Understanding Deep Neural Networks

# Chapter Five

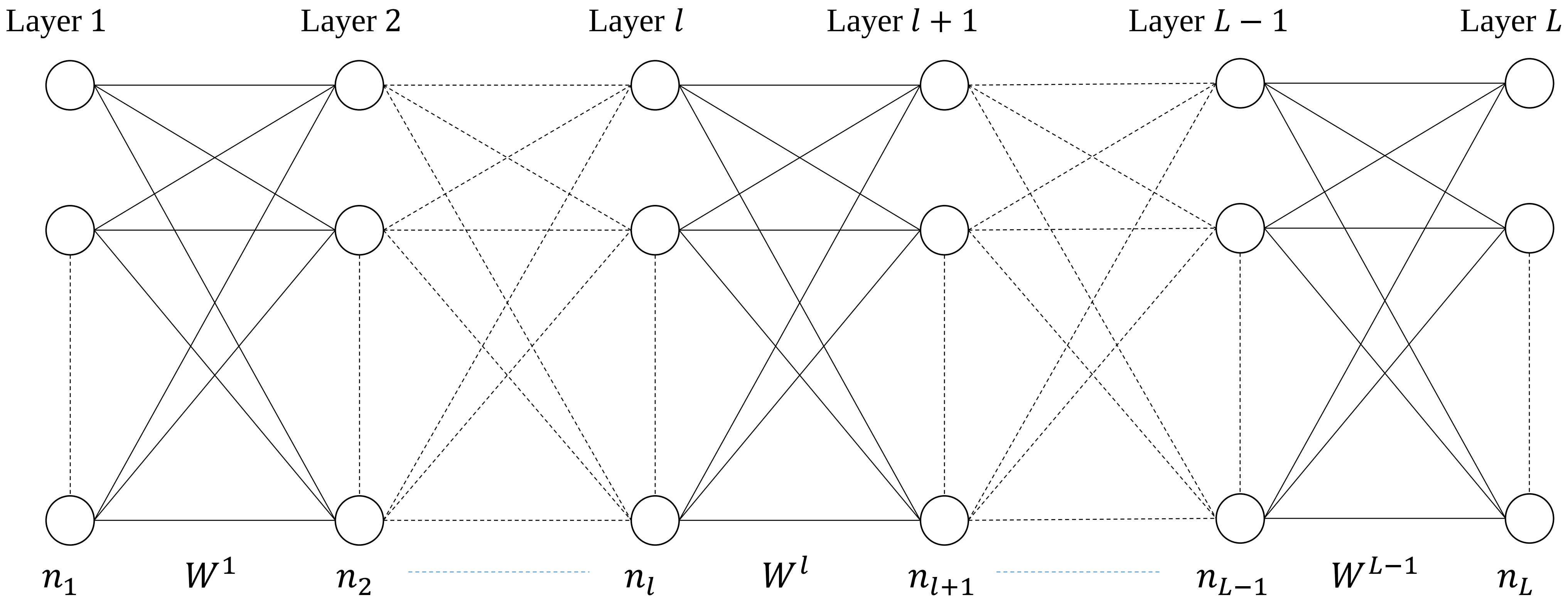
# On Some Problems of BP

Zhang Yi, *IEEE Fellow*
Autumn 2019

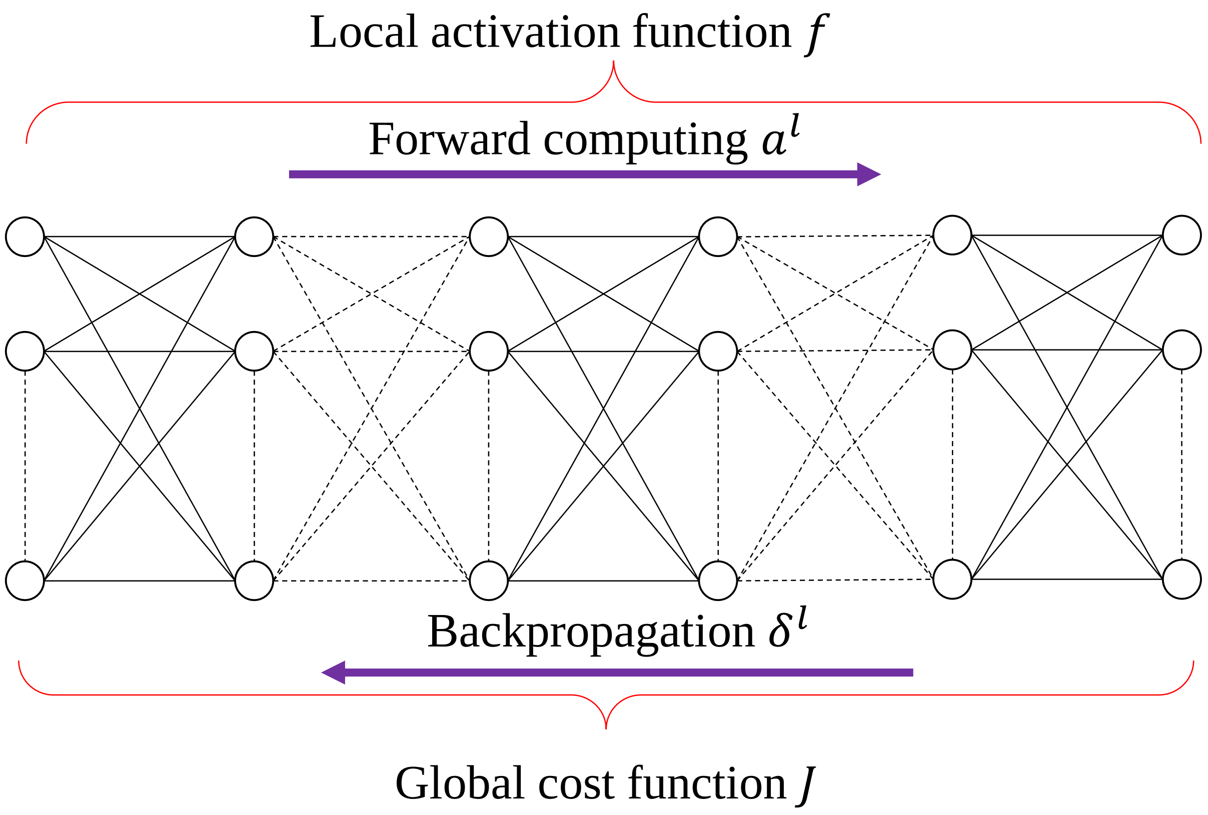
# Outline

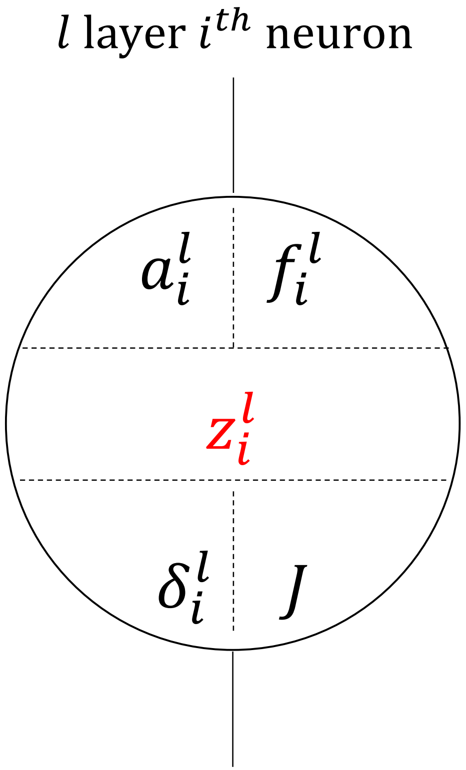
- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

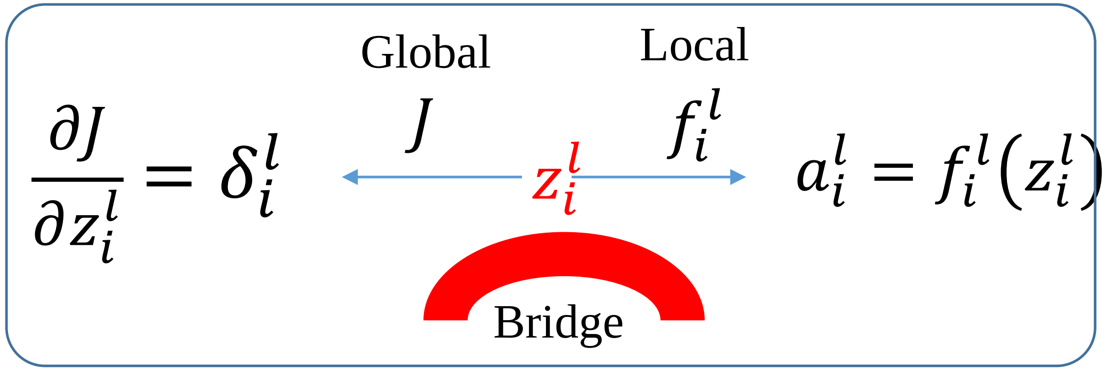
# Network Structure



# Network Concepts

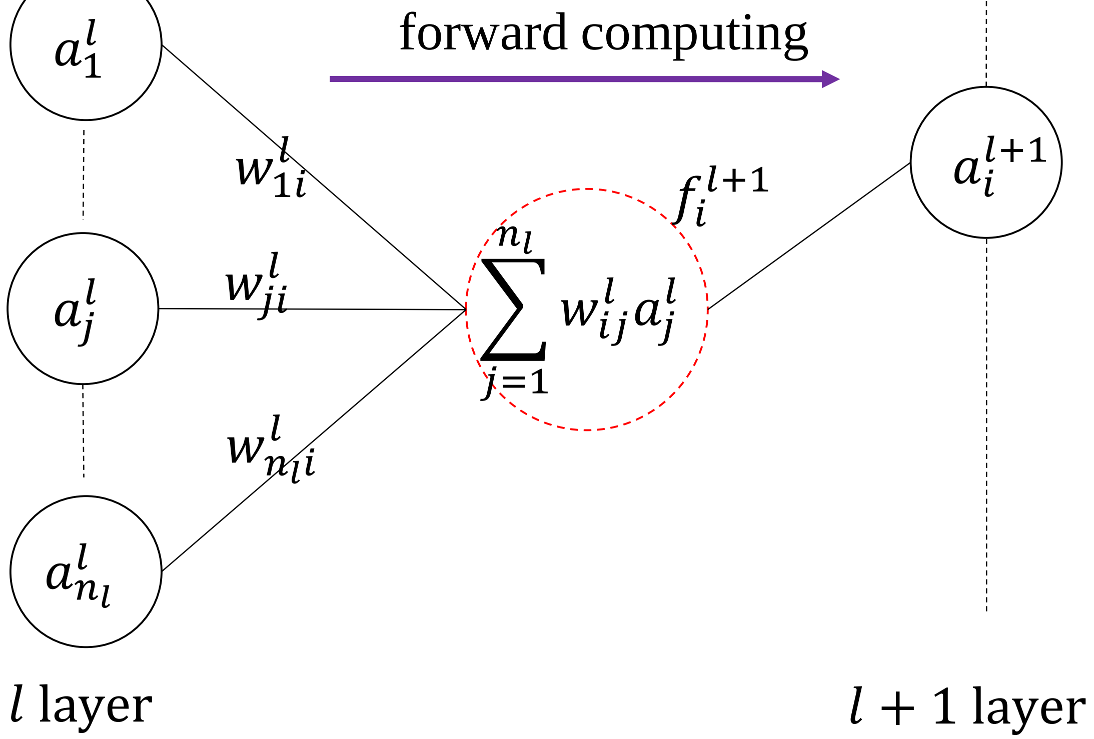


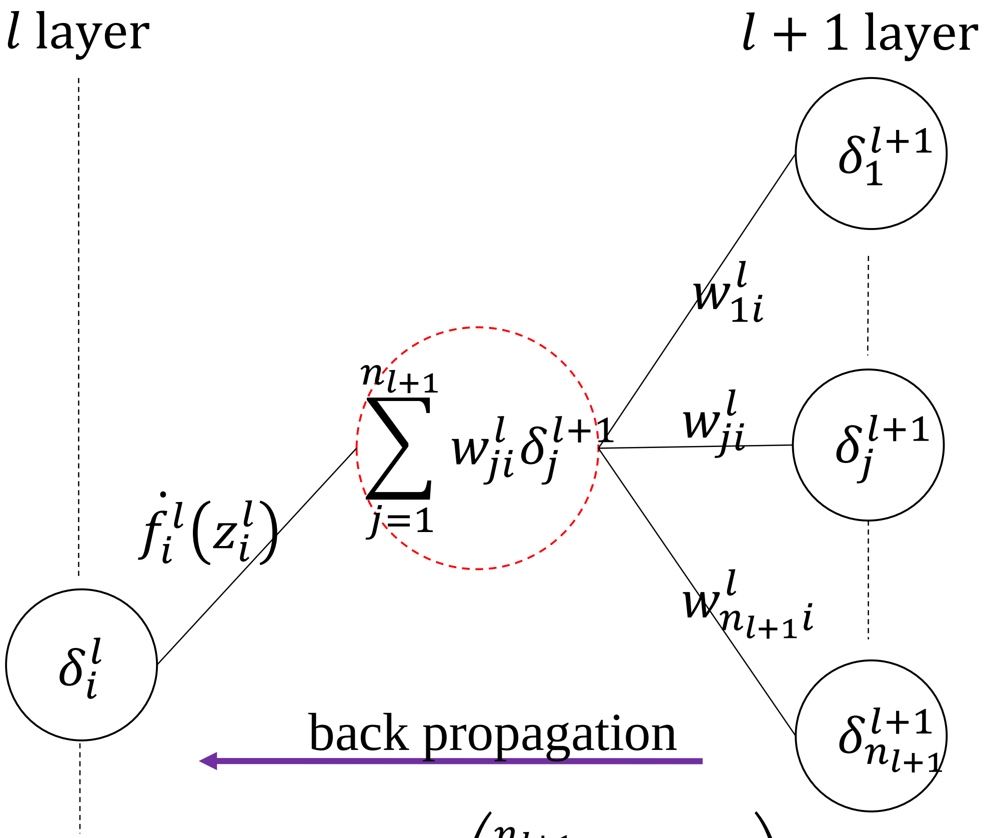



# Network Operations

$$a_i^{l+1} = f_i^{l+1}\left(\sum_{j=1}^{n_l} w_{ij}^l a_j^l\right)$$



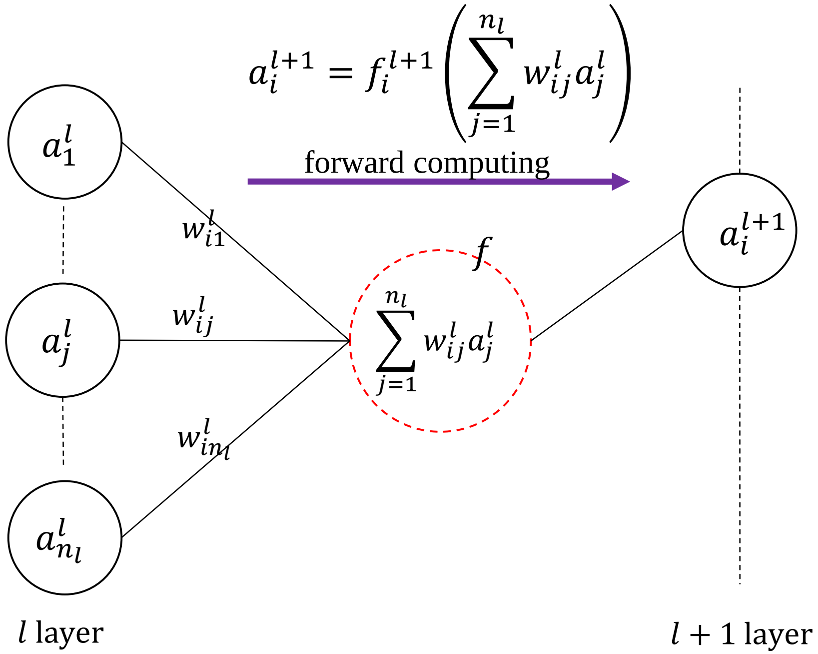



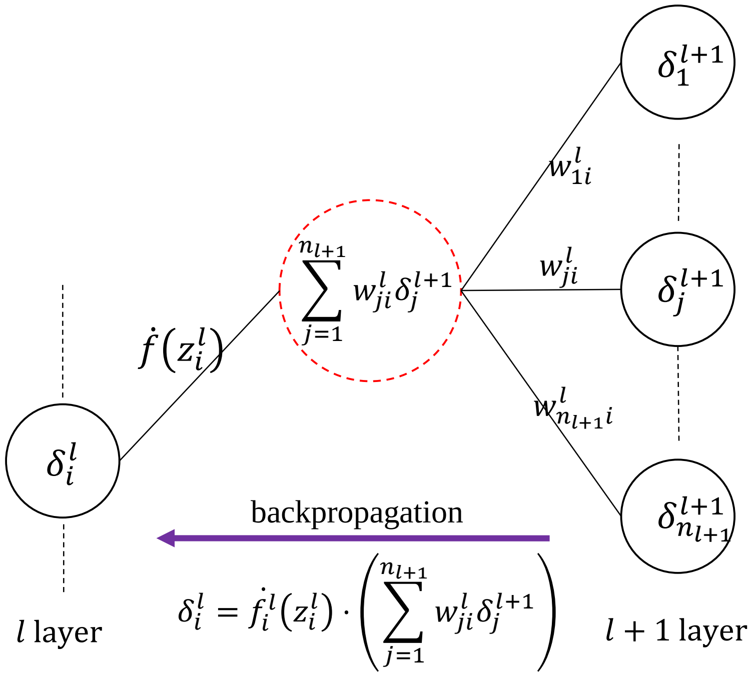

$$\delta_i^l = \dot{f}_i^l(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

# Network Functions

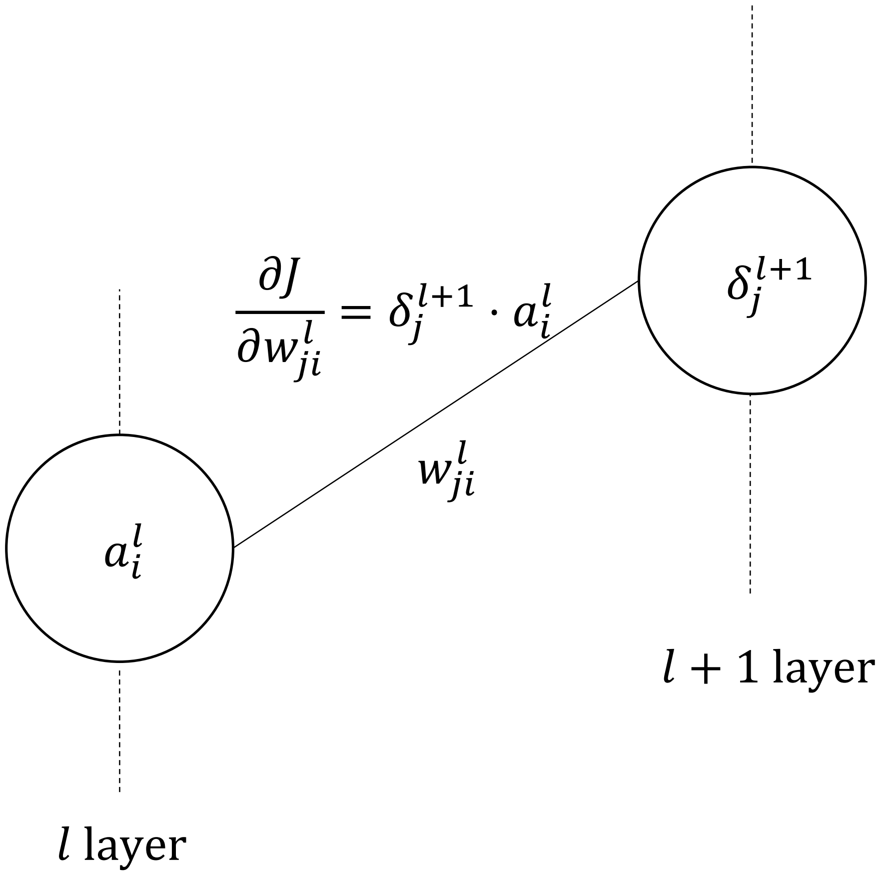
function $fc\left(w^l, a^l\right)$
$for\ i = 1{:}\,n_{l+1}$
$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$
$$a_i^{l+1} = f_i^{l+1}\left(z_i^{l+1}\right)$$
$end$

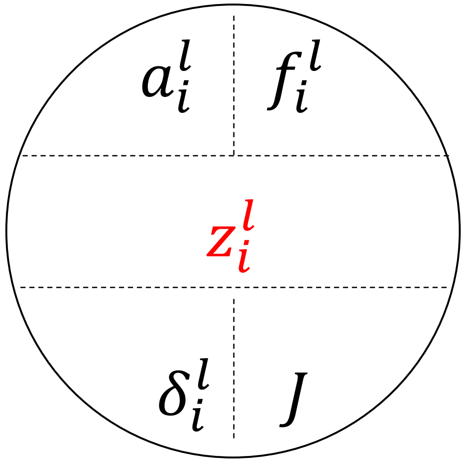
function $bc\left(w^l, \delta^{l+1}\right)$
$for\ i = 1{:}\,n_l$
$$\delta_i^l = \dot{f}_i^l\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$
$end$

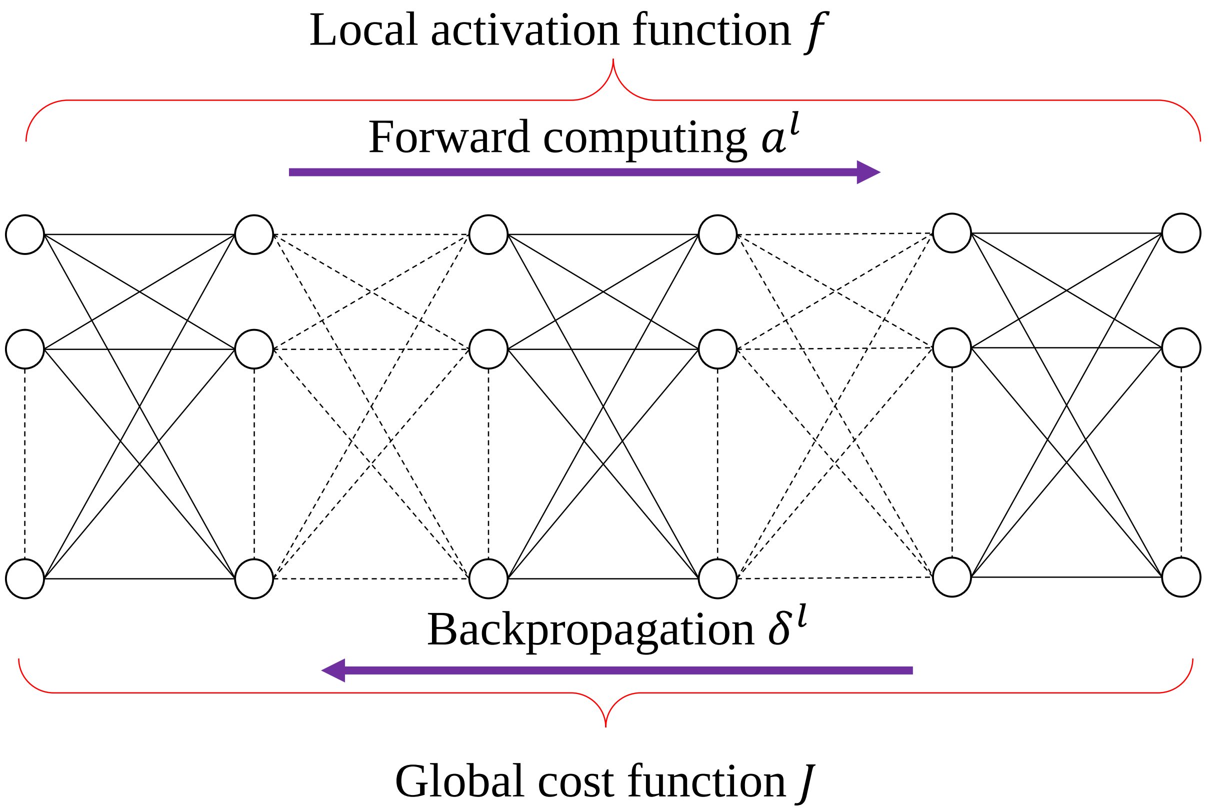


# Network Relationship



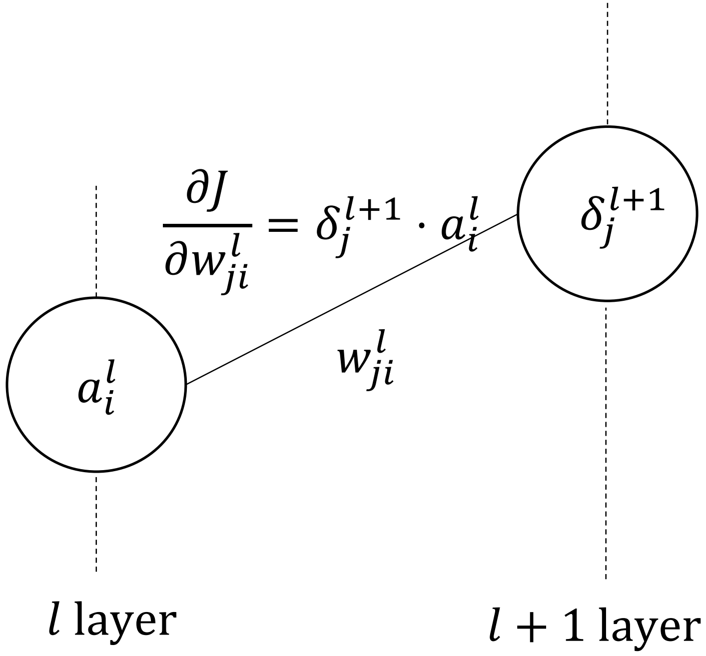



# Network Learning Rule

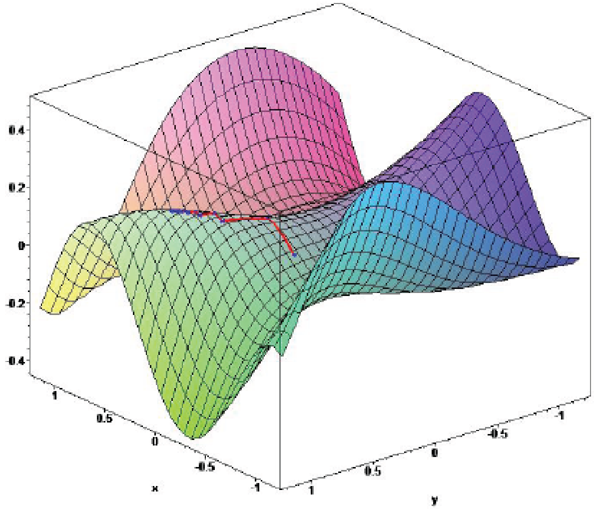
Learning rule

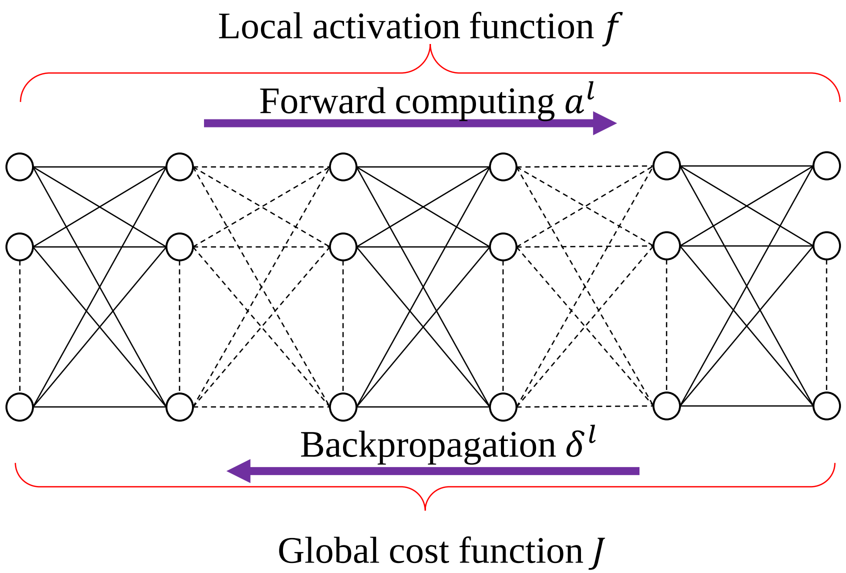
$$w_{ji}^{l} \leftarrow w_{ji}^{l} - \alpha \cdot \frac{\partial J}{\partial w_{ji}^{l}}$$

$$w_{ji}^{l} \leftarrow w_{ji}^{l} - \alpha \cdot \left(\delta_j^{l+1} \cdot a_i^{l}\right)$$




$$J = J(w^1, w^2, \cdots, w^L)$$





# The BP Algorithm

Step 1. Input the training data set $D = \{(x, y)\}$

Step 2. Initialize each $w_{ij}^l$, and choose a learning rate $\alpha$.

Step 3. for each mini-batch sample $D_m \subseteq D$

for each $x \in D_m$

$a^1 \leftarrow x \in D_m$;

for $l = 2{:}L$

$a^{l+1} \leftarrow fc(w^l, a^l)$;

end

$$\delta^L = \frac{\partial J(x)}{\partial z^L};$$

for $l = L - 1{:}2$

$\delta^l \leftarrow bc(w^l, \delta^{l+1})$;

end

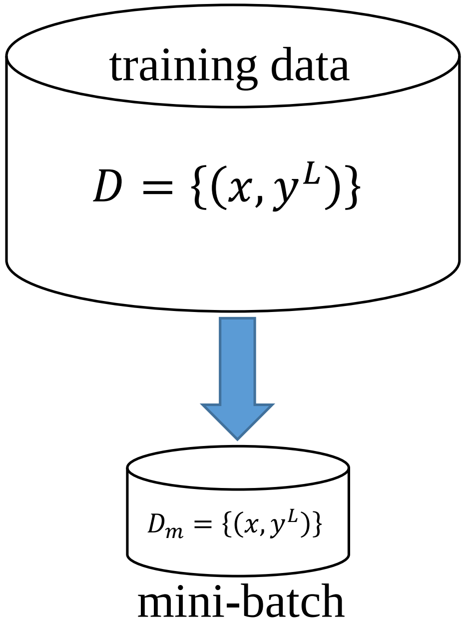
$$\frac{\partial J}{\partial w_{ji}^l} \leftarrow \frac{\partial J}{\partial w_{ji}^l} + \delta_j^{l+1} \cdot a_i^l;$$

end

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l};$$

end

Step 4. Return to Step 3 until each $w^l$converge.




function $fc(w^l, a^l)$

$for\ i = 1{:}n_{l+1}$

$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$

$$a_i^{l+1} = f_i^{l+1}(z_i^{l+1})$$

$end$

Relationship:

$$\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$$

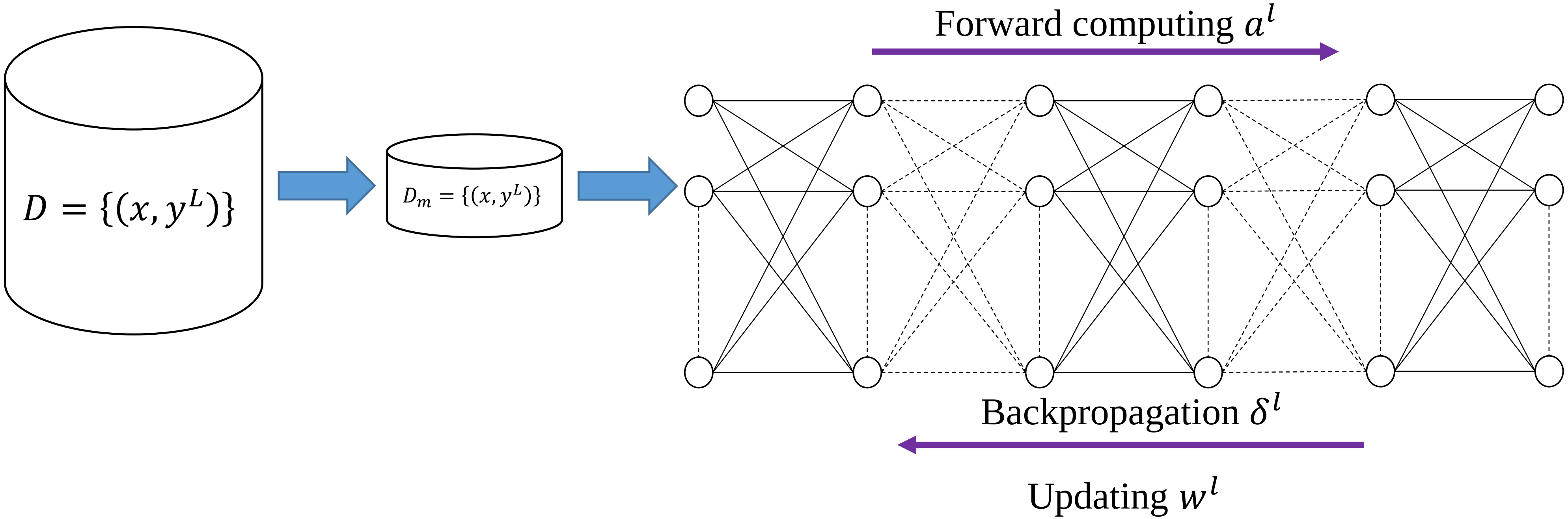
function $bc(w^l, \delta^{l+1})$

$for\ i = 1{:}n_l$

$$\delta_i^l = \dot{f}_i^l(z_i^l) \cdot \left( \sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1} \right)$$

$end$

# Network Training



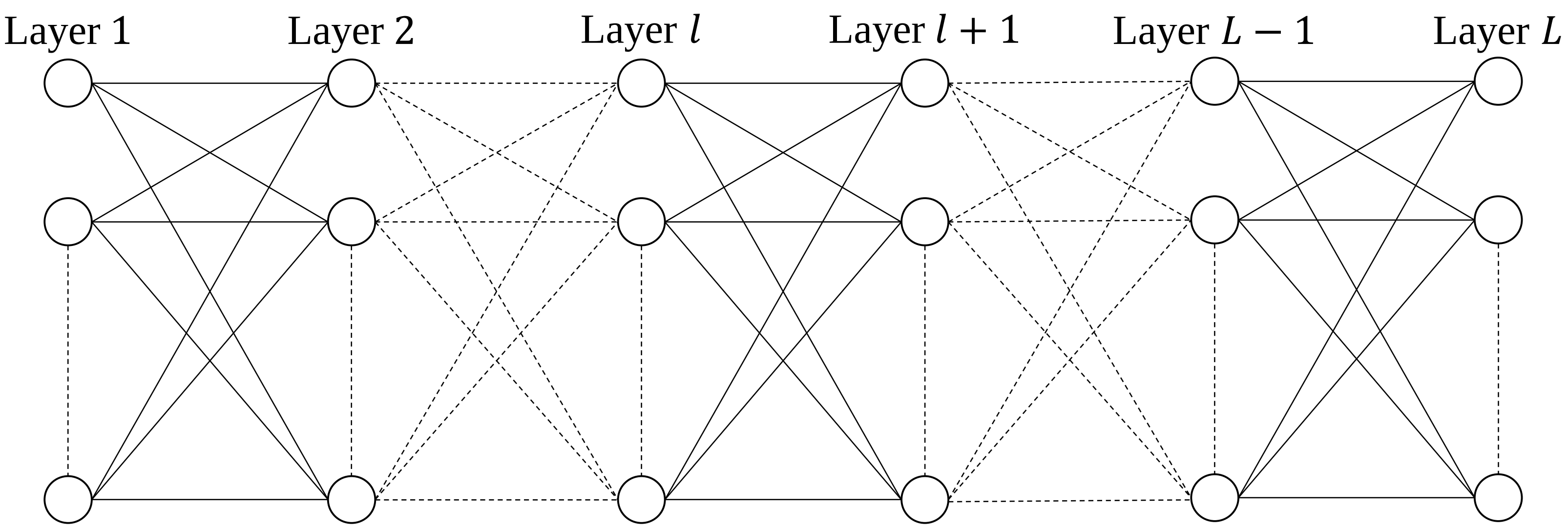
# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Output
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Network Structure

Two important characters:

- No any connection in any layer
- No any connection across any layer



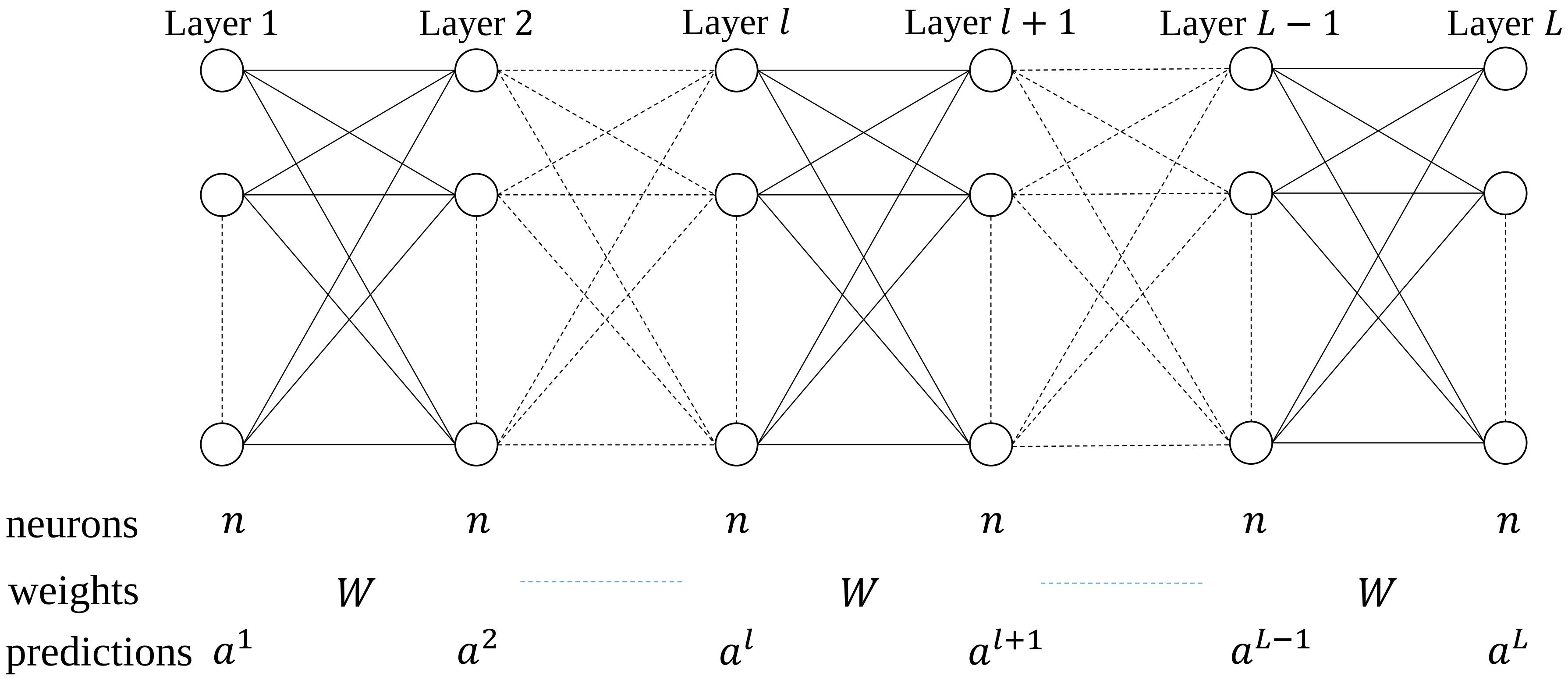

neurons $n_1$ $n_2$ $n_l$ $n_{l+1}$ $n_{L-1}$ $n_L$

weights $W^1$ $W^l$ $W^{L-1}$

predictions $a^1$ $a^2$ $a^l$ $a^{l+1}$ $a^{L-1}$ $a^L$

# On the Network Structure

## Recurrent Neural Networks

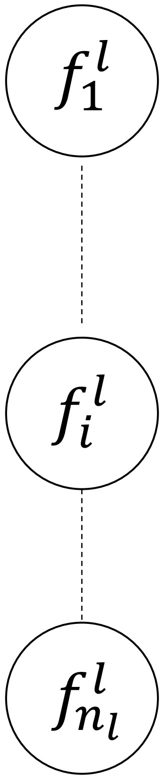



$$a^{l+1} = f\left(W a^l\right)$$

# On the Network Structure

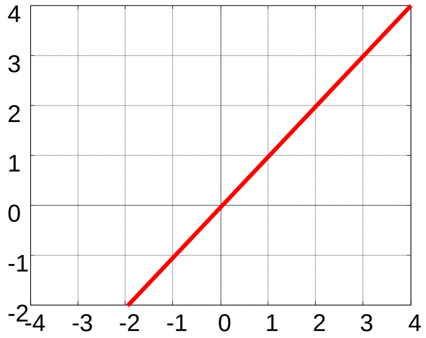
Activation functions of each neuron can be different

Layer $l$



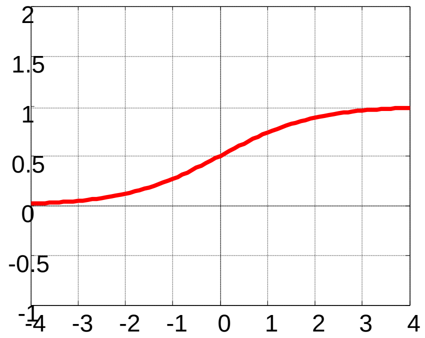

Linear function

$$f(z) = z$$



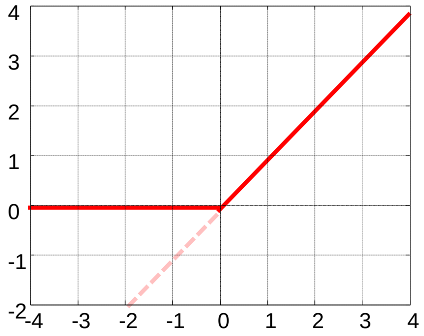
Sigmoid function

$$f(z) = \frac{1}{1+e^{-z}}$$



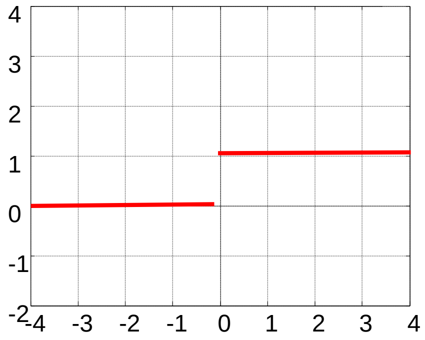
Rectifier function

$$f(z) = \begin{cases} z, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

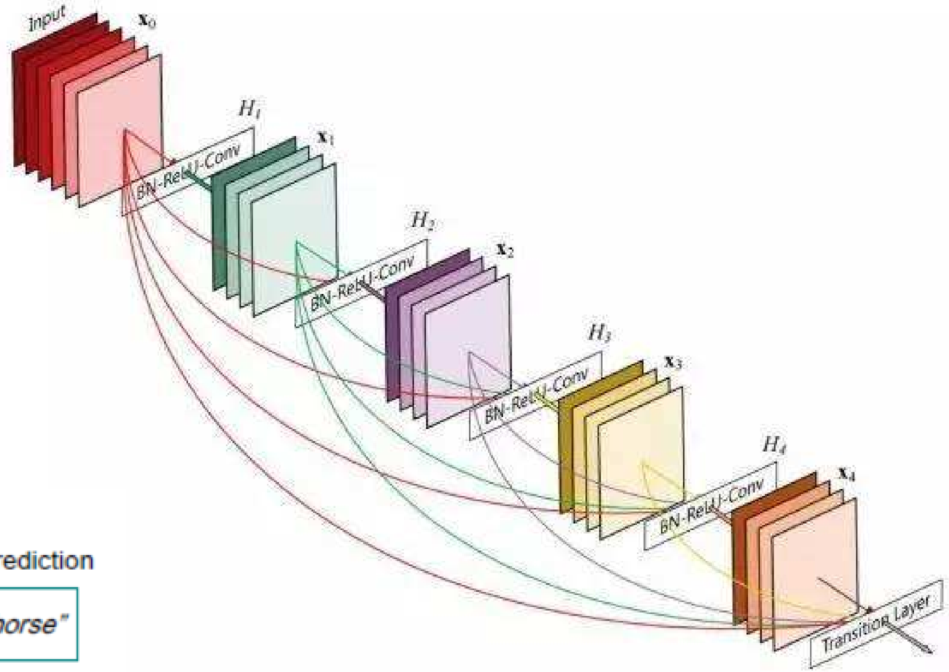


Hard-limit function

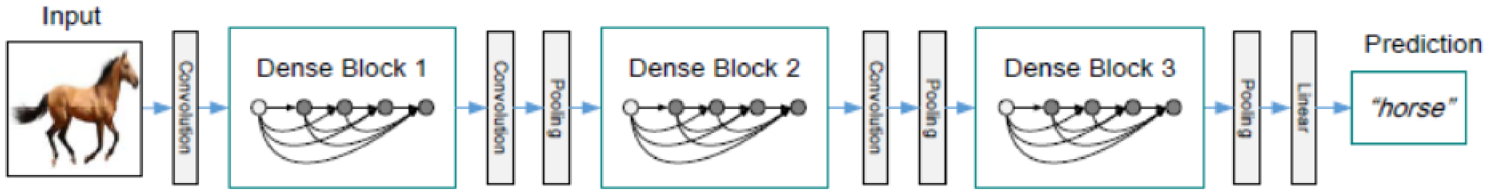
$$f(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

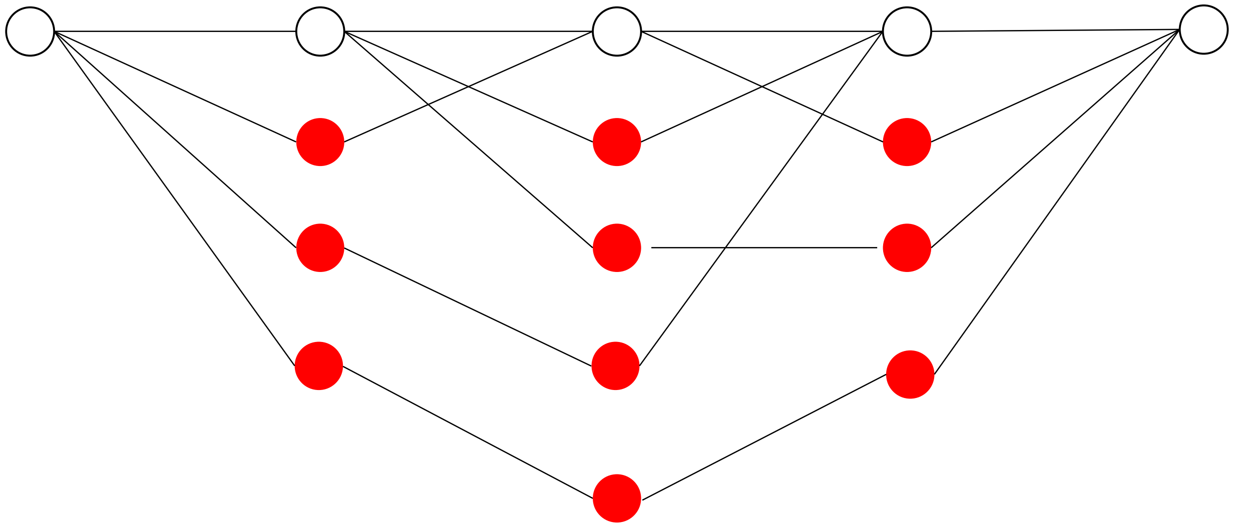


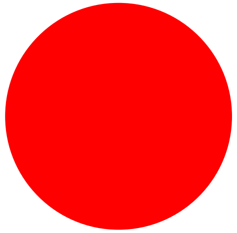
# On the Network Structure




DenseNets



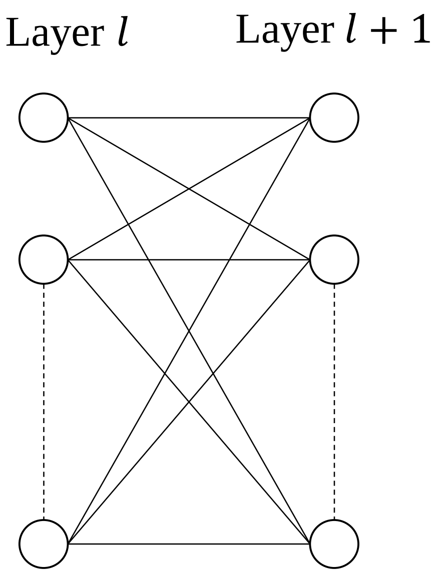





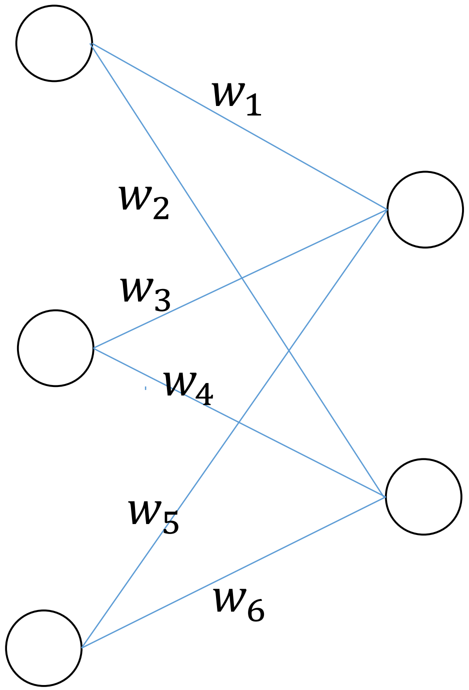
Linear neuron
$f(s) = s$

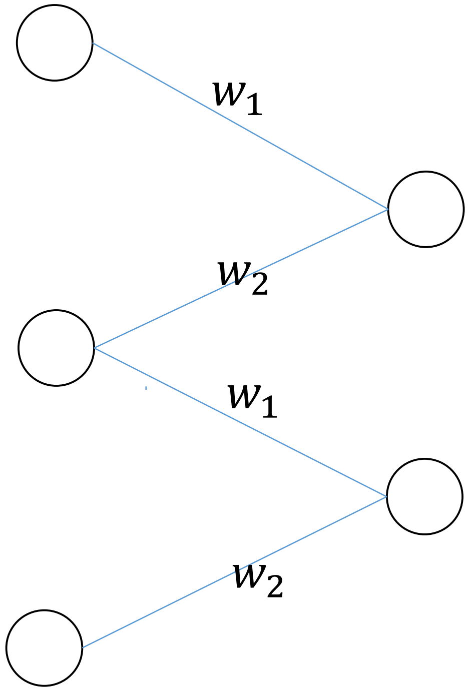
# On the Network Structure

Connection weights between two layers can share some weights




$$W^l = \left(w_{ij}^l\right)_{n_{l+1}\times n_l}$$

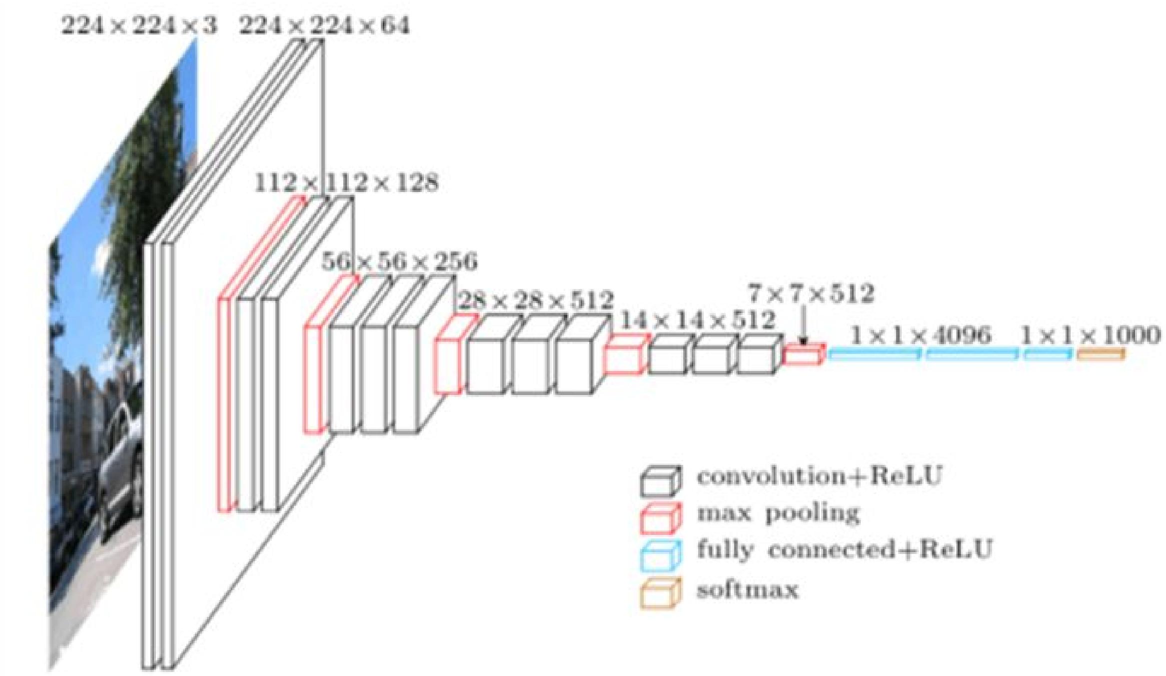


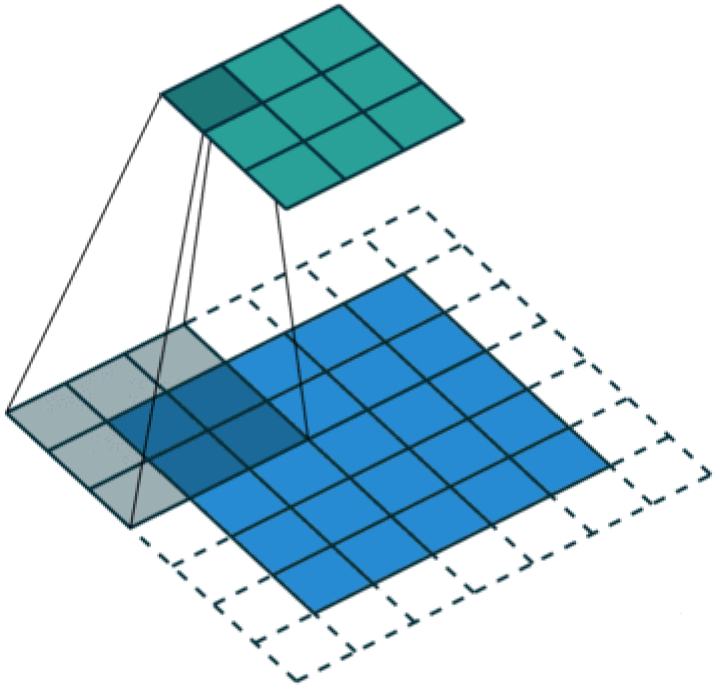



# On the Network Structure

## CNNs

Sharing of connection weights between two layers



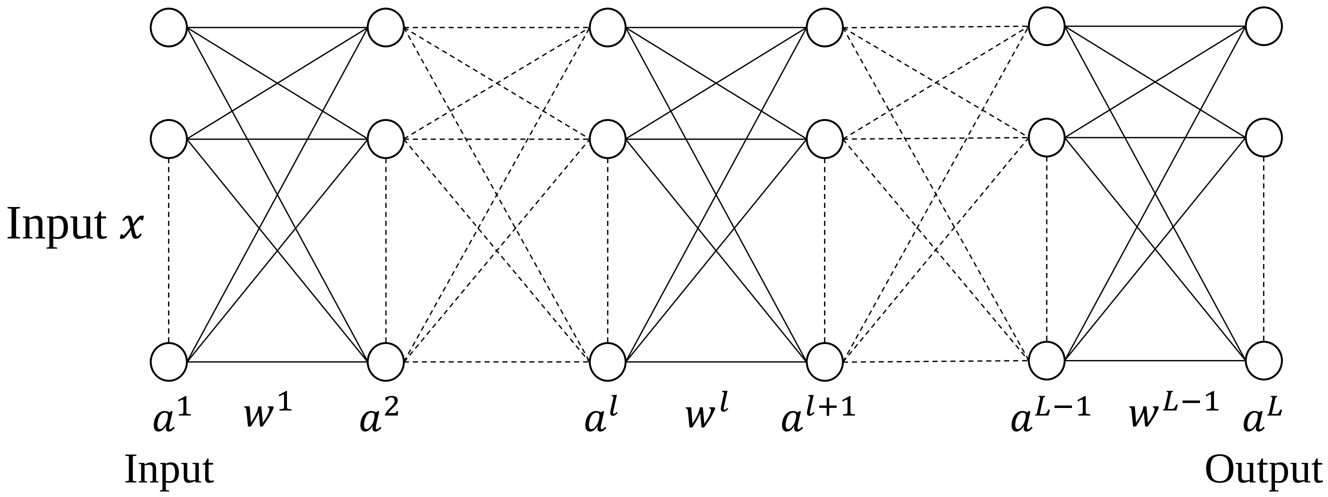
# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Output
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Target Output

Problem: How to define target output?

In principle, it can be defined in any way by users. However, it must fit the meaning of applications. Thus, it is application originated. A target output must correspond to its associated input.




Defined on the last layer
Target Output

$$\underbrace{y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix} \Longleftrightarrow \text{Input } x}_{\text{A training sample } (x,\, y^L)}$$

$$\dim(a^L) = \dim(y^L)$$

# On the Target Output

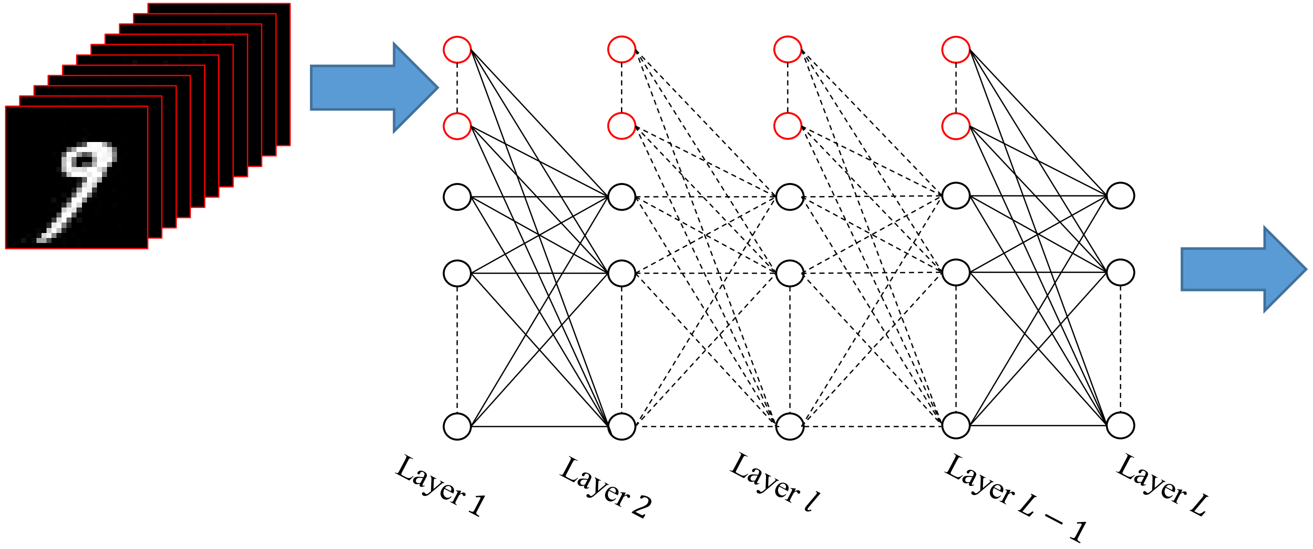
## Classification Problem

The target is to assign each input data sample to its class label. Thus, the target output can be defined by the representation of the label.

**Tip:**
The number of output neurons equals to the number of classes.

Target Output

Representation of the label associated with the input. $\Longleftrightarrow$ $y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$ $\Rightarrow$

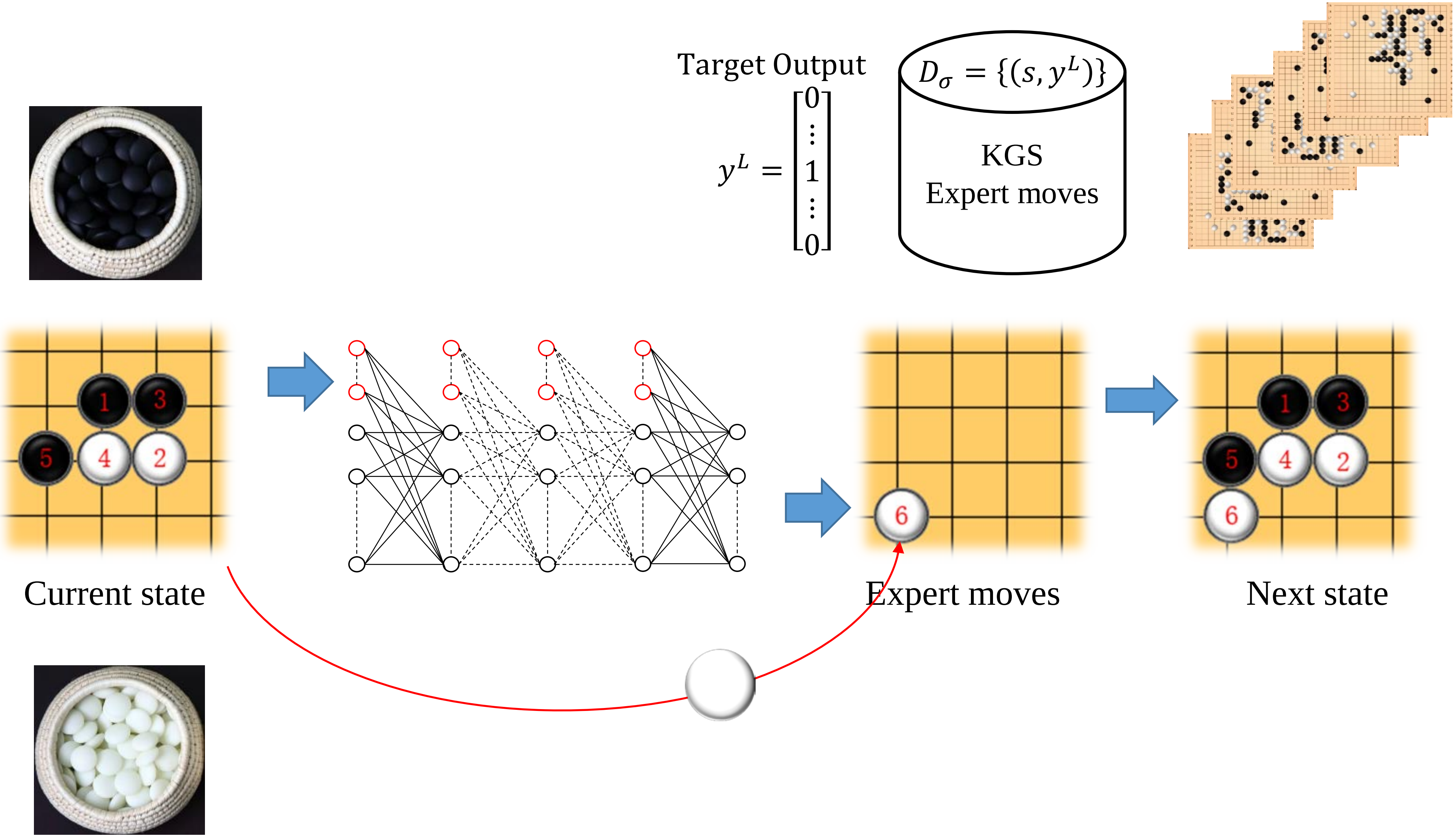



Classes Label

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Representation

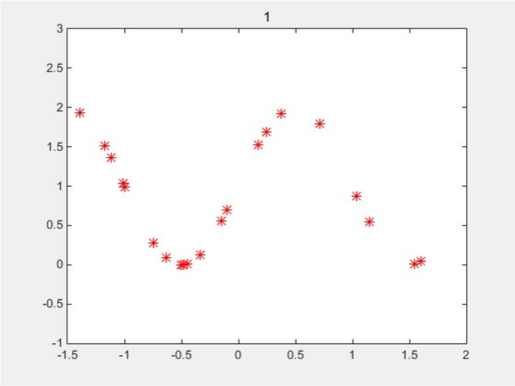
# On the Target Output



# On the Target Output

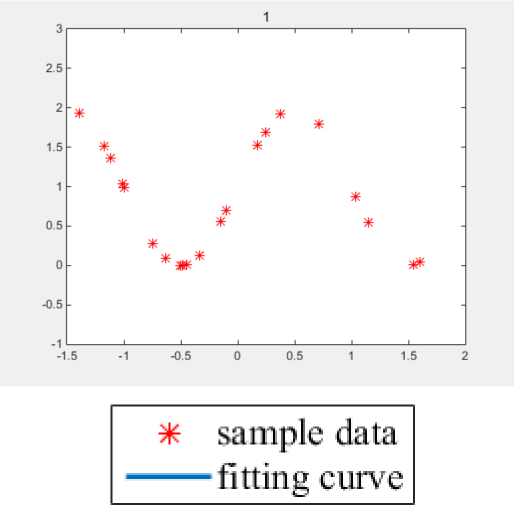
## Curve Fitting Problem

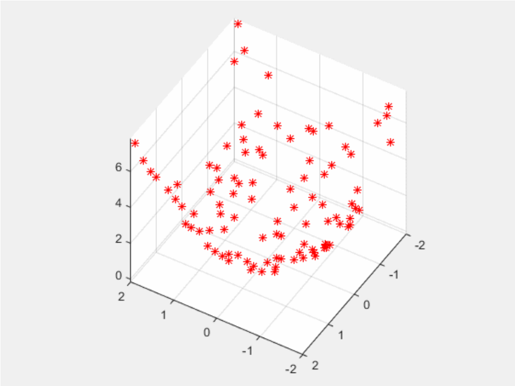
Given a set of sample data, estimates a curve that go through the samples.



Sample data

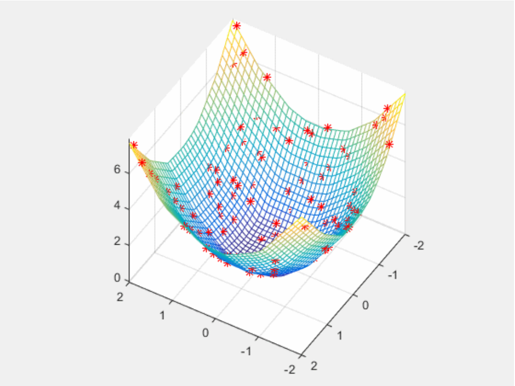
| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $x$ | -0.5000 | 0.1740 | 0.7100 | -0.9980 | -0.6340 | 1.0400 |
| $y$ | 0 | 1.5198 | 1.7902 | 0.9937 | 0.0873 | 0.8747 |



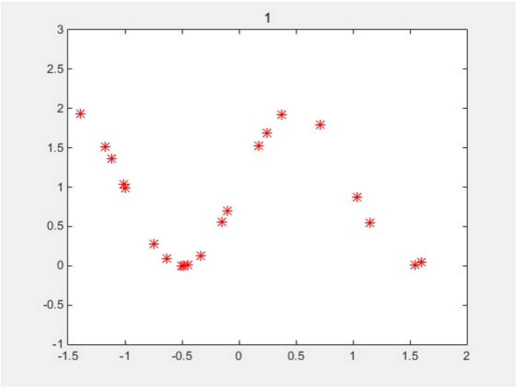



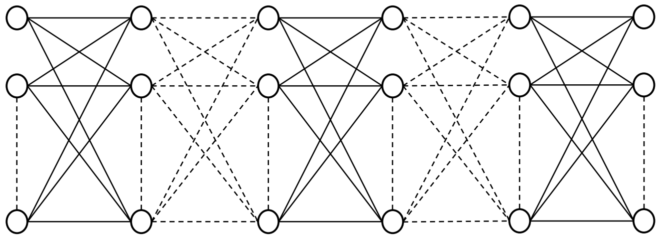
Sample data

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $x$ | -0.2000 | -1.9000 | 1.9000 | 0.4000 | -1.9000 | 0.8000 |
| | 1.4000 | -1.9000 | -1.5000 | -0.5000 | 0.3000 | -0.1000 |
| $y$ | 2.0000 | 7.2200 | 5.8600 | 0.4100 | 3.7000 | 0.6500 |

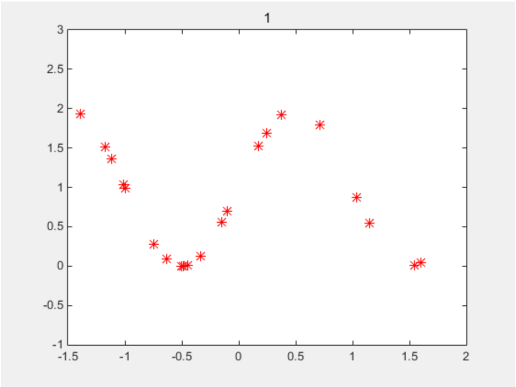


# On the Target Output

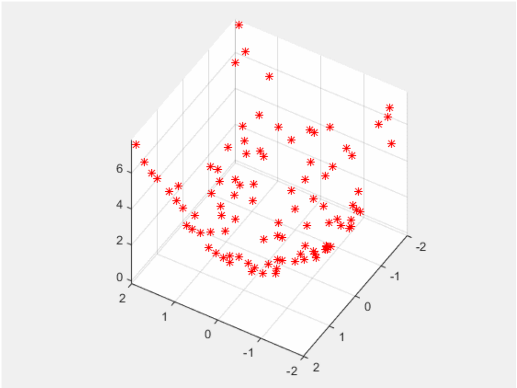


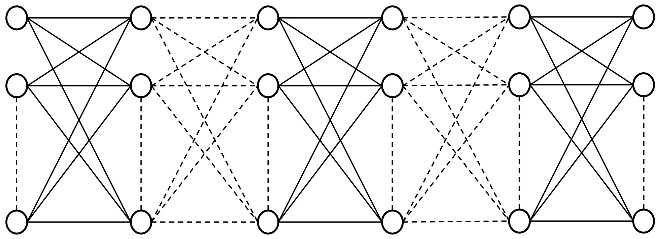



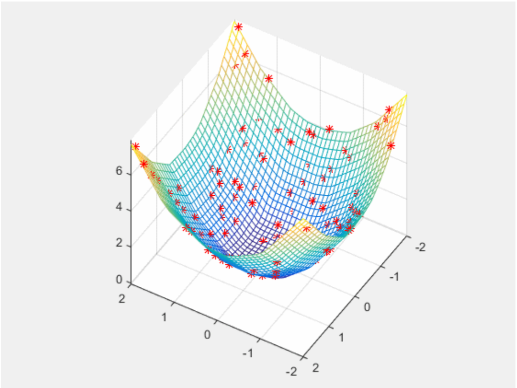
Training sample$(x, y)$




The target output is the value of $y$ corresponding to $x$ of each sample.





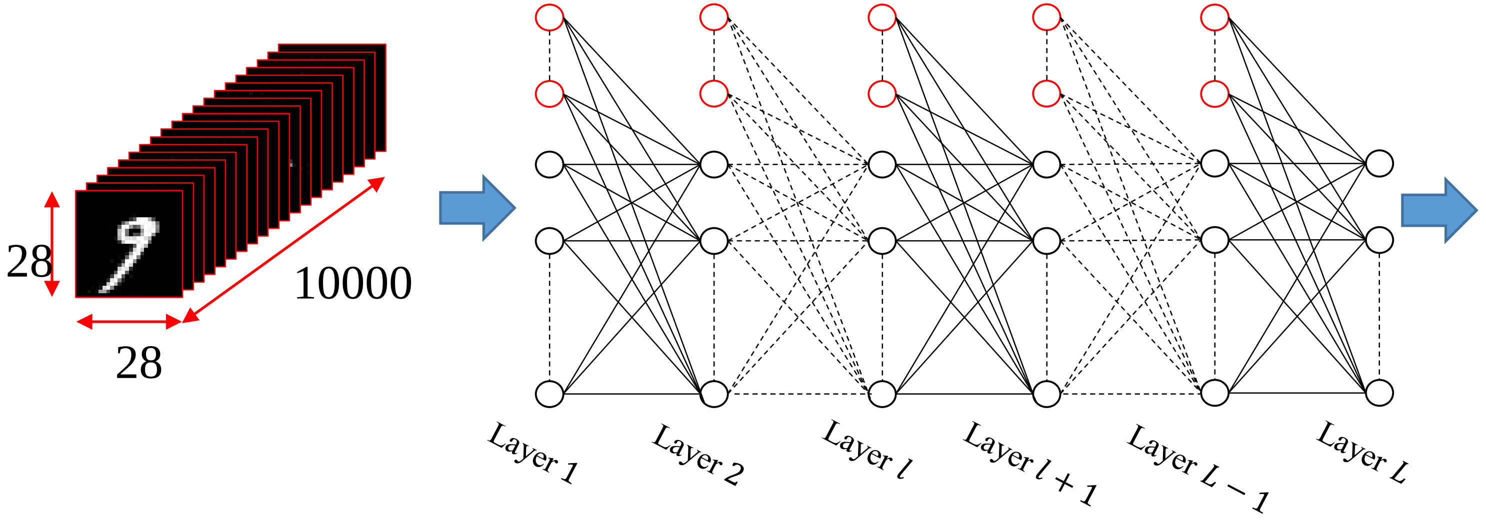
Target Output

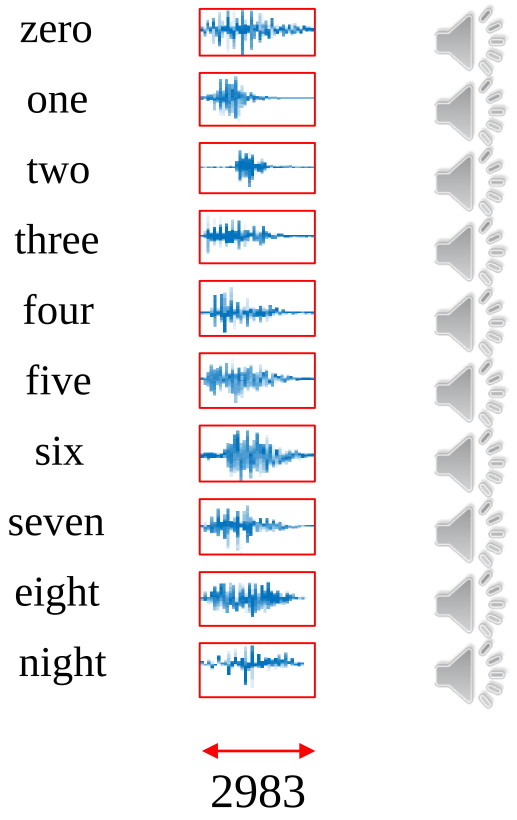
$$y^L = y$$

# On the Target Output

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ y_2^L \\ \vdots \\ y_{2983}^L \end{bmatrix}$$

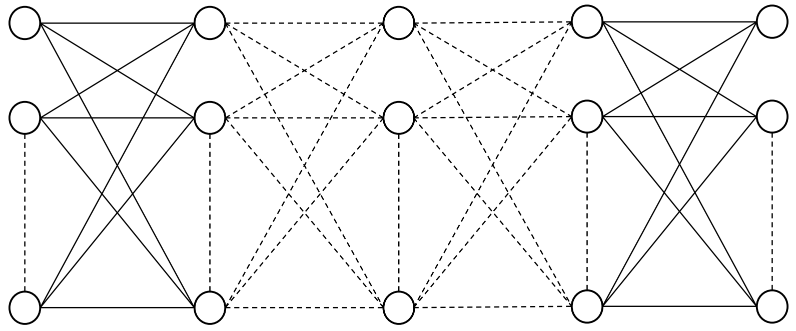


# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Network Prediction




Network Prediction

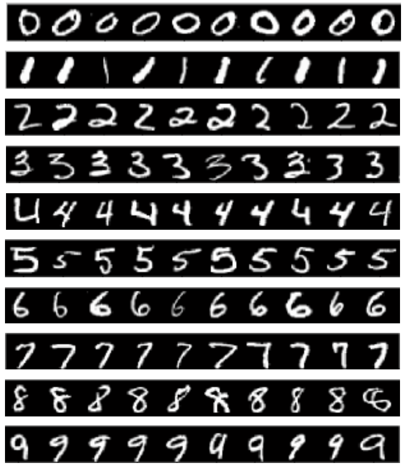
$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Define the last layer activation function $f^L$ so that the network output $a^L$ can match the target output $y^L$. Note that $f^L$ should be differentiable.

$$a_i^L = f_i^L(z_i^L)$$

Target

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

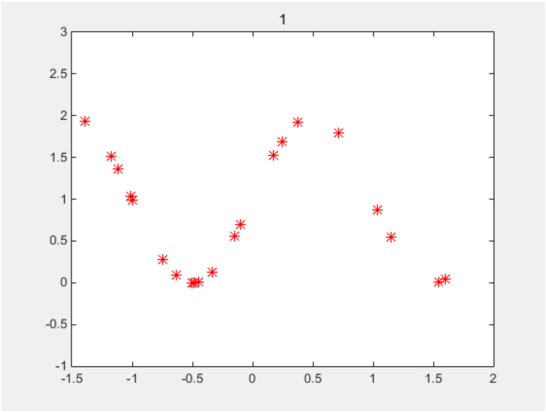


Sigmoid function

$$f(s) = \frac{1}{1 + e^{-s}} \in (0,1)$$

$$\begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix} \quad \text{Threshold } \theta \quad \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

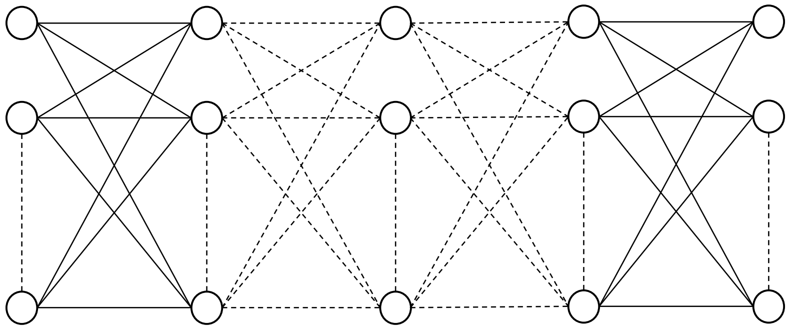
$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$




Linear function

$$f(s) = s$$

# On the Network Prediction



$a^1$ $w^1$ $a^2$ $w^l$ $a^l$ $a^{L-1}$ $w^{L-1}$ $a^L$

Input Output

Target

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

$$0 \le y_i^L \le 1$$

$$\sum_{i=1}^{n_L} y_i^L = 1$$

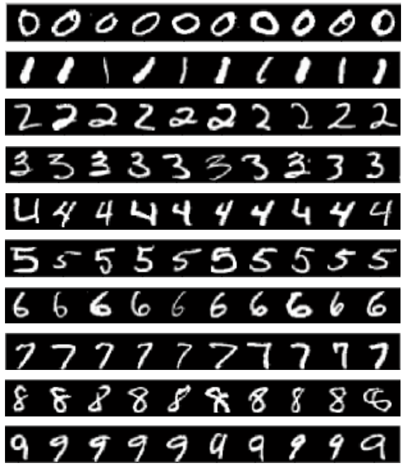
$$a_i^L = \frac{e^{z_i^L}}{e^{z_1^L} + \cdots + e^{z_{n_L}^L}}$$

Softmax function

Network Prediction

$$0 < a_i^L < 1$$

$$\sum_{i=1}^{n_L} a_i^L = 1$$



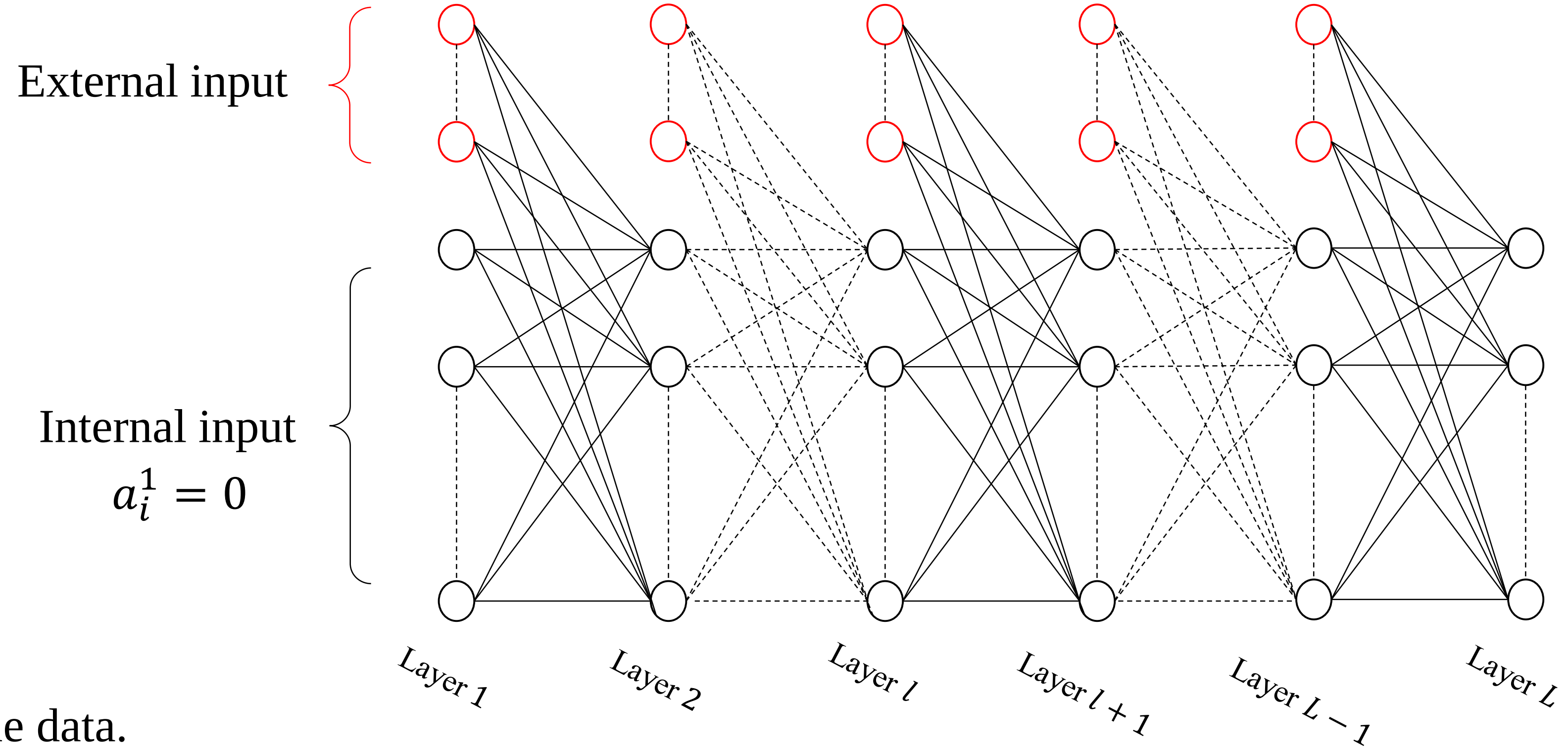
$$\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\1\end{bmatrix} & \begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\1\\0\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\1\\0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\0\\1\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\1\\0\end{bmatrix} \end{array}$$

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix} \qquad 0 \le y_i^L \le 1, \sum_{i=1}^{n_L} y_i^L = 1$$

# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Network Input




**External input:**

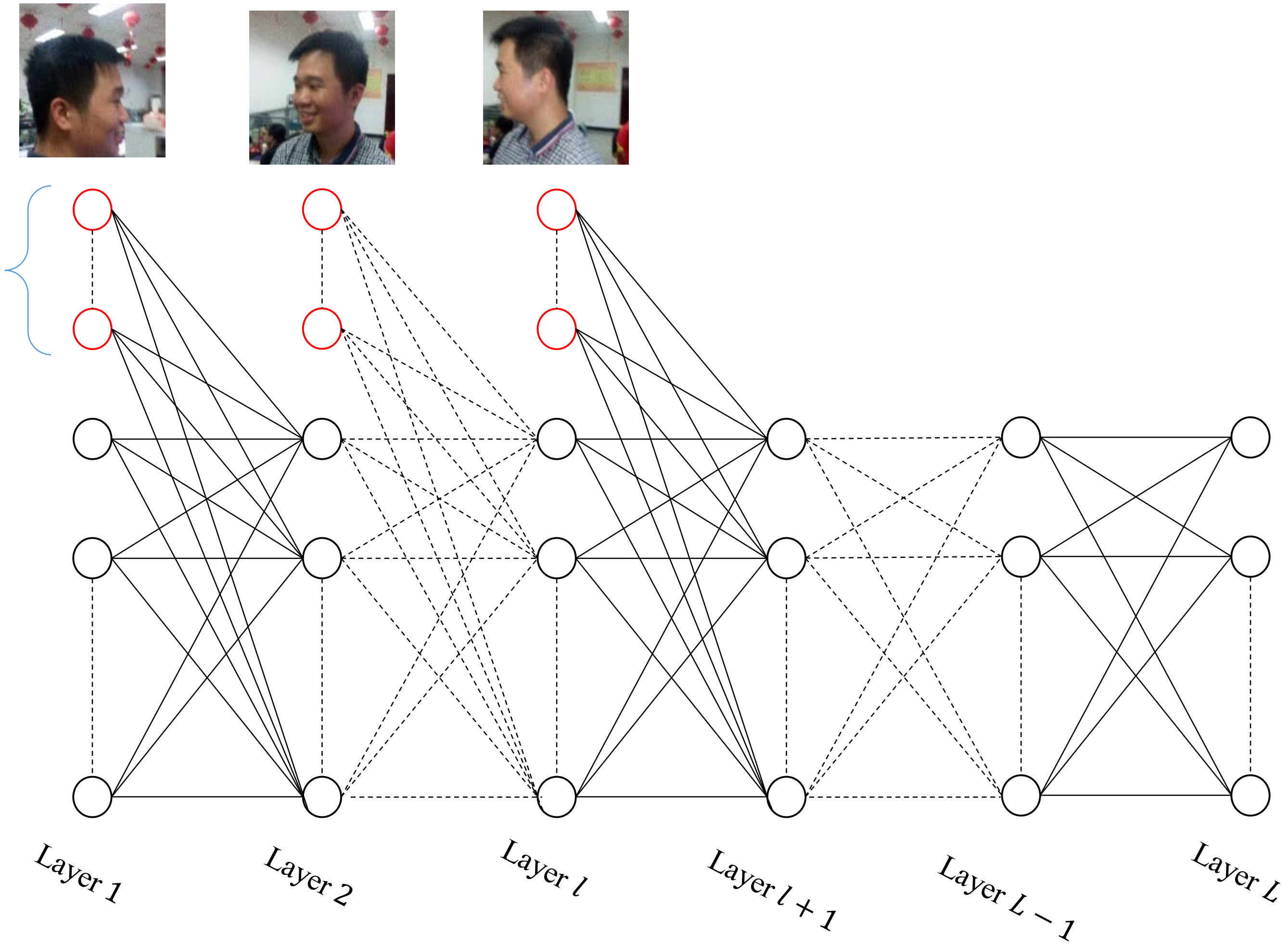
- Directly from sample data.

**Internal input:**

- Generated by former layer
- Maintain a working memory for the neural network
- The first layer internal input is generated by user

# On the Input

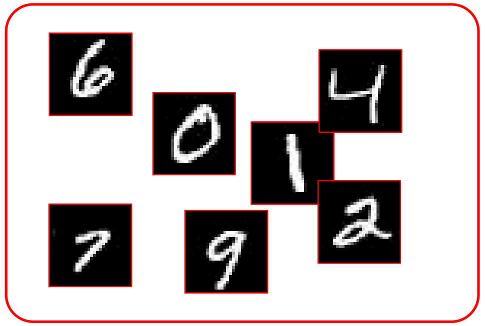
Sequence Input

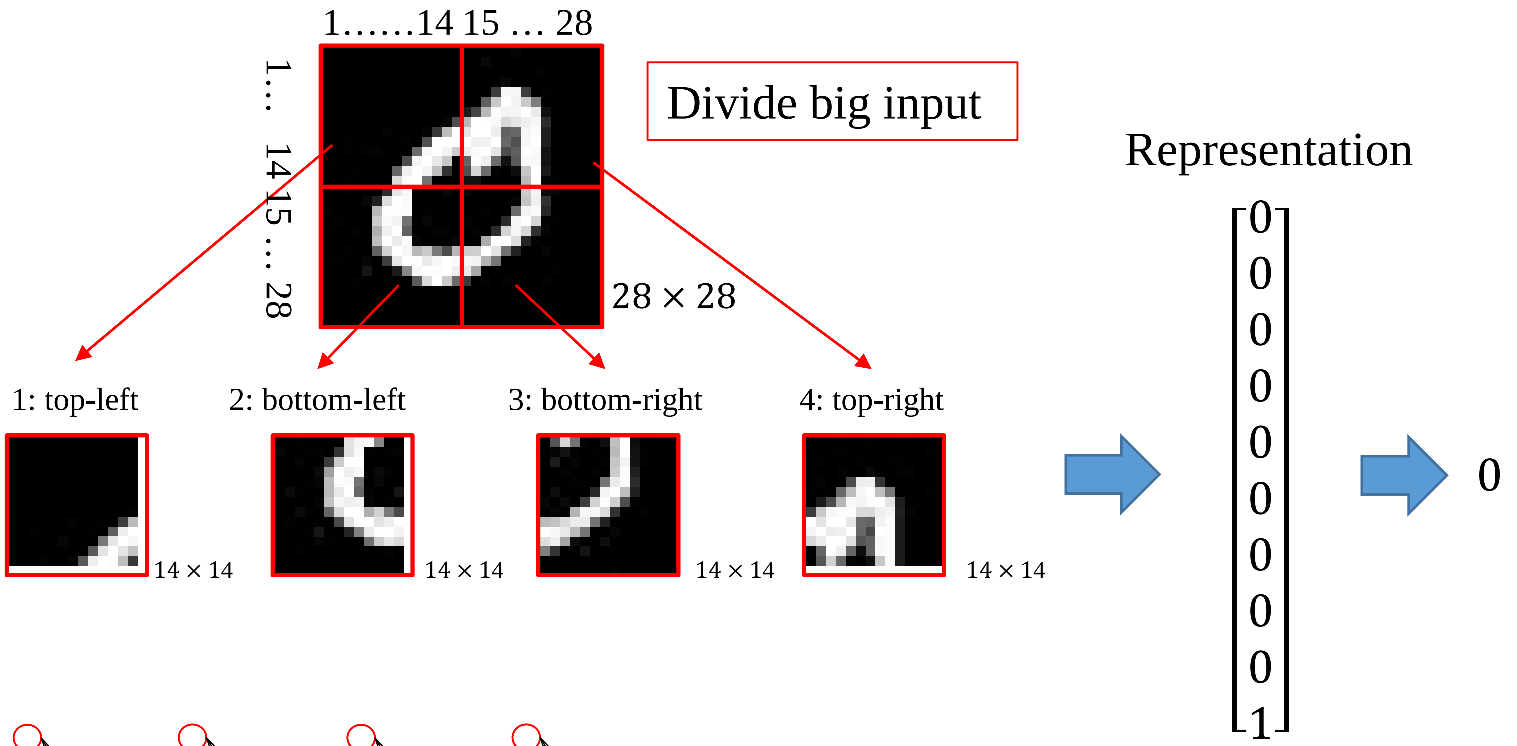


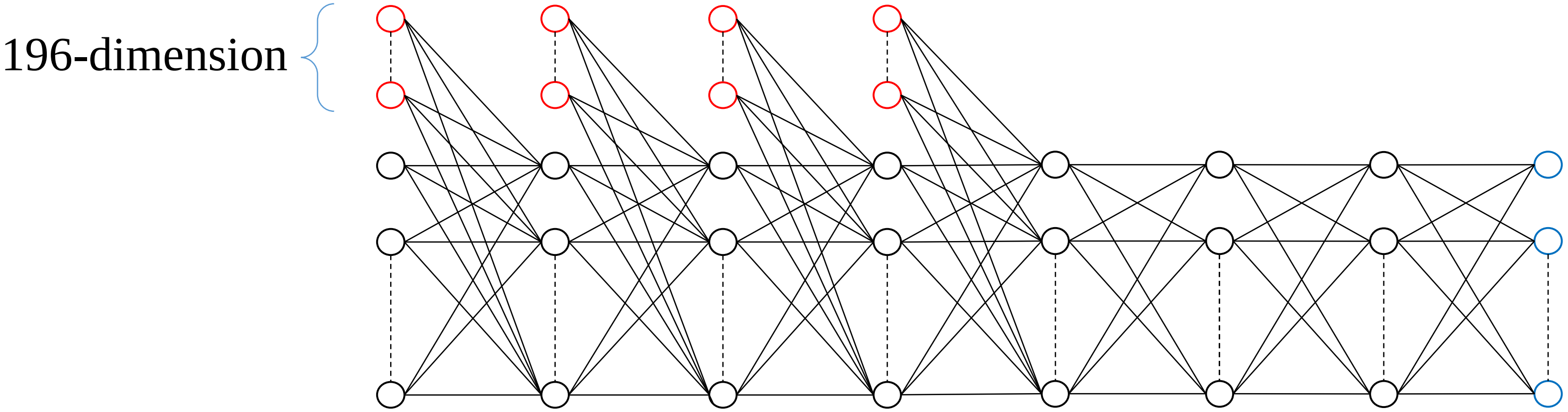

Recognize
the identity

# On the Input

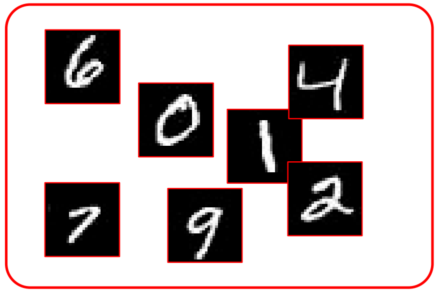
If the dimension of the input data  is too large, it can be divided into small ones.

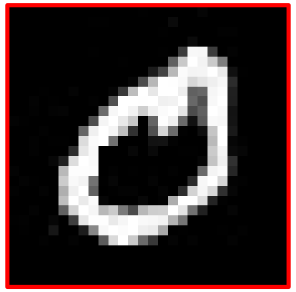




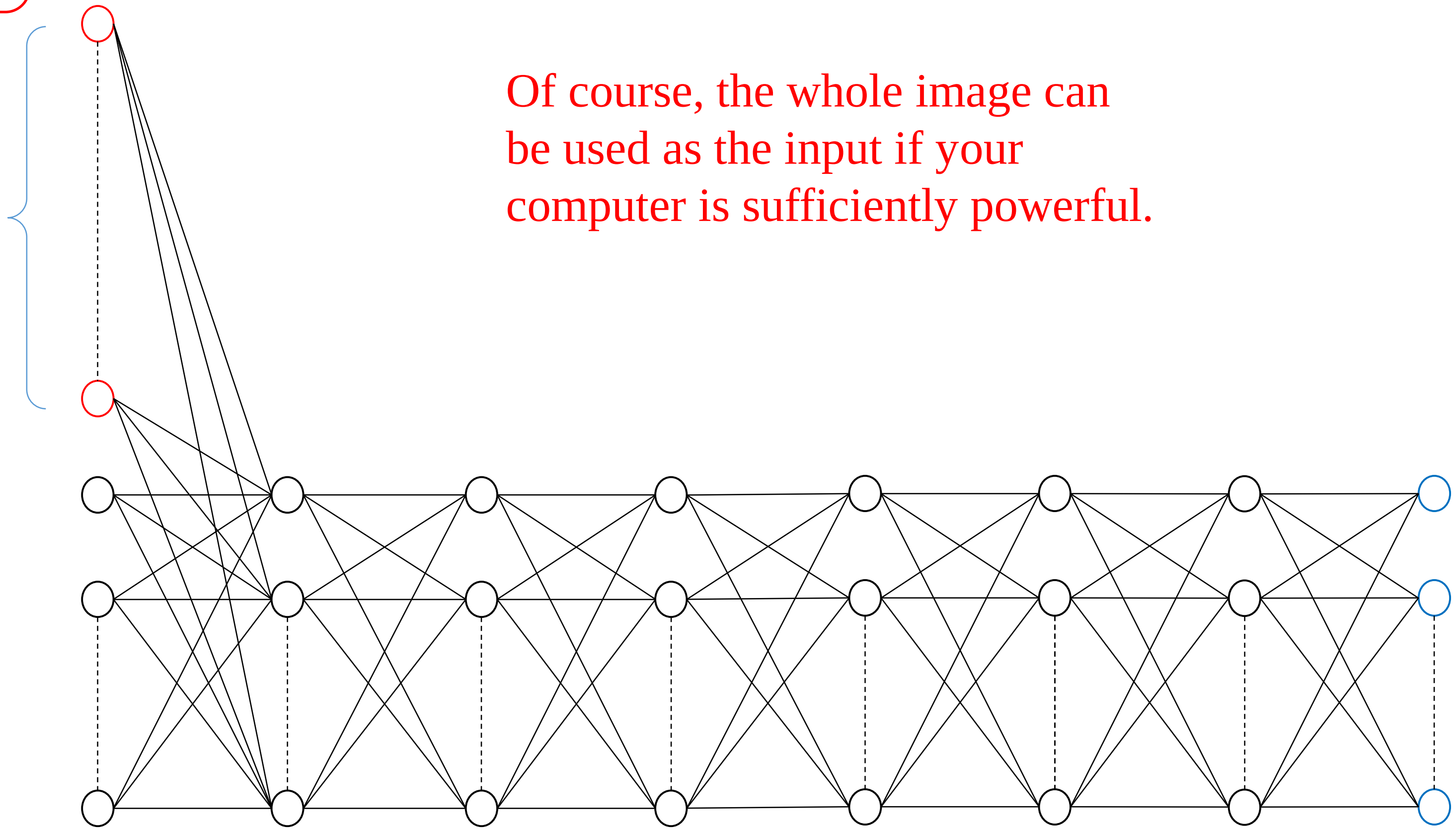



# On the Input



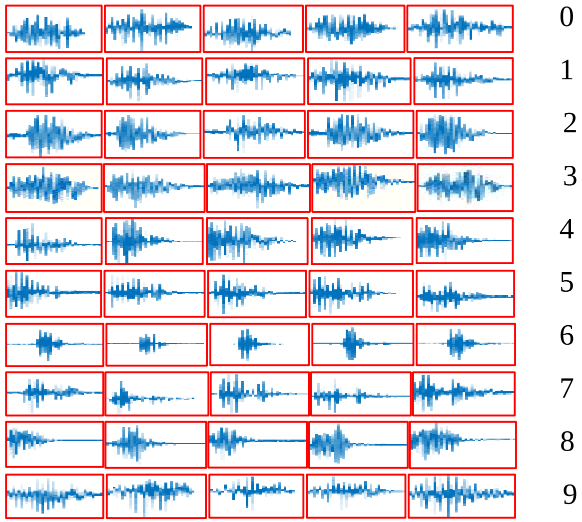


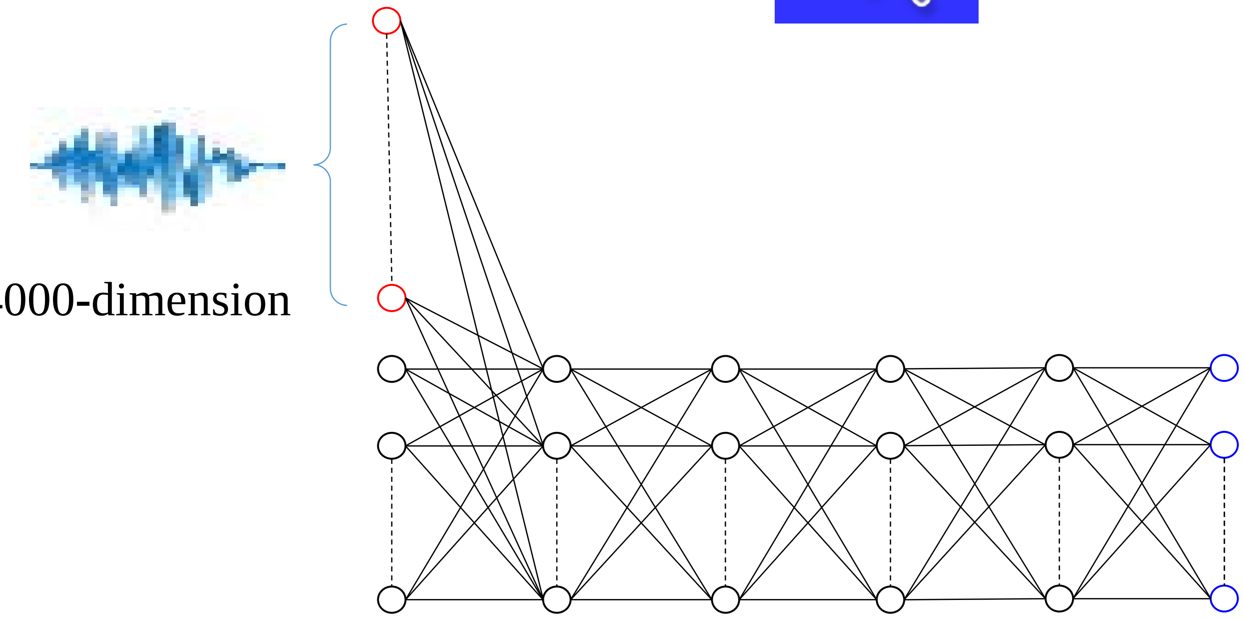
784-dimension



Of course, the whole image can be used as the input if your computer is sufficiently powerful.

# On the input

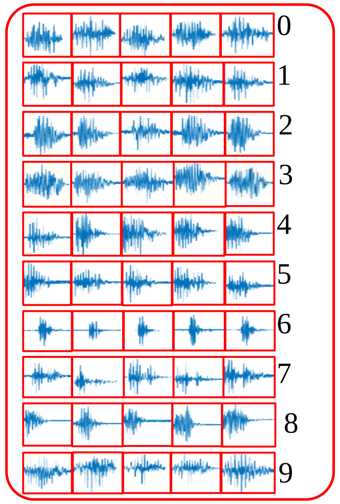


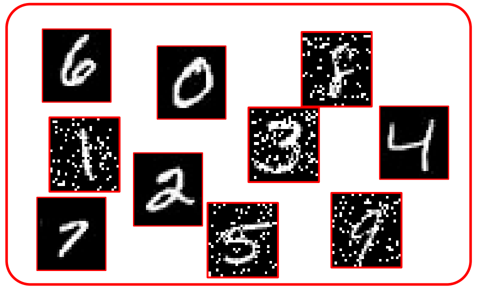



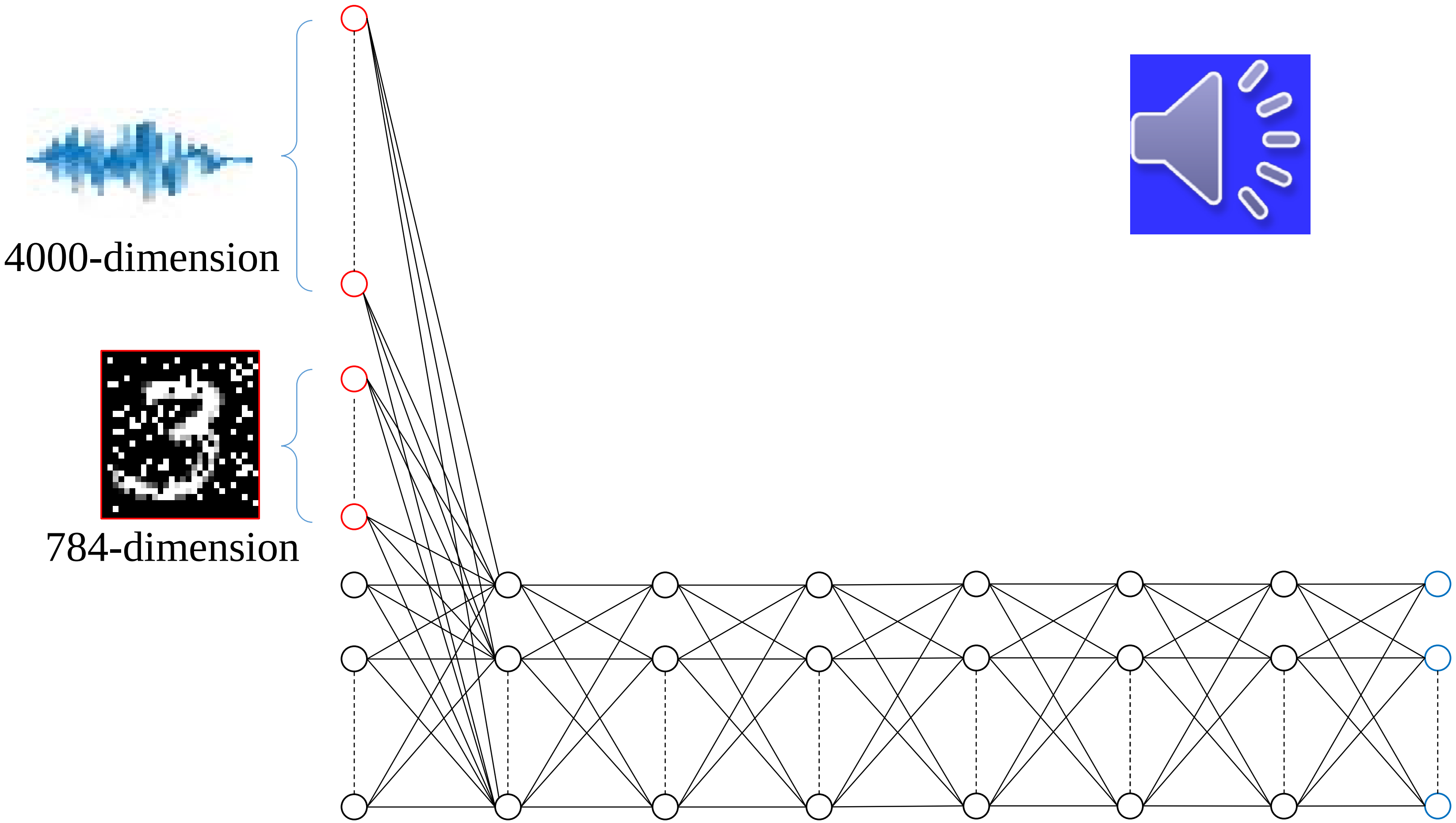

3

$$\begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

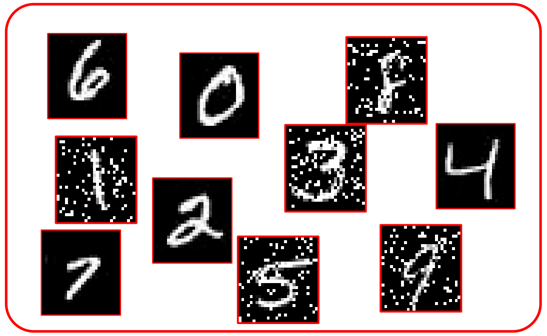
# On the input

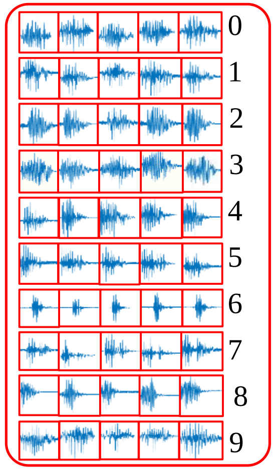


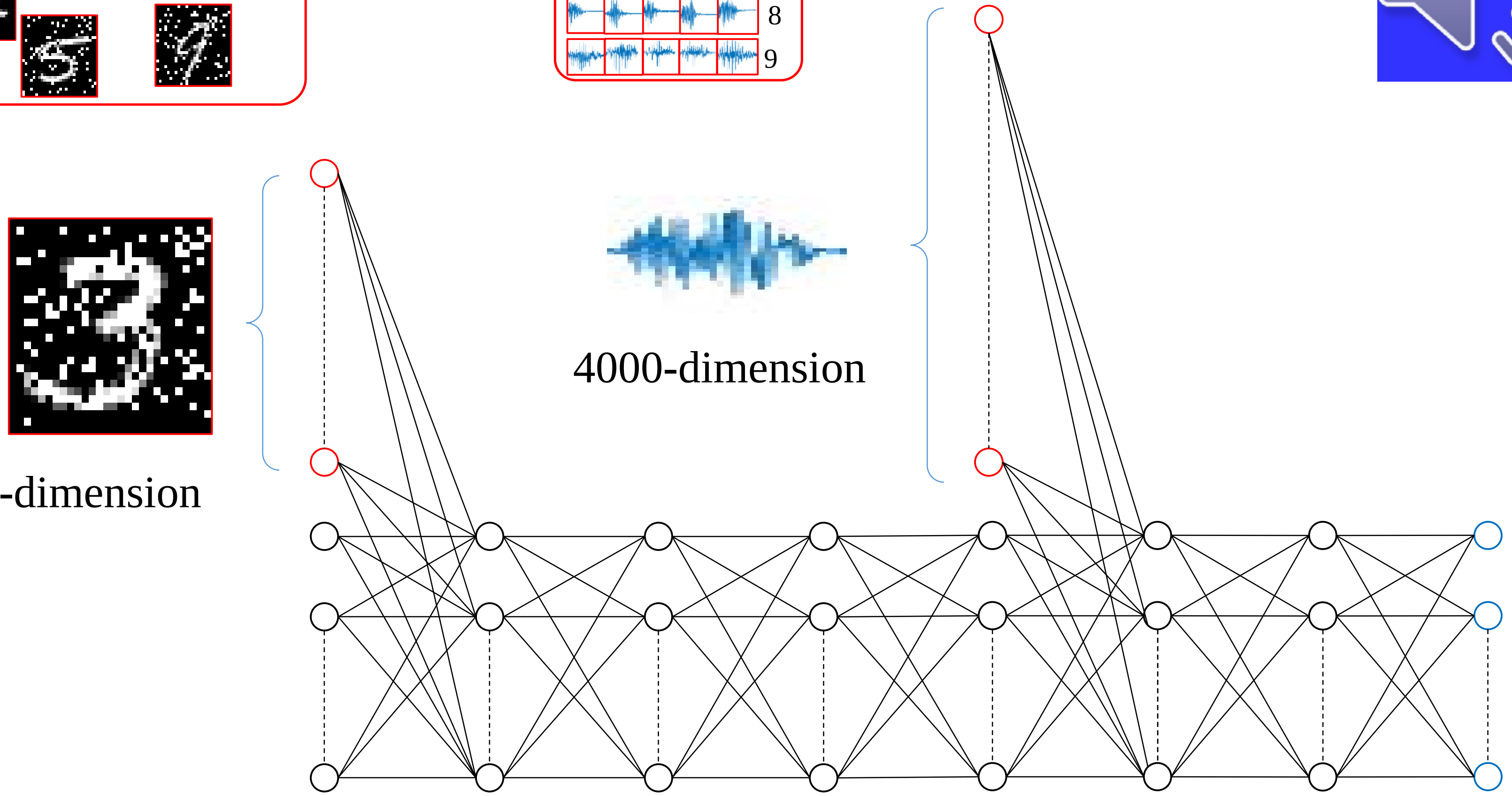





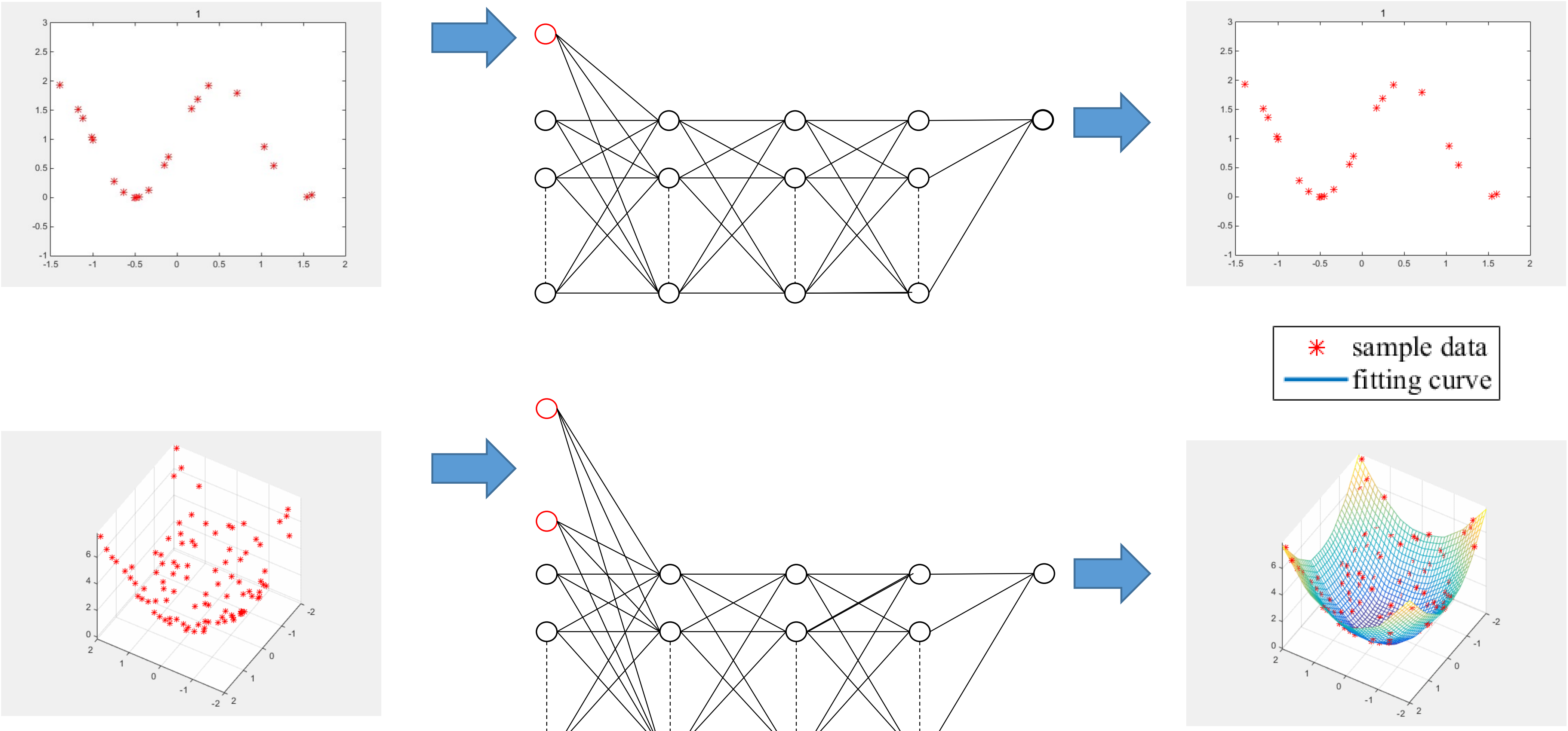
# On the Input





# On the input

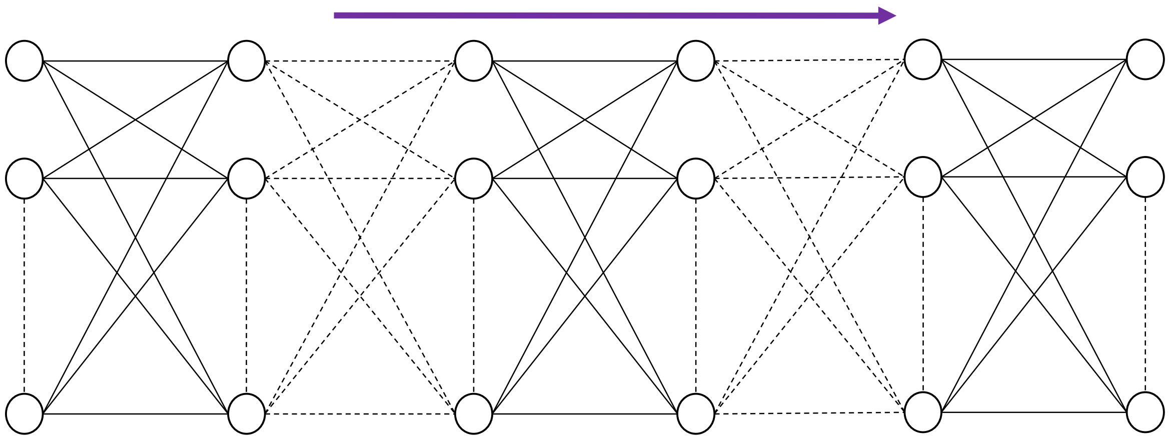


# Outline

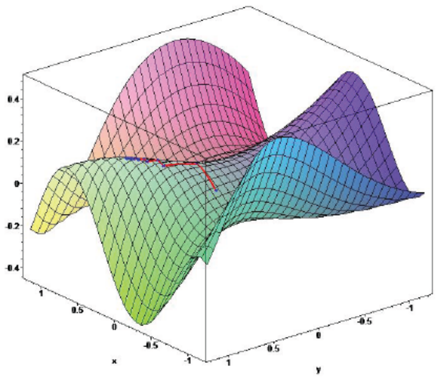
- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Output
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Cost Function

forward computing $a^l$



backpropagation $\delta^l$



**Network Output**

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

**Target Output**

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

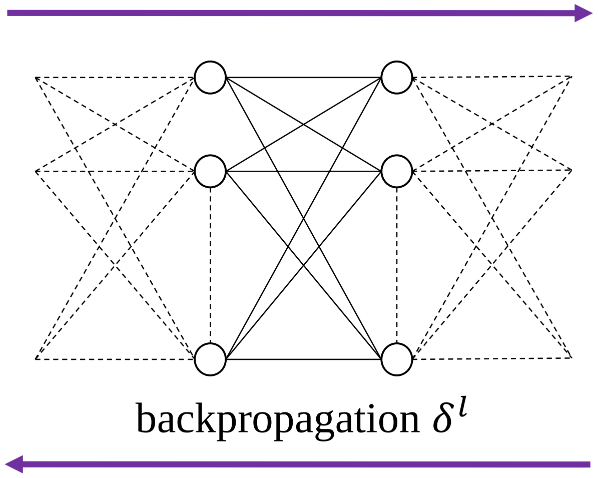
$$J(a^L, y^L)$$

Cost function $J(a^L, y^L)$ is used to describe the closeness between $a^L$ and $y^L$, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

$$J = J(w^1, \cdots, w^{L-1}).$$

# On the Cost Function

forward computing $a^l$



backpropagation $\delta^l$

$$0 \le y_i^L \le 1 \ (i = 1, \cdots, n_L)$$

$$a_i^L = f(z_i^L) = \frac{1}{1 + e^{-z_i^L}}$$

Sigmoid function

Network Output

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

$J(a^L, y^L)$

Cost function $J(a^L, y^L)$ is used to describe the closeness between $a^L$ and $y^L$, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

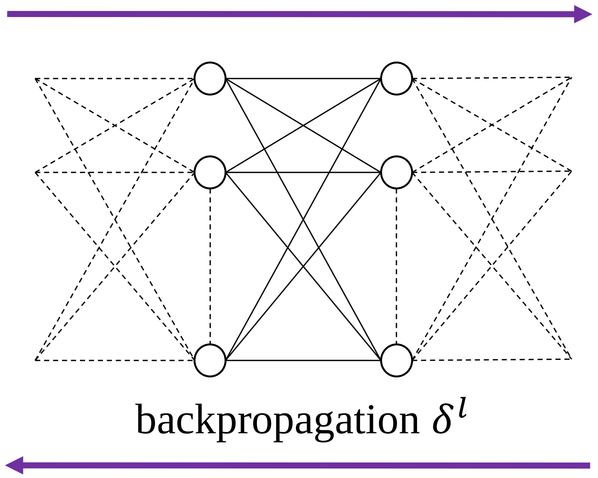
$$J = J(w^1, \cdots, w^{L-1}).$$

Square Error

$$\begin{cases} J = \dfrac{1}{2}\displaystyle\sum_{j=1}^{n^L} (a_j^L - y_j^L)^2 \\ \delta_i^L = \dfrac{\partial J}{\partial z_i^l} = (a_i^L - y_i^L) \cdot \dot{f}(z_i^L) \end{cases}$$

# On the Cost Function

forward computing $a^l$



backpropagation $\delta^l$

**Cross Entropy**

$$J = -\sum_{j=1}^{n_L} y_j^L \cdot \log(a_j^L) + \lambda \cdot \sum (w_{ij}^l)^2$$

$$a_j^L = \frac{e^{z_j^L}}{\sum_{i=1}^{n_L} e^{z_i^L}}$$

$$\delta_i^L = a_i^L - y_i^L$$

$$\sum_{j=1}^{n_L} y_i^L = 1$$

$$a_j^L = \frac{e^{z_j^L}}{e^{z_1^L} + \cdots + e^{z_{n_L}^L}}$$

Softmax function

Network Output

$$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$$

Target Output

$$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$$

$J(a^L, y^L)$

Cost function $J(a^L, y^L)$ is used to describe the closeness between $a^L$ and $y^L$, $J(a^L, y)$ is indeed a function of $(w^1, \cdots, w^{L-1})$, i.e.,

$$J = J(w^1, \cdots, w^{L-1}).$$

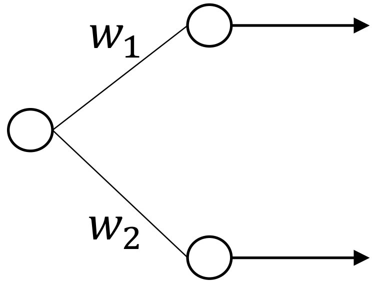
# On the Cost Function

## An example

Sample data

| | 1 | 2 |
|---|---|---|
| $x$ | 0.8000 | 0.2000 |
| $y$ | 0 | 1 |
| | 1 | 0 |

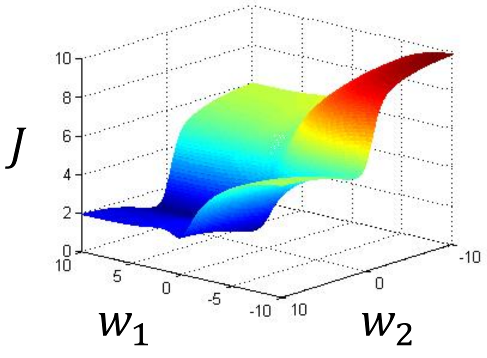
Network

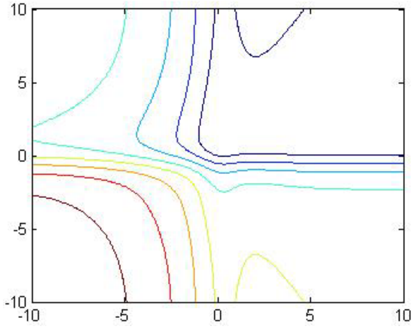



Layer 1 Layer 2

## Square Error

$$\begin{cases} J = \frac{1}{2}\sum_{j=1}^{2}(a_j - y_j)^2 \\ a_j = \frac{1}{1+\exp(-z_j)} \\ z_j = w_j \cdot x \end{cases}$$

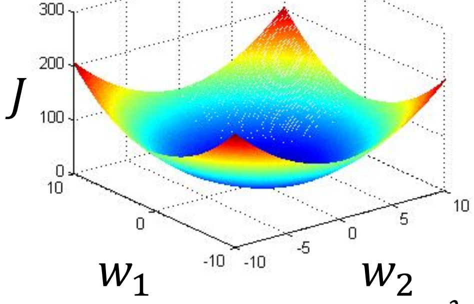


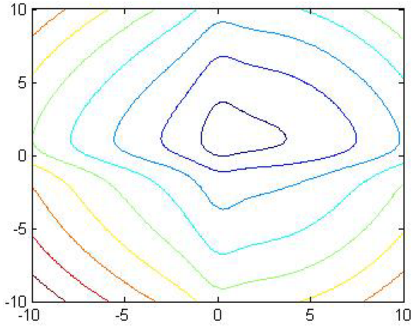



contour

## Cross Entropy

$$\begin{cases} J = -\sum_{j=1}^{2} y_j \cdot \log(a_j) + \lambda(w_1^2 + w_2^2) \\ a_j = \frac{e^{z_j}}{\sum_{i=1}^{2} e^{z_j}} \\ z_j = w_j \cdot x \end{cases}$$



$\lambda = 0.05$

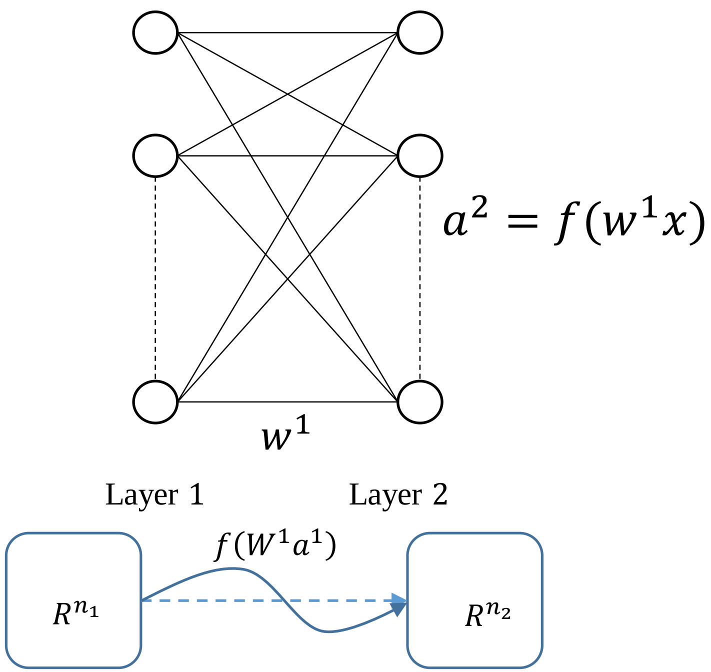
# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Depth of the Networks

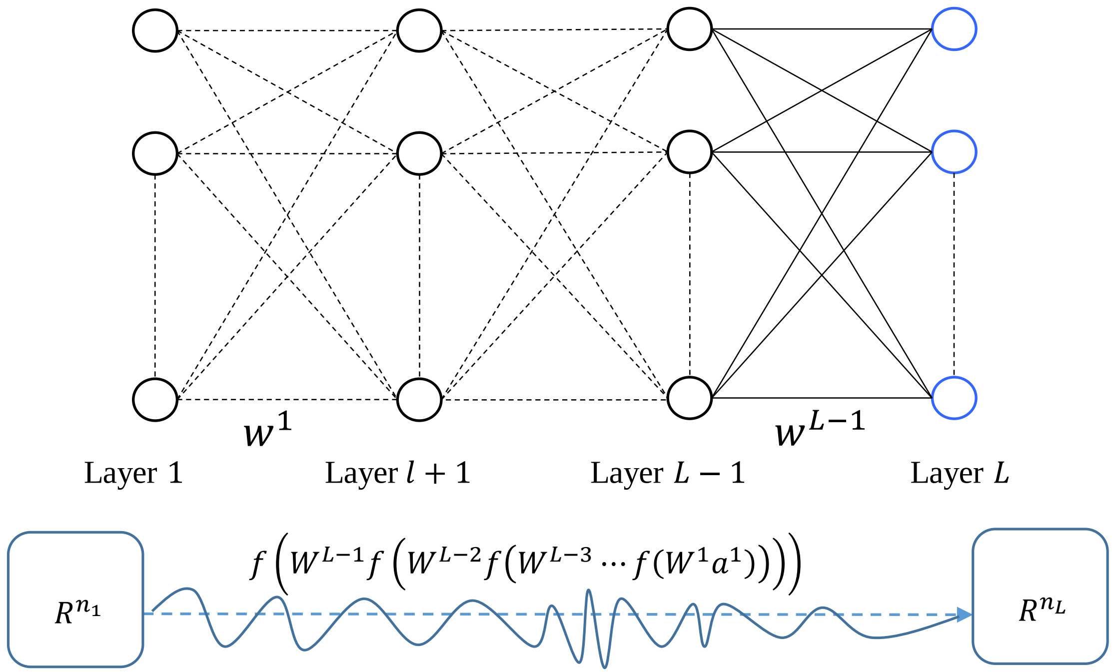
## Shallow neural network

- $L = 2$
- too shallow to learn complex mappings

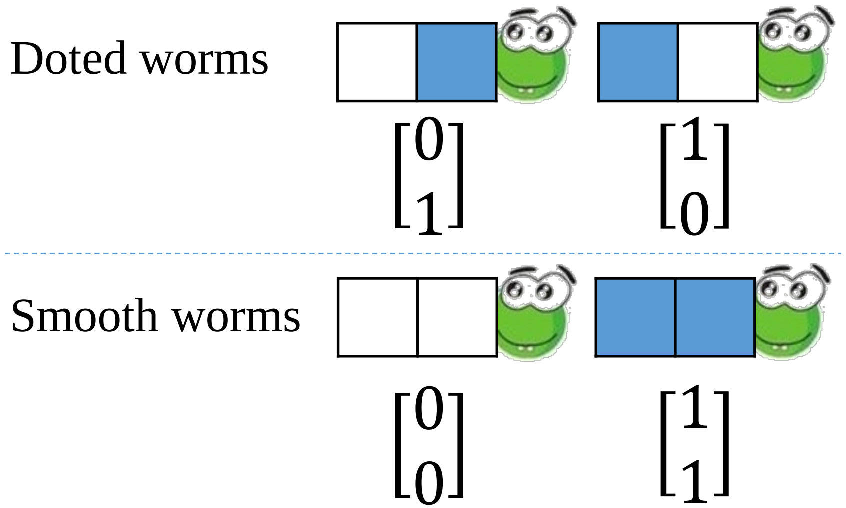


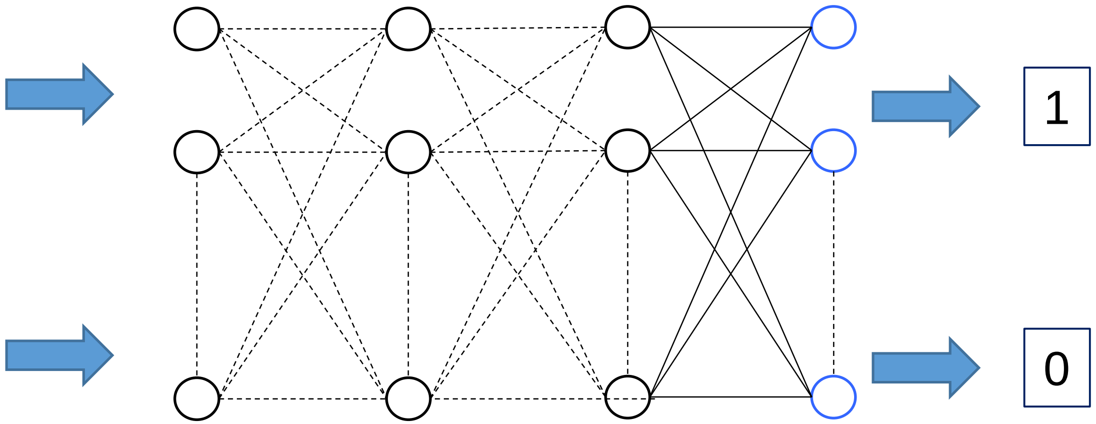

## Deep neural network

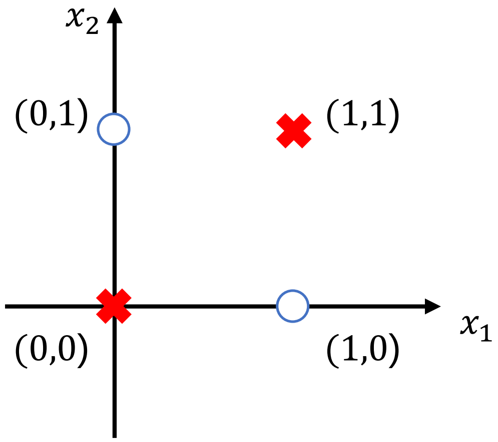
- $L > 2$
- can approximate any nonlinear mappings in any precise provided sufficient neurons in the networks



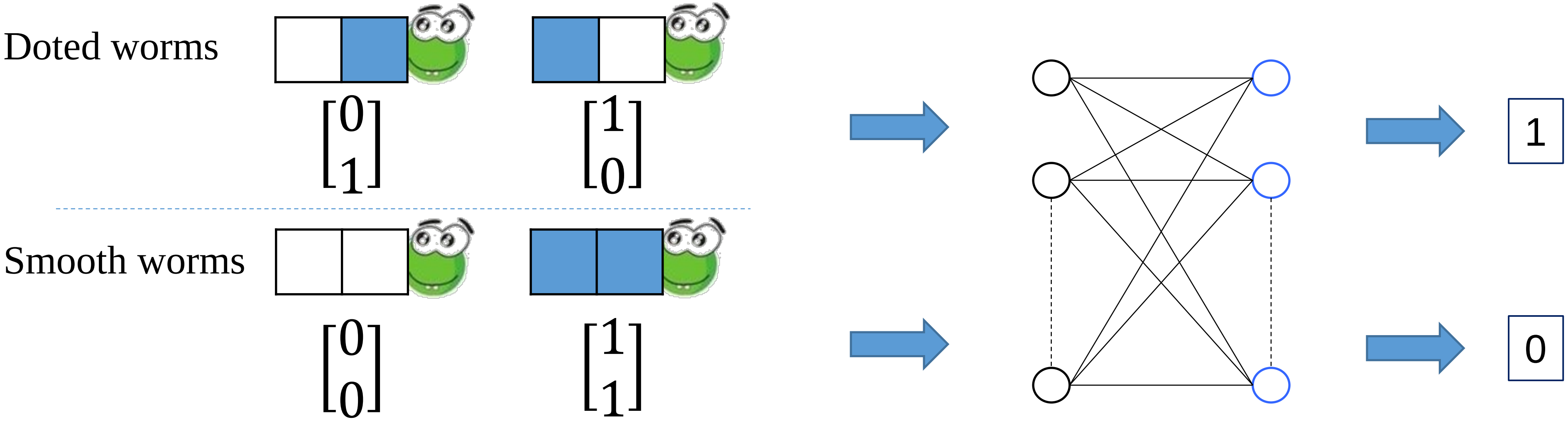
# An example: XOR problem

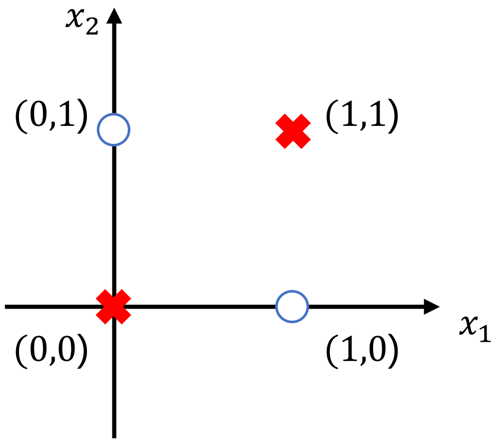


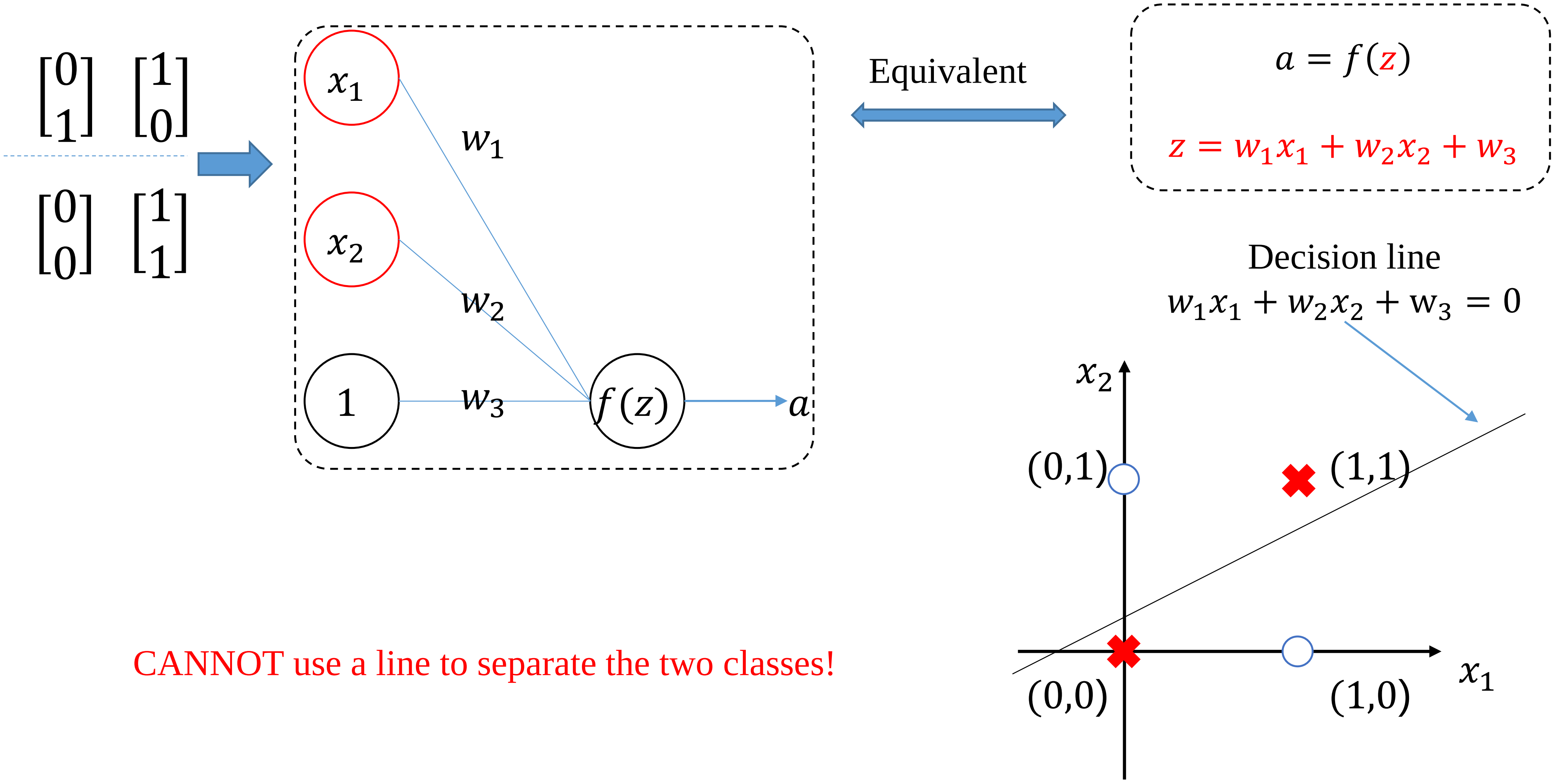
# An example: XOR problem




The classification task CANNOT be completed by using two layers network.

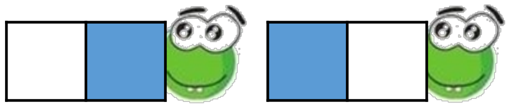
# An example: XOR problem




CANNOT use a line to separate the two classes!

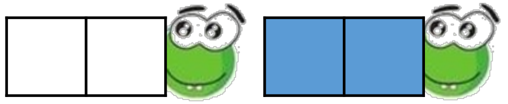
# An example: XOR problem

Doted worms

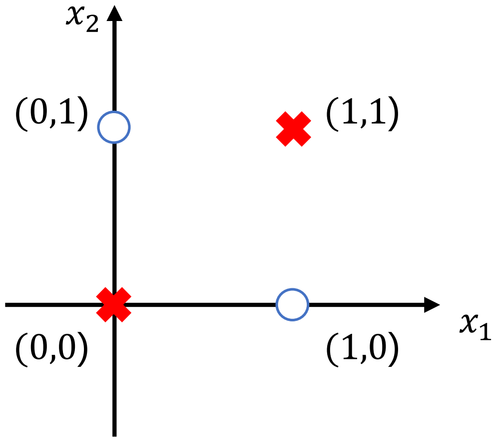


$\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$

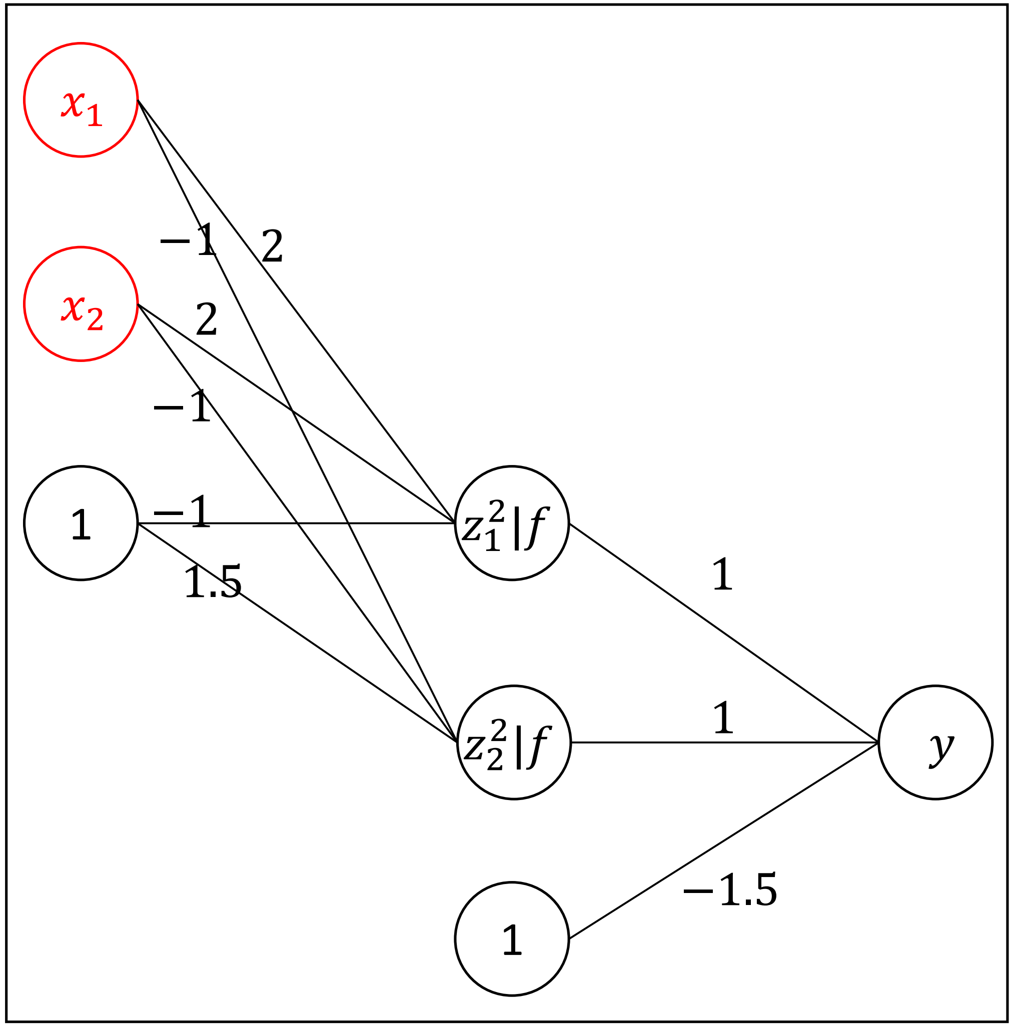
Smooth worms



$\begin{bmatrix}0\\0\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$




At least three layers are required for XOR problem.




1

0

# On the Depth of the Networks

Gradient Vanishing Problem

Cost function: $J(w^1, \cdots, w^{L-1})$

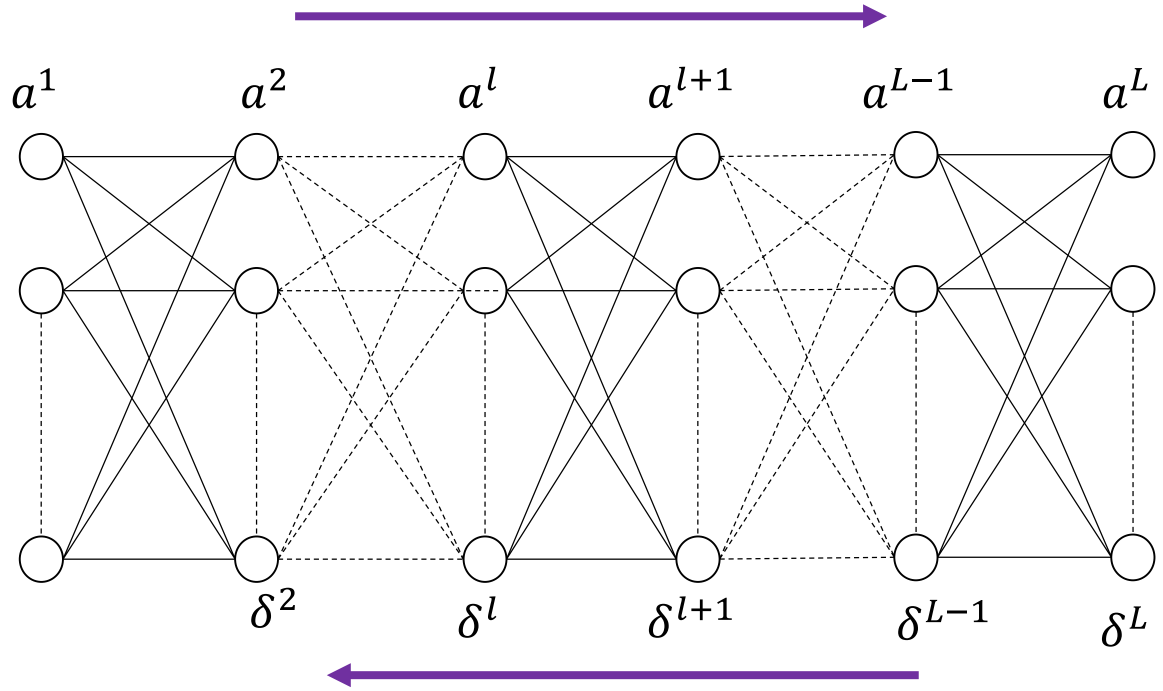
Updating rule: $w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$

Relationship: $\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$

key:

$$\delta_i^l = \dot{f}(z_i^l) \cdot \left( \sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1} \right)$$

Forward computing $a^l$




Backpropagation $\delta^l$

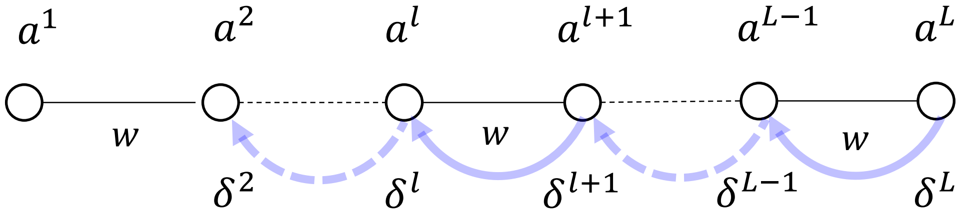
# On the Depth of the Networks

## Gradient Vanishing Problem

a simple example

$$w = w^{\,l}$$

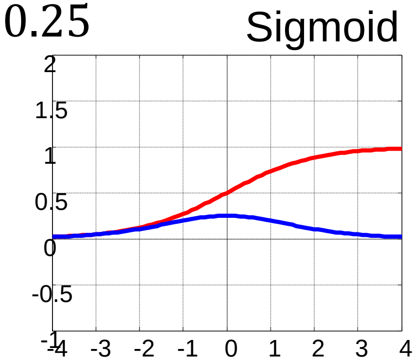
$$\delta^l = \dot{f}\left(z^l\right) \cdot w \cdot \delta^{l+1}$$




$$\begin{aligned}
\delta^l &= \dot{f}\left(z^l\right) \cdot w \cdot \delta^{l+1} \\
&= \dot{f}\left(z^l\right) \cdot w \cdot \dot{f}\left(z^{l+1}\right) \cdot w \cdot \delta^{l+2} \\
&= w \cdot \dot{f}\left(z^l\right) \cdot w \cdot \dot{f}\left(z^{l+1}\right) \cdots w \cdot \dot{f}\left(z^{L-1}\right) \cdot \delta^L \\
&= \prod_{m=L-1}^{l} \left(w \cdot \dot{f}(z^m)\right) \cdot \delta^L
\end{aligned}$$

$$\left|\frac{\partial \delta^l}{\partial \delta^L}\right| = \prod_{m=L-1}^{l} \left|w \cdot \dot{f}(z^m)\right| \leq |w|^{L-l+1} \cdot (0.25)^{L-l+1}$$

$$\dot{f}(z^m) \leq 0.25$$



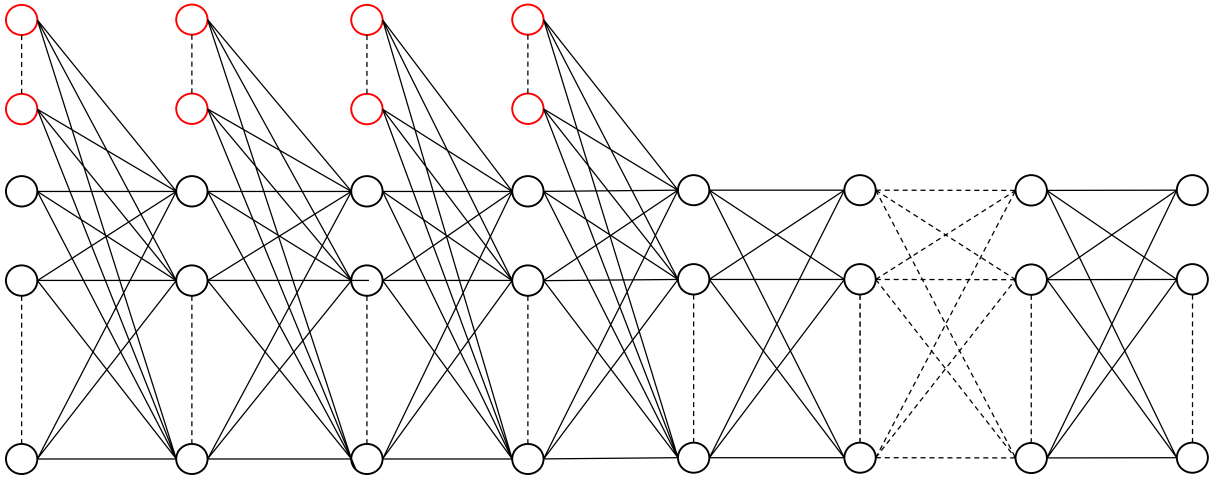

Notes：

The exponential descent of $\delta^l$ causes the gradient vanish problem.

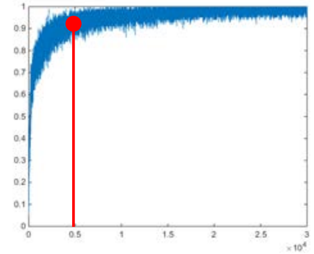
# On the Depth of the Networks

The depth of the network is correlated to the problem.

Handwritten digits recognition problem

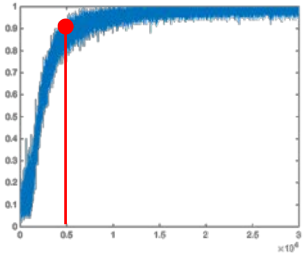


5 layers



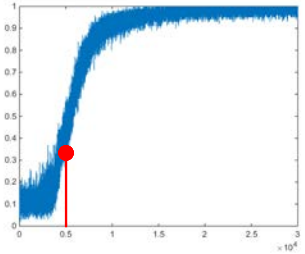
Accuracy
- Training=97.55%
- Testing=95.25%

8 layers



Accuracy
Training=98.65%
Testing=95.10%

9 layers



Accuracy
- Training=98.45%
- Testing=93.20%

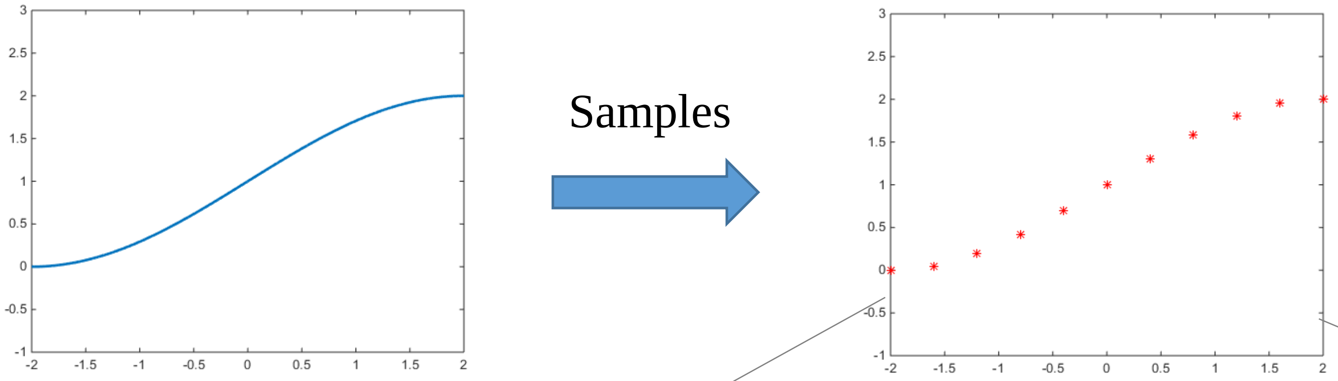
# Outline

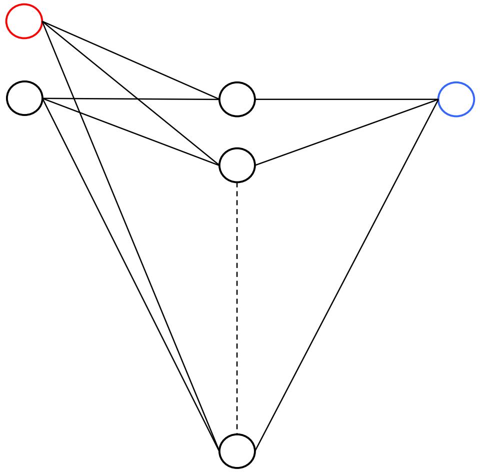
- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Output
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# On the Training Data

Using a 2-9-1 network to fit a partial sin curve

$$y = g(x) = 1 + \sin\left(\frac{\pi}{4}x\right), x \in [-2,2]$$



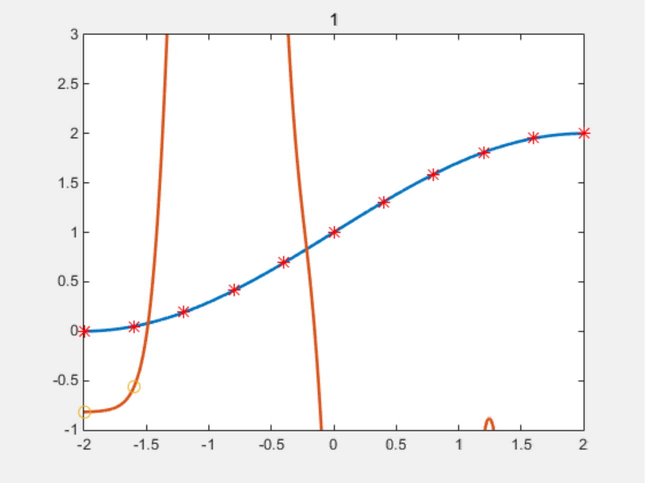
(1,1) ⟶ (0,9) ⟶ (0,1)

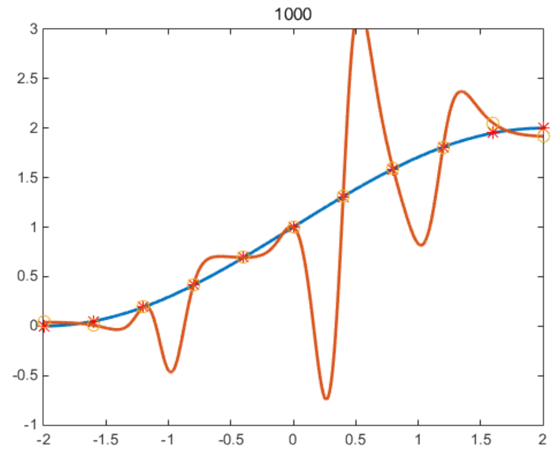
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | -2 | -1.6000 | -1.2000 | -0.8000 | -0.4000 | 0 | 0.4000 | 0.8000 | 1.2000 | 1.6000 | 2 |
| $y$ | 0 | 0.0489 | 0.1910 | 0.4122 | 0.6910 | 1 | 1.3090 | 1.5878 | 1.8090 | 1.9511 | 2 |

11 samples

# On the Training Data

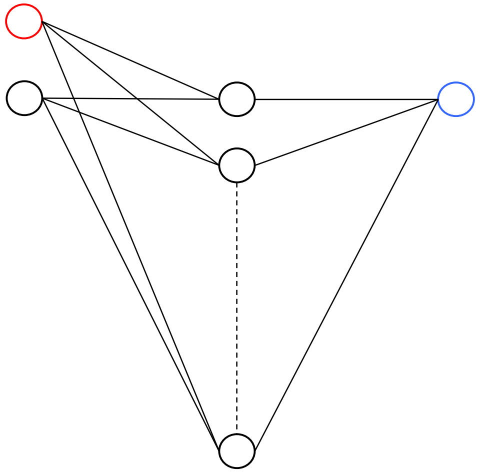
## Overfitting




11 data samples

Using a 2-9-1 network to fit a partial sin curve

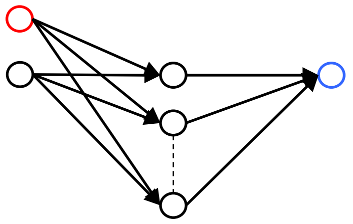


2-9-1 network has 27 weights to be tuned.
In general, we need more samples than the number of unknown parameters in a system.

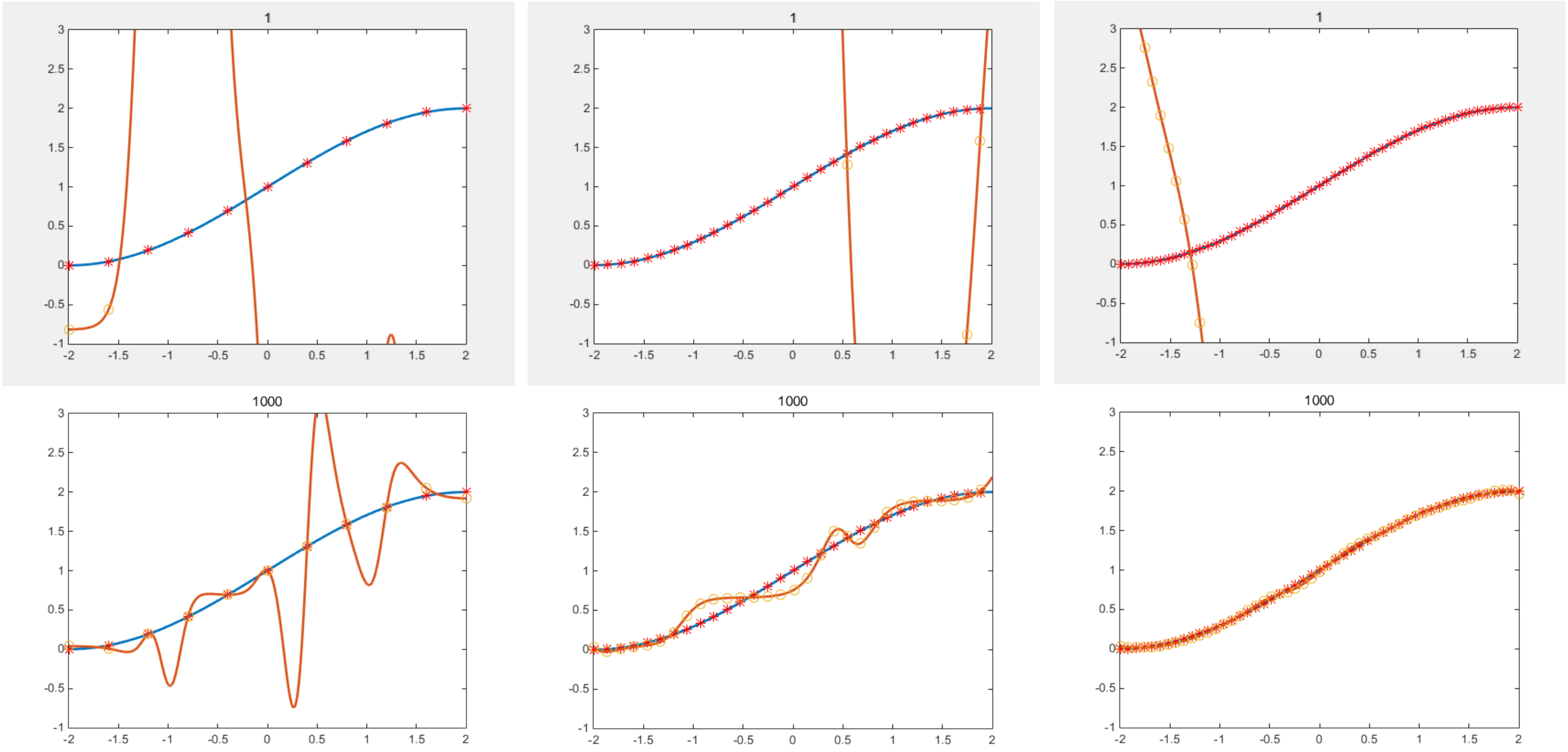
The network fit the data sample properly, but nowhere else on the curve! Overfitting!

- Fit training data well
- Cannot fit testing data
- We need **MORE** data!

# On the Training Data

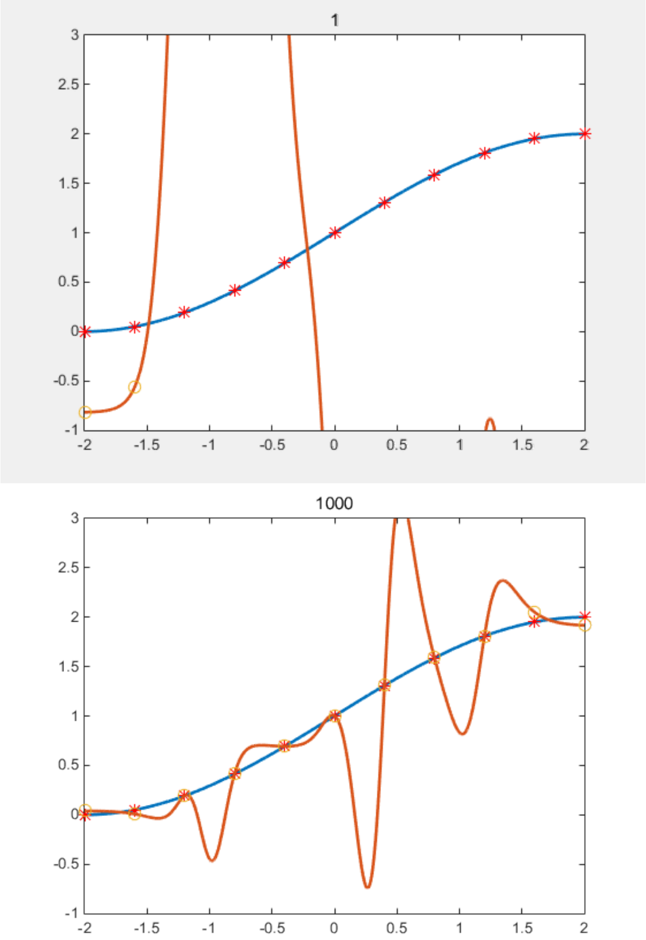


(1,1) → (0,9) → (0,1)

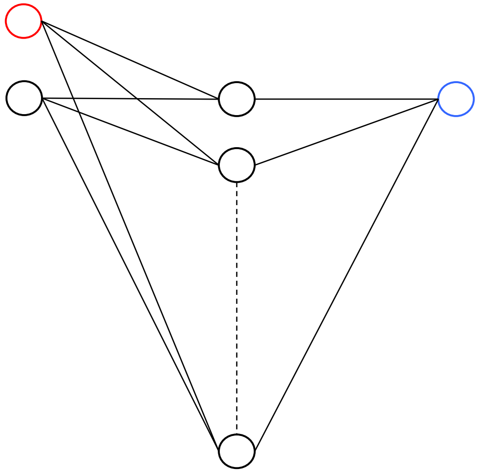



11 data samples | 23 data samples | 51 data samples

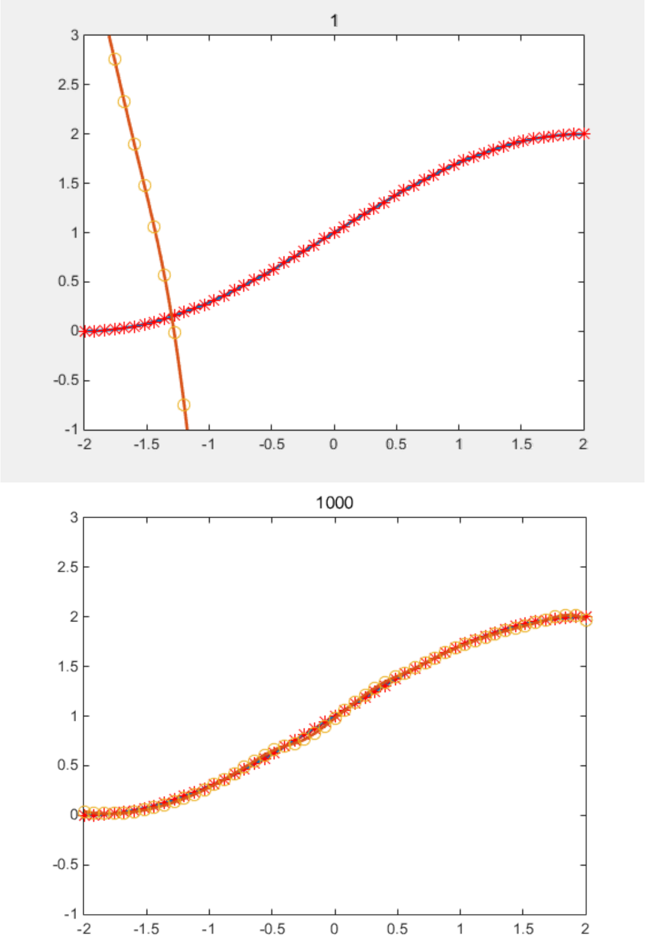
# On the Training Data



11 data samples



For a network to be able to generalize, it should have fewer parameters than there are data points in the training set.



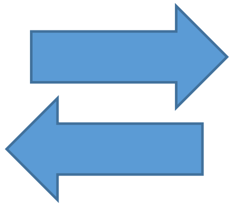
51 data samples

# On the Training Data

## Big data

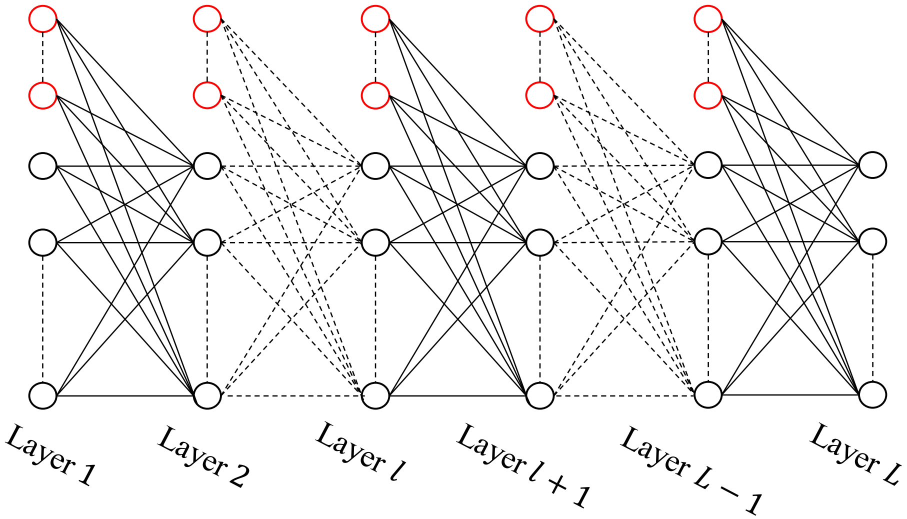

Complex patterns in **big data** need complex model to deal with.

Abundant data sample for training model (samples)



Highly nonlinear, flexible, and trainable model (complexity)

## DNN




Huge number of parameters in **DNN** models need to be determined.

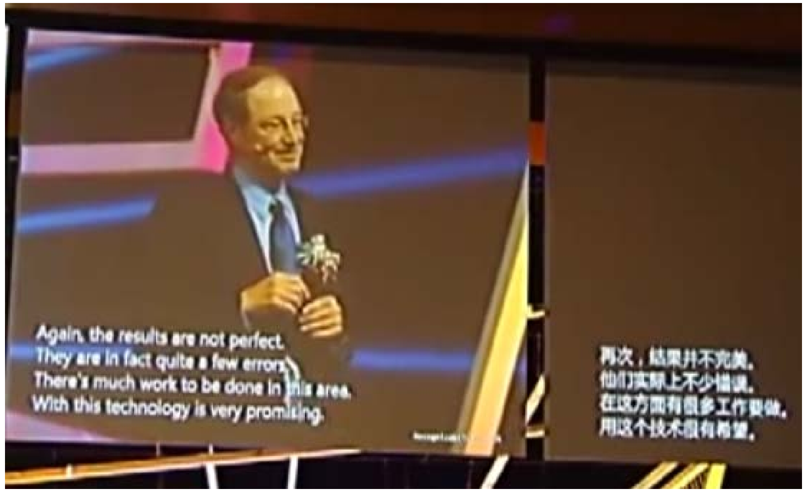
# Big data + DNN Example

**Speech Recognition**

1950s Wave of speech + pattern recognition = few words

1970s Gaussian Mixture Model + Hidden Markov Model = ~80% recognition rate

2011 Deep neural network for modeling speech = awesome real-time recognition!




English speech

English text

Voice Feature

Real-time speech translation

Chinese text

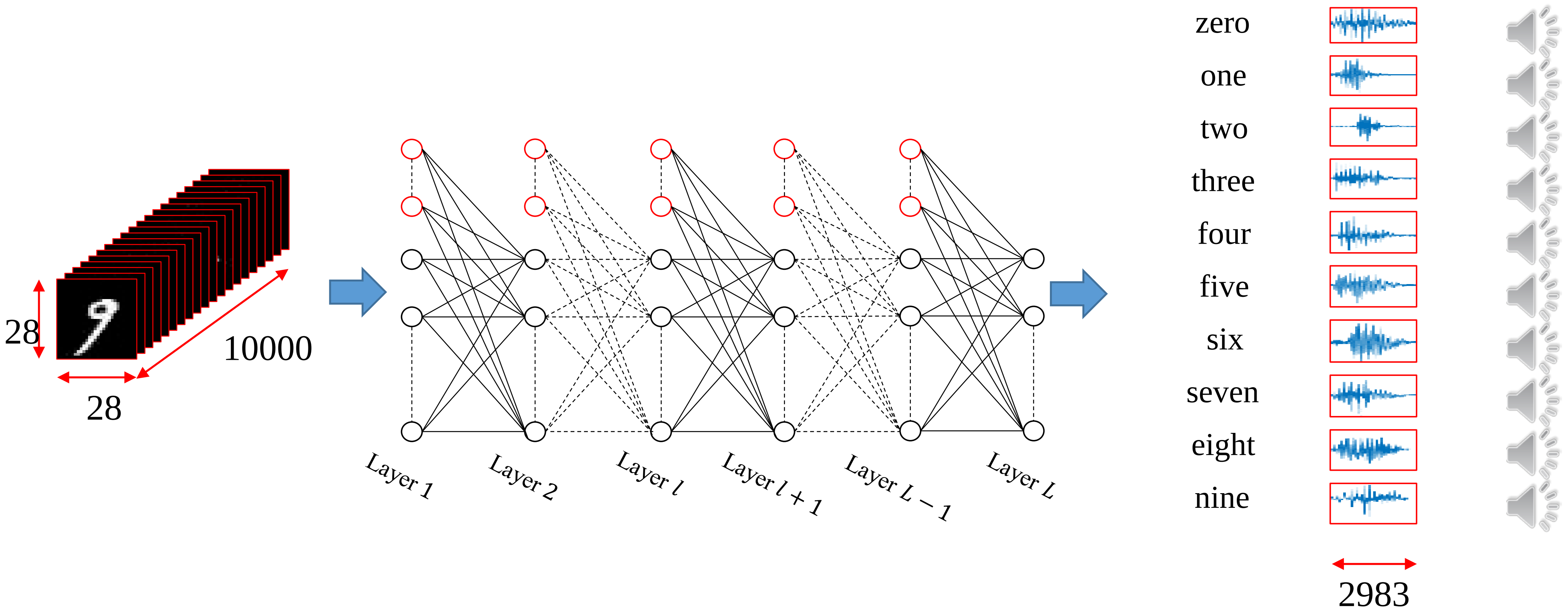
Chinese speech

# Outline

- Brief Review of Backpropagation Algorithm
- On Some Problems of BP
  - On the Network Structure
  - On the Target Output
  - On the Network Prediction
  - On the Input
  - On the Cost Function
  - On the Depth of the Network
  - On the Training Data
- Assignment

# Assignment

Implement the handwritten digits to speech convertor by MATLAB.

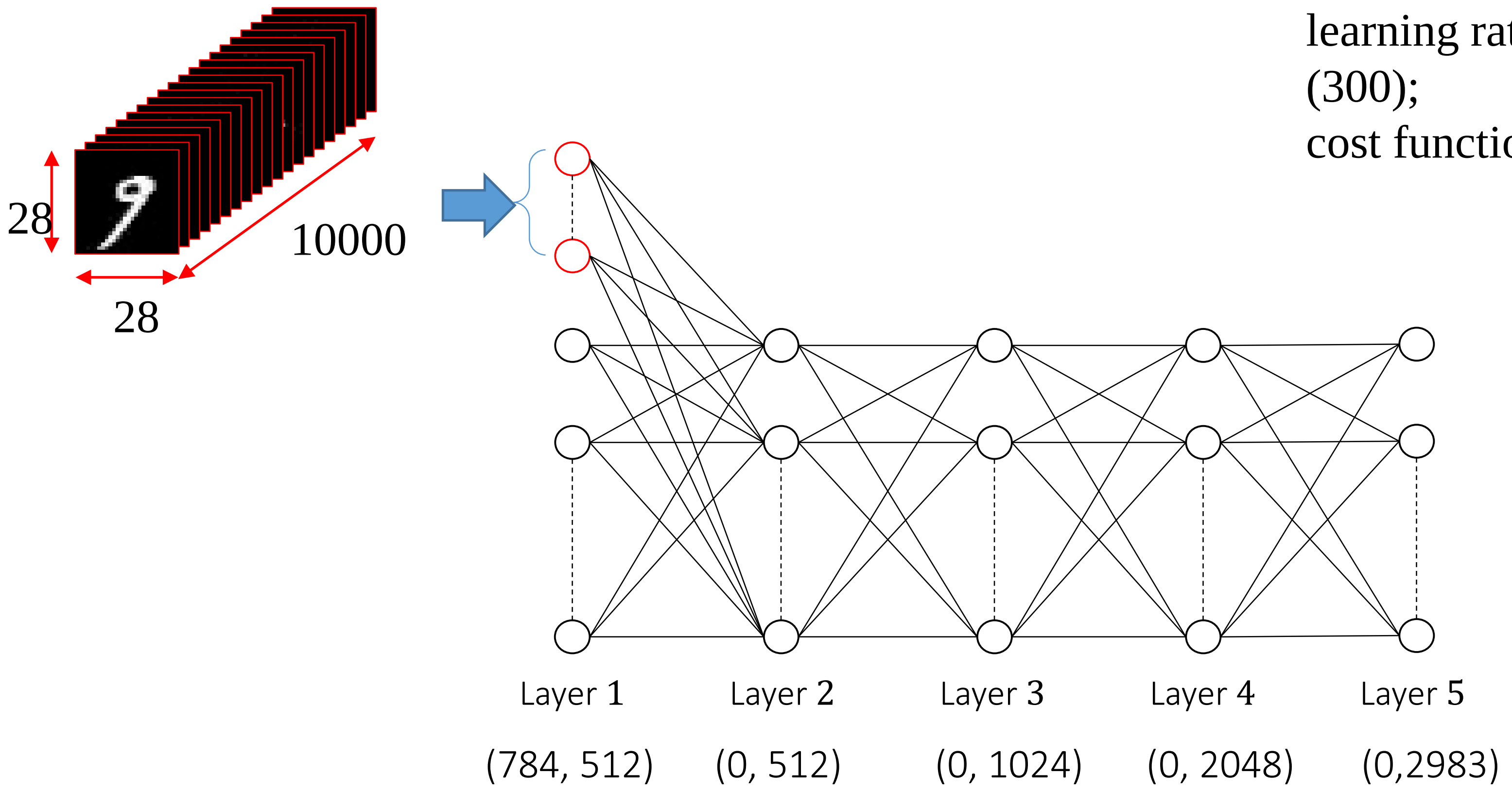


***Thanks***

# Assignment: an example

Hint: One of my student used the following parameters for the network and successfully trained the network.

learning rate (0.1); mini batch (100); iteration (300);
cost function (mean square error).



# Assignment: codes

fc.m %%forward computation file

```
function [a_next, z_next] = fc(w, a, x, f)

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code BELOW
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % forward computing (either component or vector form)
    z_next = w * [x; a];
    a_next = f(z_next);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code ABOVE
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

# Assignment: codes

bc.m %%backward computation file

```
function delta = bc(w, z, delta_next, df)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code BELOW
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % backward computing (either component or vector form)
    delta = df(z) .* (w(:, end-size(z,1)+1:end)' * delta_next);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Your code ABOVE
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

# Assignment: codes

get_audio.m %%the function to get audio file

```
function [ audio_y ] = get_audio( y, audio )

audio_y = zeros(size(audio,2), size(y,2));
for i = 1:size(y,2)
    audio_y(:,i) = audio(find(y(:,i)==1),:);
end
```

# Assignment: codes

lab5.m %%the main training function

```
% clear workspace and close plot windows
clear;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Your code BELOW
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% prepare the data set
load mnist_small_matlab.mat
input_size = 28 * 28; % size of each patch
% prepare training data
train_size = size(trainLabels,2);
X_train{1} = reshape(trainData,[],train_size);% top-left
X_train{2} = zeros(0, train_size);
X_train{3} = zeros(0, train_size);
X_train{4} = zeros(0, train_size);
X_train{5} = zeros(0, train_size);
```

```
% prepare testing data
test_size = size(testLabels,2);
X_test{1} = reshape(trainData,[],test_size);% top-left
X_test{2} = zeros(0, test_size);
X_test{3} = zeros(0, test_size);
X_test{4} = zeros(0, test_size);
X_test{5} = zeros(0, test_size);

% prepare standard speech audio
sample_rate = 4000; % shall assert they all have a same sample rate
audio = zeros(2983, 10); % we checked with the audio file and know its 2983-dim
input
for i = 1:10
    [audio(:,i), sample_rate] = audioread(fullfile('audio',sprintf('%d.wav',i-1)));
    soundsc(audio(:,i), sample_rate);
    pause(1)
end
audio = (audio+1)/2;

% choose parameters
alpha = 0.1; % learning rate
max_iter = 300;
mini_batch = 100;
```

```
layer_size = [input_size 512    % layer 1
                      0 512    % layer 2
                      0 1024    % layer 3
                      0 2048    % layer 4
                      0 2983];  % layer 5
L = size(layer_size, 1);
% define function
sigm = @(s) 1 ./ (1 + exp(-s));
dsigm = @(s) sigm(s) .* (1 - sigm(s));
lin = @(s) s;
dlin = @(s) 1;
fs  = {[],  sigm,  sigm,  sigm,  sigm,  sigm,  sigm,  sigm};
dfs = {[], dsigm, dsigm, dsigm, dsigm, dsigm, dsigm, dsigm};
% initialize weights
w = cell(L-1, 1);
for l = 1:L-1
    %w{l} = randn(layer_size(l+1,2), sum(layer_size(l,:)));
    % a tricky, but effective, initialization
    w{l} = (rand(layer_size(l+1,2), sum(layer_size(l,:))) * 2 -1) *
sqrt(6/(layer_size(l+1,2)+sum(layer_size(l,:))));
end

% train
J = [];
x = cell(L, 1);
a = cell(L, 1);
z = cell(L, 1);
delta = cell(L, 1);
```

```matlab
for iter = 1:max_iter
    ind = randperm(train_size);
    % for each mini-batch
    for k = 1:ceil(train_size/mini_batch)
        % prepare internal inputs
        a{1} = zeros(layer_size(1,2),mini_batch);
        % prepare external inputs
        for l=1:L
            x{l} = X_train{l}(:,ind((k-1)*mini_batch+1:min(k*mini_batch, train_size)));
        end
        % prepare labels
        [~, ind_label] = max(trainLabels(:,ind((k-1)*mini_batch+1:min(k*mini_batch, train_size))));
        % prepare targets
        y = audio(:,ind_label);

        % batch forward computation
        for l=1:L-1
            [a{l+1}, z{l+1}] = fc(w{l}, a{l}, x{l}, fs{l+1});
        end

        % cost function and error
        J = [J 1/2/mini_batch*sum((a{L}(:)-y(:)).^2)];
        delta{L} = (a{L} - y).* dfs{L}(z{L});

        % batch backward computation
        for l=L-1:-1:2
            delta{l} = bc(w{l}, z{l}, delta{l+1}, dfs{l});
        end
        % update weight
        for l=1:L-1
            gw = delta{l+1} * [x{l};a{l}]' / mini_batch;
            w{l} = w{l} - alpha * gw;
        end
    end
```

```
 % end loop
    if mod(iter,1) == 0
        fprintf('%i/%i epochs: J=%.4f\n', iter,
max_iter, J(end));
    end
end

% save model
save model.mat w layer_size J
```