MATH 1121 (Calculus for Engineering Technology) Course Outline

1.3 Rectangular Coordinates

- Illustrate the following concepts:
  - rectangular coordinate system,
  - x-axis,
  - y-axis,
  - origin,
  - quadrants,
  - coordinates
- Examples:
  - (Example 1) Plot $A = (2, 1)$ and $B = (-4, -3)$.
  - (Example 3) Three verticies of a rectangle are $A = (-3, -2)$, $B = (4, -2)$, $C = (4, 1)$. What is the fourth vertex?
- *HW: 1-9, 15-16, 21-24*

2.1 Some Basic Definitions

- Distance Formula
  - $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - (Example 2) Find the distance between $(3, -1)$ and $(-2, -5)$.
- Slope Formula
  - $m = \frac{y_2 - y_1}{x_2 - x_1}$
  - (Example 3) Find the slope of the line joining $(3, -5)$, $(-2, -6)$.
  - (Example 4) Find the slope of the line joining $(3, 4)$, $(4, -6)$.
  - $m = \tan\theta$
  - (Example) Find the slope of the line with inclination $120°$.
- Identify parallel/perpendicular lines by slopes.
  - Parallel: $m_1 = m_2$
  - Perpendicular: $m_1 = -\frac{1}{m_2}$

  - (Example 7) Prove that the triangle with vertices $A = (-5, 3)$, $B = (6, 0)$, and $C = (5, 5)$ is a right triangle.

- *HW: 1-20, 29-36*

2.2 The Straight Line

- Point-slope form
  - $y - y_1 = m(x - x_1)$
  - (Example 2) Find the equation of the line passing through $(2, -1)$ and $(6, 2)$.
- Slope-intercept form
  - $y = mx + b$
  - (Example 4) Find the slope and y-intercept of the straight line with equation $2y + 4x - 5 = 0$.
- *HW: 1-21, 33-40*

2.3 The Circle

- Definition
  - A circle is a collection of points equidistant from its center.
- Standard form
  - $(x - h)^2 + (y - k)^2 = r^2$
  - (Example 1) Sketch $(x - 1)^2 + (y + 2)^2 = 16$.
  - (Example 2) Find an equation for the circle with center $(2, 1)$ which passes through $(4, 8)$.
- General form
  - $x^2 + y^2 + Dx + Ey + F = 0$
  - (Example 4) Find the center and radius of the circle $x^2 + y^2 - 6x + 8y - 24 = 0$.
  - (Example) Find two functions whose graphs represent the circle with the previous equation.
- *HW: 1-32, 37-38*

2.4 The Parabola

- Definition
  - A parabola is a colleciton of points equidistant from a focus point and a directrix line.
  - The vertex of a parabola is the point closest to the focus and directix.
  - (Example 6) Find an equation for the parabola with focus $(2, 3)$ and directrix $(y = -1)$
- Standard forms with vertex at origin and horizontal/vertical directrix
  - $y^2 = 4px$ with directrix at $x = -p$ and focus at $(p, 0)$
  - $y^2 = -4px$ with directrix at $x = p$ and focus at $(-p, 0)$
  - $x^2 = 4py$ with directrix at $y = -p$ and focus at $(0, p)$
  - $x^2 = -4py$ with directrix at $y = p$ and focus at $(0, -p)$
  - (Example 2) Find an equation for the parabola with focus $(-2, 0)$ and directrix $(x = 2)$.
  - (Example 4) Find the focus and directrix of the parabola with equation $2x^2 = -9y$.
- *HW: 1-22, 25-28*

2.5 The Ellipse

- Definition
  - An ellipse is a collection of points where the sum of distances from two fixed points (called foci) is kept constant.
  - The two points furthest/closest apart from each other on an ellipse are the endpoints of the major/minor axis.
  - The sum of distances between each point and the foci is the same as the length of the major axis. The major axis passes through both foci.
- Standard form with center at the origin
  - $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with foci given by $(c, 0), (-c, 0)$, where $a^2 - b^2 = c^2$
  - $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, with foci given by $(0, c), (0, -c)$, where $a^2 - b^2 = c^2$
  - (Example 3) Sketch the ellipse with equation $4x^2 + 16y^2 = 64$, and compute the locations of its foci.

  - (Example 5) Find the equation of the ellipse centered at the origin with an end of its minor axis at $(2, 0)$ and containing the point $(-1, \sqrt{6})$.

- *HW: 1-26*

2.6 The Hyperbola

- Definition
  - A hyperbola is a collection of points where the difference of distances from two fixed points (called foci) is kept constant.
  - Hyperbolas are split into two curves. The two closest points on opposite curves are called vertices and give the transverse axis.
- Standard form with center at the origin
  - $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, with foci given by $(c, 0), (-c, 0)$ and asymptotes $y = \pm\frac{bx}{a}$, where $a^2 + b^2 = c^2$
  - $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, with foci given by $(0, c), (0, -c)$ and asymptotes $x = \pm\frac{by}{a}$, where $a^2 + b^2 = c^2$
  - (Example 2) Sketch $\frac{y^2}{4} - \frac{x^2}{16} = 1$, labeling its vertices, asymptotes, and foci.
  - (Example 3) Sketch $4x^2 - 9y^2 = 36$, labeling its vertices, asymptotes, and foci.
- Hypberbola with coordinate axis asymptotes
  - $xy = c^2$, with vertices given by (c, c) and $(-c, -c)$
  - $xy = -c^2$, with vertices given by $(c, -c)$ and $(-c, c)$
  - (Example 5) Sketch $xy = 4$.
- *HW: 1-14, 17-24*

2.7 Translation of Axes

- Vertical/horizontal translation:
  - Shift right h: replace x with $x - h$.
  - Shift up k: replace y with $y - k$.
  - (Example 1) Give an equation of the parabola with vertex $(2, 4)$ and focus $(4, 4)$.
  - (Example 2) Sketch the curve with equation $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$.
- *HW: 1-36*

1.2 Algebraic Functions

- Definition of a function $y = f(x)$.
- Types of functions
  - Polynomials $P(x) = a_0 + a_1 x + \cdots + a_n x^n$
  - Rational functions $R(x) = \frac{P(x)}{Q(x)}$ for polynomials P, Q
  - (Example 1) Voltage equals current multiplied by resistance. If the voltage at time t is given by $E(t) = 2t^2 = y + 5$ and the resistance at time t is given by $R(t) = 3t + 20$, then find a function $I(t)$ which measures the current at time t. Identify it as a polynomial and/or rational function.
- Combinations of functions
  - Addition/Subtraction/Multiplication/Division
  - Compositions $f \circ g$ and $g \circ f$
  - (Example 2) Express $f+g$, $f \circ g$, and $g \circ f$ for the functions given by $f(x) = 2x^2 - 3$ and $g(x) = \sqrt{x+2}$.
- Domain/Range
  - The domain of a function is all real numbers which may be plugged into it without causing division by zero, even roots of negatives, or any other undefined operations.
  - The range of a function is all real numbers which may possibly be attained by the function.
  - (Example 5) Find the domain and range of $f(x) = x^2 + 2$ and $g(t) = \frac{1}{t+2}$.
  - (Example 7) Find the domain of $f(x) = 16\sqrt{x} + \frac{1}{x}$.
- Piecewise functions
  - Piecewise functions are defined differently for different parts of their domains.
  - (Example 9) Find the domain for

    $$f(t) = \begin{cases} 8 - 2t & 0 \le t \le 4 \\ 0 & t > 4 \end{cases}$$

    and compute $f(3), f(6), f(-1)$ if possible.
- Exponent laws
  - $a^m a^n = a^{m+n}$

  - $\frac{a^m}{a^n} = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^m = a^m b^m$
  - $(\frac{a}{b})^m = \frac{a^m}{b^m}$
  - $a^0 = 1$
  - $a^{-n} = \frac{1}{a^n}$
  - $a^{1/n} = \sqrt[n]{a}$
  - Note $\sqrt{a^2} = |a|$ but $\sqrt[3]{a^3} = a$
  - (Example 4) Simplify

$$f(x) = \frac{(3x^2-1)^{1/3}(2x) - (2x^3)(3x^2-1)^{-2/3}}{(3x^2-1)^{2/3}}$$

- *HW: 1-18, 21-34*

1.4 The Graph of a Function

- Definition
  - The graph of a function is the collection of all ordered pairs (x, y) such that $y = f(x)$
  - Graphing Method 1: using Chapter 2
  - Graphing Method 2: using xy chart
  - Vertical line test: the graph of any function hits every vertical line at most once
- Examples
  - (Example 1) Graph $f(x) = 3x - 5$.
  - (Example 3) Graph $f(x) = 1 + \frac{1}{x}$.
  - (Example 4) Graph $f(x) = \sqrt{x+1}$.
  - (Example 6) Graph

$$f(x) = \begin{cases} 2x+1 & x \leq 1 \\ 6 - x^2 & x > 1 \end{cases}$$

- *HW: 1-12, 37-40*

3.1 Limits

- Limits
  - $\lim_{x \to a} f(x) = L$ means that the value of $f(x)$ approaches L as the value of x approaches a in the domain of f.
  - (Example) Given $f(x) = x^2$, we may write the following chart of values

x	$f(x)$
1.9	3.61
1.99	3.9601
1.999	3.996001
2.001	4.004001
2.01	4.0401
2.1	4.41

    to infer that $\lim_{x \to 2} f(x) = 4$.
  - (Example) Given
    $$g(x) = \begin{cases} x^2 & x \neq 2 \\ -5 & x = 2 \end{cases}$$
    we have the same chart of values as before, so we assume $\lim_{x \to 2} g(x) = 4$.
  - (Example) Since $h(x) = \frac{x^3 - 2x^2}{x-2}$ equals x^2 for all values of x except 2, we have the same chart of values as before, and we assume $\lim_{x \to 2} h(x) = 4$.
  - (Example) By graphing $y = f(x)$, $y = g(x)$, and $y = h(x)$, we can see that the points on the graph approach the point $(2, 4)$ in all three cases.

- Continuity
  - A continuous function satsifies the equality $f(a) = \lim_{x \to a} f(x)$ for all numbers a in its domain. (The "just plug it in" rule.)
  - Intuitively: the graph of the function can be drawn without lifting your pencil on the intervals where it is defined
  - FACT: $f(x) = x$ is continuous, and any combination of continuous functions using $+$, $-$, $\times$, $/$, $\circ$, or powers is continuous (where it is defined).
  - (Example 3) $f(x) = \frac{1}{x-2}$ is continuous for its entire domain, but undefined at its asymptote $x = 2$.
  - (Example 5) By graphing
    $$f(x) = \begin{cases} x + 2 & x < 1 \\ -\frac{x}{2} + 5 & x \geq 1 \end{cases} \quad g(x) = \begin{cases} 2x - 1 & x \leq 2 \\ -x + 5 & x > 2 \end{cases}$$
    we see that f is continuous except for when $x = 1$, and g is continuous everywhere.

- Limits to $\pm\infty$
    - $\lim_{x\to\infty} f(x) = L$ means that the value of $f(x)$ approaches L as the value of x attains arbitrarily large postive values.
    - $\lim_{x\to-\infty} f(x) = L$ means that the value of $f(x)$ approaches L as the value of x attains arbitrarily large negative values.
    - (Example) Use a chart of values to infer that $\lim_{x\to\pm\infty} \frac{1}{x} = 0$.
    - (Example 14) Use a chart of values and algebraic manipulation to show that $\lim_{x\to\pm\infty} \frac{x^2+1}{2x^2+3} = \frac{1}{2}$.
- Evaluating limits analytically
    - For continuous functions, use the "just plug it in" rule.
    - (Example 10) Evaluate $\lim_{x\to4} x^2 - 7$
    - For limits of the form $\frac{\text{nonzero}}{0}$, the limit is undefined.
    - (Example 9) Show $\lim_{x\to2} \frac{1}{x-2}$ does not exist.
    - For limits of the form $\frac{0}{0}$, the limit is indeterminate: use canceling to determine its value.
    - (Example 11) Evaluate $\lim_{x\to2} \frac{x^2-4}{x-2}$.
- *HW: 25-44*

3.3 The Derivative

- Secant and Tangent Lines
    - The slope of a secant line is given by $\frac{\Delta y}{\Delta x}$.
    - The slope of a tangent line is given by $\lim_{\Delta x\to0} \frac{\Delta y}{\Delta x}$.
    - (Example) Find the slope of a few secant lines for $y = x^2$ about the point $(2, 4)$, use this to guess the slope of the tangent line at $(2, 4)$, then calculate the tangent slope directly from the limit.
- Derivative
    - The derivative $f'(x)$ or $\frac{d}{dx}[f(x)]$ of a function gives the slope of the tangent lines for each point on the graph $(x, f(x))$.
    - $f'(x) = \lim_{\Delta x\to0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x\to0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$
    - (Example) Show that the derivative of $f(x) = x^2$ is $f'(x) = 2x$, then use this to find the slope of the tangent line at $(2, 4)$.
    - (Example 2) Prove that for $y = 6x - 2x^3$, $y' = \frac{dy}{dx} = 6 - 6x^2$.

- (Example 4) Prove that for $g(x) = x^2 + \frac{1}{x+1}$, $g'(x) = 2x - \frac{1}{(x+1)^2}$.

- *HW: 1-24*

3.5 Derivatives of Polynomials

- Derivatives of Constants and Identity
  - $\frac{d}{dx}[c] = 0$
  - (Example 1) Calculate the $\frac{dy}{dx}$ for $y = -5$.
  - $\frac{d}{dx}[x] = 1$
  - (Example 3) Prove that if $y = x$ then $y' = 1$.
- Derivatives of x^n
  - (Example) Prove that if $f(x) = x^5$ then $f'(x) = 5x^4$.
  - $\frac{d}{dx}[x^p] = px^{p-1}$
  - (Example 2) Find the derivative of $y = x^3$.
  - (Example 4) Find $\frac{dv}{dr}$ where $v = r^{10}$.
- Constant Multiple Rule
  - $\frac{d}{dx}[cf(x)] = cf'(x)$
  - (Example 5) Find the derivative of $y = 3x^2$.
- Sum/Difference Rule
  - $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
  - (Example 7) Find the slope of a line tangent to the curve $y = 4x^7 - x^4$ at the point $(1, 3)$.
- *HW: 1-18*

3.4 The Derivative as an Instantaneous Rate of Change

- Interpretation of $\frac{du}{dv}$
  - The fraction $\frac{\Delta u}{\Delta v}$ represents the change in a variable u as compared to the change in another variable v.
  - Therefore the expression $\frac{du}{dv} = \lim_{\Delta v \to 0} \frac{\Delta u}{\Delta v}$ measures the instantaneous rate of change in u with respect to the rate of change in v.

- In particular, if s is the position of an object and t is the time, then $\frac{ds}{dt}$ is the instantaneous rate of change in position with respect to time, known as its velocity.
- (Example 3) Objects at sea level fall roughly $16t^2$ feet after t seconds from release. Note that after 4 seconds, the object has fallen 256 feet. Use the following chart to approximate the instantaneous downward velocity of the object 4 seconds after release, then compute it exactly using a derivative.

t	3	3.9	3.99	3.999	4
Δt from 4	1	0.1	0.01	0.001	0
s	144	243.36	254.7216	255.872016	256
Δs from 256	112	12.64	1.2784	0.127984	0
$\frac{\Delta s}{\Delta t}$	112	126.4	127.84	127.984	(DNE)

- (Example 5) A spherical balloon is being inflated. Find a formula for the instantaneous rate of change of volume with respect to its radius, then compute it when the radius is 2 meters. (Hint: $V = \frac{4}{3}\pi r^3$.)
- (3.5 Example 8) Suppose the displacement of a piston is $t^3 - 6t^2 + 8t$ centimeters after t seconds have elapsed. Find the position and velocity of the piston in one second intervals from $t = 0$ to $t = 4$.

- *HW **in section 3.5**: 25-32, 38-42*

3.6 Derivatives of Products and Quotients of Functions

- Product Rule
  - (Example 2) Find the derivative of the function $p(x) = (x^2 + 2)(3 - 2x)$.
  - Product Rule: $\frac{d}{dx}[f(x)g(x)] = g(x)f'(x) + f(x)g'(x)$
  - (Example 2 again) Verify the product rule.
  - (Example 3) Find $\frac{dy}{dx}$ where $y = (3 - x - 2x^2)(x^4 - x)$.
- Quotient Rule
  - (Example) Find the derivative of the function $q(x) = \frac{x^2+1}{x}$.
  - Quotient Rule: $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
  - (Same Example) Verify the quotient rule.
  - (Example 4) Find the derivative of $h(x) = \frac{3-2x}{x^2+2}$.

  - (Example 5) The stress S on a hollow tube with tension T, outer diameter D, and inner diameter d is given by the equation $S = \frac{16DT}{\pi(D^4 - d^4)}$. Assume this tube has constant tension $T = 10$ and constant inner diameter $d = 1$. Find the rate of change stress increases with respect to an increasing outer diameter when $D = 2$.

- *HW: 1-28, 39-42*

3.7 The Derivative of a Power of a Function

- Chain Rule for Power Functions
  - (Example 1) Find the derivative of the function $y = (3 - 2x)^3$.
  - Chain Rule for Powers: $\frac{d}{dx}[(f(x))^p] = p(f(x))^{p-1} f'(x)$
  - (Example 1 again) Verify the chain rule.
  - (Example 5) Find the derivative of $y = 6\sqrt[3]{x^2}$.
  - (Example 4) Find the derivative of $y = \sqrt{x^2 + 1}$.
  - (Example 8) Evaluate the derivative of $y = \frac{x}{\sqrt{1-4x}}$ when $x = -2$.
- *HW: 1-24, 29-30, 35-38*

3.8 Differentiation of Implicit Functions

- Implicit Functions
  - The expression $y = f(x)$ defines an explicit function.
  - An equation with variables x, y may define y as an implicit function of x.
  - (Example 1) Manipulate the equation $3x + 4y = 5$ which defines y as an implicit function of x so that y is defined as an explicit function of x.
  - (Example) Give an explicit function which describes the part of the hyperbola centered at the origin with focus $(0, 5)$ and vertex $(0, 3)$ passing through the point $(-16/3, 5)$.
- Implicit Differentiation
  - (Example) Find the slope of the line tangent to the hyperbola from the previous example at the point $(-16/3, 5)$.
  - Implicit functions may be differentated directly by using the chain rule: differentiate y terms as you would x, but tack on a $\frac{dy}{dx}$ term each time you do.
  - (Example) Find the slope of the line described in Example 1 using both implicit and explicit differentation.

- (Example) Solve the hyperbola problem using implicit differentiation.
- (Example 3) Find $\frac{dy}{dx}$ in terms of x, y where $3y^4 + xy^2 = 6 - 2x^3$.
- (Example 5) Find the slope of a line tangent to the graph of $2y^3 + xy + 1 = 0$ at the point $(-3, 1)$.

- *HW: 1-25, 28-30*

3.9 Higher Derivatives

- Higher Derivatives
  - The derivative of a derivative is its second derivative. The derivative of a second derivative is its third derivative, etc.
  - (Example 1) Find all higher derivatives of $y = 5x^3 - 2x$.
  - (Example 3) Find the second derivative of $y = \frac{2}{1-x}$ when $x = -2$.
  - (Example 4) Find y'' for the implicit function defined by $2x^2 + 3y^2 = 6$ in terms of x, y.
- Acceleration
  - If s is position defined in terms of t (time), then $s' = \frac{ds}{dt}$ is velocity and $s'' = \frac{d^2s}{dt^2}$ is acceleration.
  - (Example) The height of an object launched upward from the ground with an initial velocity of v_0 m/s is roughly $s = -4.9t^2 + v_0 t$ meters after t seconds. Find the velocity and acceleration of this object after 1 second given its initial velocity $v_0 = 10$ meters per second.
- *HW: 1-34, 37-38*

4.4 Related Rates

- Related Rates as Implicit Differentation
  - If the variables in an equation are functions of time, then we may use implicit differentiation to compare their rates of change with respect to time.
  - (Example 1) The voltage E of a certain thermocouple may be measured as $E = 2.8T + 0.006T^2$ where T is its temperature in Celcius. If the temperature of the thermocouple is increasing at a rate of $1°$ C/min, then how fast is the voltage increasing when the temperature is $100°$ C?

  - (Example 3) A spherical balloon is being filled at a rate of 2 cubic feet per minute. How fast is its radius growing when the radius is 3 feet long?
  - (Example 5) Two ships leave port at noon. Ship A travels west at 12 km/h, and ship B travels south at 16 km/h. Show that their relative speed is 20 km/h at 2pm. (Note: it's actually always 20 km/h, but it's easier to solve at a specific time.)
  - (Example) A 13 foot ladder is slipping down a vertical wall. The top of the vertical ladder is $13 - 2t^2$ feet high after t seconds of slipping. How fast is the bottom of the ladder extending from the base of the wall after 2 seconds?
  - (Example) A football is spiralling directly towards the ground at a rate of 2 yards per second from a height of 10 yards. A 50 yard tall lightpost shines on the ball. The base of the lightpost is currently 100 yards away from the ball's shadow (but of course, that will change with time). How fast is the ball's shadow moving along the ground?

- *HW: 1-24*

4.7 Applied Maximum and Minimum Problems

- Optimizing functions
  - The optimal (max or min) value of a function must occur either at a critical value or at an endpoint of its domain.
  - (Example 3) Maximize the area of a rectangular corral built with 1600 feet of fencing.
  - (Example 2) Find the number which exceeds its square by the greatest amount.
  - (Example 5) Find the point on the parabola $y = x^2$ closest to the point $(6, 3)$.
  - (Example 6) A company sells 1000 widgets a month when sold at a price of \$5 per widget. For every \$0.01 reduction in price, the company will sell 10 more units a month. What price should the company set for each widget in order to maximize sales in dollars?

- *HW: 1-18,23-27*

4.8 Differentials and Linear Approximations

- Linear approximations
  - The line tangent to a function approximates the function near that point.

  - $L_a(x) = f(a) + f'(a)(x-a)$ approximates the function $f(x)$ near $x = a$.
  - (Example 8) Approximate the value of $\sqrt{9.06}$ using the linear approximation $L_4(x)$ of $f(x) = \sqrt{2x+1}$ at $x = 4$. (A calculator gives 3.00998.)
  - (Example) Find the linear function $L_0(x)$ which approximates $g(x) = 3x^3 - 4x + 7$ near $x = 0$, then approximate $g(0.1)$. (The exact value is 6.603.)
  - (Example) Use $f(x) = \frac{1}{\sqrt{3+x^2}}$ to show that $\frac{1}{\sqrt{4.21}} \approx 0.4875$. (A calculator gives 0.48737.)

- *HW: 21-24, 32-34*

5.2 The Indefinite Integral

- Antiderivatives
  - If $F'(x) = f(x)$, then $F(x)$ is an antiderivative of $f(x)$.
  - (Example) Find a few antiderivatives of $f(x) = 5x^4$.
- Indefinite Integral
  - $\int f(x)\,dx$ represents all antiderivatives of $f(x)$.
  - $\int f(x)\,dx = F(x) + C$.
  - (Example 2) Find $\int 5x^4\,dx$.
- Reverse Power Rule
  - $\int x^p\,dx = \frac{1}{p+1}x^{p+1}$ **except** when $p = -1$
  - (Example) Find $\int x^7\,dx$.
- Constant Multiple and Sum/Difference Rules
  - $\int cf(x)\,dx = cF(x) + C$
  - (Example 3) Find $\int 6x\,dx$.
  - $\int f(x) \pm g(x)\,dx = F(x) \pm G(x) + C$
  - (Example 4) Find $\int 5x^3 - 6x^2 + 1\,dx$.
  - (Example 5) Find $\int \sqrt{r} - \frac{1}{r^3}\,dr$.
- Substitution
  - Complicated integrals may be evaluated by substituting $u = u(x)$ and $du = u'(x)dx$.
  - (Example 6) Find $\int (x^2+1)^3(2x)\,dx$.

  - (Example 7) Find $\int x^2\sqrt{x^3+2}\,dx$.

- Finding the constant of integration
  - While $\int f(x)\,dx = F(x)+C$ represents all antiderivatives of $f(x)$, we sometimes need to solve for C to get a specific antiderivative.
  - (Example 8) Find an equation for y in terms of x where $\frac{dy}{dx} = 3x-1$ and its graph passes through $(1,4)$.
  - (Example 9) Find an equation for the displacement of an object with velocity given by $v = t\sqrt{9-t^2}$ assuming $s=0$ when $t=0$.

- *HW: 1-18, 21-27, 33-36*

5.3 The Area Under a Curve

- Derivation
  - Let ΔA be the area under an increasing curve $y=f(x)$ from x to $x+\Delta$.
  - Then $f(x)\Delta x \leq \Delta A \leq f(x+\Delta x)\Delta x$ and $f(x) \leq \frac{\Delta A}{\Delta x} \leq f(x+\Delta x)$.
  - As $\Delta x \to 0$, we get $\frac{dA}{dx} = f(x)$.
  - Thus $A(x) = \int f(x)\,dx$ can be used to measure the area under a curve between points.

- Formula
  - Let $F(x)$ be any antiderivative of $f(x)$ (usually assuming $C=0$). Then the definite integral $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b)-F(a)$ measures the area under the curve $y=f(x)$ from $x=a$ to $x=b$.
  - (Example 2) Find the area under the straight line $y=2x$ between $x=0$ and $x=4$.
  - (Example 4) Find the area under the curve $y=x^3$ for $1\leq x\leq 2$.
  - (Example 5) Find the area in the first quadrant between the coordinate axes and $y=4-x^2$.

- *HW: 11-19*

7 Trigonometric Functions

- Angles
  - A full counter-clockwise rotation is represented by 2π radians or 360° degrees.

- Smaller angles are measured as a ratio: e.g. a right angle is $2\pi/4 = \pi/2$ radians or 90° degrees.
- Angles/degrees may be converted by multiplying by the ratio $\frac{\pi}{180^\circ}$ or $\frac{180^0}{\pi}$.
- (7.1 Example 2) Convert 45° to radians, and $\frac{3\pi}{4}$ to degrees.

• Trig Functions

- The trigonometric functions measure the relationship between sides of a right triangle in standard position on the xy plane:
  - $\sin\theta = \frac{opp}{hyp}$
  - $\cos\theta = \frac{adj}{hyp}$
  - $\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$
  - $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$
  - $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$
  - $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$
- (Example) Compute $\sin 60^\circ$.
- (Example) Compute $\cos -\pi$.
- The unit circle is often used as a reference for common angles.

• Derivatives of Trig Functions

- Using the graphs of $\sin\theta$, $\cos\theta$, we may infer that $\frac{d}{d\theta}[\sin\theta] = \cos\theta$ and $\frac{d}{d\theta}[\cos\theta] = -\sin\theta$.
- (7.3 Example 2) Compute the derivative of $y = 2\sin(x^2)$.
- (7.3 Example 6) Compute the derivative of $y = \sqrt{1+\cos 2x}$.
- (Example) A 3 foot wide door is shutting such that its angle away from the wall is reducing at a rate of $\pi/4$ radians per second. How fast is the edge of the door moving towards the wall at the moment it is slammed shut?
- This full table of trig derivatives may be proven using the quotient rule:

Original	Derivative
$\sin\theta$	$\cos\theta$
$\cos\theta$	$-\sin\theta$
$\tan\theta$	$\sec^2\theta$
$\cot\theta$	$-\csc^2\theta$
$\sec\theta$	$\sec\theta\tan\theta$
$\csc\theta$	$-\csc\theta\cot\theta$

- (Example) Use the identity $\sin^2 x + \cos^2 x = 1$ to prove that $\frac{d}{dx}[\tan x] = \sec^2 x$.
- (7.4 Example 2) Find the derivative of $y = 2\tan 8x$.
- (7.4 Example 4) Find the derivative of $y = (\tan 2x + \sec 2x)^3$.

  - (7.4 Example 6) Find $\frac{dy}{dx}$ where $\cot 2x - 3\csc xy = y^2$.

- *HW 7.1: 5-12, 21-22, 25-26*
- *HW 7.3: 1-10*
- *HW 7.4: 1-10*

Remaining Topics

- 8.1 Exponential and Logarithmic Functions
- 8.2 Derivative of the Logarithmic Functions
- 8.3 Derivative of the Exponentials Function
- 9.1 The General Power Formula
- 9.2 Basic Logarithmic Form
- 9.3 Exponential Form
- 9.4 Basic Trigonometric Forms