MATH 1121 - Fall 2015 - Dr. Clontz - Test 3

Name: Answers Section: MW 1100 (001) / TR 1530 (002)

- This test is worth 250 points toward your overall grade. Each problem is labeled with its value toward this total. Points earned beyond 250 will be counted as bonus.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the instructor.
- This exam is open notes, provided that these notes are completely in your own handwriting. The professor may take up notes you use with your test and return them after the test is graded.
- Calculators are not necessary to solve any questions on the test and are not allowed. Notes on electronic devices must be approved by the instructor prior to the test day (e.g. for accomodations) and should be in airplane mode.
- Tests submitted after the end of 70 minutes will be deducted 25 points, with 25 more points deducted every following minute.

Multiple Choice (160 points total)

1. (20 points) The kinetic energy K of an object is given by $K = \frac{1}{2}mv^2$ where m is its mass and v is its velocity? Which of these expressions represents the rate of change in kinetic energy with respect to time?

   - ○ 0
   - ○ $\frac{3}{2}v$
   - ◉ $\frac{1}{2}v^2\frac{dm}{dt} + mv\frac{dv}{dt}$
   - ○ $2\frac{dm}{dt}\frac{dv}{dt}$

$$\frac{d}{dt}[K] = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$$

$$\frac{dK}{dt} = v^2\left(\frac{1}{2}\frac{dm}{dt}\right) + \frac{1}{2}m\left(2v\frac{dv}{dt}\right)$$

$$= \frac{1}{2}v^2\frac{dm}{dt} + mv\frac{dv}{dt}$$

2. (20 points) What is the minimum value of the function $f(x) = x^2 + 2x + 8$?

   - ○ $f(x) = -1$
   - ○ $f(x) = 4$
   - ◉ $f(x) = 7$
   - ○ $f(x) = 10$
   - ○ None of these.

$$f'(x) = 2x + 2 = 0$$

$$2x = -2$$

$$x = -1$$

$$f(-1) = (-1)^2 + 2(-1) + 8$$

$$= 1 - 2 + 8$$

$$= 7$$

3. (20 points) Find $\int x(x+4)\,dx$.

   - ◉ $\frac{1}{3}x^3 + 2x^2 + C$
   - ○ $x^4 + 4x^3 + C$
   - ○ $2x + C$
   - ○ $\ln|x+4| + C$
   - ○ None of these.

$$\int x^2 + 4x\,dx$$

$$= \frac{1}{3}x^3 + 4\left(\frac{1}{2}x^2\right) + C$$

$$= \frac{1}{3}x^3 + 2x^2 + C$$

4. (20 points) Find $\int 3x^2\sqrt{x^3+1}\,dx$.

- (●) $\frac{2}{3}(x^3+1)^{3/2}+C$
- ( ) $\sqrt{3x^2+1}+C$
- ( ) $\frac{1}{4}x^4+x^3+x+C$
- ( ) $\frac{x^3}{(3x^2+1)^{1/2}}+C$
- ( ) None of these.

Let $u = x^3+1$
$du = 3x^2\,dx$

$= \int \sqrt{x^3+1}\,(3x^2\,dx)$
$= \int \sqrt{u}\,du$
$= \int u^{1/2}\,du$
$= \frac{1}{3/2}u^{3/2}+C$
$= \frac{2}{3}(x^3+1)^{3/2}+C$

5. (20 points) Which of these expressions gives the area bounded above by the curve $2x\sqrt{x^2+1}$ and below by the x-axis between $x=0$ and $x=6$?

- ( ) $\frac{d}{dx}[2x\sqrt{x^2+1}]$
- ( ) $\tan(2x\sqrt{x^2+1})$
- (●) $\int_0^6 2x\sqrt{x^2+1}\,dx$
- ( ) $e^{2x\sqrt{x^2+1}}$
- ( ) None of these.

$A = \int_a^b f(x)\,dx$
$= \int_0^6 2x\sqrt{x^2+1}\,dx$

6. (20 points) Find the derivative $\frac{dy}{dx}$ of $y = 3\cos(4x)$.

- ( ) $3\sec^2(4x)$
- (●) $-12\sin(4x)$
- ( ) $6\csc(4x)\cot(4x)$
- ( ) $4\sin\cos(4x)$
- ( ) None of these.

$\frac{dy}{dx} = 3(-\sin(4x))(4)$
D of out D of In
$= -12\sin(4x)$

7. (20 points) Find the derivative $\frac{dy}{dx}$ of $y = e^{(x^2+1)}$.

- (●) $2xe^{x^2+1}$
- ( ) $\ln|x^2+1|$
- ( ) $(x^2+1)e^{2x}$
- ( ) $\frac{1}{x^2+1}$
- ( ) None of these.

$\frac{dy}{dx} = \underbrace{e^{(x^2+1)}}_{\text{D of Out}} \underbrace{(2x+0)}_{\text{D of In}}$

$= 2x e^{(x^2+1)}$

8. (20 points) Find $\int \frac{2x}{3x^2+4}\,dx$.

- (●) $\frac{1}{3}\ln|3x^2+4| + C$
- ( ) $e^{3x^2+4} + C$
- ( ) $\frac{1}{2}\sin(3x^2+4) + C$
- ( ) $x^3 + x^2 + 4x + C$
- ( ) None of these.

Let $u = 3x^2+4$

$du = 6x\,dx$

$\frac{1}{6}du = x\,dx$

$= \int 2\left(\frac{1}{3x^2+4}\right)(x\,dx)$

$= \int 2\left(\frac{1}{u}\right)\left(\frac{1}{6}du\right)$

$= \int \frac{1}{3}\frac{1}{u}\,du$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|3x^2+4| + C$

Full Response (100 points total)

9. (20 points) A cylindrical tank with radius 10 meters is being filled with water at a rate of 1 cubic meter per second. How fast is the water rising in the tank? Leave your answer in terms of π.

   dV/dt Want dh/dt

   Hint: the formula for the volume of a cylinder is $V = \pi r^2 h$ where r is its radius and h is its height.

$$\frac{d}{dt}[V] = \frac{d}{dt}[\pi r^2 h]$$

Constant Constant

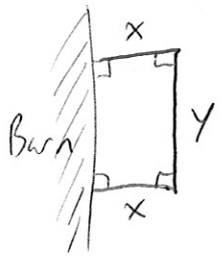
$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$1 = \pi (10)^2 \frac{dh}{dt}$$

$$1 = 100\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \boxed{\frac{1}{100\pi}} \text{ meters per second}$$

10. (20 points) What's the maximum area you can enclose by a rectangle which uses part of an existing barn wall for one side and 200 feet of fencing for the other three sides? (You may assume the barn wall is as long as you need.)



$2x + y = 200$

$A = xy$

$y = 200 - 2x$

$A = x(200 - 2x)$

$A = 200x - 2x^2$

$\frac{dA}{dx} = 200 - 4x = 0$

$200 = 4x$

$50 = x$

$A = 50(200 - 2(50))$

$= 50(200 - 100)$

$= 50(100)$

$= \boxed{5000}$ square feet

11. (20 points) Use $L_4(x)$ where $f(x) = (8+2x)^{1/2}$ to show that $\sqrt{16.8} \approx 4.1$.

$L_4(x) = f(4) + f'(4)(x-4)$

$f(4) = (8+2(4))^{1/2}$
$= (16)^{1/2} = 4$

$f'(x) = \frac{1}{2}(8+2x)^{-1/2}(2)$
$= \frac{1}{\sqrt{8+2x}}$
$f'(4) = \frac{1}{\sqrt{8+2(4)}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

$L_4(x) = 4 + \frac{1}{4}(x-4)$

$\sqrt{16.8} = (16.8)^{1/2}$
$= (8+8.8)^{1/2}$
$= (8+2(4.4))^{1/2}$
$= f(4.4)$

$f(4.4) \approx L_4(4.4)$
$= 4 + \frac{1}{4}(4.4-4)$
$= 4 + \frac{1}{4}(0.4)$
$= 4 + 0.1$
$= \boxed{4.1}$

12. (20 points) Find a function $f(x)$ such that $f'(x) = 8x + 1$ and $f(-1) = 4$.

$$f(x) = \int f'(x)\,dx$$
$$= \int 8x+1\,dx$$
$$= 8\left(\tfrac{1}{2}x^2\right) + x + C$$
$$= 4x^2 + x + C$$

$$f(-1) = 4(-1)^2 + (-1) + C = 4$$
$$4 - 1 + C = 4$$
$$3 + C = 4$$
$$C = 1$$

$$\boxed{f(x) = 4x^2 + x + 1}$$

13. (20 points) Find the derivative of $y = \log_{10}(x^2 + 1)$.

$$\frac{dy}{dx} = \left(\frac{1}{x^2+1} \log_{10} e\right)(2x)$$

$$= \boxed{\frac{2x}{x^2+1} \log_{10} e}$$