Calorimetry at a Future Linear Collider

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Abstract

This thesis describes the optimisation of the calorimeter design for collider experiments at the future Compact LInear Collider (CLIC) and the International Linear Collider (ILC). The detector design of these experiments is built around high-granularity Particle Flow Calorimetry that, in contrast to traditional calorimetry, uses the energy measurements for charged particles from the tracking detectors. This can only be realised if calorimetric energy deposits from charged particles can be separated from those of neutral particles. This is made possible with fine granularity calorimeters and sophisticated pattern recognition software, which is provided by the PandoraPFA algorithm. This thesis presents results on Particle Flow calorimetry performance for a number of detector configurations. To obtain these results a new calibration procedure was developed and applied to the detector simulation and reconstruction to ensure optimal performance was achieved for each detector configuration considered.

This thesis also describes the development of a software compensation technique that vastly improves the intrinsic energy resolution of a Particle Flow Calorimetry detector. This technique is implemented within the PandoraPFA framework and demonstrates the gains that can be made by fully exploiting the information provided by the fine granularity calorimeters envisaged at a future linear collider.

A study of the sensitivity of the CLIC experiment to anomalous gauge couplings that effect vector boson scattering processes is presented. These anomalous couplings provide insight into possible beyond standard model physics. This study, which utilises the excellent jet energy resolution from Particle Flow Calorimetry, was performed at centre-of-mass energies of 1.4 TeV and 3 TeV with integrated luminosities of 1.5ab^{-1}

and 2ab^{-1} respectively. The precision achievable at CLIC is shown to be approximately one to two orders of magnitude better than that currently offered by the LHC.

In addition, a study into various technology options for the CLIC vertex detector is described.

Declaration

This dissertation is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university. This dissertation does not exceed the word limit for the respective Degree Committee.

Steven Green

Acknowledgements

Of the many people who deserve thanks, some are particularly prominent, such as my supervisor…

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"Writing in English is the most ingenious torture ever devised for sins committed in previous lives."
— James Joyce

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Chapter 1

Anomalous Gauge Coupling Theory

“"Meaningless! Meaningless!" says the Teacher. "Utterly meaningless! Everything is meaningless."”
— Ecclesiastes 1:2

Presented in chapter **??** is an analysis of the sensitivity of the CLIC experiment to the anomalous gauge couplings α_4 and α_5 through the vector boson scattering process. Here a brief description of the Standard Model of particle physics and a deeper discussion of the anomalous coupling theory studied in chapter **??** is given.

1.1 The Standard Model

The Standard Model is a non-abelian gauge theory of the $\mathrm{SU}(3) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)$ symmetry group. It provides a description of three of the four fundamental forces of nature: the electromagnetic, weak and strong nuclear forces [1,2]. The Standard Model contains a total of 24 fermion fields: six flavours of quark, each with three colours, and six leptons. A summary of the properties of these particles is given in table 1.1 and 1.2. As these fields, ψ, are spin-$\frac{1}{2}$, they obey the Dirac equation

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi \ , \tag{1.1}$$

where $\mathcal{L}$ is the Lagrangian density and m is a mass term. The derivative term, $\partial\!\!\!/ = \gamma^{\mu}\partial_{\mu}$, represents a summation over the partial derivate, $\partial^{\mu} = (\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, of the field ψ and the gamma matrices, γ^{μ}. Each of the gauge transformations of the Standard Model are defined by a unitary operator U, which acts to transform the vector space, Ψ, formed from a combination of fermion fields, ψ, in the following way

$$\Psi \rightarrow \Psi' = \mathrm{U}\Psi \ . \tag{1.2}$$

In the Standard Mode, the Lagrangian density describing the fermion fields is invariant

Generation	Particle	Mass [MeV]	Spin	Q/e
1	e^-	$548.579909070 \pm 0.000000016$	1/2	-1
	ν_e	-	1/2	0
2	μ^-	$105.6583745 \pm 0.0000024$	1/2	-1
	ν_μ	-	1/2	0
3	τ^-	1776.86 ± 0.12	1/2	-1
	ν_τ	-	1/2	0

Table 1.1: The mass, spin and electric charge (Q) of the leptons found in the Standard Model [3]. Neutrino masses have not been included in the above table as precise measurements are yet to be made. However, oscillations between different neutrino flavour states have been observed, which indicates that the flavour and mass eigenstates differ and that the neutrinos have a non-zero mass. The current upper bound on neutrino mass measurements is 2 eV.

Generation	Particle	Mass [MeV]	Spin	Q/e
1	u	$2.2^{+0.6}_{-0.4}$	1/2	$+2/3$
	d	$4.7^{+0.5}_{-0.4}$	1/2	$-1/3$
2	c	1270 ± 30	1/2	$+2/3$
	s	98^{+8}_{-4}	1/2	$+2/3$
3	t	$173210 \pm 510 \pm 710$	1/2	$+2/3$
	b	4180^{+40}_{-30}	1/2	$-1/3$

Table 1.2: The mass, spin and electric charge (Q) of the quarks found in the Standard Model [3]. Each of the particles in the above table corresponds to three fermion fields, one for each of the three colours of the SU(3) symmetry.

under a SU(3), $\mathrm{SU(2)_L}$ and U(1) gauge transformations. The $\mathrm{SU}(2)_L$ gauge symmetry acts on doublets formed of pairs of left handed chiral components of the fermion fields, $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, while the right handed components, $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$, transform trivially as singlets [4]. Similarly, the SU(3) symmetry acts on triplets formed of the fermion fields for each flavour of quark. All fields transform under the fundamental representation of U(1). The invariance of the Standard Model Lagrangian to these gauge transformations is established by introducing 12 gauge fields, summarised in table 1.3, through the covariant derivate of the fermion fields

$$\partial^{\mu} \rightarrow D^{\mu} = \partial^{\mu} + ig_1 Y B^{\mu} + ig_2 \mathbf{T} \cdot \mathbf{W}^{\mu} + ig_3 \mathbf{X} \cdot \mathbf{G}^{\mu} \ , \tag{1.3}$$

where B^{μ} is the gauge field for the U(1) symmetry, $\mathbf{W}^{\mu}$ ($\mathrm{W}_j^{\mu}, j = 1, 2, 3$) are the fields of the $\mathrm{SU(2)_L}$ symmetry and $\mathbf{G}^{\mu}$ ($\mathrm{G}_j^{\mu}, j = 1, .., 8$) are the fields of the SU(3). Y is the weak hypercharge, which relates to the chirality and flavour of the fermion field that it is associated to. The three coefficients g_1, g_2 and g_3 are coupling constants related to the three gauged symmetry groups in the Standard Model. Mixing of the gauge fields for the U(1) and SU(2) symmetry of the form

$$\mathrm{Z}_{\mu} = \cos\theta_W \mathrm{W}_{\mu}^3 - \sin\theta_W \mathrm{B}_{\mu} \ , \tag{1.4}$$

$$\mathrm{A}_{\mu} = \sin\theta_W \mathrm{W}_{\mu}^3 + \cos\theta_W \mathrm{B}_{\mu} \ , \tag{1.5}$$

$$\mathrm{W}_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(\mathrm{W}_{\mu}^1 \mp i\mathrm{W}_{\mu}^2) \ , \tag{1.6}$$

where

$$\cos\theta_W = \frac{g_2}{g_1 + g_2} \text{ and } \sin\theta_W = \frac{g_1}{g_1 + g_2} \ , \tag{1.7}$$

gives the electroweak gauge bosons; $\mathrm{W}^{\pm}$, Z and γ. This mixing ensures that the $\mathrm{W}^{\pm}$ and Z bosons become massive, while the γ remains massless. The G_j^{μ} fields are the eight massless gluons of the strong force. **T** and **X** are the generators for the SU(2) and SU(3) symmetries, which are typically chosen as

$$T_i = \frac{1}{2}\tau_i \ , \tag{1.8}$$

$$X_i = \frac{1}{2}\lambda_i \ , \tag{1.9}$$

$$\tag{1.10}$$

where τ and λ are the Pauli and the Gell-Mann matrices respectively. The gauge fields

Force	Particle	Mass [GeV]	Spin	Q/e
Electromagnetic	γ	0	1	0
Weak Nuclear	W$^{\pm}$	80.385 ± 0.015	1	± 1
	Z	91.1876 ± 0.0021	1	0
Strong Nuclear	g ($\times$ 8 colours)	0	1	0
Higgs	H	125.1 ± 0.3	0	0

Table 1.3: The mass, spin and electric charge (Q) of the gauge bosons found in the Standard Model [3]. The γ and gs theoretically have zero mass, which is consistent with measurements. The upper bound on the γ mass has been measured at 10^{-18} eV, while gluon masses of up to a few MeV have not been precluded. The upper bound on the magnitude of the charge of the γ is measured at 10^{-35}.

of the Standard Model, B_μ, $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$, transform under the gauge transformations as

$$K_\mu \rightarrow K'_\mu = UK_\mu U^\dagger + \frac{i}{g}(\partial^\mu U)U^\dagger \ , \tag{1.11}$$

where K_μ is any of B_μ, $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$ and g is the coupling constants associated to the relevant gauged symmetry group. As the B_μ, $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$ gauge fields are spin-1, they are described by the Proca Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_i^{\mu\nu}F_{\mu\nu i} + \frac{1}{2}m_K^2 K_{i\mu}K_i^\mu \ , \tag{1.12}$$

where

$$F_i^{\mu\nu} = \partial^\mu K_i^\nu - \partial^\nu K_i^\mu - g f_{ijk} K_j^\mu K_k^\nu \ , \tag{1.13}$$

f_{ijk} are the fully anti-symmetric structure constants of the group, K_i^μ is the i^{th} gauge field of the group and m_K is a mass term for the gauge boson. The structure constants are defined from the commutation relations between generators of the symmetry group

$$[T_i, T_j] = i f_{ijk} T_k. \tag{1.14}$$

These structure constants govern the self-interactions for the gauge bosons. There is only one structure constant for the U(1) symmetry, which is zero, as the U(1) symmetry is abelian. The SU(2) symmetry structure constants are $f_{ijk} = \epsilon_{ijk}$, where ϵ_{ijk} is the Levi-Civita tensor. Due to the symmetries that are present in the Standard Model, $m_K = 0$ for all the gauge fields, however, it is clear that is is not the case. Therefore, to generate gauge boson mass terms a Higgs field is introduced that undergoes spontaneous symmetry breaking, as described in section 1.2.

1.2 Higgs Physics

Mass terms are generated in the Standard Model by introducing a Higgs field that undergoes spontaneous symmetry breaking. This allows the gauge bosons, as well as the quarks and leptons, to obtain a mass, while still respecting the gauge symmetries found in the Standard Model.

1.2.1 Spontaneous Symmetry Breaking

To illustrate spontaneous symmetry breaking, consider a complex scalar field ψ with the Klein-Gordon Lagrangian

$$\mathcal{L} = \partial^{\mu}\psi^{*}\partial_{\mu}\psi - m^{2}|\psi|^{2} = \partial^{\mu}\psi^{*}\partial_{\mu}\psi - V(\psi) \ , \tag{1.15}$$

where m is a mass term and $V(\psi)$ is the potential the field ψ. This Lagrangian density is invariant under the global symmetry $\psi \rightarrow e^{i\alpha}\psi$. By adding extra terms to the Lagrangian, which retain the invariance to this global symmetry, it is possible to modify the interactions of this scalar field. For example, consider modifying the potential of the scalar field to the following

$$\mathrm{V}(\psi) = m^{2}|\psi|^{2} + \lambda|\psi|^{4} \ , \tag{1.16}$$

If $m^2 > 0$, the potential has a minima at zero, however, if $m^2 < 0$ then the minima exists on a circle in the complex ψ plane, which is centred at $(0, 0)$ and has radius $v = \sqrt{-m^2/\lambda}$. To quantise this theory it is necessary to expand about the minima of the potential. However, in the case of $m^2 < 0$ there are an infinite number of choices of minima to expand about. Irrespective of the choice of minima used to expand the field about, the symmetry $\psi \rightarrow e^{i\alpha}\psi$ is broken. Fluctuations about the minima along the degenerate direction leave the potential unchanged, which is a consequence of the breaking of the $\psi \rightarrow e^{i\alpha}\psi$ symmetry; this is known as spontaneous symmetry breaking. Goldstone's theorem [5] implies that, for Lorentz-invariant theories, spontaneous symmetry breaking always leads to the existence a massless particles known as Goldstone bosons. For example, consider expanding the complex scalar ψ about the minima. In that case, ψ takes the form

$$\psi = \frac{1}{\sqrt{2}}(v + \psi_1 + i\psi_2) \ , \tag{1.17}$$

where ψ_1 and ψ_2 are real fields and $v = \sqrt{-m^2/\lambda}$. Applying this parameterisation to the Lagrangian yields a mass term of $\sqrt{-m^2}$ for the ψ_1 field. However, there is no corresponding mass term for the ψ_2 field, which indicates that it is massless as predicated by Goldstone's theorem

$$\mathcal{L} = \frac{1}{2}\partial^\mu \psi_1 \partial_\mu \psi_1 + \frac{1}{2}\partial^\mu \psi_2 \partial_\mu \psi_2 - m^2|\psi_1|^2 + ... \ , \tag{1.18}$$

Spontaneous symmetry breaking is the origin of the gauge boson mass terms when applied to local symmetries instead of global ones. For example consider the global symmetry, $\psi \to e^{i\alpha}\psi$ that exists in equation 1.15. If this global symmetry is promoted to a local symmetry by letting $\alpha \to \alpha(x)$ and $\partial^\mu \to D^\mu = \partial^\mu + iA^\mu$, where A^μ a the gauge field that transforms as $A^\mu \to A^\mu - \partial^\mu \alpha(x)$, the Lagrangian becomes

$$\mathcal{L} = (D^\mu \psi)^*(D_\mu \psi) - m^2|\psi|^2 - \lambda|\psi|^4 \ . \tag{1.19}$$

If the ψ field is expanded about a non-zero minima in the potential, i.e. $m^2 < 0$ and $v = \sqrt{-m^2/\lambda}$, as was done in equation 1.17, then a gauge boson mass term, $+\frac{v^2}{2}A^\mu A_\mu$, is generated from the $(D^\mu\psi)^*(D_\mu\psi)$ term.

1.2.2 Electroweak Interactions

The electroweak sector of the Standard Model is that related to the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)$ symmetry [6]. In this sector, spontaneous symmetry breaking must occur in such a way as to give three massive gauge bosons, $\mathrm{W}^\pm$ and Z, and one massless gauge boson, the γ. This can be achieved through a Higgs field, H, that transforms as a doublet under the $\mathrm{SU}(2)_\mathrm{L}$ symmetry. The Lagrangian for this field is

$$\mathcal{L}_{Higgs} = (D_\mu \mathrm{H})^\dagger D^\mu \mathrm{H} - \mathrm{V}(\mathrm{H}) \ . \tag{1.20}$$

The Higgs potential, V(H), is

$$\mathrm{V(H)} = -\mu^2 \mathrm{H}^\dagger \mathrm{H} + \lambda(\mathrm{H}^\dagger \mathrm{H})^2 \ , \tag{1.21}$$

where μ and λ are constants. The covariant derivative of this Higgs field must satisfy the $\mathrm{SU(2)_L} \times \mathrm{U(1)}$ gauge symmetry meaning it takes the form

$$D_\mu \mathrm{H} = (\partial_\mu + ig_1 Y B_\mu + ig_2 \frac{\tau^i}{2} W^i_\mu)\mathrm{H} \ , \tag{1.22}$$

where g_1 and g_2 are coupling constants for the U(1) and $\mathrm{SU(2)_L}$ gauged symmetries respectively, $Y = \frac{1}{2}$ is the weak hypercharge of the Higgs and τ^i are the Pauli matrices. B_μ and W^i_μ are the gauge fields for the U(1) and $\mathrm{SU(2)_L}$ gauged symmetries respectively.

Consider spontaneously breaking the symmetry in the Higgs sector by expanding the Higgs field about a non-zero vacuum expectation value (vev)

$$\langle \mathrm{H} \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \ , \tag{1.23}$$

where the minima of the field is defined as

$$\frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}} \ , \tag{1.24}$$

where v real. In that case, the kinematic term in the Higgs Lagrangian, $D^\mu \mathrm{H}^\dagger D_\mu \mathrm{H}$, contains mass terms for the gauge bosons

$$D^\mu \mathrm{H}^\dagger D_\mu \mathrm{H} \subset \frac{v^2}{2}(ig_1 Y B^\mu + ig_2 \frac{\tau^i}{2} W^{i\mu})(ig_1 Y B_\mu + ig_2 \frac{\tau^i}{2} W^i_\mu) \ . \tag{1.25}$$

If there is mixing of the $\mathrm{SU(2)_L}$ and U(1) fields of the form

$$\mathrm{Z}_\mu = \cos\theta_W \mathrm{W}^3_\mu - \sin\theta_W \mathrm{B}_\mu \ , \tag{1.26}$$

$$\mathrm{A}_\mu = \sin\theta_W \mathrm{W}^3_\mu + \cos\theta_W \mathrm{B}_\mu \ , \tag{1.27}$$

$$\mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}}(\mathrm{W}^1_\mu \mp i\mathrm{W}^2_\mu) \ , \tag{1.28}$$

then the following gauge boson mass terms are generated

$$\frac{(gv)^2}{4}\mathrm{W}^+_\mu \mathrm{W}^{-\mu} + \frac{(g^2 + g'^2)v^2}{8}\mathrm{Z}_\mu \mathrm{Z}^\mu \ . \tag{1.29}$$

The gauge boson masses generated by spontaneous symmetry breaking of the Higgs field are

$$\begin{aligned} m_{\mathrm{W}} =& \frac{gv}{2} \ , \\ m_{\mathrm{Z}} =& \frac{v\sqrt{g^2+g'^2}}{2} = \frac{m_W}{\cos\theta_W} \ , \\ m_{\mathrm{A}} =& 0 \ , \end{aligned} \tag{1.30}$$

where θ_W is the Weinberg angle. This mixing produces a massless gauge boson, the γ, and three massive gauge bosons, the $W^\pm$ and Z. By acquiring a non-zero vev, the Higgs field breaks the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)$ symmetry that was present in the Lagrangian to the $\mathrm{U}(1)_{em}$ symmetry of electromagnetism.

The ratio of the masses of the $W^\pm$ and Z bosons is predicted when spontaneous symmetry breaking occurs in the Higgs sector. This prediction sets the ρ parameter to unity, where the ρ parameter is defined as

$$\rho = \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2 \cos\theta_W{}^2} = 1 \ . \tag{1.31}$$

This is a consequence of the Higgs potential containing custodial symmetry [3]. As the ρ parameter has been experimentally measured to be 1.00040 ± 0.00024 [7], it is clear that any extension to the Standard Model should retain this result.

1.2.2.1 Custodial Symmetry

The Standard Model Higgs field is defined by the Lagrangian

$$\mathcal{L}_{Higgs} = (D_\mu \mathrm{H})^\dagger D^\mu \mathrm{H} - \mathrm{V}(\mathrm{H}), \tag{1.32}$$

where

$$\mathrm{V}(\mathrm{H}) = -\mu^2 \mathrm{H}^\dagger \mathrm{H} + \lambda(\mathrm{H}^\dagger \mathrm{H})^2 \ , \tag{1.33}$$

and μ and λ are constants. By construction, the Higgs sector of the Standard Model is invariant under local $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)$ gauge transformations. However, a larger global symmetry also exists in this sector, which can be seen by considering the Higgs doublet [8]

$$\mathrm{H} = \begin{pmatrix} \psi^+ \\ \psi^0 \end{pmatrix} = \begin{pmatrix} \psi_1 + i\psi_2 \\ \psi_3 + i\psi_4 \end{pmatrix} . \tag{1.34}$$

All the terms in the Higgs potential involve $\mathrm{H}^\dagger\mathrm{H} = \psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2$, which is invariant under any rotation of these four components and hence under a SO(4) global symmetry. In general, SO(4) $\cong$ SU(2) $\times$ SU(2), where $\cong$ denotes an isomorphism. In the case of the Higgs sector SO(4) $\cong$ SU$(2)_L \times$ SU$(2)_R$ where the SU$(2)_L$ symmetry is the gauged symmetry of the Standard Model. This symmetry can be manifested using an alternative parameterisation [9] of the Higgs field

$$\Phi = (i\tau_2\mathrm{H}, \mathrm{H}) = \begin{pmatrix} \psi^{0*} & \psi^+ \\ -\psi^{+*} & \psi^0 \end{pmatrix} . \tag{1.35}$$

In this parametrisation the Higgs Lagrangian, $\mathcal{L}_{Higgs}$, becomes

$$\mathcal{L}_{Higgs} = \frac{1}{2}\mathrm{Tr}[(D_\mu\Phi)^\dagger D^\mu\Phi] + \mu^2\mathrm{Tr}[\Phi^\dagger\Phi] - \lambda\mathrm{Tr}[\Phi^\dagger\Phi\Phi^\dagger\Phi] , \tag{1.36}$$

which is invariant under transformations of the form

$$\Phi \to U_L\Phi U_R^\dagger , \tag{1.37}$$

where U_L and U_R are transformations of the SU$(2)_L$ and SU$(2)_R$ symmetry groups respectively.

When the Higgs field acquires a non-zero vev the SU$(2)_L \times$ SU$(2)_R$ symmetry of the Higgs potential is broken to a SU$(2)_C$ symmetry, which is known as custodial symmetry [10]. As SO(3) $\cong$ SU(2), symmetry breaking in the Higgs sector is equivalent to a SO(4) symmetry being broken to a SO(3) symmetry. This becomes clear when considering the form of the Higgs potential after symmetry breaking. Prior to symmetry breaking a SO(4) global symmetry is present, however, after expanding the Higgs about a non-zero vev, defined in equation 1.23, the terms in the Higgs potential involve $\mathrm{H}^\dagger\mathrm{H} = (\psi_3 - v)^2 + \psi_1^2 + \psi_2^2 + \psi_4^2$, which is only invariant to rotations between the ψ_1, ψ_2 and ψ_4 fields, which is a SO(3) symmetry.

The Higgs field, H, transforms a singlet under this SU$(2)_C$ custodial symmetry, while the SU$(2)_L$ gauge boson fields, W_μ^i, transform as a triplet. It is the transformation of the

W_μ^i fields under the $\mathrm{SU}(2)_C$ symmetry that enforces the relationship between the masses of the $W^\pm$ and Z gauge bosons and that ρ should equal unity. It should be noted that the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry only exists in the Higgs sector of the Standard Model. The $\mathrm{SU}(2)_R$ symmetry is the Standard Model is broken by Yukawa couplings of the Higgs to quarks and leptons and by a non-zero coupling to the U(1) gauge symmetry of the Standard Model, g_1. However, this breaking of the $\mathrm{SU}(2)_R$ symmetry is weak, which means the deviations of ρ from unity are minimal [10].

1.3 Effective Field Theory

There are a number of features in the observable universe that cannot be accounted for using the Standard Model of particle physics. However, the Standard Model is a very good description of the interactions between particles at the energies being probed at modern particle collider experiments. Any underlying theory governing the interactions of particles must, therefore, behave like the Standard Model over these energies, or distance scales. Above such energies the theory will deviate from the Standard Model to account for the full underlying theory. Effective field theories (EFTs) work from this premise by assuming that the complete theory has a momentum scale, Λ, below which Standard Model behaviour is replicated [11, 12].

Quantum field theories must be renormalizable to ensure that non-infinite predictions of the coefficients in the Lagrangian can be made and tested [13]. Infinities arise from non-renormalizable theories due to divergent integrals from loop diagrams that assume the theory being applied is valid at all energy and length scales. Effective field theories act to avoid such problems by only integrating up to the momentum scale Λ and not above it. At the energy scale being considered, any infinities arising from the loop calculations in the EFT can be absorbed into a finite number of parameters. This methodology avoids the assumption that the theory in question is applicable to all energy scales and allows measurable predictions to be made.

As the Standard Model should be replicated at the low energy scale, it is appropriate when creating an EFT Lagrangian to append new operators to the Standard Model Lagrangian to account for areas of new physics. This gives the general form for an EFT

Lagrangian as [11]

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_{\text{dimension d>4}} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} \ , \tag{1.38}$$

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian, $c_i^{(d)}$ are free parameters, $\mathcal{O}_i^{(d)}$ is the i^{th} unique operator with dimension d in the EFT and Λ is the EFT momentum scale. The sum runs over all unique operators with dimension greater than four. The presence of the Λ^{d-4} in the denominator is required to ensure correct dimensionality of the new terms being added to the Lagrangian.

New physics is introduced by the operators $\mathcal{O}_i^{(d)}$, but suppressed by the momentum scale Λ. It is assumed that Λ is large with respect to the momentum scales that have been examined at preexisting particle collider experiments, therefore, any new physics is suppressed. Under this assumption, new operators with dimension less than, or equal to, four can be vetoed from the EFT as their effects would be readily observed at preexisting particle collider experiments, due to the Λ^{4-d} coefficient. At energies below the momentum scale, Λ, it is possible to find the dominant new physics terms in the EFT and consider these as corrections to the Standard Model. Above this scale the EFT breaks down as operator $\mathcal{O}_i^{(d)}$ in $\mathcal{L}_{EFT}$ has a non-negligible coefficient. In the extremal limit, $\Lambda \rightarrow \infty$, the Standard Model is recovered as new physics is too far out of reach to have any impact on observables.

1.4 Electroweak Chiral Lagrangian

The introduction of a Higgs field undergoing spontaneous symmetry breaking is able to produce mass terms in the Lagrangian for the $\mathrm{W}^{\pm}$ and Z bosons. However, it is possible to introduce these terms by parameterising the Higgs field using the gauge boson fields of the $\mathrm{SU(2)_L}$ Standard Model symmetry [14]. In this approach, the pattern of spontaneous symmetry breaking mirrors that found in the Higgs sector of the Standard Model i.e. a global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry is broken to a $\mathrm{SU}(2)_C$ symmetry. This will ensure that the ρ parameter, introduced in section 1.2.2, retains a value of unity, which is consistent with experimental measurements. The Standard Model spontaneous symmetry breaking pattern can be replicated using a field, $\Sigma(x)$, which transforms under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetries as

$$\Sigma \to U_L \Sigma U_R^\dagger \; , \tag{1.39}$$

where U_L and U_R are transformations of the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ symmetry groups respectively and $\Sigma(x)$ is

$$\Sigma(x) = \exp\left(\frac{-i}{v}\Sigma_{a=1}^{3}\pi^a\tau^a\right) \; , \tag{1.40}$$

where π^a are the three would-be Goldstone bosons that exist when the $\mathrm{SU(2)_L} \times \mathrm{U(1)}$ symmetry is broken to $\mathrm{U}(1)_{em}$ [15]. The $\mathrm{SU(2)_L}$ and U(1) symmetries of the Standard Model are gauged in the usual way by defining the covariant derivate of the Σ field

$$\mathcal{D}_\mu\Sigma(x) = \partial_\mu\Sigma(x) + \frac{ig_2}{2}\mathrm{W}_\mu^a\tau^a\Sigma(x) - \frac{ig_1}{2}\mathrm{B}_\mu\tau^3\Sigma(x) \; , \tag{1.41}$$

where g_1 and g_2 are coupling constants for the U(1) and $\mathrm{SU(2)_L}$ symmetries respectively and τ^a are the Pauli spin matrices. The lowest order derivative term for this Σ field that could appear in the Lagrangian is

$$\mathcal{L}_\Sigma = \frac{v^2}{4}\mathrm{Tr}(\mathcal{D}^\mu\Sigma^\dagger\mathcal{D}_\mu\Sigma) = -\frac{v^2}{4}\mathrm{Tr}(V_\mu V^\mu) \; , \tag{1.42}$$

where $V_\mu = (\mathcal{D}_\mu\Sigma)\Sigma^\dagger$. This terms respects all the symmetries present in the Higgs sector of that Standard Model, including the custodial symmetry in the limit $g_1 \to 0$. Furthermore, by expanding this field about a non-zero vev, the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry is broken to a $\mathrm{SU}(2)_C$ symmetry exactly as it is in the Standard Model. For example, if this field is expanded about the point $\Sigma = \not{\mathbb{1}}$, i.e. the unitary gauge, mass terms for the electroweak gauge bosons are generated that match those produced from spontaneous symmetry breaking of the Higgs field as described in section 1.2.1

$$\frac{v^2}{4}\mathrm{Tr}[V^\mu V_\mu] = -\frac{(gv)^2}{4}\mathrm{W}_\mu^+\mathrm{W}^{-\mu} - \frac{(g^2+g'^2)v^2}{8}\mathrm{Z}_\mu\mathrm{Z}^\mu \tag{1.43}$$

$$\begin{aligned} m_\mathrm{A} &= 0 \; , \\ m_\mathrm{W} &= \frac{gv}{2} \; , \\ m_\mathrm{Z} &= \frac{v\sqrt{g^2+g'^2}}{2} = \frac{m_\mathrm{W}}{\cos\theta_W} \; , \end{aligned} \tag{1.44}$$

So far, all that has been done is a parameterisation of the Higgs field, however, it was shown by Longhitano [15] that there are several relevant operators involving the Σ field that are $\mathrm{SU(2)_L} \times \mathrm{U(1)}$ invariant. As these operators obey the same symmetries as those found in the Standard Model they should be considered. This can be done using EFT approach, as discussed in section 1.3. Of the operators introduced by Longhitano, only two involve quartic massive gauge boson vertices and preserve the custodial symmetry [16]. They are

$$\alpha_4 \mathrm{Tr}[V^\mu V_\nu]\mathrm{Tr}[V^\nu V_\mu] \quad \text{and} \quad \alpha_5 \mathrm{Tr}[V^\mu V_\mu]^2 \, . \tag{1.45}$$

These terms contribute to the massive gauge boson quartic vertices shown in figure 1.1. The Standard Model already contains triple and quartic vertices involving the electroweak gauge bosons, shown in figure 1.2, and these are also present in this EFT approach. These vertices originate from the kinematic terms in the Proca Lagrangian density $\mathcal{L}_{kin} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu}$. Of the vertices showing sensitivity to α_4 and α_5, only that shown in figure 1.1c is not present in the Standard Model.

Both terms shown in equation 1.45 contain dimension 8 operators [11] and, with respect to the EFT approach, i.e. equation 1.38, their coefficients are proportional to Λ^{-4}, where Λ is the momentum scale of the new physics being modelled. In the limit that the momentum scale of new physics is beyond experimental reach, i.e. $\Lambda \to \infty$, these terms do not contribute to measurable observables and the Standard Model is recovered. It should be noted that in this case, the Standard Model has been parameterised using the Σ field, so in the limit $\Lambda \to \infty$, the gauge boson mass terms generated from $\mathcal{L}_\Sigma$ do not vanish.

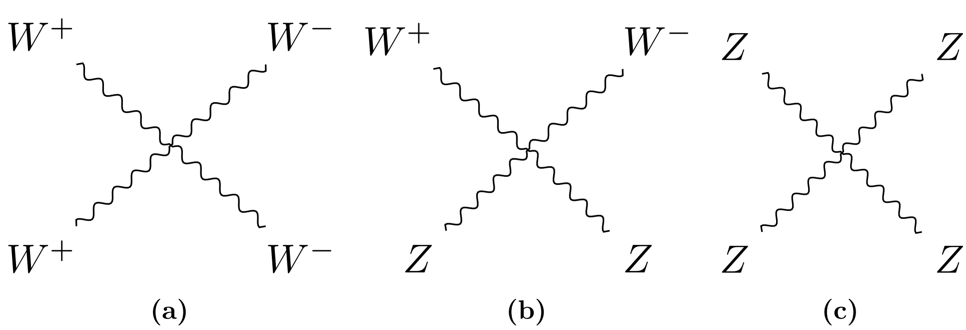

Figure 1.1: Gauge boson self-coupling vertices that are sensitive to the anomalous gauge couplings α_4 and α_5.

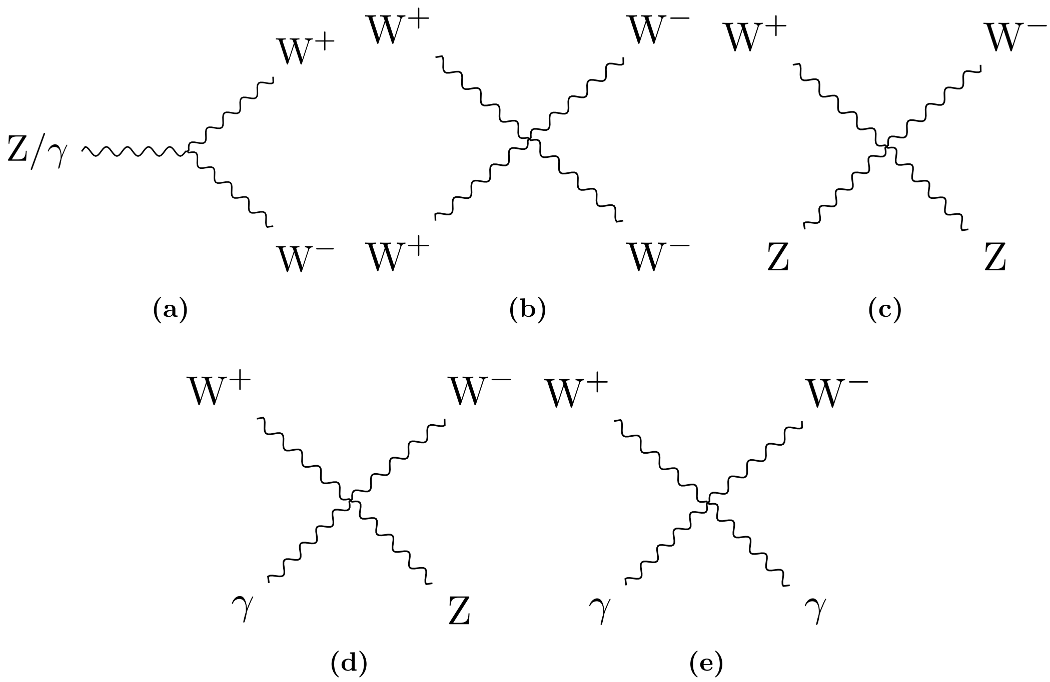

Figure 1.2: Gauge boson self-coupling vertices in the Standard Model.

Colophon

This thesis was made in LaTeX 2_ε using the "hepthesis" class [17].

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