MATH 2242-090 — Spring 2016 — Dr. Clontz — Quiz 5

Name: Solutions

- Each quiz question is labeled with its worth toward your total quiz grade for the semester.
- On multiple choice problems, you do not need to show your work. No partial credit will be given.
- On full response problems, show all of your work and give a complete solution. When in doubt, don't skip any steps. Partial credit will be given at the discretion of the professor.
- This quiz is open notes and open book.
- This quiz is due at the end of class. Quizzes submitted over one minute late will be penalized by 50%.

1. (10 points) Prove that $\mathbf{c}(t) = (t^2, 2t-1, \sqrt{t})$ is a flow line for the vector field $\mathbf{F}(x, y, z) = (y+1, 2, \frac{1}{2z})$.

$$\underline{c}'(t) = \left(2t, 2, \tfrac{1}{2}t^{-1/2}\right)$$
$$= \left(2t, 2, \frac{1}{2\sqrt{t}}\right)$$

$$\underline{F}(\underline{c}(t)) = \left((2t\cancel{-1)+1}, 2, \frac{1}{2\sqrt{t}}\right)$$
$$= \left(2t, 2, \frac{1}{2\sqrt{t}}\right)$$

Since $\underline{c}' = \underline{F} \circ \underline{c}$, $\underline{c}$ is a flow line for $\underline{F}$.

2. (10 points) For $f : \mathbb{R}^2 \to \mathbb{R}$ and $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$, prove that $\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \nabla f$. (Hint: $f\mathbf{F} = (fF_1, fF_2)$, so use the product rule to compute $\frac{\partial}{\partial x}[fF_1]$ and $\frac{\partial}{\partial y}[fF_2]$.)

$$\operatorname{div}(f\underline{F}) = \frac{\partial}{\partial x}\left[fF_1\right] + \frac{\partial}{\partial y}\left[fF_2\right]$$
$$= F_1\frac{\partial f}{\partial x} + f\frac{\partial F_1}{\partial x} + F_2\frac{\partial f}{\partial y} + f\frac{\partial F_2}{\partial y}$$

$$f \operatorname{div}\underline{F} + \underline{F}\cdot\nabla f = f\left(\frac{\partial F_1}{\partial x}, \frac{\partial F_2}{\partial y}\right) + (F_1, F_2)\cdot\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$
$$= f\frac{\partial F_1}{\partial x} + f\frac{\partial F_2}{\partial y} + F_1\frac{\partial f}{\partial x} + F_2\frac{\partial f}{\partial y}$$

Thus $\operatorname{div}(f\underline{F}) = f\operatorname{div}(\underline{F}) + \underline{F}\cdot\nabla f$