MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 8

Name: Solutions

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This quiz is open notes and open book.
- This quiz is due at the end of class. Quizzes submitted over one minute late will be penalized by 50%.

1. (10 points) Find a polar coordinate $\mathbf{p}(r, \theta)$ coresponding to the Cartesian coordinate $(-1, \sqrt{3})$.

$x^2 + y^2 = r^2$
$(-1)^2 + (\sqrt{3})^2 = r^2$
$1 + 3 = r^2$
$4 = r^2$
$r = 2$

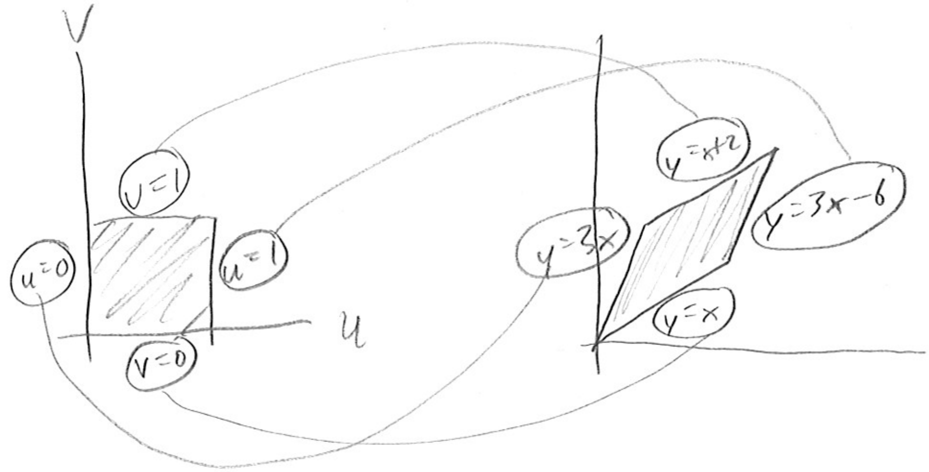
$\tan\theta = \frac{y}{x}$
$= -\frac{\sqrt{3}}{1}$
$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{-1/2}$
$\theta = 2\pi/3$

$\boxed{p(2, 2\pi/3)}$

2. (10 points) Which of these equations using cylindrical/spherical coordinates describes a plane in $\mathbb{R}^3$?

- ○ $r = 3$ $\quad x^2 + y^2 = 9$
- ● $\theta = \frac{\pi}{4}$ $\quad \tan \pi/4 = y/x \implies y - x + 0z = 0$ ✓
- ○ $\rho = 2$ $\quad x^2 + y^2 + z^2 = 4$
- ○ $\phi = \frac{2\pi}{3}$ $\quad \tan 2\pi/3 = \frac{\sqrt{x^2+y^2}}{z} \implies -\sqrt{3}z = \sqrt{x^2+y^2}$ $\implies 3z^2 = x^2 + y^2$

3. (10 points) Find an affine transformation $\mathbf{T}(u, v) = (x, y)$ mapping the unit square $[0, 1] \times [0, 1]$ in the uv plane to the parallelogram with sides given by the lines $y = x$, $y = x + 2$, $y = 3x$, $y = 3x - 6$ in the xy plane.



$u=0 \Rightarrow y=3x$

$u=1 \Rightarrow y=3x-6$

$u \in [0,1] \Rightarrow y = 3x - 6u$

$v=0 \Rightarrow y=x$

$v=1 \Rightarrow y=x+2$

$v \in [0,1] \Rightarrow y = x + 2v$

$3x - 6u = x + 2v$

$2x = 6u + 2v$

$x = 3u + v$

$y = (3u + v) + 2v$

$y = 3u + 3v$

$$\boxed{\begin{aligned} T(u,v) &= (3u+v,\ 3u+3v) \\ &= \begin{bmatrix} 3 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}}$$