

Calculus II - Fall 2014 - Mr. Clontz - Midterm Exam

Name: _____ 9am / 10am

- If you completed the practice midterm, turn it in before beginning this exam.
- This exam is closed-note and closed-book.
- The withdrawal deadline is the evening of Tuesday, October 7. If you need me to post your grade to Canvas before the deadline, please mark this circle:
☐ POST GRADE BEFORE WITHDRAWAL DEADLINE

Good luck! Here are the series tests in case you need them:

Test	When to Use	Conclusion
Geometric Series	$\sum_{k=1}^{\infty} ar^k$	Converges to $\frac{a}{1-r}$ if $ r < 1$; diverges if $ r \geq 1$.
Divergence Test	All Series	If $\lim_{k \rightarrow \infty} a_k \neq 0$, the series diverges.
Integral Test	$\sum_{k=1}^{\infty} a_k$ where $f(k) = a_k$ and f is continuous, decreasing, and $f(x) \geq 0$	$\sum_{k=1}^{\infty} a_k$ and $\int_1^{\infty} f(x) dx$ both converge or both diverge.
p-series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	Converges for $p > 1$, diverges for $p \leq 1$.
Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $0 \leq a_k \leq b_k$	If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges. If $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} b_k$ diverges.
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $a_k, b_k > 0$ and $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L > 0$	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both converge or both diverge.
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ where $a_k > 0$ for all k	If $\lim_{k \rightarrow \infty} a_k = 0$ and $a_{k+1} \leq a_k$ for all k , then the series converges.
Absolute Convergence	Series with some positive and some negative terms (including alternating series)	If $\sum_{k=1}^{\infty} a_k $ converges, then $\sum_{k=1}^{\infty} a_k$ converges (absolutely).
Ratio Test	Any Series (especially those involving exponentials and/or factorials)	For $\lim_{k \rightarrow \infty} \left \frac{a_{k+1}}{a_k} \right = L$, if $L < 1$, $\sum_{k=1}^{\infty} a_k$ converges absolutely, if $L > 1$, $\sum_{k=1}^{\infty} a_k$ diverges, if $L = 1$, no conclusion.
Root Test	Any Series (especially those involving exponentials)	For $\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } = L$, if $L < 1$, $\sum_{k=1}^{\infty} a_k$ converges absolutely, if $L > 1$, $\sum_{k=1}^{\infty} a_k$ diverges, if $L = 1$, no conclusion.

Multiple Choice (10 points total)

Please only mark the correct choice for each question.

1. (3 points) Nick Saban wrote the following¹:

“Since $\lim_{n \rightarrow \infty} \frac{n}{n^2 + 1} = 0$, the series $\sum_{n=0}^{\infty} \frac{n}{n^2 + 1}$ converges.”

Why is this horribly wrong?

- ☐ The limit $\lim_{n \rightarrow \infty} \frac{n}{n^2 + 1}$ is $\frac{1}{2}$, not 0.
 - ☐ Since $\lim_{n \rightarrow \infty} \frac{n}{n^2 + 1} = 0$, the series $\sum_{n=0}^{\infty} \frac{n}{n^2 + 1}$ diverges.
 - ☐ The Divergence Test requires that the limit be different from 0, and cannot prove that a series converges.
 - ☐ The Divergence Test doesn't work on a series with only positive terms.
2. (3 points) Integration by parts is the reverse version of which rule?
- ☐ Chain Rule $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$
 - ☐ Power Rule $\frac{d}{dx}[x^p] = px^{-1}$
 - ☐ Product Rule $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
 - ☐ Exponential Rule $\frac{d}{dx}[b^x] = b^x \ln b$
3. (4 points) Since $\sin(x)$ has the MacLaurin Series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, which of these is the best approximating polynomial for the value of $\sin(x)$ when x is close to 0?
- ☐ $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$
 - ☐ $1 + x^2 + x^3 + x^4$
 - ☐ $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$
 - ☐ $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

¹I can't back that up, but I feel like he would, y'know?

Full Solutions (90 points total)

Please show all work and draw a box around your final answer, if appropriate. Solutions will be graded according to the rubrics given in the practice midterm.

1. (10 points) Find a general formula for the sequence $\left\{\frac{3}{2}, -\frac{4}{4}, \frac{5}{8}, -\frac{6}{16}, \frac{7}{32}, \dots\right\}$.

2. (10 points) Does the series $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^n}$ converge or diverge? If it converges, give its sum.

3. (10 points) Determine whether or not $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is absolutely convergent, conditionally convergent, or divergent.

4. (10 points) Determine whether the series $\sum_{n=0}^{\infty} \frac{3n^2}{n^3 + 1}$ converges or diverges.

5. (10 points) Determine whether the series $\sum_{n=2}^{\infty} \frac{\sqrt{n-1}}{2+n^2}$ converges or diverges.

6. (10 points) For what values of x is the series $\sum_{n=0}^{\infty} \frac{(1-x)^n}{n+1}$ convergent? What is its radius of convergence?

7. (10 points) Give a power series representing the function $f(x) = \frac{1}{1+3x}$ and its radius of convergence.

8. (10 points) Find the Maclaurin series representing the “hyperbolic cosine” function

$$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

9. (10 points) Evaluate $\int 4xe^x dx$.