MATH 2242-090 — Spring 2016 — Dr. Clontz — Midterm

Name: ______________________________

- Each question is labeled with its worth toward the grade for this midterm out of 100.
- **Choose SEVEN of the eight problems to solve. Mark the one you don't want graded by marking it with the word "SKIP" in the upperleft corner of the page.** If you fail to do so, your highest score from all eight problems will not be counted. Note that this means you can earn up to 105/100 for the midterm.
- Show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- You may use at most three pages (front and back) of 8.5×11 inch paper for notes.
- You may use a calculator no more powerful than a TI-89 (in particular, no cell phones are allowed). None of the questions require the use of a calculator.
- This midterm is due after 70 minutes. Midterms submitted over one minute late will be penalized by 50%.

1. (15 points) Recall that $\det(AB) = (\det A)(\det B)$. Evaluate

$$\det\left(\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}\right).$$

2. (15 points) Compute the partial derivative matrix for

$$\mathbf{f}(x, y, z) = (2x + 3y, e^z, \sin(yz)).$$

3. (15 points) Let $\mathbf{f}(u,v) = (u^2 + v^3, 2uv)$, $\mathbf{g}(x,y) = (e^{xy}, x+y)$. It follows that

$$\mathbf{Df}(u,v) = \begin{bmatrix} 2u & 3v^3 \\ 2v & 2u \end{bmatrix} \quad \text{and} \quad \mathbf{Dg}(x,y) = \begin{bmatrix} ye^{xy} & xe^{xy} \\ 1 & 1 \end{bmatrix}.$$

Use the above matrices and the chain rule to compute $\mathbf{D}(\mathbf{f} \circ \mathbf{g})(0,0)$.

4. (15 points) Give an approximate value of $f(1.1, -2.1)$ given the following information about f:

$$f(1,-2) = 2 \qquad \frac{\partial f}{\partial x}(1,-2) = 0 \qquad \frac{\partial f}{\partial y}(1,-2) = -1$$

$$\frac{\partial^2 f}{\partial x^2}(1,-2) = -2 \qquad \frac{\partial^2 f}{\partial y^2}(1,-2) = 4 \qquad \frac{\partial^2 f}{\partial x \partial y}(1,-2) = 3$$

5. (15 points) Prove that $\mathbf{c}(t) = (t^2, 2, t)$ is a flow line for the vector field $\mathbf{F}(x, y, z) = (yz, x - z^2, \frac{1}{2}y)$.

6. (15 points) Evaluate $\int_0^2 \int_{-1}^2 2xy + 3x^2 \; dydx$.

7. (15 points) Evaluate $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} 2x^2 \cos(xy)\ dxdy$.

8. (15 points) Express the volume of the pyramid with vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ as either a double or triple integral. (Hint: the sides of the pyramid have equations $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.) Do not evaluate the integral.