Autoregression for Time Series

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What is a time series?

Definition

A *time series* is a sequence of measurements of a quantity (e.g. temperature, pressure) over time.

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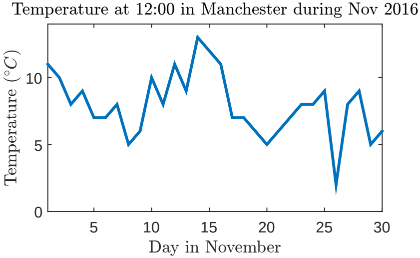

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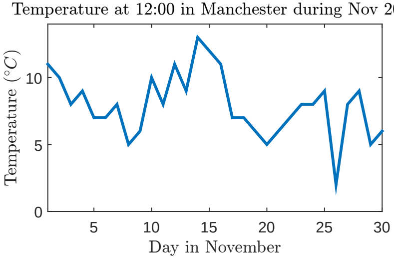
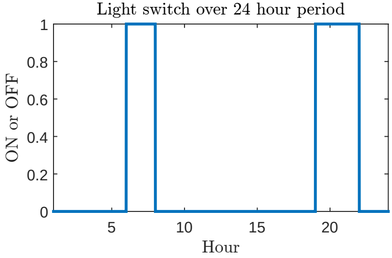
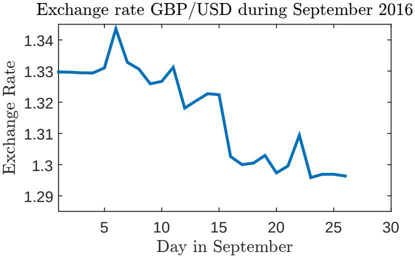


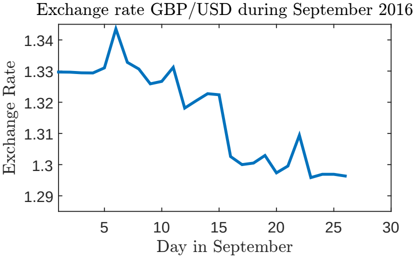


Prediction



Exchange rate GBP/USD during September 2016
Exchange Rate
1.34
1.33
1.32
1.31
1.3
1.29
5
10
15
20
25
30
Day in September

Prediction



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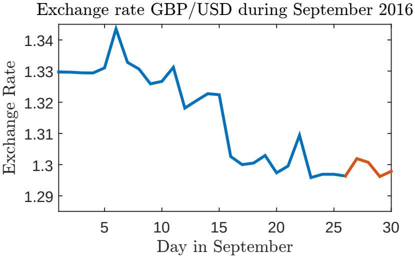

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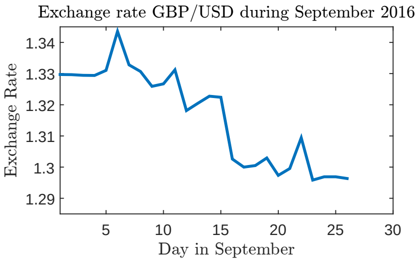

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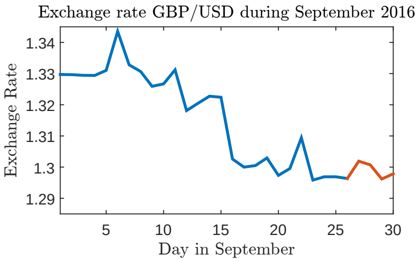

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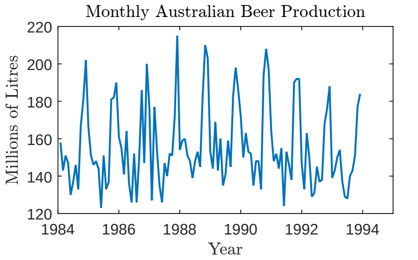
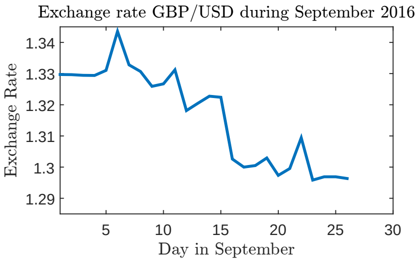
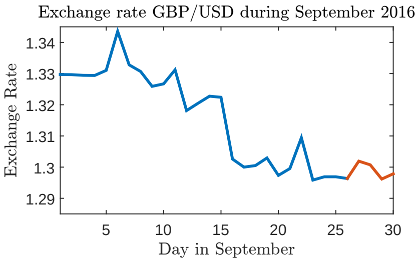
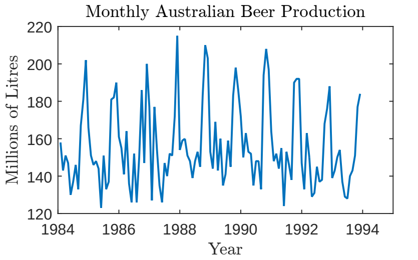


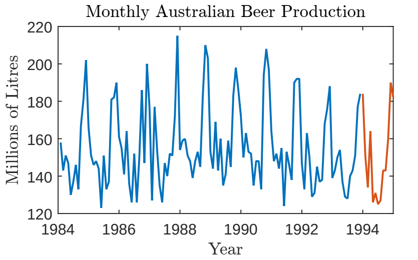
Monthly Australian Beer Production
Millions of Litres
220
200
180
160
140
120
1984
1986
1988
1990
1992
1994
Year

Prediction









http://users.ecs.soton.ac.uk/jn2/teaching/timeSeries.pdf

Autoregressive (AR) Model

The $AR(p)$ model is defined as

$$t_n = \nu + \sum_{i=1}^{p} \phi_i t_{n-i} + \epsilon_n.$$

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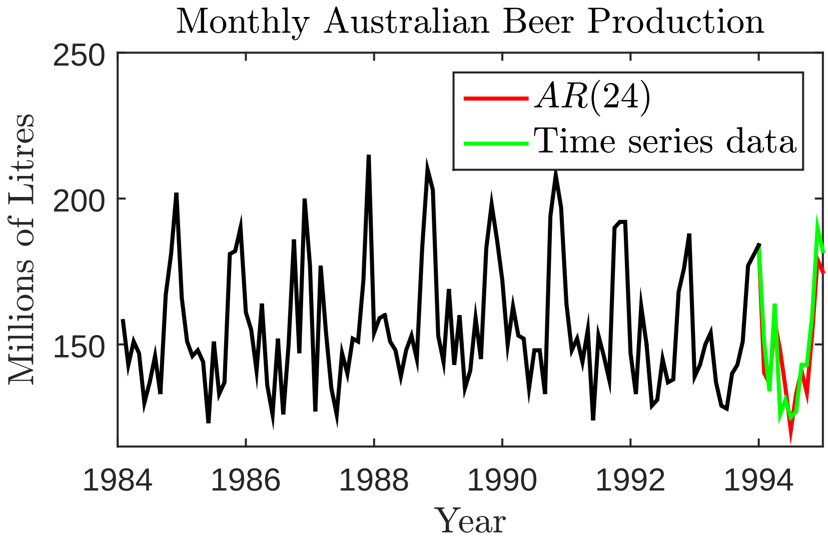
If $p = 3$ then

$$\begin{aligned} t_n &\approx \nu + \phi_1 t_{n-1} + \phi_2 t_{n-2} + \phi_3 t_{n-3}, \\ t_{n-1} &\approx \nu + \phi_1 t_{n-2} + \phi_2 t_{n-3} + \phi_3 t_{n-4}, \\ &\vdots \\ t_4 &\approx \nu + \phi_1 t_3 + \phi_2 t_2 + \phi_3 t_1, \end{aligned}$$

and we can find the values ϕ_1, ϕ_2 and ϕ_3 by solving a least squares problem using historical time series data.

Example



Monthly Australian Beer Production
250
200
150
1984
1986
1988
1990
1992
1994
Year
Millions of Litres
$AR(24)$
Time series data

Vector Autoregressive (VAR) Model

The $VAR(p)$ model is defined as

$$\mathbf{t}_n = \boldsymbol{\nu} + \sum_{i=1}^{p} A_i \mathbf{t}_{n-i} + \boldsymbol{\epsilon}_n,$$

where A_i are matrices and $\mathbf{t}_n, \mathbf{t}_{n-i}, \boldsymbol{\nu}$ and $\boldsymbol{\epsilon}_n$ are vectors.

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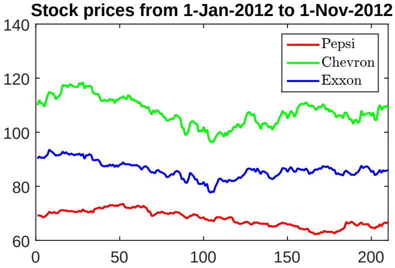
where A_i are matrices and $\mathbf{t}_n, \mathbf{t}_{n-i}, \boldsymbol{\nu}$ and $\boldsymbol{\epsilon}_n$ are vectors. If $p = 2$ and we have two time series, then

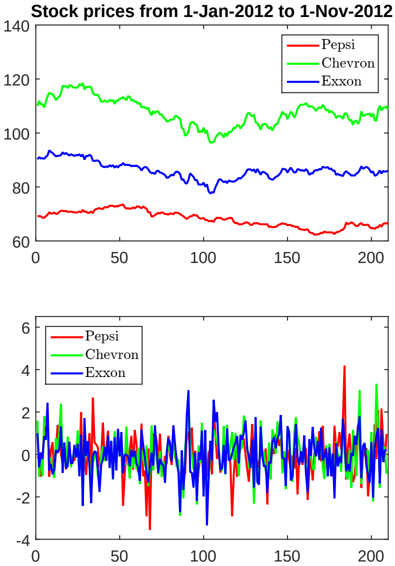
$$\begin{pmatrix} s_n \\ t_n \end{pmatrix} = \begin{pmatrix} \nu_1 + a_{11}^{(1)} s_{n-1} + a_{12}^{(1)} t_{n-1} + a_{11}^{(2)} s_{n-2} + a_{12}^{(2)} t_{n-2} \\ \nu_2 + a_{21}^{(1)} s_{n-1} + a_{22}^{(1)} t_{n-1} + a_{21}^{(2)} s_{n-2} + a_{22}^{(2)} t_{n-2} \end{pmatrix}$$

Example



Stock prices from 1-Jan-2012 to 1-Nov-2012
Pepsi
Chevron
Exxon
140
120
100
80
60
0
50
100
150
200

Example



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200
Pepsi
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Exxon
6
4
2
0
-2
-4
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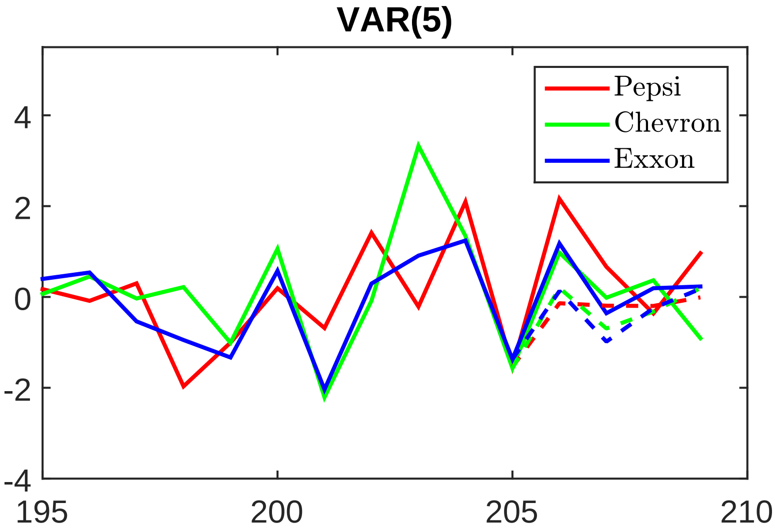
Example



Summary and Future Work

- Vector Autoregression is a multivariate generalisation of Autoregression.
- As well as prediction, it can be used to capture the linear interdependencies among several time series.

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- As well as prediction, it can be used to capture the linear interdependencies among several time series.
- I am looking at an iterative linear algebra approach to Vector Autoregression.