{0}} = \vec{\mathbf{0}} \cdot \vec{\mathbf{v}} = 0$$

Theorem 34. By the Law of Cosines:

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle = u_1 v_1 + u_2 v_2$$

$$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Definition 35. Two vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ are **orthogonal** if $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 0$.

Theorem 36. Two non-zero vectors are orthogonal if the angle θ between them is $\frac{\pi}{2}$ radians.

Theorem 37. The following properties hold for any three vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$ and scalar c.

- $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{v}} \cdot \vec{\mathbf{u}}$
- $(c\vec{\mathbf{u}}) \cdot \vec{\mathbf{v}} = \vec{\mathbf{u}} \cdot (c\vec{\mathbf{v}}) = c(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})$
- $\vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} + \vec{\mathbf{w}}) = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{u}} \cdot \vec{\mathbf{w}}$
- $\vec{\mathbf{u}} \cdot \vec{\mathbf{u}} = |\vec{\mathbf{u}}|^2$

Problem 38. Solve for $\cos\theta$ for the following pairs of vectors.

- $\vec{\mathbf{u}} = \langle 4, -3 \rangle$
  $\vec{\mathbf{v}} = \langle 5, 12 \rangle$
- $\vec{\mathbf{u}} = \langle 1, 4, 2 \rangle$
  $\vec{\mathbf{v}} = \langle 4, 1, -2 \rangle$
- $\vec{\mathbf{u}} = \langle 0, 5, -11 \rangle$
  $\vec{\mathbf{v}} = \langle 2, 0, 0 \rangle$

Definition 39. The work W done by a force vector $\vec{\mathbf{F}}$ over a displacement vector $\vec{\mathbf{D}}$ is given by

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}} = |\vec{\mathbf{F}}||\vec{\mathbf{D}}| \cos\theta$$

Suggested Homework: Section 12.3 numbers 3, 5, 6, 7, 8, 9, 10, 11, 15, 17, 21, 27, 41, 42, 44

12.4 The Cross Product

Definition 40. For any two non-parallel vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$ in $\mathbb{R}^3$, the **Right-Hand Rule** gives a specific direction orthogonal to both: position $\vec{\mathbf{u}}$ with your right thumb and $\vec{\mathbf{v}}$ with your right index finger, and let your middle finger extend orthogonal to both to give this direction.

Definition 41. Let θ be the angle between two non-zero vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$ in $\mathbb{R}^3$, and let $\vec{\mathbf{n}}$ be the direction given by the Right-Hand Rule. The **cross product** $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$ is the vector orthogonal to both which follows the Right-Hand Rule and has magnitude equal to the area of the parallelogram formed from both.

$$\vec{\mathbf{u}} \times \vec{\mathbf{v}} = (|\vec{\mathbf{u}}||\vec{\mathbf{v}}| \sin\theta)\vec{\mathbf{n}}$$

$$|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = |\vec{\mathbf{u}}||\vec{\mathbf{v}}| \sin\theta$$

Definition 42. The cross product with a zero vector is always the zero vector:

$$\vec{\mathbf{v}} \times \vec{\mathbf{0}} = \vec{\mathbf{0}} \times \vec{\mathbf{v}} = \vec{\mathbf{0}}$$

Theorem 43. The following properties hold for any three vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$ and scalars a,b.

- $(a\vec{\mathbf{u}}) \times (b\vec{\mathbf{v}}) = (ab)(\vec{\mathbf{u}} \times \vec{\mathbf{v}})$
- $\vec{\mathbf{u}} \times (\vec{\mathbf{v}} + \vec{\mathbf{w}}) = \vec{\mathbf{u}} \times \vec{\mathbf{v}} + \vec{\mathbf{u}} \times \vec{\mathbf{w}}$
- $(\vec{\mathbf{v}} + \vec{\mathbf{w}}) \times \vec{\mathbf{u}} = \vec{\mathbf{v}} \times \vec{\mathbf{u}} + \vec{\mathbf{w}} \times \vec{\mathbf{u}}$
- $\vec{\mathbf{v}} \times \vec{\mathbf{u}} = -(\vec{\mathbf{u}} \times \vec{\mathbf{v}})$

Definition 44. Two vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ are **parallel** if $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = 0$.

Theorem 45. Two non-zero vectors are parallel if the angle θ between them is 0 or π radians.

Definition 46. The cross products of the standard unit vectors are given as follows:

- $\widehat{\mathbf{i}} \times \widehat{\mathbf{j}} = \widehat{\mathbf{k}}$
- $\widehat{\mathbf{j}} \times \widehat{\mathbf{k}} = \widehat{\mathbf{i}}$
- $\widehat{\mathbf{k}} \times \widehat{\mathbf{i}} = \widehat{\mathbf{j}}$

Definition 47. A **determinant** is a short hand for writing certain commonly occuring algebraic expressions:

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Theorem 48. By breaking up $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$ into standard unit vectors:

$$\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \begin{vmatrix} \widehat{\mathbf{i}} & \widehat{\mathbf{j}} & \widehat{\mathbf{k}} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \left\langle \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right\rangle$$

Problem 49. Use the cross product to find a vector normal to both $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

- $\vec{\mathbf{u}} = \langle 4, -3, 0 \rangle$
  $\vec{\mathbf{v}} = \langle 2, 6, -3 \rangle$
- $\vec{\mathbf{u}} = \langle 1, 4, 2 \rangle$
  $\vec{\mathbf{v}} = \langle 4, 1, -2 \rangle$
- $\vec{\mathbf{u}} = \langle 0, 5, -11 \rangle$
  $\vec{\mathbf{v}} = \langle 2, 0, 0 \rangle$

Definition 50. The torque τ done by a force vector $\vec{\mathbf{F}}$ on an arm given by $\vec{\mathbf{D}}$ is given by

$$\tau = |\vec{\mathbf{F}} \times \vec{\mathbf{D}}| = |\vec{\mathbf{F}}||\vec{\mathbf{D}}| \sin\theta$$

Theorem 51. The volume of a parallelpiped determined by the vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$, is given by the **triple scalar product**

$$(\vec{\mathbf{u}} \times \vec{\mathbf{v}}) \cdot \vec{\mathbf{w}} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Suggested Homework: Section 12.4 numbers $1-3$, 17, 19, 28, 29, 33, 35

12.5 Lines and Planes in Space

Theorem 52. Let L be the line in $\mathbb{R}^2$ normal to the vector $\overrightarrow{\mathbf{N}} = \langle A, B\rangle$ and passing through the point $P_0 = (x_0, y_0)$. Then every point $P = (x, y)$ on the line L must satisfy the following equations:

$$\overrightarrow{\mathbf{N}} \cdot \overrightarrow{\mathbf{P_0P}} = 0$$

$$A(x - x_0) + B(y - y_0) = 0$$

Let M be the plane in $\mathbb{R}^3$ normal to the vector $\overrightarrow{\mathbf{N}} = \langle A, B, C\rangle$ and passing through the point $P_0 = (x_0, y_0, z_0)$. Then every point $P = (x, y, z)$ on the plane M must satisfy the following equations:

$$\overrightarrow{\mathbf{N}} \cdot \overrightarrow{\mathbf{P_0P}} = 0$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Problem 53. Sketch and find equations for the following lines and planes:

- The line passing through $(1, -2)$ and parallel to the line with equation $2x - y = 3$.
- The plane passing through $(1, 3, -2)$ and normal to the vector $\langle 3, 0, 1\rangle$.
- The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.

Definition 54. Parametric equations $x(t), y(t)$ for a curve in $\mathbb{R}^2$ assign a point $(x(t), y(t))$ of the curve to each value of t.

Parametric equations $x(t), y(t), z(t)$ for a curve in $\mathbb{R}^3$ assign a point $(x(t), y(t), z(t))$ of the curve to each value of t.

Problem 55. Sketch the curves given by the following parametric equations.

- $x(t) = t$, $y(t) = t^2$
- $x(t) = \sin t$, $y(t) = \frac{t}{\pi}$
- $x(t) = 1 - t$, $y(t) = 3t$, $z(t) = 2t - 3$
- $x(t) = -t^2$, $y(t) = 2$, $z(t) = t$

Theorem 56. Let L be the line in $\mathbb{R}^2$ parallel to the vector $\vec{\mathbf{v}} = \langle a, b \rangle$ and passing through the point $P_0 = (x_0, y_0)$. Then every point $P = (x, y)$ on the line L must satisfy the following vector equation for some t:

$$\vec{\mathbf{P}} = \vec{\mathbf{v}}t + \overrightarrow{\mathbf{P_0}}$$

Thus the line is given by the parametric equations

$$x(t) = at + x_0$$

$$y(t) = bt + y_0$$

Let L be the line in $\mathbb{R}^3$ parallel to the vector $\vec{\mathbf{v}} = \langle a, b, c \rangle$ and passing through the point $P_0 = (x_0, y_0, z_0)$. Then every point $P = (x, y, z)$ on the line L must satisfy the following vector equation for some t:

$$\vec{\mathbf{P}} = \vec{\mathbf{v}}t + \overrightarrow{\mathbf{P_0}}$$

Thus the line is given by the parametric equations

$$x(t) = at + x_0$$

$$y(t) = bt + y_0$$

$$z(t) = ct + z_0$$

Problem 57. Sketch and give parametric equations for the following lines.

- The line with equation $y = -3x + 1$ in the xy plane.
- The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1\rangle$.
- The line normal to the plane with equation $x+y+2z = 4$ and passing through $(1, 1, 1)$.

Suggested Homework: Section 12.5 numbers 3, 4, 6, 7, 17, 19, 24, 27, 31, 32