Crash Course in Mathematical Induction:

IDEA: I want to prove a statement $S(n)$ for every natural number $n \in \{1, 2, 3, \ldots\}$

$$\boxed{\text{(For Example) } S(n) \text{ is the statement } 1+2+3+\cdots+n = \frac{n(n+1)}{2}.}$$

Suppose that $S(1)$ is true ("base case") (usually obvious). What if I can use $S(n)$ to prove $S(n+1)$ to be true?... Then I know this:

- $S(1)$ is true.
- For $n=1$, $S(1)$ is true, so $S(1+1)=S(2)$ is true.
- For $n=2$, $S(2) \Rightarrow S(2+1)$ aka $S(3)$.
- For $n=3$, $S(3) \Rightarrow S(3+1)$ aka $S(4)$.
- And so on...

(Example) Prove that for $n \in \{1, 2, 3, \dots\}$,

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}.$$

Proof: Suppose $n = 1$. Then

$$1 + \dots + n = 1 = \frac{1(2)}{2} = \frac{1(1+1)}{2} = \frac{n(n+1)}{2}$$

so the theorem holds for this base case.

So we may assume that

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

holds by induction, and use it to prove that

$$1 + 2 + 3 + \dots + n + (n+1) = \frac{(n+1)((n+1)+1)}{2}.$$

This formula holds because:

$$\begin{aligned}
(1 + 2 + 3 + \dots + n) + (n+1) &= \left(\frac{n(n+1)}{2}\right) + (n+1) \\
&= \frac{n^2 + n}{2} + \frac{2n + 2}{2} \\
&= \frac{n^2 + 3n + 2}{2} \\
&= \frac{(n+1)(n+2)}{2} \\
&= \frac{(n+1)((n+1)+1)}{2}.
\end{aligned}$$

$\square$

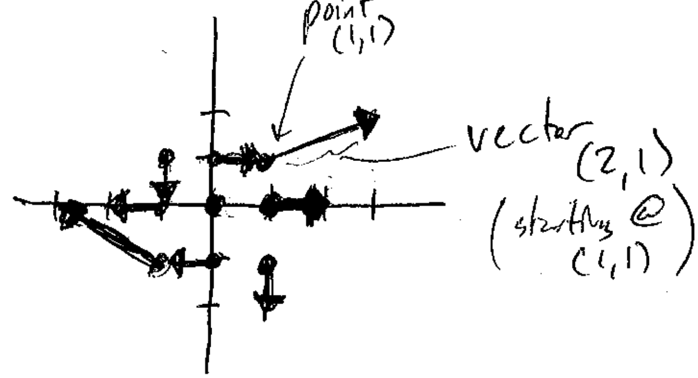
4.3 Vector Fields

commonly (not always) capitalized

A vector field is a function $\underline{F}: \mathbb{R}^n \to \mathbb{R}^n$ which assigns a vector (in $\mathbb{R}^n$) to each point (in $\mathbb{R}^n$).

For example: $\underline{F}(x,y) = (x+y, xy)$ is a 2D vector field:

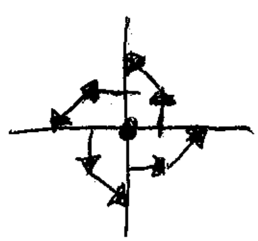
$\underline{F}(0,0) = (0,0)$
$\underline{F}(1,0) = (1,0)$
$\underline{F}(1,1) = (2,1)$
$\underline{F}(0,1) = (1,0)$

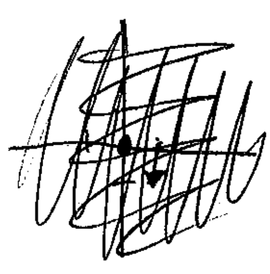


(Example 1) The velocity field of a fluid (e.g. wind or current) may be modeled with a vector field.

(Example 2) Sketch the rotary motion of the vector field $\underline{V}(x,y) = (-y, x)$.

$\underline{V}(0,0) = (0,0)$
$\underline{V}(1,0) = (0,1)$
$\underline{V}(1,1) = (-1,1)$
$\underline{V}(0,1) = (-1,0)$
$\underline{V}(-1,1) = (-1,-1)$
etc.



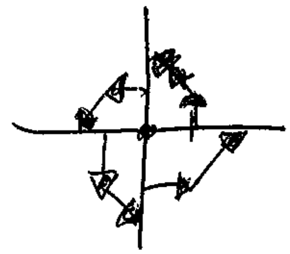


ML

If the numbers in a vector field grow too large, you may draw a scaled down multiple of each vector.

For example: $\underline{V}(x,y) = (-10y, 10x)$

$\underline{V}(1,0) = (0,10)$
$\underline{V}(1,1) = (-10,10)$



(That's All for today...
stupid fire alarm...)