MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 10

Name: Solutions

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This quiz is open notes and open book.
- This quiz is due at the end of class. Quizzes submitted over one minute late will be penalized by 50%.

1. (10 points) Which of these is a parametrization of the line segment joining the points $(1, 0, 3)$ and $(2, -2, 5)$?

   ● $\mathbf{r}(t) = (1 + t, -2t, 3 + 2t); 0 \leq t \leq 1$

   ○ $\mathbf{r}(t) = (\cos t, 2\sin t, 3\cos t); 0 \leq t \leq 2\pi$

   ○ $\mathbf{r}(t) = (t^2, -2e^t, 3 + 5t); -1 \leq t \leq 1$

   ○ $\mathbf{r}(t) = (\cos t, -2\sin t, 3\cos t); 0 \leq t \leq \pi$

   ○ None of these.

2. (10 points) Prove $\int_C \sqrt{y} - x + 3 \; ds = \frac{\sqrt{125}-1}{6}$, where C is parametrized by the vector function $\mathbf{r}(t) = (3 - t, t^2)$ for $t \in [0, 1]$.

$$= \int_0^1 f(\underline{r}(t)) \left\| \frac{d\underline{r}}{dt} \right\| dt$$

$$\frac{d\underline{r}}{dt} = (-1, 2t)$$

$$\left\| \frac{d\underline{r}}{dt} \right\| = \sqrt{1+4t^2}$$

$$= \int_0^1 \left(\sqrt{t^2} - (3-t) + 3\right) \sqrt{1+4t^2}\, dt$$

$$= \int_0^1 2t\sqrt{1+4t^2}\, dt$$

$$= \int_1^5 \tfrac{1}{4} u^{1/2}\, du$$

$$= \left[\tfrac{1}{6} u^{3/2}\right]_1^5$$

$$= \frac{5^{3/2} - 1}{6}$$

$$= \frac{\sqrt{125} - 1}{6} \qquad \square$$

3. (10 points) Calculate the work done by the force $\mathbf{F} = (y, z + x, -2x)$ around the curve parametrized by the vector function $\mathbf{r}(t) = (\sin t, 2\sin t, \cos t)$ for $t \in [0, \pi]$.

$$\text{Work} = \int_C \underline{F} \cdot d\underline{r}$$

$$= \int_0^\pi \underline{F}(\underline{r}(t)) \cdot \frac{d\underline{r}}{dt}\, dt$$

$$= \int_0^\pi (2\sin t, \cos t + \sin t, -2\sin t) \cdot (\cos t, 2\cos t, -\sin t)\, dt$$

$$= \int_0^\pi 2\sin t\cos t + 2\cos^2 t + 2\sin t\cos t + 2\sin^2 t\, dt$$

$$= \int_0^\pi 4\sin t\cos t + 2\, dt$$

$$= \left[2\sin^2 t + 2t\right]_0^\pi$$

$$= (\cancel{2\sin^2\pi} + 2\pi) - (\cancel{2\sin^2 0} + \cancel{0})$$

$$= \boxed{2\pi}$$