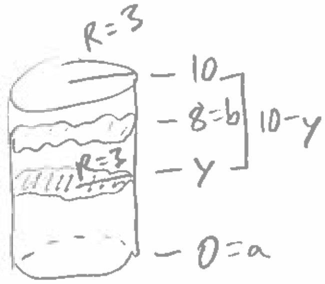
7. Assume salt water weighs 10 kilonewtons per cubic meter. A cylindrical tank with a radius of 3 m and a height of 10 m holds 8 m of salt water. Show that the work required to pump out the salt water to the top of the tank is 4320π kN-m (kJ).

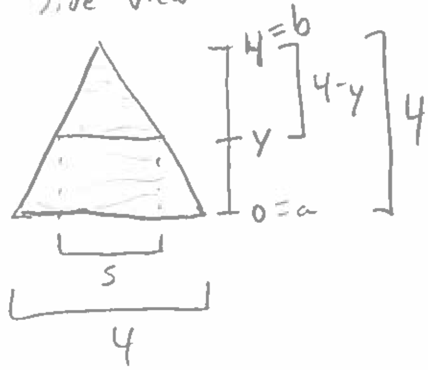


$$A = \pi R^2$$
$$= 9\pi$$
$$dV = A\,dy$$
$$= 9\pi\,dy$$
$$dF = \rho\,dV$$
$$= 10\,dV$$
$$= 90\pi\,dy$$
$$dW = h\,dF$$
$$= (10-y)\,dF$$
$$= 90\pi(10-y)\,dy$$

$$W = \int_{y=a}^{y=b} dW$$
$$= 90\pi\int_0^8 (10-y)\,dy$$
$$= 90\pi\left[10y - \tfrac{1}{2}y^2\right]_0^8$$
$$= 90\pi\left[80-32\right]$$
$$= 90\pi(48)$$
$$= \boxed{4320\pi}$$

8. Assume salt water weighs 10,000 N/m^3. A pyramid-shaped tank of height 4m is pointed upward, with a square base of side length 4m, and is completely filled with salt water. Show that the work done in completely pumping all the water in this tank up to the point of the pyramid is $10000\int_0^4 (4-y)^3\,dy$ J.

Side view:



Similar triangles:

$$\frac{4-y}{s}=\frac{4}{4}$$

$$4-y=s$$

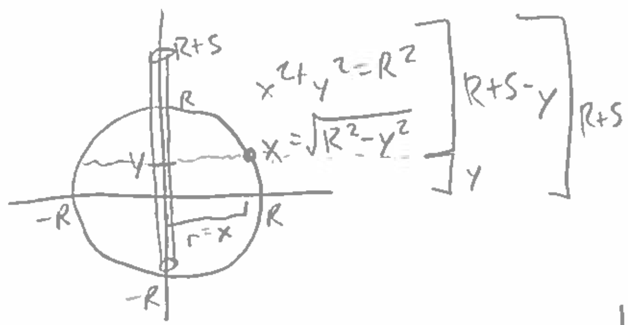
$$A=s^2$$
$$=(4-y)^2$$
$$dV=A\,dy$$
$$=(4-y)^2\,dy$$
$$dF=\rho\,dV$$
$$=10000(4-y)^2\,dy$$
$$dW=h\,dF$$
$$=10000(4-y)^3\,dy$$

$$W=\int_{y=a}^{y=b} dW$$
$$=\boxed{10000\int_0^4 (4-y)^3\,dy}$$

(Let $u=4-y$)

$$=10000\int_0^4 u^3\,du$$
$$=10000\left[\tfrac{1}{4}u^4\right]_0^4$$
$$=640{,}000 \text{ J}$$

(9) Assume that a cubic inch of Juicy Juice™ weighs D oz. Suppose a perfectly spherical coconut-shaped cup with radius R inches is completely filled with Juicy Juice™. Show that drinking the entire beverage using a straw which extends S inches above the top of the container requires $\frac{4}{3} D\pi R^3(R+S)$ in-oz of work.



$$A = \pi r^2$$
$$= \pi\left(\sqrt{R^2-y^2}\right)^2$$
$$= \pi\left(R^2-y^2\right)$$
$$dV = \pi(R^2-y^2)\,dy$$
$$dF = D\pi(R^2-y^2)\,dy$$
$$dW = D\pi(R^2-y^2)(R+S-y)\,dy$$
$$W = \int_{-R}^{R} D\pi(R^2-y^2)(R+S-y)\,dy$$

$$W = D\pi\int_{-R}^{R}\left(R^3+R^2S-R^2y-Ry^2-Sy^2+y^3\right)dy$$
$$= D\pi\left[R^3y+R^2Sy-\cancel{\tfrac{1}{2}R^2y^2}-\tfrac{1}{3}Ry^3-\tfrac{1}{3}Sy^3+\cancel{\tfrac{1}{4}y^4}\right]_{-R}^{R}$$
$$= 2D\pi\left[R^3y+R^2Sy-\tfrac{1}{3}Ry^3-\tfrac{1}{3}Sy^3\right]_0^R$$
$$= 2D\pi\left[R^4+R^3S-\tfrac{1}{3}R^4-\tfrac{1}{3}SR^3\right]$$
$$= \tfrac{4}{3}D\pi R^3(R+S) \quad \square$$

(10) What is the work required to push a heavy box 3 m over an irregular surface, assuming it requires $F(x) = 3 + 2x - x^2$ newtons of force to move at x meters?

$$W = \int_0^3 F(x)\,dx = \int_0^3 (3 + 2x - x^2)\,dx = \left[3x + x^2 - \tfrac{1}{3}x^3\right]_0^3$$

$$= \left[(9 + \cancel{9 - 9}) - \cancel{(0 + 0 - 0)}\right] = \boxed{9} \text{ N-m or J}$$

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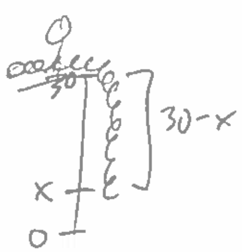
(11) What integral gives the work in ft-lbs required to pull up a hanging 30-lb 15-ft chain?

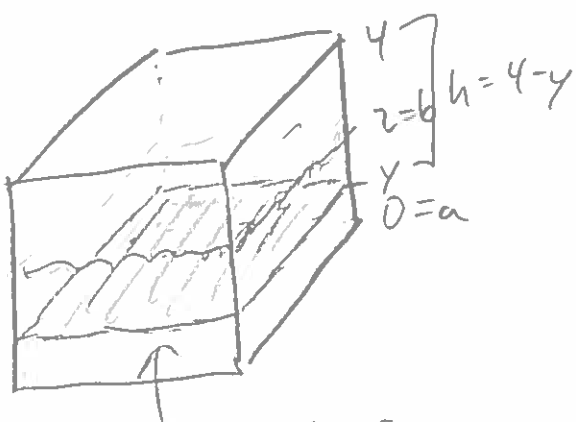


$$F(x) = 2\frac{30 \text{ lbs}}{15 \cancel{\text{ ft}}}(30 - x \cancel{\text{ ft}})$$

$$= 60 - 2x \text{ lbs}$$

$$\boxed{W = \int_0^{30} (60 - 2x)\,dx}$$

(12) What integral gives the work in kN-m required to pump out all saltwater to the top of a cubical tank with side length 4m, if it is initially half-full? Assume that salt water weighs 10 kN per cubic meter.



$$A = (4)^2 = 16$$

$$dV = A\,dy = 16\,dy$$

$$dF = \rho\,dV = 10(16\,dy) = 160\,dy$$

$$dW = h\,dF = (4-y)(160\,dy) = 160(4-y)\,dy$$

$$\boxed{W = \int_0^2 160(4-y)\,dy}$$