

## MATH 1121 (Calculus for Engineering Technology) Course Outline

### 1.3 Rectangular Coordinates

- Illustrate the following concepts:
  - rectangular coordinate system,
  - $x$ -axis,
  - $y$ -axis,
  - origin,
  - quadrants,
  - coordinates
- Examples:
  - (Example 1) Plot  $A = (2, 1)$  and  $B = (-4, -3)$ .
  - (Example 3) Three vertices of a rectangle are  $A = (-3, -2)$ ,  $B = (4, -2)$ ,  $C = (4, 1)$ . What is the fourth vertex?
- HW: 1-9, 15-16, 21-24

### 2.1 Some Basic Definitions

- Distance Formula
  - $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - (Example 2) Find the distance between  $(3, -1)$  and  $(-2, -5)$ .
- Slope Formula
  - $m = \frac{y_2 - y_1}{x_2 - x_1}$
  - (Example 3) Find the slope of the line joining  $(3, -5)$ ,  $(-2, -6)$ .
  - (Example 4) Find the slope of the line joining  $(3, 4)$ ,  $(4, -6)$ .
  - $m = \tan \theta$
  - (Example) Find the slope of the line with inclination  $120^\circ$ .
- Identify parallel/perpendicular lines by slopes.
  - Parallel:  $m_1 = m_2$
  - Perpendicular:  $m_1 = -\frac{1}{m_2}$

- (Example 7) Prove that the triangle with vertices  $A = (-5, 3)$ ,  $B = (6, 0)$ , and  $C = (5, 5)$  is a right triangle.
- HW: 1-20, 29-36

## 2.2 The Straight Line

- Point-slope form
  - $y - y_1 = m(x - x_1)$
  - (Example 2) Find the equation of the line passing through  $(2, -1)$  and  $(6, 2)$ .
- Slope-intercept form
  - $y = mx + b$
  - (Example 4) Find the slope and  $y$ -intercept of the straight line with equation  $2y + 4x - 5 = 0$ .
- HW: 1-21, 33-40

## 2.3 The Circle

- Definition
  - A circle is a collection of points equidistant from its center.
- Standard form
  - $(x - h)^2 + (y - k)^2 = r^2$
  - (Example 1) Sketch  $(x - 1)^2 + (y + 2)^2 = 16$ .
  - (Example 2) Find an equation for the circle with center  $(2, 1)$  which passes through  $(4, 8)$ .
- General form
  - $x^2 + y^2 + Dx + Ey + F = 0$
  - (Example 4) Find the center and radius of the circle  $x^2 + y^2 - 6x + 8y - 24 = 0$ .
  - (Example) Find two functions whose graphs represent the circle with the previous equation.
- HW: 1-32, 37-38

## 2.4 The Parabola

- Definition
  - A parabola is a collection of points equidistant from a focus point and a directrix line.
  - The vertex of a parabola is the point closest to the focus and directrix.
  - (Example 6) Find an equation for the parabola with focus  $(2, 3)$  and directrix  $(y = -1)$
- Standard forms with vertex at origin and horizontal/vertical directrix
  - $y^2 = 4px$  with directrix at  $x = -p$  and focus at  $(p, 0)$
  - $y^2 = -4px$  with directrix at  $x = p$  and focus at  $(-p, 0)$
  - $x^2 = 4py$  with directrix at  $y = -p$  and focus at  $(0, p)$
  - $x^2 = -4py$  with directrix at  $y = p$  and focus at  $(0, -p)$
  - (Example 2) Find an equation for the parabola with focus  $(-2, 0)$  and directrix  $(x = 2)$ .
  - (Example 4) Find the focus and directrix of the parabola with equation  $2x^2 = -9y$ .
- HW: 1-22, 25-28

## 2.5 The Ellipse

- Definition
  - An ellipse is a collection of points where the sum of distances from two fixed points (called foci) is kept constant.
  - The two points furthest/closest apart from each other on an ellipse are the endpoints of the major/minor axis.
  - The sum of distances between each point and the foci is the same as the length of the major axis. The major axis passes through both foci.
- Standard form with center at the origin
  - $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , with foci given by  $(c, 0), (-c, 0)$ , where  $a^2 - b^2 = c^2$
  - $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ , with foci given by  $(0, c), (0, -c)$ , where  $a^2 - b^2 = c^2$
  - (Example 3) Sketch the ellipse with equation  $4x^2 + 16y^2 = 64$ , and compute the locations of its foci.

- (Example 5) Find the equation of the ellipse centered at the origin with an end of its minor axis at  $(2, 0)$  and containing the point  $(-1, \sqrt{6})$ .
- HW: 1-26

## 2.6 The Hyperbola

- Definition
  - A hyperbola is a collection of points where the difference of distances from two fixed points (called foci) is kept constant.
  - Hyperbolas are split into two curves. The two closest points on opposite curves are called vertices and give the transverse axis.
- Standard form with center at the origin
  - $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , with foci given by  $(c, 0), (-c, 0)$  and asymptotes  $y = \pm \frac{bx}{a}$ , where  $a^2 + b^2 = c^2$
  - $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ , with foci given by  $(0, c), (0, -c)$  and asymptotes  $x = \pm \frac{by}{a}$ , where  $a^2 + b^2 = c^2$
  - (Example 2) Sketch  $\frac{y^2}{4} - \frac{x^2}{16} = 1$ , labeling its vertices, asymptotes, and foci.
  - (Example 3) Sketch  $4x^2 - 9y^2 = 36$ , labeling its vertices, asymptotes, and foci.
- Hyperbola with coordinate axis asymptotes
  - $xy = c^2$ , with vertices given by  $(c, c)$  and  $(-c, -c)$
  - $xy = -c^2$ , with vertices given by  $(c, -c)$  and  $(-c, c)$
  - (Example 5) Sketch  $xy = 4$ .
- HW: 1-14, 17-24

## 2.7 Translation of Axes

- Vertical/horizontal translation:
  - Shift right  $h$ : replace  $x$  with  $x - h$ .
  - Shift up  $k$ : replace  $y$  with  $y - k$ .
  - (Example 1) Give an equation of the parabola with vertex  $(2, 4)$  and focus  $(4, 4)$ .
  - (Example 2) Sketch the curve with equation  $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$ .
- HW: 1-36

## 1.2 Algebraic Functions

- Definition of a function  $y = f(x)$ .
- Types of functions
  - Polynomials  $P(x) = a_0 + a_1x + \cdots + a_nx^n$
  - Rational functions  $R(x) = \frac{P(x)}{Q(x)}$  for polynomials  $P, Q$
  - (Example 1) Voltage equals current multiplied by resistance. If the voltage at time  $t$  is given by  $E(t) = 2t^2 = y + 5$  and the resistance at time  $t$  is given by  $R(t) = 3t + 20$ , then find a function  $I(t)$  which measures the current at time  $t$ . Identify it as a polynomial and/or rational function.
- Combinations of functions
  - Addition/Subtraction/Multiplication/Division
  - Compositions  $f \circ g$  and  $g \circ f$
  - (Example 2) Express  $f+g$ ,  $f \circ g$ , and  $g \circ f$  for the functions given by  $f(x) = 2x^2 - 3$  and  $g(x) = \sqrt{x+2}$ .
- Domain/Range
  - The domain of a function is all real numbers which may be plugged into it without causing division by zero, even roots of negatives, or any other undefined operations.
  - The range of a function is all real numbers which may possibly be attained by the function.
  - (Example 5) Find the domain and range of  $f(x) = x^2 + 2$  and  $g(t) = \frac{1}{t+2}$ .
  - (Example 7) Find the domain of  $f(x) = 16\sqrt{x} + \frac{1}{x}$ .
- Piecewise functions
  - Piecewise functions are defined differently for different parts of their domains.
  - (Example 9) Find the domain for

$$f(t) = \begin{cases} 8 - 2t & 0 \leq t \leq 4 \\ 0 & t > 4 \end{cases}$$

and compute  $f(3)$ ,  $f(6)$ ,  $f(-1)$  if possible.

- Exponent laws
  - $a^m a^n = a^{m+n}$

- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $a^{1/n} = \sqrt[n]{a}$
- Note  $\sqrt{a^2} = |a|$  but  $\sqrt[3]{a^3} = a$
- (Example 4) Simplify

$$f(x) = \frac{(3x^2 - 1)^{1/3}(2x) - (2x^3)(3x^2 - 1)^{-2/3}}{(3x^2 - 1)^{2/3}}$$

- HW: 1-18, 21-34

## 1.4 The Graph of a Function

- Definition
  - The graph of a function is the collection of all ordered pairs  $(x, y)$  such that  $y = f(x)$
  - Graphing Method 1: using Chapter 2
  - Graphing Method 2: using  $xy$  chart
  - Vertical line test: the graph of any function hits every vertical line at most once
- Examples
  - (Example 1) Graph  $f(x) = 3x - 5$ .
  - (Example 3) Graph  $f(x) = 1 + \frac{1}{x}$ .
  - (Example 4) Graph  $f(x) = \sqrt{x+1}$ .
  - (Example 6) Graph

$$f(x) = \begin{cases} 2x + 1 & x \leq 1 \\ 6 - x^2 & x > 1 \end{cases}$$

- HW: 1-12, 37-40

### 3.1 Limits

- Limits

- $\lim_{x \rightarrow a} f(x) = L$  means that the value of  $f(x)$  approaches  $L$  as the value of  $x$  approaches  $a$  in the domain of  $f$ .
- (Example) Given  $f(x) = x^2$ , we may write the following chart of values

$x$	$f(x)$
1.9	3.61
1.99	3.9601
1.999	3.996001
2.001	4.004001
2.01	4.0401
2.1	4.41

to infer that  $\lim_{x \rightarrow 2} f(x) = 4$ .

- (Example) Given

$$g(x) = \begin{cases} x^2 & x \neq 2 \\ -5 & x = 2 \end{cases}$$

we have the same chart of values as before, so we assume  $\lim_{x \rightarrow 2} g(x) = 4$ .

- (Example) Since  $h(x) = \frac{x^3 - 2x^2}{x - 2}$  equals  $x^2$  for all values of  $x$  except 2, we have the same chart of values as before, and we assume  $\lim_{x \rightarrow 2} h(x) = 4$ .
- (Example) By graphing  $y = f(x)$ ,  $y = g(x)$ , and  $y = h(x)$ , we can see that the points on the graph approach the point  $(2, 4)$  in all three cases.

- Continuity

- A continuous function satisfies the equality  $f(a) = \lim_{x \rightarrow a} f(x)$  for all numbers  $a$  in its domain. (The “just plug it in” rule.)
- Intuitively: the graph of the function can be drawn without lifting your pencil on the intervals where it is defined
- FACT:  $f(x) = x$  is continuous, and any combination of continuous functions using  $+$ ,  $-$ ,  $\times$ ,  $/$ ,  $\circ$ , or powers is continuous (where it is defined).
- (Example 3)  $f(x) = \frac{1}{x-2}$  is continuous for its entire domain, but undefined at its asymptote  $x = 2$ .
- (Example 5) By graphing

$$f(x) = \begin{cases} x + 2 & x < 1 \\ -\frac{x}{2} + 5 & x \geq 1 \end{cases} \quad g(x) = \begin{cases} 2x - 1 & x \leq 2 \\ -x + 5 & x > 2 \end{cases}$$

we see that  $f$  is continuous except for when  $x = 1$ , and  $g$  is continuous everywhere.

- Limits to  $\pm\infty$ 
  - $\lim_{x \rightarrow \infty} f(x) = L$  means that the value of  $f(x)$  approaches  $L$  as the value of  $x$  attains arbitrarily large positive values.
  - $\lim_{x \rightarrow -\infty} f(x) = L$  means that the value of  $f(x)$  approaches  $L$  as the value of  $x$  attains arbitrarily large negative values.
  - (Example) Use a chart of values to infer that  $\lim_{x \rightarrow \pm\infty} \frac{1}{x} = 0$ .
  - (Example 14) Use a chart of values and algebraic manipulation to show that  $\lim_{x \rightarrow \pm\infty} \frac{x^2+1}{2x^2+3} = \frac{1}{2}$ .
- Evaluating limits analytically
  - For continuous functions, use the “just plug it in” rule.
  - (Example 10) Evaluate  $\lim_{x \rightarrow 4} x^2 - 7$
  - For limits of the form  $\frac{\text{nonzero}}{0}$ , the limit is undefined.
  - (Example 9) Show  $\lim_{x \rightarrow 2} \frac{1}{x-2}$  does not exist.
  - For limits of the form  $\frac{0}{0}$ , the limit is indeterminate: use canceling to determine its value.
  - (Example 11) Evaluate  $\lim_{x \rightarrow 2} \frac{x^2-4}{x-2}$ .
- HW: 25-44

## Remaining Topics

- 3.2 The Slope of a Tangent to a Curve
- 3.3 The Derivative
- 3.4 The Derivative as an Instantaneous Rate of Change
- 3.5 Derivatives of Polynomials
- 3.6 Derivatives of Products and Quotients of Functions
- 3.7 The Derivative of a Power of a Function
- 3.8 Differentiation of Implicit Functions
- 3.9 Higher Derivatives
- 4.1 Tangents and Normals
- 4.4 Related Rates



- 4.5 Using Derivatives in Curve Sketching
- 4.6 More on Curve Sketching
- 4.7 Applied Maximum and Minimum Problems
- 4.8 Differentials and Linear Approximations
- 5.1 Antiderivatives
- 5.2 The Indefinite Integral
- 5.3 The Area Under a Curve
- 5.4 The Definite Integral
- 7.1 The Trigonometric Functions
- 7.2 Basic Trigonometric Relations
- 7.3 Derivatives of the Sine and Cosine Functions
- 7.4 Derivatives of the Other Trigonometric Functions
- 8.1 Exponential and Logarithmic Functions
- 8.2 Derivative of the Logarithmic Functions
- 8.3 Derivative of the Exponentials Function
- 9.1 The General Power Formula
- 9.2 Basic Logarithmic Form
- 9.3 Exponential Form
- 9.4 Basic Trigonometric Forms