MATH 2242-090 — Spring 2016 — Dr. Clontz — Midterm

Name: Solutions

- Each question is labeled with its worth toward the grade for this midterm out of 100.
- **Choose SEVEN of the eight problems to solve. Mark the one you don't want graded by marking it with the word "SKIP" in the upperleft corner of the page.** If you fail to do so, your highest score from all eight problems will not be counted. Note that this means you can earn up to 105/100 for the midterm.
- Show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- You may use at most three pages (front and back) of 8.5 $\times$ 11 inch paper for notes.
- You may use a calculator no more powerful than a TI-89 (in particular, no cell phones are allowed). None of the questions require the use of a calculator.
- This midterm is due after 70 minutes. Midterms submitted over one minute late will be penalized by 50%.

1. (15 points) Recall that $\det(AB) = (\det A)(\det B)$. Evaluate

$$\det\left(\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}\right).$$

$$= \det\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} \det\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$= (6-(-4))\cancel{(5-4)}$$

$$= \boxed{10}$$

2. (15 points) Compute the partial derivative matrix for

$$\mathbf{f}(x, y, z) = (2x + 3y, e^z, \sin(yz)).$$

$$\underline{D}\,\underline{f}(\underline{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_3}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 0 \\ 0 & 0 & e^z \\ 0 & z\cos(yz) & y\cos(yz) \end{bmatrix}$$

3. (15 points) Let $\mathbf{f}(u,v) = (u^2+v^3, 2uv)$, $\mathbf{g}(x,y) = (e^{xy}, x+y)$. It follows that

$$\mathbf{Df}(u,v) = \begin{bmatrix} 2u & 3v^3 \\ 2v & 2u \end{bmatrix} \quad \text{and} \quad \mathbf{Dg}(x,y) = \begin{bmatrix} ye^{xy} & xe^{xy} \\ 1 & 1 \end{bmatrix}.$$

Use the above matrices and the chain rule to compute $\mathbf{D}(\mathbf{f}\circ\mathbf{g})(0,0)$.

$$\underline{D}(\underline{f}\circ\underline{g}) = \underline{D}\underline{f}(\underline{g}(0,0))\,\underline{D}\underline{g}(0,0)$$

$$= \underline{D}\underline{f}(1,0)\,\underline{D}\underline{g}(0,0)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

4. (15 points) Give an approximate value of $f(1.1, -2.1)$ given the following information about f:

$$f(1,-2) = 2 \qquad \frac{\partial f}{\partial x}(1,-2) = 0 \qquad \frac{\partial f}{\partial y}(1,-2) = -1$$

$$\frac{\partial^2 f}{\partial x^2}(1,-2) = -2 \qquad \frac{\partial^2 f}{\partial y^2}(1,-2) = 4 \qquad \frac{\partial^2 f}{\partial x \partial y}(1,-2) = 3$$

$$f(1.1,-2.1) \approx f(1,-2) + \frac{\partial f}{\partial x}(1,-2)(1.1-1) + \frac{\partial f}{\partial y}(1,-2)(-2.1+2)$$

$$+ \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(1,-2)(1.1-1)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}(1,-2)(-2.1+2)^2$$

$$+ \frac{\partial^2 f}{\partial x \partial y}(1,-2)(1.1-1)(-2.1+2)$$

$$= 2 + 0 + (-1)(-0.1)$$

$$+ \frac{1}{\cancel{2}}(-\cancel{2})(0.1)^2 + \frac{1}{\cancel{2}}(\overset{2}{\cancel{4}})(-0.1)^2$$

$$+ 3(0.1)(-0.1)$$

$$= 2 + 0.1 - 0.01 + 0.02 - 0.03$$

$$= 2.1 - 0.02 = \boxed{2.08}$$

5. (15 points) Prove that $\mathbf{c}(t) = (t^2, 2, t)$ is a flow line for the vector field $\mathbf{F}(x, y, z) = (yz, x - z^2, \frac{1}{2}y)$.

$$\underline{c}'(t) = (2t, 0, 1)$$

$$\underline{F}(\underline{c}(t)) = ((2)(t), t^2 - (t)^2, \tfrac{1}{2}(2))$$
$$= (2t, 0, 1)$$

Since $\underline{F} \circ \underline{c} = \underline{c}'$, $\underline{c}$ is a flow line for $\underline{F}$.

6. (15 points) Evaluate $\int_0^2 \left[\int_{-1}^2 2xy + 3x^2 \, dy\right] dx$.

$$= \int_0^2 \left[xy^2 + 3x^2y\right]_{-1}^{2} dx$$

$$= \int_0^2 (4x + 6x^2) - (x - 3x^2)\, dx$$

$$= \int_0^2 3x + 9x^2\, dx$$

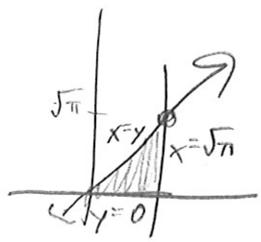
$$= \left[\frac{3}{2}x^2 + 3x^3\right]_0^2$$

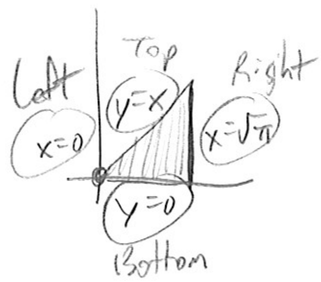
$$= \frac{3}{\cancel{2}}(\overset{2}{\cancel{4}}) + 3(8)$$

$$= 6 + 24 = \boxed{30}$$

7. (15 points) Evaluate $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} 2x^2 \cos(xy)\, dx dy$.

Need to swap...





$$= \int_0^{\sqrt{\pi}} \left[\int_0^{x} 2x^2 \cos(xy)\, dy \right] dx$$

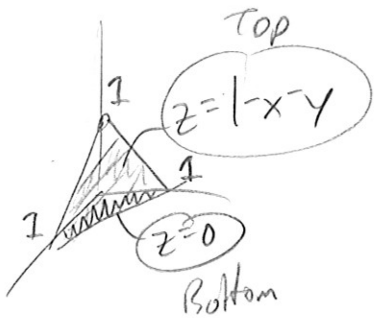
$$= \int_0^{\sqrt{\pi}} \Big[2x \sin(xy) \Big]_0^{x} dx$$

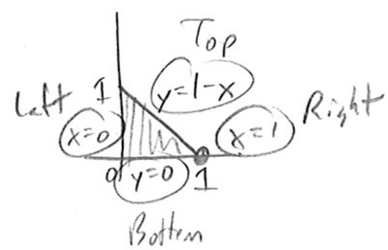
$$= \int_0^{\sqrt{\pi}} 2x \sin(x^2)\, dx$$

$$= \Big[-\cos(x^2) \Big]_0^{\sqrt{\pi}}$$

$$= -\cos(\pi) + \cos(0) = 1 + 1 = \boxed{2}$$

8. (15 points) Express the volume of the pyramid with vertices at $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$ as either a double or triple integral. (Hint: the sides of the pyramid have equations $x=0$, $y=0$, $z=0$, and $x+y+z=1$.) Do not evaluate it.





$$\boxed{\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1 \, dz\,dy\,dx}$$

OR

$$\boxed{\int_0^1 \int_0^{1-x} 1-x-y \; dy\,dx}$$