MATH 2242 - Fall 2015 - Dr. Clontz - Test 2

Name: Answer Key

- This test is worth 250 points toward your overall grade. Each problem is labeled with its value toward this total. ~~Points earned beyond 250 will be counted as bonus.~~
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This exam is open notes, provided that these notes are completely in your own handwriting. The professor may take up notes you use with your test and return them after the test is graded.
- Calculators are not necessary to solve any questions on the test and are not allowed. Notes on electronic devices must be approved by the professor prior to the test day (e.g. for accomodations) and should be in airplane mode.
- Tests submitted after the end of 70 minutes will be deducted 25 points, with 25 more points deducted every following minute.

Multiple Choice (100 points total)

1. (20 points) Evaluate $\displaystyle\int_0^1 \left[\int_{2x}^{3x-1} x\, dy \right] dx.$ $= \displaystyle\int_0^1 \Big[xy \Big]_{2x}^{3x-1} dx$

$$= \int_0^1 (3x^2 - x) - (2x^2)\, dx$$

$$= \int_0^1 x^2 - x\, dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{2} \right) - (0 - 0)$$

$$= -\frac{1}{6}$$

- ○ $\frac{3}{2}$
- ● $-\frac{1}{6}$
- ○ 3
- ○ 0
- ○ None of these

2. (20 points) Suppose for all values of $(x, y) \in \overbrace{[-2, 0]}^{2} \times \overbrace{[-1, 2]}^{3}$ we can show that $1 \le f(x, y) \le 2$. Which of these inequalities gives the best bounds for the value of $\iint_D f(x, y)\, dA$?

- ○ $1 \le \iint_D f(x, y)\, dA \le 2$
- ○ $\frac{1}{2} \le \iint_D f(x, y)\, dA \le 4$
- ● $6 \le \iint_D f(x, y)\, dA \le 12$
- ○ $-12 \le \iint_D f(x, y)\, dA \le -6$
- ○ None of these inequalities must hold.

$$1 \le f(x,y) \le 2$$

$$\text{Area}(D) \le \iint_D f(x,y)\, dA \le 2\, \text{Area}(D)$$

$$6 \le \iint_D f(x,y)\, dA \le 12$$

3. (20 points) Evaluate $\int_0^{\pi}\int_0^{\pi/2}\left[\int_0^{3}\rho^2\sin\phi\,d\rho\right]d\phi\,d\theta$. $= \int_0^{\pi}\int_0^{\pi/2}\left[\frac{1}{3}\rho^3\sin\phi\right]_0^3 d\phi\,d\theta$

● 9π

○ $3\cos 1$

○ $-\frac{3\pi}{2}$

○ -3

○ None of these.

$$= \int_0^{\pi}\int_0^{\pi/2} 9\sin\phi\,d\phi\,d\theta$$

$$= \int_0^{\pi}\left[-9\cos\phi\right]_0^{\pi/2} d\theta$$

$$= \int_0^{\pi} 0-(-9)\,d\theta$$

$$= \int_0^{\pi} 9\,d\theta = 9\pi$$

4. (20 points) Which of these Cartesian points is the same as the spherical coordinate $\mathbf{s}(\rho,\theta,\phi) = \mathbf{s}(2,\pi/2,\pi/3)$?

● $(x,y,z) = (0,\sqrt{3},1)$

○ $(x,y,z) = (\pi,2,-1)$

○ $(x,y,z) = (\sqrt{2},0,-\sqrt{2})$

○ $(x,y,z) = (4,2\sqrt{3},2)$

○ None of these

$$(x,y,z) = (\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)$$

$$= \left(\cancel{2\sin\tfrac{\pi}{3}\cos\tfrac{\pi}{2}},\ 2\sin\tfrac{\pi}{3}\cancel{\sin\tfrac{\pi}{2}},\ 2\cos\tfrac{\pi}{3}\right)$$

$$= \left(0,\ \cancel{2}\left(\tfrac{\sqrt{3}}{\cancel{2}}\right),\ \cancel{2}\left(\tfrac{1}{\cancel{2}}\right)\right)$$

$$= (0,\sqrt{3},1)$$

5. (20 points) The transformation $\mathbf{T}:\mathbb{R}^2\to\mathbb{R}^2$ defined by $\mathbf{T}(x,y) = (2x+y-1,3-x)$ is which of the following?

● Onto and one-to-one

○ Onto, but not one-to-one

○ One-to-one, but not onto

○ Not onto and not one-to-one

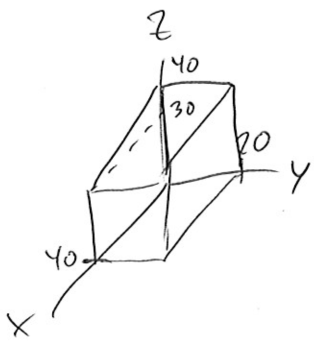
$$= \underbrace{\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}}\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}-1\\ 3\end{bmatrix}$$

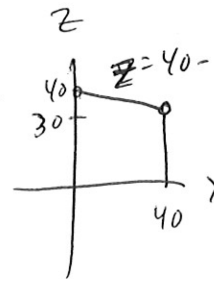
$$\det\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix} = 0-(-1)$$

$$= 1 \neq 0$$

Full Response (150 points total)

6. (30 points) A barn has a rectangular base measuring 20 feet on the front and back, and 40 feet on the sides. The front wall is 30 feet high, and the back wall is 40 feet high. Express the volume of this barn as a double or triple iterated integral, then show that the volume is 28,000 cubic feet.





$z = 40 - \frac{10}{40}x = 40 - \frac{1}{4}x$

$$V = \boxed{\int_0^{40}\int_0^{20}\int_0^{40-\frac{1}{4}x} 1\,dz\,dy\,dx} = \boxed{\int_0^{40}\int_0^{20} 40 - \frac{1}{4}x\,dy\,dx}$$

$$= \int_0^{40} 800 - 5x\,dx$$

$$= \left[800x - \frac{5}{2}x^2\right]_0^{40}$$

$$= 800(40) - \frac{5}{\cancel{2}}(\cancel{1600}\ 800)$$

$$= 32000 - 4000$$

$$= \boxed{28000}$$

$\int e^{x^2}\,dx = ???$

Try switching...

7. (30 points) Evaluate $\displaystyle\int_{y=0}^{y=4}\int_{x=y/2}^{x=2} e^{x^2}\,dx\,dy$.

$x = y/2 \rightarrow y = 2x$

$$= \int_0^2\int_0^{2x} e^{x^2}\,dy\,dx$$

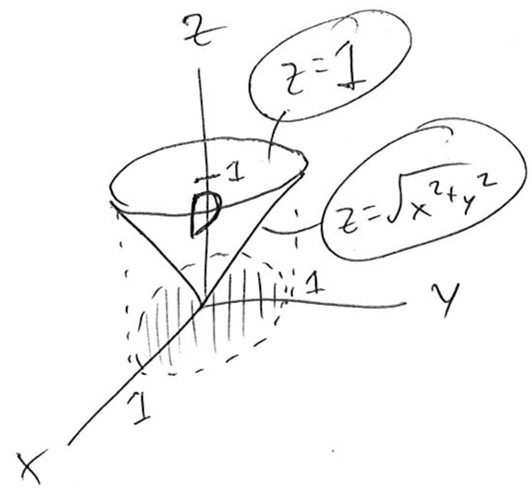
$$= \int_0^2 2xe^{x^2}\,dx$$

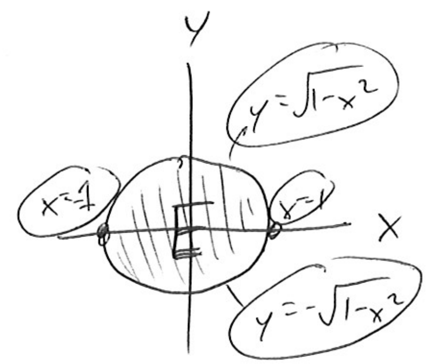
$$= \left[e^{x^2}\right]_0^2$$

$$= \boxed{e^4 - e^0}$$

$$= \boxed{e^4 - 1}$$

8. (30 points) Write the volume of the cone with height 1 and radius 1 as an iterated integral of x, y, z. (Hint: use the equation $z = \sqrt{x^2 + y^2}$). Do not evaluate the integral.



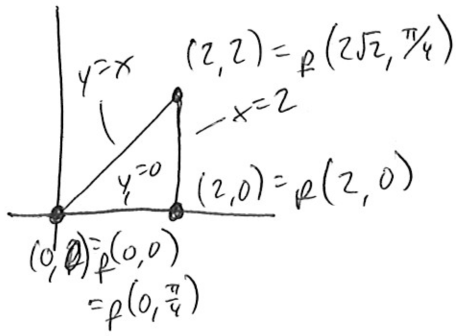


$$\boxed{\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} 1\,dz\,dy\,dx}$$

OR

$$\boxed{4\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} 1\,dz\,dy\,dx}$$

9. (30 points) Use equations of the polar coordinates r, θ to describe the sides of the triangle with vertices at $(x, y) = (0, 0)$, $(x, y) = (2, 0)$, and $(x, y) = (2, 2)$.



$y=0$: $r\sin\theta = 0$

$\sin\theta = 0$

$$\boxed{\boxed{\theta = 0} \quad \text{for } \boxed{0 \le r \le 2}}$$

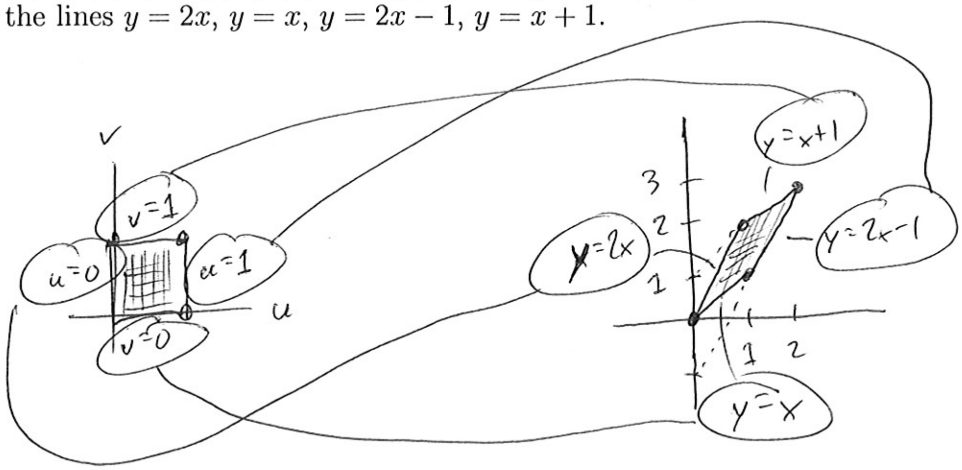
$x=2$: $$\boxed{\begin{array}{c} r\cos\theta = 2 \\ \text{OR} \\ r = 2\sec\theta \\ \text{for } 0 \le \theta \le \frac{\pi}{4} \end{array}}$$

$y=x$: $r\sin\theta = r\cos\theta$

$\tan\theta = 1$

$$\boxed{\begin{array}{c} \theta = \frac{\pi}{4} \\ \text{for } 0 \le r \le 2\sqrt{2} \end{array}}$$

10. (30 points) Define an affine transformation taking the unit square with vertices at $(u, v) = (0, 0)$, $(u, v) = (1, 0)$, $(u, v) = (1, 1)$, and $(u, v) = (0, 1)$ to the parallelogram bounded by the lines $y = 2x$, $y = x$, $y = 2x - 1$, $y = x + 1$.



For $u=0$: $y=2x$

For $u=1$: $y=2x-1$

$\Rightarrow y = 2x - u$

For $v=0$: $y=x$

For $v=1$: $y=x+1$

$\Rightarrow y = x + v$

$2x - u = x + v$

$x = u + v$

$y = (u+v) + v$

$y = u + 2v$

$$\boxed{T(x,y) = (u+v,\ u+2v)}$$

OR

$$\boxed{T(\underline{x}) = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

(Other solutions possible.)