MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 1

Name: Answers

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This quiz is open notes and open book.
- This quiz is due at the end of class. Quizzes submitted over one minute late will be penalized by 50%.

1. (10 points) Recall from the homework that $\det(AB) = (\det A)(\det B)$. Evaluate

$$\det\left(\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}\right) = \det\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix} \det\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= (2-(-3))(0-(-2))$$

$$= (5)(2)$$

$$= 10$$

- ○ -9
- ○ 0
- ○ 6
- ● 10
- ○ None of these

2. (10 points) Verify the triangle inequality $\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$ for $\mathbf{x} = \langle 1, -2, 0, 2 \rangle$, $\mathbf{y} = \langle 0, 4, -3, 0 \rangle$.

$$\|\underline{x}+\underline{y}\| = \|\langle 1,-2,0,2\rangle + \langle 0,4,-3,0\rangle\|$$

$$= \|\langle 1,2,-3,2\rangle\|$$

$$= \sqrt{1+4+9+4}$$

$$= \sqrt{18} = 3\sqrt{2}$$

$$\|\underline{x}\| = \sqrt{1+4+0+4}$$

$$= \sqrt{9} = 3$$

$$\|\underline{y}\| = \sqrt{0+16+9+0}$$

$$= \sqrt{25} = 5$$

So

$$\|\underline{x}+\underline{y}\| = 3\sqrt{2}$$

$$\leq 3(2) = 6$$

$$< 8 = 3+5$$

$$= \|\underline{x}\| + \|\underline{y}\|$$

is verified.

3. (10 points) Use the "identity of Lagrange"

$$\left(\sum_{i=1}^{n} x_i y_i\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right) - \sum_{1\leq i<j\leq n} (x_i y_j - x_j y_i)^2$$

to prove the Cauchy-Schwarz inequality $|\mathbf{x}\cdot\mathbf{y}| \leq \|\mathbf{x}\|\|\mathbf{y}\|$ in $\mathbb{R}^n$. (Hint: the identity of Lagrange involves the terms $\|\mathbf{x}\|^2$, $\|\mathbf{y}\|^2$, $|\mathbf{x}\cdot\mathbf{y}|^2$.)

$$\begin{aligned} |\underline{x}\cdot\underline{y}|^2 &= \|\underline{x}\|^2\|\underline{y}\|^2 - \sum_{1\leq i<j\leq n} (x_i y_j - x_j y_i)^2 \\ &\leq \|\underline{x}\|^2\|\underline{y}\|^2 - 0 \\ &= \|\underline{x}\|^2\|\underline{y}\|^2. \end{aligned}$$

Since $|\underline{x}\cdot\underline{y}|^2 \leq \|\underline{x}\|^2\|\underline{y}\|^2$, we conclude $|\underline{x}\cdot\underline{y}| \leq \|\underline{x}\|\|\underline{y}\|$. $\square$