MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 3

Name: Solutions

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This quiz is open notes and open book.
- This quiz is due at the end of class. Quizzes submitted over one minute late will be penalized by 50%.

1. (10 points) The partial derivative matrix of the differentiable function

$$\mathbf{f}(x, y, z) = (e^x, \sqrt{yz}, x + 3z)$$

at the point $(0, 12, 3)$ is

$$\mathbf{Df}(0, 12, 3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

Explain why $\mathbf{f}(0.1, 11.9, 3.1) \approx (1.1, 6.075, 9.4)$ using the linear approximation $\mathbf{L}(x, y, z)$ for $\mathbf{f}$ at $(0, 12, 3)$.

$$\underline{f}(0.1, 11.9, 3.1) \approx \underline{L}(0.1, 11.9, 3.1) \quad \text{since } (0.1, 11.9, 3.1) \approx (0,12,3)$$

$$= \underline{f}(0,12,3) + D\underline{f}(0,12,3)\big((0.1, 11.9, 3.1) - (0,12,3)\big)$$

$$= (1, \sqrt{36}, 9) + D\underline{f}(0,12,3)(0.1, -0.1, 0.1)$$

$$= \begin{bmatrix} 1 \\ 6 \\ 9 \end{bmatrix} + \begin{bmatrix} \cancel{1} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0.25} & \cancel{1} \\ \cancel{1} & \cancel{0} & \cancel{3} \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 6 \\ 9 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.025 + 0.1 \\ 0.1 + 0.3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 6 \\ 9 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.075 \\ 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 1.1 \\ 6.075 \\ 9.4 \end{bmatrix}.$$

2. (10 points) Let $\mathbf{f}(u,v) = (\tan(u-1) - e^v, u^2 - v^2)$, $\mathbf{g}(x,y) = (e^{x-y}, x-y)$. It follows that

$$\mathbf{Df}(u,v) = \begin{bmatrix} \sec^2(u-1) & -e^v \\ 2u & -2v \end{bmatrix} \quad \text{and} \quad \mathbf{Dg}(x,y) = \begin{bmatrix} e^{x-y} & -e^{x-y} \\ 1 & -1 \end{bmatrix}.$$

Use the above matrices and the chain rule to compute $\mathbf{D}(\mathbf{f} \circ \mathbf{g})(0,0)$. HINT: don't forget to plug $\mathbf{g}(0,0)$ into $\mathbf{Df}$ rather than just plugging in $(0,0)$.

$$\left(\underline{g}(0,0) = (e^0, 0) = (1,0) \right)$$

$$\underline{D}(\underline{f} \circ \underline{g})(0,0) = \underline{Df}(\underline{g}(0,0))\,\underline{D}\,\underline{g}(0,0)$$

$$= \underline{Df}(1,0)\,\underline{D}\,\underline{g}(0,0)$$

$$= \begin{bmatrix} \sec^2 0 & -e^0 \\ 2(1) & -2(0) \end{bmatrix} \begin{bmatrix} e^0 & -e^0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{1} & \cancel{-1} \\ \cancel{2} & \cancel{0} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}$$