MATH 1121 - Fall 2015 - Dr. Clontz - Test 2

Name: Answer Key _______________ Section: MW 1100 (001) / TR 1530 (002)

- This test is worth 250 points toward your overall grade. Each problem is labeled with its value toward this total. Points earned beyond 250 will be counted as bonus.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the instructor.
- This exam is open notes, provided that these notes are completely in your own handwriting. The professor may take up notes you use with your test and return them after the test is graded.
- Calculators are not necessary to solve any questions on the test and are not allowed. Notes on electronic devices must be approved by the instructor prior to the test day (e.g. for accomodations) and should be in airplane mode.
- Tests submitted after the end of 70 minutes will be deducted 25 points, with 25 more points deducted every following minute.

Multiple Choice (160 points total)

1. (20 points) Evaluate $\lim_{x\to 2} \frac{4x^2 - 8x}{x-2}$. $= \frac{16-16}{2-2} = \frac{0}{0}$ indeterminate

   $= \lim_{x\to 2} \frac{4x\cancel{(x-2)}}{\cancel{x-2}} = 4(2) = 8$

   ○ $\frac{0}{0}$
   ○ $8x$
   ● 8
   ○ 0
   ○ None of these.

2. (20 points) Differentiate $y = 4x^3 - 2x^4 + 7x - 3$.

   $y' = 4(3x^2) - 2(4x^3) + 7(1) - 0$
   $= 12x^2 - 8x^3 + 7$

   ○ $y' = 3x^4 - 4x^2 + x$
   ● $y' = 12x^2 - 8x^3 + 7$
   ○ $y' = 8x^7 - 21x$
   ○ $y' = 6x^3 - 8x^4 + 7x - 3$
   ○ None of these.

3. (20 points) Find a formula for the velocity v of an object whose position s is given by the formula $s = 7 + 4t - t^3$ for a given time t.

   $v = \frac{ds}{dt} = 0 + 4(1) - 3t^2$
   $= 4 - 3t^2$

   ○ $v = 7 - 3t^2$
   ● $v = 4 - 3t^2$
   ○ $v = 7t^3$
   ○ $v = 28 - t^2$
   ○ None of these.

4. (20 points) Which of the below choices is the (unsimplified) derivative of $f(x) = (\underset{\text{1st}}{3x-2})(\overset{\text{2nd}}{4x^2+3})$?

- ● $f'(x) = (\underset{\text{2nd}}{4x^2+3})(\underset{\text{D 1st}}{3}) + (\underset{\text{1st}}{3x-2})(\underset{\text{D 2nd}}{8x})$
- ○ $f'(x) = (3)(8x)$
- ○ $f'(x) = (3x-2)(3) + (4x^2+3)(8x)$
- ○ $f'(x) = (24x)(3x-4x^2)$
- ○ None of these.

5. (20 points) Which of the below choices is the (unsimplified) derivative of $f(x) = \dfrac{3x^3 - x \leftarrow \text{High}}{2x^2-5x+4 \leftarrow \text{Low}}$?

- ○ $f'(x) = \dfrac{(3x^3-x)(2x^2-5x+4)-(9x^2-1)(4x-5)}{(3x^3-x)^2}$
- ○ $f'(x) = \dfrac{(2x^2-5x+4)(9x^2)-(3x^3-x)(4x-5)}{(3x^3-x)^2(2x^2-5x+4)^2}$
- ○ $f'(x) = \dfrac{(3x^3-x)(4x^2-5)-(2x^2-5x+4)(27x-1)}{(3x^3-x)^2}$
- ● $f'(x) = \dfrac{(\overset{\text{Low}}{2x^2-5x+4})(\overset{\text{D High}}{9x^2-1})-(\overset{\text{High}}{3x^3-x})(\overset{\text{D Low}}{4x-5})}{\underset{\text{Low}^2}{(2x^2-5x+4)^2}}$
- ○ None of these.

6. (20 points) Which of the below choices is equal to $\frac{dy}{dx}$ given $y = (2x-3)^3$?

- ● $6(4x^2 - 12x + 9)$
- ○ $8x^3 - 36x^2 + 54x - 27$
- ○ $\dfrac{3}{(2x-3)^2}$
- ○ $8(2x^2 - 3x)^2$
- ○ None of these.

$$\frac{dy}{dx} = 3(2x-3)^2(2)$$
$$= 6(2x-3)^2$$
$$= 6(4x^2-12x+9)$$

Use FOIL or
$(a+b)^2 = a^2+2ab+b^2$

7. (20 points) Which of the below choices is equal to $\frac{dy}{dx}$ at the point $(x, y) = (2, 0)$ for the equation $3xy - y^3 = 2 - x$ implicitly defining y as a function of x?

- ○ 3
- ○ 0
- ○ $\frac{2}{7}$
- ● $-\frac{1}{6}$
- ○ None of these.

$$\frac{d}{dx}\Big[\underbrace{3x}_{1st}\underbrace{y}_{2nd} - y^3\Big] = \frac{d}{dx}[2-x]$$

$$\Big(\underbrace{y}_{2nd}\underbrace{(3)}_{D1st} + \underbrace{3x}_{1st}\underbrace{\left(\tfrac{dy}{dx}\right)}_{D2nd}\Big) - 3y^2\frac{dy}{dx} = 0 - 1$$

Plug in $(x,y) = (2,0)$:

$$0 + 6\frac{dy}{dx} - 0 = -1$$
$$\frac{dy}{dx} = -\frac{1}{6}$$

8. (20 points) Exactly which higher order derivatives of $f(x) = 2x - 4x^3 + 7$ are the zero function?

- ○ $f^{(n)}(x) = 0$ for all $n \geq 2$
- ○ $f^{(n)}(x) = 0$ for all $n \geq 3$
- ● $f^{(n)}(x) = 0$ for all $n \geq 4$
- ○ $f^{(n)}(x) = 0$ for all $n \geq 5$
- ○ None of these.

$f'(x) = 2 - 12x^2$
$f''(x) = -24x$
$f^{(3)}(x) = -24$
$f^{(4)}(x) = 0$
$f^{(5)}(x) = 0$
$\vdots$
etc.

Full Response (100 points total)

9. (20 points) Show how to evaluate $\lim_{x\to\infty} \frac{6x^2}{2-3x^2} = -2$. (Hint: remember that $\lim_{x\to\pm\infty} \frac{a}{x^p} = 0$ for constant numbers a and positive constant numbers p.)

$$\lim_{x\to\infty} \frac{6x^2}{2-3x^2} = \lim_{x\to\infty} \frac{6x^2}{2-3x^2} \frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty} \frac{6}{\frac{2}{x^2}-3} = \frac{6}{0-3} = \frac{6}{-3} = \boxed{-2}.$$

10. (20 points) Prove using the limit definition

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(what the book calls the "delta-process") that the derivative of

$$f(x) = \frac{2}{1 - x}$$

is

$$f'(x) = \frac{2}{(1 - x)^2}$$

(No credit will be given for using shortcuts like the quotient rule or chain rule.)

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2}{1-(x+\Delta x)} - \frac{2}{1-x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{2}{1-(x+\Delta x)} - \frac{2}{1-x}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{2(1-x) - 2[1-(x+\Delta x)]}{(1-(x+\Delta x))(1-x)}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\cancel{\Delta x}}\left(\frac{\cancel{2} - \cancel{2x} - \cancel{2} + \cancel{2x} + 2\cancel{\Delta x}}{(1-(x+\Delta x))(1-x)}\right)$$

$$= \lim_{\Delta x \to 0} \frac{2}{(1-(x+\Delta x))(1-x)}$$

$$= \frac{2}{(1-(x))(1-x)} = \boxed{\frac{2}{(1-x)^2}}.$$

11. (20 points) The formula for the volume of water in a conical tank whose point has an angle of 90° is given by $V = \frac{1}{3}\pi h^3$, where h is the height of water in the tank. Find the rate of change of the water's volume with respect to the height of the water when the water is 2 units high. (You should leave your answer in terms of the constant π.)

$$\frac{dV}{dh} = \cancel{\frac{1}{3}}\pi(\cancel{3}h^2)$$

$$= \pi h^2$$

at $h=2$:

$$\frac{dV}{dh} = \pi(2)^2 = \boxed{4\pi}$$

12. (20 points) Find the derivative of $y = \frac{4}{\sqrt{1-3x}}$. For full credit, write the derivative as a simplified fraction with denominator $(1-3x)^{3/2}$.

$$y = 4(1-3x)^{-1/2}$$

$$y' = 2\cancel{4}\left(+\cancel{\tfrac{1}{2}}\right)(1-3x)^{-3/2}(+3)$$

$$= 6(1-3x)^{-3/2}$$

$$= \boxed{\frac{6}{(1-3x)^{3/2}}}$$

OR

$$y' = \frac{\overset{\text{Low}}{\sqrt{1-3x}}\,\overset{\text{D High}}{(0)} - \overset{\text{High}}{(4)}\overset{\text{D Low}}{\left(\frac{d}{dx}\sqrt{1-3x}\right)}}{\underset{\text{Low}^2}{(1-3x)}}$$

$$\frac{d}{dx}\sqrt{1-3x} = \frac{d}{dx}(1-3x)^{1/2}$$

$$= \frac{1}{2}(1-3x)^{-1/2}(-3$$

$$= -\frac{3}{2}(1-3x)^{-1/2}$$

$$= \frac{+\overset{2}{\cancel{4}}\left(+\frac{3}{\cancel{2}}\right)(1-3x)^{-1/2}}{(1-3x)}$$

$$= \frac{6(1-3x)^{-1/2}}{(1-3x)}$$

$$= \boxed{\frac{6}{(1-3x)^{3/2}}}$$

13. (20 points) Using the methods of chapter 2, we can show that the circle with center $(3, 4)$ and radius 5 has the equation $(x-3)^2 + (y-4)^2 = 25$. Use this equation to prove that the slope of the line tangent to this circle at the origin $(0, 0)$ is $-\frac{3}{4}$.

$$\frac{d}{dx}\left[(x-3)^2+(y-4)^2\right] = \frac{d}{dx}\left[25\right]$$

$$2(x-3)^1(1) + 2(y-4)^1\left(\frac{dy}{dx}\right) = 0$$

At $(x,y)=(0,0)$:

$$2(-3)(1) + 2(-4)\frac{dy}{dx} = 0$$

$$-6 - 8\frac{dy}{dx} = 0$$

$$-8\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = -\frac{6}{8}$$

$$= \boxed{-\frac{3}{4}}$$