MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 12 (Take-home)

Name: ______________________________

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This take-home quiz is open notes and open book. You may work with others as long as you don't plagiarize their answers.
- This quiz is due at the beginning of class on Monday, May 2. Late submissions will not be accepted.

1. (10 points) Which of these is a parametrization of the portion of the surface $z = x^2 + y^2$ above the unit circle in the xy plane?

   - ○ $\mathbf{\Phi}(u, v) = (u^2, v^2, u + v); 0 \leq u, v \leq 1$
   - ○ $\mathbf{\Phi}(x, y) = (x + y, x + y, z^2); 0 \leq x, y \leq 1$
   - ○ $\mathbf{\Phi}(r, \theta) = (r \cos \theta, r \sin \theta, r^2); 0 \leq r \leq 1, 0 \leq \theta \leq 2\pi$
   - ○ $\mathbf{\Phi}(u, v) = 2u\mathbf{i} - 2v\mathbf{j}; 0 \leq u, v \leq 1$
   - ○ None of these.

2. (10 points) Prove that the area of the of the triangle with vertices $(0, 0, 0)$, $(1, 2, -2)$, and $(0, 3, 3)$ is $\frac{9\sqrt{2}}{2}$ by using the formula $A = \iint_S 1dS = \iint_D \|\frac{\partial \mathbf{\Phi}}{\partial u} \times \frac{\partial \mathbf{\Phi}}{\partial v}\| dA$ with the parametrization $\mathbf{\Phi}(u, v) = (u, 2u + 3v, -2u + 3v)$. (Hint: You need to find the domain D for this parametrization mapping onto the surface; this will give you the bounds for the double integral.)

3. (10 points) Let S be the oriented surface with an orientation-preserving parametrization $\mathbf{\Phi}(u,v) = (u, u+v, v^2)$ for $0 \leq u, v \leq 1$. If $\mathbf{F} = (y, x, z)$ is the velocity field of a fluid, then show that the flux of the fluid moving through S with respect to its orientation is 1; that is, verify that $\iint_S \mathbf{F} \cdot d\mathbf{S} = 1$.