MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 12 (Take-home)

Name: Solutions

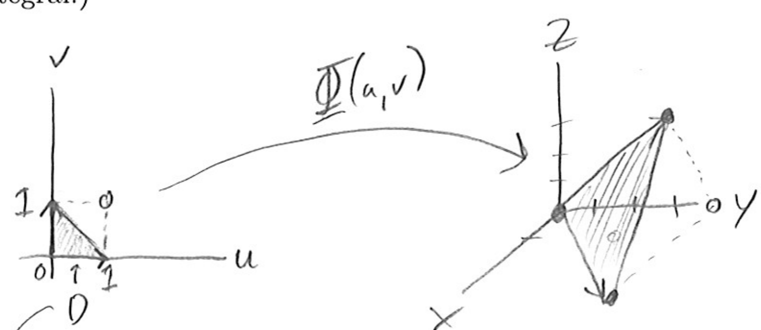
- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This take-home quiz is open notes and open book. You may work with others as long as you don't plagiarize their answers.
- This quiz is due at the beginning of class on Monday, May 2. Late submissions will not be accepted.

$0 \le r \le 1$
$0 \le \theta \le 2\pi$

$z = r^2$ in cylindrical coordinates

1. (10 points) Which of these is a parametrization of the portion of the surface $z = x^2 + y^2$ above the unit circle in the xy plane?
   - ○ $\Phi(u, v) = (u^2, v^2, u + v); 0 \le u, v \le 1$
   - ○ $\Phi(x, y) = (x + y, x + y, z^2); 0 \le x, y \le 1$
   - ● $\Phi(r, \theta) = (r\cos\theta, r\sin\theta, r^2); 0 \le r \le 1, 0 \le \theta \le 2\pi$
   - ○ $\Phi(u, v) = 2u\mathbf{i} - 2v\mathbf{j}; 0 \le u, v \le 1$
   - ○ None of these.

2. (10 points) Prove that the area of the of the triangle with vertices $(0, 0, 0)$, $(1, 2, -2)$, and $(0, 3, 3)$ is $\frac{9\sqrt{2}}{2}$ by using the formula $A = \iint_S 1 dS = \iint_D \|\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}\| dA$ with the parametrization $\Phi(u, v) = (u, 2u + 3v, -2u + 3v)$. (Hint: You need to find the domain D for this parametrization mapping onto the surface; this will give you the bounds for the double integral.)



$$\frac{\partial\Phi}{\partial u} = (1, 2, -2)$$
$$\times \; \frac{\partial\Phi}{\partial v} = (0, 3, 3)$$
$$(12, -3, 3)$$

$$= \int_0^1 \int_0^{1-u} \left\|\frac{\partial\Phi}{\partial u} \times \frac{\partial\Phi}{\partial v}\right\| dv\,du$$

$$= \int_0^1 \int_0^{1-u} \sqrt{12^2 + 3^2 + 3^2}\, dv\,du$$

$$= \int_0^1 \int_0^{1-u} 3\sqrt{4^2 + 1 + 1}\, dv\,du$$

$$= \int_0^1 \int_0^{1-u} 3\sqrt{18}\, dv\,du$$

$$= 9\sqrt{2} \int_0^1 \int_0^{1-u} 1\, dv\,du = 9\sqrt{2}\left(\tfrac{1}{2}\right) = \boxed{\frac{9\sqrt{2}}{2}}$$

(Area of D)

$x \quad y \quad z$

3. (10 points) Let S be the oriented surface with an orientation-preserving parametrization $\mathbf{\Phi}(u, v) = (u, u + v, v^2)$ for $0 \le u, v \le 1$. If $\mathbf{F} = (y, x, z)$ is the velocity field of a fluid, then show that the flux of the fluid moving through S with respect to its orientation is 1; that is, verify that $\iint_S \mathbf{F} \cdot d\mathbf{S} = 1$.

$$\iint_S \underline{F} \cdot d\underline{S} = \iint_D (y, x, z) \cdot \left(\frac{\partial \underline{\Phi}}{\partial u} \times \frac{\partial \underline{\Phi}}{\partial v}\right) dA$$

$$\frac{\partial \underline{\Phi}}{\partial u} = (1, 1, 0)$$

$$\times \frac{\partial \underline{\Phi}}{\partial v} = (0, 1, 2v)$$

$$(2v, -2v, 1)$$

$$= \int_0^1 \int_0^1 (u+v, u, v^2) \cdot (2v, -2v, 1)\, dv\, du$$

$$= \int_0^1 \int_0^1 \cancel{2uv} + 2v^2 - \cancel{2uv} + v^2 \, dv\, du$$

$$= \int_0^1 \int_0^1 3v^2 \, dv\, du$$

$$= \int_0^1 1 \, du$$

$$= \boxed{1}$$