

Something something something physics

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Abstract

This thesis describes the optimisation of the calorimeter design for collider experiments at the future Compact Linear Collider (CLIC) and the International Linear Collider (ILC). The detector design of these experiments is built around high-granularity Particle Flow Calorimetry that, in contrast to traditional calorimetry, uses the energy measurements for charged particles from the tracking detectors. This can only be realised if calorimetric energy deposits from charged particles can be separated from those of neutral particles. This is made possible with fine granularity calorimeters and sophisticated pattern recognition software, which is provided by the PandoraPFA algorithm. This thesis presents results on Particle Flow calorimetry performance for a number of detector configurations. To obtain these results a new calibration procedure was developed and applied to the detector simulation and reconstruction to ensure optimal performance was achieved for each detector configuration considered.

This thesis also describes the development of a software compensation technique that vastly improves the intrinsic energy resolution of a Particle Flow Calorimetry detector. This technique is implemented within the PandoraPFA framework and demonstrates the gains that can be made by fully exploiting the information provided by the fine granularity calorimeters envisaged at a future linear collider.

A study of the sensitivity of the CLIC experiment to anomalous gauge couplings that effect vector boson scattering processes is presented. These anomalous couplings provide insight into possible beyond standard model physics. This study, which utilises the excellent jet energy resolution from Particle Flow Calorimetry, was performed at centre-of-mass energies of 1.4 TeV and 3 TeV with integrated lumi-

nosities of 1.5ab^{-1} and 2ab^{-1} respectively. The precision achievable at CLIC is shown to be approximately one to two orders of magnitude better than that currently offered by the LHC.

Finally, a study into various technology options for the CLIC vertex detector is described.

Declaration

This dissertation is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university. This dissertation does not exceed the word limit for the respective Degree Committee.

Andy Buckley

Acknowledgements

Of the many people who deserve thanks, some are particularly prominent, such as my supervisor...

Preface

This thesis describes my research on various aspects of the LHCb particle physics program, centred around the LHCb detector and LHC accelerator at CERN in Geneva.

For this example, I'll just mention Chapter ?? and Chapter ??.

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*“Writing in English is the most ingenious torture
ever devised for sins committed in previous lives.”*

— James Joyce

Chapter 1.

Anomalous Gauge Coupling Theory

“There, sir! that is the perfection of vessels!”

— Jules Verne, 1828–1905

1.1. The Standard Model

The Standard Model is a non-abelian gauge theory of the $SU(3) \times SU(2)_L \times U(1)$ symmetry group. It provides a description of three of the four fundamental forces of nature: the electromagnetic, weak and strong nuclear forces. CITE. The Standard Model contains a total of 24 fermion fields: six flavours of quark, each with three colours, and six leptons. A summary of the properties of these particles is given in table 1.1 and 1.2. As these fields are spin- $\frac{1}{2}$, the fields obey the Dirac equation:

$$\mathcal{L} = \bar{\psi}(i\partial - m)\psi, \quad (1.1)$$

where \mathcal{L} is the Lagrangian density and m is a mass term. The derivative term, $\partial = \gamma^\mu \partial_\mu$, represents a summation over the partial derivate, $\partial^\mu = (\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, of the scalar field ψ with respect to the position 4-vector, $x^\mu = (t, x, y, z)$, and the gamma matrices, γ^μ . Each of the gauge transformations of the Standard Model act are defined by a unitary operator U , which acts to transform the vector space, Ψ , formed from a combination of fermion fields, ψ , in the following way:

$$\Psi \rightarrow \Psi' = U\Psi. \quad (1.2)$$

The Lagrangian density describing the fermion fields in the Standard Model is invariant under a $SU(3)$, a $SU(2)_L$ and a $U(1)$ gauge transformations. The $SU(2)_L$ gauge symmetry acts on doublets formed, in the fundamental representation, of pairs of left handed chiral components, $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$. The fundamental representation of a $SU(N)$ theory allows the symmetry to be expressed using $N \times N$ matrices acting on a N -dimensional vector. The right handed components, $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$, transform trivially as a singlet. Similarly, the $SU(3)$ symmetry acts on triplets formed of the fermion fields for each flavour of quark. All fields transform under the fundamental representation of $U(1)$. The Standard Model is a non-abelian theory as gauge transformations from a given symmetry group, with the exception of the $U(1)$ symmetry, do not commute with each other. The invariance of the Standard Model Lagrangian to these gauge transformations is established by introducing 12 gauge fields, which are summarised in table 1.3, via the covariant derivate of the fermion fields:

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + ig_1 Y B^\mu + ig_2 \mathbf{T} \cdot \mathbf{W}^\mu + ig_3 \mathbf{X} \cdot \mathbf{G}^\mu, \quad (1.3)$$

where B^μ is the gauge field for the $U(1)$ symmetry, \mathbf{W}^μ ($W_j^\mu, j = 1, 2, 3$) are the fields of the $SU(2)_L$ symmetry and \mathbf{G}^μ ($G_j^\mu, j = 1, \dots, 8$) are the fields of the $SU(3)$. Y is the weak hypercharge, which is related to the chirality and flavour of the fermion it relates to. g_1, g_2 and g_3 are three coupling constants related to the three symmetry groups of the Standard Model. Mixing of the gauge fields for the $U(1)$ and $SU(2)$ symmetry of the form:

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu, \quad (1.4)$$

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, \quad (1.5)$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad (1.6)$$

where:

$$\cos\theta_W = \frac{g_2}{g_1 + g_2} \text{ and } \sin\theta_W = \frac{g_1}{g_1 + g_2}, \quad (1.7)$$

produce the W^\pm , Z and γ gauge bosons. This mixing ensures that the W^\pm and Z bosons are massive, while the γ remains massless. The G_j^μ fields are the eight massless gluons of the strong force. \mathbf{T} and \mathbf{X} are the generators for the $SU(2)$ and $SU(3)$

symmetries, which are typically chosen as:

$$T_i = \frac{1}{2} \tau_i, \quad (1.8)$$

$$X_i = \frac{1}{2} \lambda_i, \quad (1.9)$$

$$(1.10)$$

where τ and λ are the Pauli and the Gell-Mann matrices respectively. The gauge fields of the Standard Model, B_μ , \mathbf{W}_μ and \mathbf{G}_μ , transform under the gauge transformations as:

$$K_\mu \rightarrow K'_\mu = UK_\mu U^\dagger + \frac{i}{g} (\partial^\mu U) U^\dagger, \quad (1.11)$$

where K_μ is any of B_μ , \mathbf{W}_μ and \mathbf{G}_μ and g is the coupling constants associated to the relevant gauge field. As the B_μ , \mathbf{W}_μ and \mathbf{G}_μ gauge fields are spin-1, they are described by the Proca action:

$$\mathcal{L} = -\frac{1}{4} F_i^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_K^2 K_{i\mu} K_i^\mu, \quad (1.12)$$

where:

$$F_i^{\mu\nu} = \partial^\mu K_i^\nu - \partial^\nu K_i^\mu - g f_{ijk} K_j^\mu K_k^\nu, \quad (1.13)$$

where f_{ijk} are the fully anti-symmetric structure constants of the symmetry group, K_i^μ is the i^{th} gauge field of the group and m_K is a mass term for the gauge bosons. The structure constants are defined from the commutation relations between generators of the symmetry group:

$$[T_i, T_j] = i f_{ijk} T_k. \quad (1.14)$$

These structure constants that govern the self-interactions for the gauge bosons. There is only one structure constant for the U(1) symmetry, which is zero as the U(1) symmetry is abelian. The SU(2) symmetry structure constants are $f_{ijk} = \epsilon_{ijk}$, where ϵ_{ijk} is the Levi-Civita tensor. Due to the symmetries that are enforced in the Standard Model $m_K = 0$ for all the gauge fields present, however, it is clear that is is not the case in nature. The gauge boson mass terms are introduced through the Higgs field, as described in section 1.2.

Generation	Particle	Mass [MeV]	Spin	Q/e
1	e^-	$548.579909070 \pm 0.000000016$	1/2	-1
	ν_e	-	1/2	0
2	μ^-	$105.6583745 \pm 0.0000024$	1/2	-1
	ν_μ	-	1/2	0
3	τ^-	1776.86 ± 0.12	1/2	-1
	ν_τ	-	1/2	0

Table 1.1.: The mass, spin and electric charge (Q) of the leptons found in the Standard Model [3]. Neutrino masses have not been included in the above table as precise measurements are yet to be made. However, oscillations between different neutrino flavour states have been observed, which indicates that the flavour and mass eigenstates differ and that the neutrinos have a non-zero mass. The current upper bound on neutrino mass measurements is 2 eV.

Generation	Particle	Mass [MeV]	Spin	Q/e
1	u	$2.2^{+0.6}_{-0.4}$	1/2	+2/3
	d	$4.7^{+0.5}_{-0.4}$	1/2	-1/3
2	c	1270 ± 30	1/2	+2/3
	s	98^{+8}_{-4}	1/2	+2/3
3	t	$173210 \pm 510 \pm 710$	1/2	+2/3
	b	4180^{+40}_{-30}	1/2	-1/3

Table 1.2.: The mass, spin and electric charge (Q) of the quarks found in the Standard Model [3]. Each of the particles in the above table corresponds to three fermion fields, one for each of the three colours of the SU(3) symmetry.

1.2. Higgs Physics

The gauge symmetries present in the Standard Model forbid a mass term for the gauge bosons, however, the electroweak gauge bosons are known to have mass, which indicates the theory as it stands is incomplete. Similarly, mass terms, $m\bar{\psi}\psi$, for the leptons and quarks are forbidden as they violate the SU(2)_L symmetry as the symmetry applied only to left hand chiral components. The mass terms are generated in the Standard Model by introducing a Higgs field, which undergoes spontaneous symmetry breaking.

Force	Particle	Mass [GeV]	Spin	Q/e
Electromagnetic	γ	0	1	0
Weak Nuclear	W^{\pm}	80.385 ± 0.015	1	± 1
	Z	91.1876 ± 0.0021	1	0
Strong Nuclear	g ($\times 8$ colours)	0	1	0
Higgs	H	125.1 ± 0.3	0	0

Table 1.3.: The mass, spin and electric charge (Q) of the gauge bosons found in the Standard Model [3]. The γ and g s theoretically have zero mass, which is consistent with measurements. The upper bound on the γ mass has been measured at 10^{-18} eV, while gluon masses of up to a few MeV have not been precluded. The upper bound on the magnitude of the charge of the γ is measured at 10^{-35} .

1.2.1. Spontaneous Symmetry Breaking

To illustrate spontaneous symmetry breaking, consider a complex scalar field ψ with the Klein-Gordon Lagrangian:

$$\mathcal{L} = \partial^\mu \psi^* \partial_\mu \psi - m^2 |\psi|^2 = \partial^\mu \psi^* \partial_\mu \psi - V(\psi) , \quad (1.15)$$

where \mathcal{L} is the Lorentz invariant Lagrangian density, $\partial^\mu = (\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ is the partial derivative of the scalar field ψ with respect to the position 4-vector $x^\mu = (t, x, y, z)$, m is a mass term and $V(\psi)$ is the potential of the field ψ . This Lagrangian density is invariant under the global symmetry $\psi \rightarrow e^{i\alpha} \psi$. By adding extra terms to the Lagrangian, which retain the invariance to the global symmetry, it is possible to modify the interactions of this scalar field. For example consider modifying the potential of the scalar field to the following:

$$V(\psi) = m^2 |\psi|^2 + \lambda |\psi|^4 , \quad (1.16)$$

If $m^2 > 0$, the potential has a minimum at zero, however, if $m^2 < 0$ then the minimum exists on a circle in the complex ψ plane. This complex circle is centred at $(0,0)$ and has radius $v = \sqrt{-m^2/\lambda}$. To quantise this theory it is necessary to expand about the minimum of the potential. However, in the case of $m^2 < 0$ there are an infinite number of choices of minima to expand about and irrespective of the choice of minima for this configuration the symmetry $\psi \rightarrow e^{i\alpha} \psi$ is broken. Fluctuations about the minima along the degenerate direction leave the potential unchanged, which is a consequence

of the breaking of the $\psi \rightarrow e^{i\alpha}\psi$ symmetry; this is known as spontaneous symmetry breaking.

Goldstone's theorem [7] implies that for Lorentz-invariant theories spontaneous symmetry breaking always leads to the existence a massless particle. This can be seen in this example when expanding the complex scalar theory example about the minima.

$$\psi = \frac{1}{\sqrt{2}}(v + \psi_1 + i\psi_2), \quad (1.17)$$

where ψ_1 and ψ_2 are real fields and $v = \sqrt{-m^2/\lambda}$. Applying this parameterisation to the Lagrangian yields a mass term of $\sqrt{-m^2}$ for the ψ_1 field, however, there is no corresponding mass term for the ψ_2 field. This indicates the ψ_2 field describes massless particles:

$$\mathcal{L} = \frac{1}{2}\partial^\mu\psi_1\partial_\mu\psi_1 + \frac{1}{2}\partial^\mu\psi_2\partial_\mu\psi_2 - m^2|\psi_1|^2 + \dots, \quad (1.18)$$

This procedure is the origin of the gauge boson mass terms when applied to local symmetries instead of global ones. For example consider the global symmetry, $\psi \rightarrow e^{i\alpha}\psi$, that exists in equation 1.15. Consider promoting this global symmetry to a local symmetry by letting $\alpha \rightarrow \alpha(x)$ and $\partial^\mu \rightarrow D^\mu = \partial^\mu + iA^\mu$, where A^μ a the gauge field that transforms as $A^\mu \rightarrow A^\mu - \partial^\mu\alpha(x)$. In that case the new Lagrangian becomes:

$$\mathcal{L} = (D^\mu\psi)^*(D_\mu\psi) - m^2|\psi|^2 - \lambda|\psi|^4. \quad (1.19)$$

If there is a non-zero minima in the potential, i.e. $m^2 < 0$ and $v = \sqrt{-m^2/\lambda}$, then a gauge boson mass term appears of the form $+\frac{v^2}{2}A^\mu A_\mu$.

1.2.2. Electroweak Interactions

The electroweak sector of the Standard Model is that related to the $SU(2)_L \times U(1)$ symmetry [5]. In this sector, spontaneous symmetry breaking must occur in such a way as to give three massive gauge bosons, W^\pm and Z , and one massless gauge boson, the photon. This can be achieved through a Higgs field, H , that transforms as a

doublet under the $SU(2)_L$ symmetry. The potential of the Higgs field, $V(H)$, is:

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \quad (1.20)$$

where μ and λ are constants. The minima of this field is at:

$$\sqrt{H^\dagger H} = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}}. \quad (1.21)$$

Without loss of generality we may choose to expand this field around the a non-zero vacuum expectation value (vev):

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad (1.22)$$

where v is real. In this case, consider the kinematic term in the Lagrangian, $\partial^\mu H^\dagger \partial_\mu H$. The covariant derivative of this Higgs field must satisfy the $SU(2)_L \times U(1)$ gauge symmetry meaning it takes the form:

$$D_\mu H = (\partial_\mu + ig_1 Y B_\mu + ig_2 \frac{\tau^i}{2} W_\mu^i) H, \quad (1.23)$$

where g_1 and g_2 are coupling constants for the relevant gauge symmetries, $Y = \frac{1}{2}$ is the weak hypercharge of the Higgs, τ^i are the Pauli matrices and B_μ is the gauge field related to the $U(1)$ symmetry and W_μ^i are the gauge fields related to the $SU(2)_L$ symmetry. If there is mixing of the $SU(2)_L$ and $U(1)$ fields of the form:

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu, \quad (1.24)$$

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, \quad (1.25)$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad (1.26)$$

then non-zero mass terms for the electroweak bosons arise:

$$\frac{(gv)^2}{4} W_\mu^+ W^{-\mu} + \frac{(g^2 + g'^2)v^2}{8} Z_\mu Z^\mu. \quad (1.27)$$

$$(1.28)$$

This leads to the following boson masses:

$$\begin{aligned} m_W &= \frac{gv}{2} , \\ m_Z &= \frac{v\sqrt{g^2 + g'^2}}{2} = \frac{m_W}{\cos\theta_W} , \\ m_A &= 0 , \end{aligned} \tag{1.29}$$

where θ_W is known as the Weinberg angle. This mixing produces a massless gauge boson, the γ , and three massive gauge bosons, the W^\pm and Z . By acquiring a non-zero vev, the Higgs field breaks the $SU(2)_L \times U(1)$ symmetry that was present in the Lagrangian to the $U(1)_{em}$ symmetry of electromagnetism.

The ratio of the masses of the W^\pm and Z bosons gives the ρ parameter, which is defined as:

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 . \tag{1.30}$$

This is a consequence of the Higgs potential, used in spontaneous symmetry breaking, containing custodial symmetry [3]. As the ρ parameter has been experimentally measured to be 1.00040 ± 0.00024 [11], it is clear that any extension to the Standard Model should maintain this result.

Custodial Symmetry

The Higgs Lagrangian:

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger D^\mu H - V(H) \tag{1.31}$$

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 , \tag{1.32}$$

is, by construction, invariant under local $SU(2)_L \times U(1)$ gauge transformations. However, a larger global symmetry also exists in this sector, which can be seen by considering the Higgs doublet [6]:

$$H = \begin{pmatrix} \psi^+ \\ \psi^0 \end{pmatrix} = \begin{pmatrix} \psi_1 + i\psi_2 \\ \psi_3 + i\psi_4 \end{pmatrix} . \tag{1.33}$$

All the terms in the Higgs potential involve $H^\dagger H = \psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2$, which is invariant under any rotation of these four components and hence under a $SO(4)$ global symmetry. This $SO(4)$ symmetry is isomorphic to the $SU(2)_L \times SU(2)_R$ symmetry where the $SU(2)_L$ symmetry is the gauged symmetry of the Standard Model. It is possible to reconstruct the Higgs potential to make this symmetry more evident [1, 8]:

$$\Phi = (i\tau_2 H, H) = \begin{pmatrix} \psi^{0*} & \psi^+ \\ -\psi^{+*} & \psi^0 \end{pmatrix}. \quad (1.34)$$

Using this parametrisation the Higgs Lagrangian, \mathcal{L}_{Higgs} , becomes:

$$\mathcal{L}_{Higgs} = \frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \mu^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda \text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi], \quad (1.35)$$

which is invariant under transformations of the form:

$$\Phi \rightarrow U_L \Phi U_R^\dagger, \quad (1.36)$$

where U_L and U_R are transformations of the $SU(2)_L$ and $SU(2)_R$ symmetry groups respectively.

When the Higgs field acquires a non-zero vev, the $SU(2)_L \times U(1)$ symmetry was broken to the $U(1)_{em}$ symmetry of electromagnetism. Analogously, the $SU(2)_L \times SU(2)_R$ symmetry of the Higgs potential is broken to a $SU(2)_C$ symmetry, which is known as custodial symmetry. As a $SU(2)$ symmetry is isomorphic to a $SO(3)$ symmetry, in effect, symmetry breaking in the Higgs sector is equivalent to an $SO(4)$ symmetry being broken to an $SO(3)$. This becomes clear when considering the form of the Higgs potential after symmetry breaking. After spontaneous symmetry breaking the terms in the Higgs potential involve $H^\dagger H = (\psi_3^2 - v^2) + \psi_1^2 + \psi_2^2 + \psi_4^2$, which is only invariant to rotations between the ψ_1 , ψ_2 and ψ_4 fields i.e. an $SO(3)$ symmetry.

The Higgs field, H , transforms a singlet under this $SU(2)_C$ custodial symmetry, while the $SU(2)_L$ gauge boson fields transform as a triplet. It is the transformation of the $SU(2)_L$ gauge boson fields that enforce the relationship between the masses of the W^\pm and Z gauge bosons and hence guarantee that $\rho = 1$. It should be noted that the $SU(2)_L \times SU(2)_R$ symmetry only exists in the Higgs sector of the standard model. The $SU(2)_R$ symmetry in the Standard Model is broken by Yukawa couplings of the Higgs to quarks and leptons and by a non-zero coupling to the $U(1)$ gauge symmetry of the

Standard Model, g_1 . This breaking of the $SU(2)_R$ symmetry is relatively weak, which means the deviations of ρ from unity are minimal.

1.3. Effective Field Theory

There are a number of features in the observable universe that cannot be accounted for using the Standard Model of particle physics. However, the Standard Model is a very good description of the interactions between particles at the energies being probed at modern particle collider experiments. Any underlying theory governing the interactions of particles must, therefore, behave like the Standard Model over these energies, or distance scales. Above such energies the theory will deviate from the Standard Model to account for the full underlying theory. Effective field theories (EFTs) work from this premise by assuming that the complete theory has a momentum scale, Λ , below which Standard Model behaviour is replicated.

Quantum field theories must be renormalizable to ensure that non-infinite predictions of the coefficients in the Lagrangian can be made and tested. Infinities arise from non-renormalizable theories due to divergent integrals from loop diagrams that assume the theory being applied is valid at all energy and length scales. Effective field theories act to avoid such problems by only integrating up to the momentum (mass) scale Λ (Δ) and not above it. At the energy scale being considered, any infinities arising from the loop calculations in the EFT can be absorbed into a finite number of parameters. This methodology avoids the assumption that the theory in question is applicable to all energy scales and allows measurable predictions to be made.

In the EFT framework, it is no longer necessary to enforce renormalization in the choice of operators appearing in the Lagrangian. As the Standard Model should be replicated at the low energy scale it is appropriate when creating the EFT Lagrangian to append new operators to the Standard Model Lagrangian that account for new physics. These new terms do not necessarily have to obey the same gauge symmetries as those found in the Standard Model. This gives the general form for an EFT Lagrangian as:

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_{\text{dimension } d} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \quad (1.37)$$

where $c_i^{(d)}$ are the coefficient of the operators $\mathcal{O}_i^{(d)}$ and the summations run over all the dimensions, d , of the operator and all unique operators, i , of dimension, d . The presence of the Λ^{d-4} in the denominator is needed to ensure correct dimensionality of the new terms being added to the Lagrangian, but it also sets the scale, Λ , of new physics being added. At energies below this scale it is possible to find the dominant terms for the EFT and consider these as corrections to the Standard Model, while above this scale the EFT breaks down as each term in the Lagrangian has a non-negligible coefficient. In the extremal limit, $\Lambda \rightarrow \infty$, the Standard Model is recovered as new physics is too far out of reach to have any impact on observables.

1.4. Electroweak Chiral Lagrangian

The introduction of a Higgs field transforming appropriately under the relevant gauge transformations is able to produce mass terms in the Lagrangian for the W^\pm and Z bosons. However, it is possible to introduce these terms via an EFT approach. The pattern of spontaneous symmetry breaking in this approach should mirror that found in the Higgs sector of the Standard Model i.e. a global $SU(2)_L \times SU(2)_R$ is broken to $SU(2)_C$. This will ensure that the ρ parameter, introduced in section 1.2.2, retains a value of unity, which is consistent with experimental measurements. The Standard Model spontaneous symmetry breaking pattern can be replicated by an EFT approach using a field, $\Sigma(x)$, that transforms under the $SU(2)_L \times SU(2)_R$ global symmetries as:

$$\Sigma \rightarrow U_L \Sigma U_R^\dagger, \quad (1.38)$$

where U_L and U_R are transformations of the $SU(2)_L$ and $SU(2)_R$ symmetry groups respectively and $\Sigma(x)$ is:

$$\Sigma(x) = \exp\left(\frac{-i}{v} \sum_{a=1}^3 W^a \tau^a\right), \quad (1.39)$$

where W^a are the Goldstone bosons generated from the spontaneous symmetry breaking of the electroweak symmetry. The $SU(2)_L$ and $U(1)$ symmetries of the Standard Model are gauged in the usual way by defining the covariant derivative of the Σ field:

$$\mathcal{D}_\mu \Sigma(x) = \partial_\mu \Sigma(x) + \frac{ig}{2} W_\mu^a \tau^a \Sigma(x) - \frac{ig'}{2} B_\mu \tau^3 \Sigma(x), \quad (1.40)$$

where g and g' are coupling constants for the $U(1)$ and $SU(2)_L$ symmetries respectively and τ^a are the Pauli spin matrices, which are the generators for the $SU(2)$ symmetry. Using the Σ field, it is possible to replace the Higgs field, and the associated terms in the Lagrangian \mathcal{L}_{Higgs} , with:

$$\mathcal{L}_{EFT} = \frac{v^2}{4} \text{Tr}(\mathcal{D}^\mu \Sigma^\dagger \mathcal{D}_\mu \Sigma) = -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu), \quad (1.41)$$

This term respects all the gauge symmetries present in the Standard Model, including the custodial symmetry in the limit $g' \rightarrow 0$. Furthermore, by expanding this field about a non-zero vev, the $SU(2)_L \times SU(2)_R$ global symmetry is broken to a $SU(2)_C$ symmetry exactly as it is in the Standard Model. For example, in the unitary gauge if this field is expanded about the point $\Sigma = \mathbb{1}$ i.e. the unitary gauge, the lowest order derivative term appearing in the Lagrangian contains:

$$\frac{v^2}{4} \text{Tr}(\mathcal{D}^\mu \Sigma^\dagger \mathcal{D}_\mu \Sigma) = -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu), \quad (1.42)$$

where $V_\mu = (\mathcal{D}_\mu \Sigma) \Sigma^\dagger$. This term contains the mass terms for the electroweak gauge bosons that match those produced from the introduction of a Higgs field as shown in section 1.2.1:

$$\frac{v^2}{4} \text{Tr}[V^\mu V_\mu] = -\frac{(gv)^2}{4} W_\mu^+ W^{-\mu} - \frac{(g^2 + g'^2)v^2}{8} Z_\mu Z^\mu \quad (1.43)$$

$$m_W = \frac{gv}{2}, \quad (1.44)$$

$$m_Z = \frac{v\sqrt{g^2 + g'^2}}{2} = \frac{m_W}{\cos\theta_W},$$

It was shown by Longhitano [9, 10] that there are several relevant operators that are $SU(2)_L \times U(1)$ invariant, up to dimension 4, that should be considered in this theory. Of those operators only two involve quartic massive gauge boson vertices and that preserve the custodial symmetry [2] shown in equation 1.30. These are:

$$\alpha_4 \text{Tr}[V^\mu V_\mu] \text{Tr}[V^\nu V_\nu] \text{ and } \alpha_5 \text{Tr}[V^\mu V_\mu]^2, \quad (1.45)$$

The full expansion of these terms in terms of the massive gauge bosons W^\pm and Z is given in the appendices A.1. These terms contribute to the vector boson scattering

processes involving W^\pm and Z bosons. A study into the sensitivity of the CLIC experiment to the anomalous gauge couplings α_4 and α_5 is presented in section ??.

Appendix A.

Pointless extras

*“Le savant n’étudie pas la nature parce que cela est utile;
il l’étudie parce qu’il y prend plaisir,
et il y prend plaisir parce qu’elle est belle.”*
— Henri Poincaré, 1854–1912

Appendixes (or should that be “appendices”?) make you look really clever, ’cos it’s like you had more clever stuff to say than could be fitted into the main bit of your thesis. Yeah. So everyone should have at least three of them...

A.1. Anomalous Gauge Coupling Quartic Vertices Of Relevance in Vector Boson Scattering

The anomalous gauge couplings involving α_4 and α_5 arise in EFT through the addition of the following terms to the Lagrangian.

$$\text{Tr}(V^\mu V_\nu) \text{Tr}(V^\nu V_\mu) \text{ and } [\text{Tr}(V^\mu V_\mu)]^2 \quad (\text{A.1})$$

Where V_μ is defined in the following way.

$$V_\mu = \Sigma(D_\mu \Sigma)^\dagger \quad (\text{A.2})$$

and Σ , the Higgs field matrix, is defined as.

$$\Sigma = \exp\left(-\frac{i}{v}\mathbf{w}\right) \quad (\text{A.3})$$

Where $\mathbf{w} = w^a \sigma^a$. w^a are the ... and σ^a are the Pauli spin matrices. The covariant derivative of the Higgs field matrix is

$$D_\mu \Sigma = \left(\partial_\mu + \frac{ig}{2}W_\mu - \frac{ig'}{2}B_\mu \sigma^3\right)\Sigma \quad (\text{A.4})$$

For clarity consider the unitarity gauge where $\mathbf{w} = 0$, which implies $\Sigma = 1$. In this gauge V_μ takes the following form.

$$\begin{aligned} V_\mu &= \frac{i}{2}(gW_\mu^i \sigma^i - g'B_\mu \sigma^3) = \frac{i}{2} \begin{pmatrix} gW_\mu^3 - g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \\ &= \frac{i}{2} \begin{pmatrix} \sqrt{g^2 + g'^2}Z_\mu & g\sqrt{2}W_\mu^+ \\ g\sqrt{2}W_\mu^- & \sqrt{g^2 + g'^2}Z_\mu \end{pmatrix} \end{aligned}$$

Where the relationship between the mass and gauge symmetry basis are as follows.

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) \quad (\text{A.5})$$

$$W_\mu^- = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \quad (\text{A.6})$$

$$Z_\mu = c_w W_\mu^3 - s_w B_\mu \quad (\text{A.7})$$

$$A_\mu = s_w W_\mu^3 + c_w B_\mu \quad (\text{A.8})$$

With $c_w = \frac{g}{\sqrt{g^2 + g'^2}}$ and $s_w = \frac{g'}{\sqrt{g^2 + g'^2}}$. Consider the expansion of the terms to be included in the Lagrangian.

$$V^\mu V_\nu = \frac{-1}{4} \begin{pmatrix} \sqrt{g^2 + g'^2} Z^\mu & g\sqrt{2} W^{+\mu} \\ g\sqrt{2} W^{-\mu} & \sqrt{g^2 + g'^2} Z^\mu \end{pmatrix} \begin{pmatrix} \sqrt{g^2 + g'^2} Z_\nu & g\sqrt{2} W_\nu^+ \\ g\sqrt{2} W_\nu^- & \sqrt{g^2 + g'^2} Z_\nu \end{pmatrix} \quad (\text{A.9})$$

$$\text{Tr}[V^\mu V_\nu] = \frac{-1}{2} ((g^2 + g'^2) Z^\mu Z_\nu + g^2 W^{+\mu} W_\nu^- + g^2 W^{-\mu} W_\nu^+) \quad (\text{A.10})$$

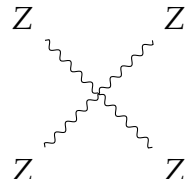
$$\text{Tr}[V^\mu V_\nu] \text{Tr}[V_\mu V^\nu] = \frac{(g^2 + g'^2)^2}{4} (Z^\mu Z_\mu)^2 + g^2 (g^2 + g'^2) (Z^\mu Z^\nu W_\mu^- W_\nu^+) \quad (\text{A.11})$$

$$+ \frac{g^4}{2} (W^{-\mu} W_\mu^+)^2 + \frac{g^4}{2} (W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+) \quad (\text{A.12})$$

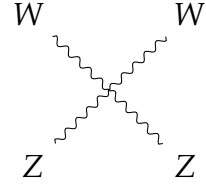
$$\text{Tr}[V^\mu V_\mu]^2 = \frac{(g^2 + g'^2)^2}{4} (Z^\mu Z_\mu)^2 + g^2 (g^2 + g'^2) (Z^\mu Z^\nu W_\mu^- W_\nu^+) \quad (\text{A.13})$$

$$+ g^4 (W^{-\mu} W_\mu^+)^2 \quad (\text{A.14})$$

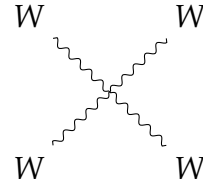
These two terms change the cross section for the vector boson scattering processes at CLIC that involve $ZZ \rightarrow ZZ$, $W^+ W^- \rightarrow ZZ$, $ZZ \rightarrow W^+ W^-$ and $W^+ W^- \rightarrow W^+ W^-$.



$$\subset (\alpha_4 + \alpha_5) \frac{(g^2 + g'^2)^2}{4} \quad (\text{A.15})$$



$$\subset (\alpha_4 + \alpha_5)g^2(g^2 + g'^2) \quad (\text{A.16})$$



$$\subset (\alpha_4 + 2\alpha_5)\frac{g^4}{2} \text{ and } \frac{g^4}{2}\alpha_4 \quad (\text{A.17})$$

Colophon

This thesis was made in $\text{\LaTeX}2_\epsilon$ using the “hepthesis” class [\[4\]](#).

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