Chapter 12

Vectors and the Geometry of Space

12.1 Two and Three Dimensional Space

Definition 1. Let $\mathbb{R}$ be the collection of real numbers, let $\mathbb{R}^2$ be the collection of all **ordered pairs** of real numbers, and let $\mathbb{R}^3$ be the collection of all **ordered triples** of real numbers.

$\mathbb{R}$ is known as the **real line**, $\mathbb{R}^2$ is known as the **real plane** or the xy**-plane**, and $\mathbb{R}^3$ is known as **real (3D) space** or xyz**-space**.

Definition 2. The **distance** between two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ in $\mathbb{R}^2$ is given by the formula

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **distance** between two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ in $\mathbb{R}^3$ is given by the formula

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Problem 3. Plot and find the distance between the following pairs of points:

- $(-2, 6)$ and $(3, -6)$
- $(0, 0, 0)$ and $(4, 2, 4)$
- $(3, 7, -2)$ and $(-1, 7, 1)$
- $(8, 2, 1)$ and $(4, -2, 7)$

Definition 4. **Simple lines** in $\mathbb{R}^2$ are given by the relations $x = a$, and $y = b$ for real numbers a, b.

Simple planes in $\mathbb{R}^3$ are given by the relations $x = a$, $y = b$, $z = c$ for real numbers a, b, c.

Definition 5. A **circle** in $\mathbb{R}^2$ is the set of all points a fixed distance (called its **radius**) from a fixed point (called its **center**). For a center (a, b) and radius r, the equation for a circle is

$$(x - a)^2 + (y - b)^2 = r^2$$

A **sphere** in $\mathbb{R}^3$ is the set of all points a fixed distance (called its **radius**) from a fixed point (called its **center**). For a center (a, b, c) and radius r, the equation for a sphere is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

Question 6. Sketch the following curves and surfaces.

- $x = 3$ in the xy-plane and xyz-space.
- $y = -1$ in the xy-plane and xyz-space.
- $z = 0$ in xyz-space.
- $(x - 2)^2 + (y + 1)^2 = 9$ in the xy-plane.
- $x^2 + y^2 + z^2 = 4$ in xyz-space.
- $x^2 + (y - 1)^2 + z^2 = 1$ in xyz-space.

Suggested Homework: Section 12.1 numbers 4, 6, 7, 8, 10, 11, 12, 14, 15, 16