A couple notes on 3.2 HW (3-7, 12)...

① log is the natural logarithm $\ln = \log_e$. (not $\log_{10}$ ... at least in our book), so $\frac{d}{dx}[\log x] = \frac{1}{x}$.

② Answers in back of book are weird...

For #5...

$$f(h_1, h_2) = 1 + h_1 + h_2 + h_1^2/2 + h_1 h_2 + h_2^2/2 + \underbrace{R_2(\underline{0}, \underline{h})}_{\text{error in approximation}}$$

$(x-x_0)$ $(y-y_0)$ $(x-x_0)$ $(y-y_0)$

so the "real" answer is...

$$f(x,y) \approx 1 + (x-x_0) + (y-y_0) + \frac{1}{2}(x-x_0)^2 + (x-x_0)(y-y_0) + \frac{1}{2}(y-y_0)^2$$

since the problem says $x_0 = 0 = y_0$, this simplifies to

$$f(x,y) \approx 1 + x + y + x^2/2 + xy + y^2/2.$$

③ For #12, it says to approximate $f(-1,-1)$ using the polynomials from #3 & #7. These are bad approximations, because you were told to use $\underline{x}_0 = (0,0)$ OR $(1,0)$.

not close to $(-1,-1)$.

To fix this, approximate $f(-0.1, -0.1)$ instead.

A note on 2.5:

What if I don't need all of $\underline{D}(f \circ \underline{g})$?

Example 2.5 exercise 9

Find $\dfrac{\partial(f \circ \underline{T})}{\partial s}(1,0)$ where $f(u,v) = \cos u \sin v$ and

$\underline{T}(s,t)$ is defined by $\underline{T}(s,t) = (\underbrace{\cos(t^2 s)}_{u}, \underbrace{\log\sqrt{1+s^2}}_{v})$. $\log\sqrt{1+s^2} \rightarrow \frac{1}{2}\log(1+s^2)$

Direct way:

$$f \circ \underline{T} = \cos(\cos(t^2 s)) \sin(\log\sqrt{1+s^2})$$

$$\frac{\partial(f \circ \underline{T})}{\partial s} = -\sin(\cos(t^2 \sim\sim\sim$$

awful, awful product rule plus cal 1 chain rule

Chain Rule:

$$\begin{bmatrix} \frac{\partial(f \circ \underline{T})}{\partial s} & \frac{\partial(f \circ \underline{T})}{\partial t} \end{bmatrix} = \underline{D}(f \circ \underline{T}) = \underline{D}f(\underline{T})\,\underline{D}\,\underline{T} = \begin{bmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \end{bmatrix}(\underline{T}) \begin{bmatrix} \frac{\partial T_1}{\partial s} & \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial s} & \frac{\partial T_2}{\partial t} \end{bmatrix}$$

Don't need this for $\frac{\partial(f \circ \underline{T})}{\partial s}$

$$\frac{\partial(f \circ \underline{T})}{\partial s} = \begin{bmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \end{bmatrix}(\underline{T}) \begin{bmatrix} \frac{\partial T_1}{\partial s} \\ \frac{\partial T_2}{\partial s} \end{bmatrix}$$

$$= \begin{bmatrix} -\sin u \sin v & \cos u \cos v \end{bmatrix}(\underline{T}) \begin{bmatrix} -\sin(t^2 s) \\ \cancel{\frac{1}{2}} \frac{\cancel{2}s}{1+s^2} \end{bmatrix}$$

$$\frac{\partial(f\circ I)}{\partial s}(\underset{s}{\underset{\uparrow}{1}},\underset{t}{\underset{\uparrow}{0}}) = \begin{bmatrix} -\sin u \sin v & \cos u \cos v \end{bmatrix}\left(I(1,0)\right)\begin{bmatrix} -\sin(0) \\ \frac{1}{1+1} \end{bmatrix}$$

$$= \begin{bmatrix} -\sin u \sin v & \cos u \cos v \end{bmatrix}(\cos 0, \log 1)\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} -\sin u \sin v & \cos u \cos v \end{bmatrix}(\underset{u}{\underset{\uparrow}{1}},\underset{v}{\underset{\uparrow}{0}})\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{-\sin 1 \sin 0} & \cos 1 \cancel{\cos 0} \end{bmatrix}\begin{bmatrix} \cancel{0} \\ 1/2 \end{bmatrix}$$

$$= 0 + (\cos 1)(1/2)$$

$$= \frac{1}{2}\cos 1$$

4.3 cont.

Let $f: \mathbb{R}^n \to \mathbb{R}$, then

Gradient Vector Field: $\nabla f(\underbrace{x, y, z}_{n=3}) = (\underline{D} f)^T$

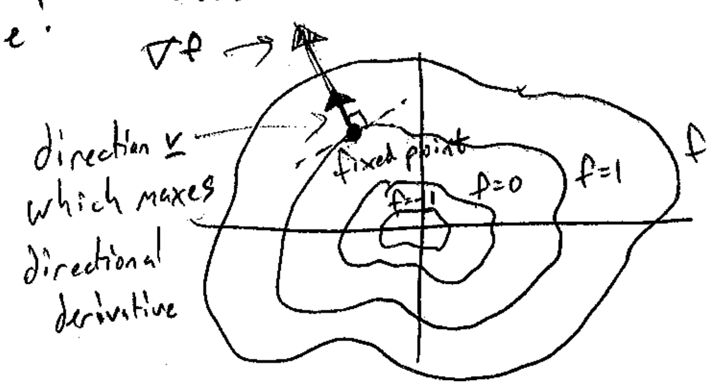
$$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

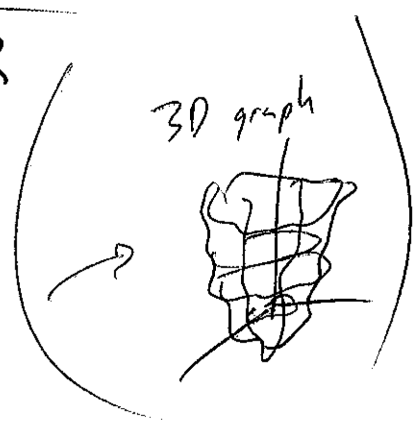
$$= \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n} \right)$$

in general

(Example) The derivative of a scalar function $f: \mathbb{R}^n \to \mathbb{R}$ in the direction of a unit vector $\underline{v}$ is given by $\nabla f \cdot \underline{v}$. Show that the max value of a directional derivative for a fixed point and variable direction $\underline{v}$ is given by $\|\nabla f\|$ when $\underline{v} = \frac{1}{\|\nabla f\|} \nabla f$.

Picture: Level curves for some $f: \mathbb{R}^2 \to \mathbb{R}$





~~In $\mathbb{R}^2$ it's easy...~~

~~$\nabla f \cdot v$~~

$$\nabla f \cdot \underline{v} \le \underbrace{\nabla f \cdot \begin{pmatrix} \text{vector of magnitude } \overset{1}{\|\underline{v}\|} \\ \text{in } \cancel{\text{the}}\text{direction of } \nabla f \end{pmatrix}}_{\text{dot prod maximized when vectors are parallel}}$$

$$= \nabla f \cdot \begin{pmatrix} \text{unit vector in the} \\ \text{direction of } \nabla f \end{pmatrix}$$

$$= \nabla f \cdot \left(\frac{1}{\|\nabla f\|} \nabla f \right)$$

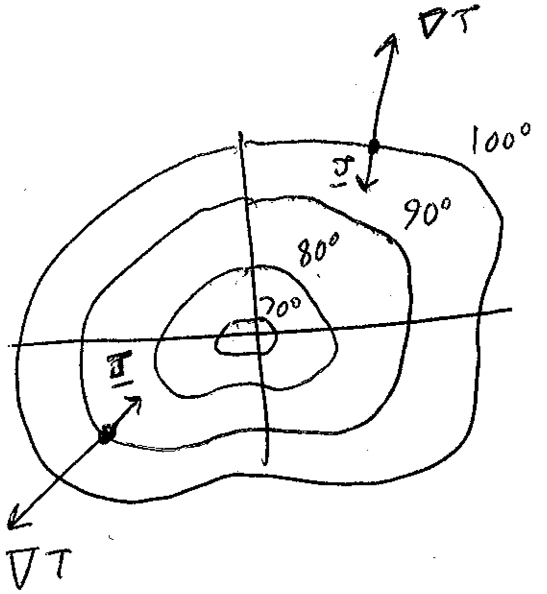
So $\nabla f \cdot \underline{v}$ is maximized when $\underline{v} = \frac{1}{\|\nabla f\|} \nabla f$.

FACT

If $\underline{u}$ is fixed, and $\|\underline{v}\|$ is fixed, but its direction is variable, then $\underline{u} \cdot \underline{v}$ is maximized when $\underline{u}$ and $\underline{v}$ are parallel.

(Example 4) If temperature is given by $T(x,y,z)$ for each point (x,y,z) in a room, then the energy flux or heat flux is given by $\underline{J} = -\underbrace{k}_{\text{scalar}}\underbrace{\nabla T}_{\text{vector}}$ where $k^{>0}$ is the conductivity of the air in the room.

$\underline{J} = \nabla T$ scaled & reflected



$\underline{J}$ measures the "movement" of energy or heat.