# Mathematics

# Study Notes

*Steven Bryan Greve*

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**Abstract**

This document pertains to mathematical concepts which I have studied. It includes my notes, work, and solutions when dealing with various problems. The source code for this document can be found on my *GitHub* at any time.

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# 1 Definitions

## 1.1 Binomial Theorems

**Definition 1**

$$(a+b)^2 = a^2 + 2ab + b^2 \tag{1}$$

$$(a-b)^2 = a^2 - 2ab + b^2 \tag{2}$$

$$(a+b)\,(a-b) = a^2 - b^2 \tag{3}$$

For higher exponentiations:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \tag{4}$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \tag{5}$$

$$(-a-b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3 \tag{6}$$

Binomial Formula:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \tag{7}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \tag{8}$$

## 1.2 Fractions

**Definition 2**

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \tag{9}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b} \tag{10}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \tag{11}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd} \tag{12}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd} \tag{13}$$

Inverse:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \tag{14}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot b}{d \cdot c} \tag{15}$$

$$\frac{a \cdot \frac{b}{c}}{\frac{d}{e}} = \frac{\frac{a \cdot c + b}{c}}{\frac{d}{c}} = \frac{a \cdot c + b}{c} \cdot \frac{c}{d} = \frac{(a \cdot c + b) \cdot c}{c \cdot d} \tag{16}$$

$$\frac{ab}{c} = \frac{a}{c} \cdot b \tag{17}$$

$$\frac{a}{b} = \frac{1}{b} \cdot a \tag{18}$$

$$\frac{a \div b}{c} = \frac{a}{c} \div b \tag{19}$$

## 1.3 Parentheses Rules

**Definition 3**

$$+(a + b) = a + b \tag{20}$$

$$+(-a - b) = -a - b \tag{21}$$

$$-(a - b) = -a + b \tag{22}$$

$$-(-a + b) = +a - b \tag{23}$$

$$-(a + b) = -a - b \tag{24}$$

Associative properties:

$$(a + b) + c = a + (b + c) \tag{25}$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) \tag{26}$$

Distributive properties:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c) \tag{27}$$

$$(a + b) \cdot c = (a \cdot c) + (b \cdot c) \tag{28}$$

Commutative properties:

$$a + b = b + a \tag{29}$$

$$a \cdot b = b \cdot a \tag{30}$$

## 1.4 Multiply with -1

**Definition 4** *Mathematical operators may be swapped by multiplying with -1, because the result does not change.*

$$a + b = c \quad \leftrightarrow \quad -1 \cdot (-a - b) = c \tag{31}$$

**Example:**

$$(a - b)^2 = (b - a)^2 \tag{32}$$

## 1.5 Square Roots

**Definition 5**

$$\sqrt[1]{a} = a \tag{33}$$

$$\sqrt[2]{a} = \sqrt{a} \tag{34}$$

$$\sqrt{a^2} = a \tag{35}$$

$$\left(\sqrt{a}\right)^2 = a \tag{36}$$

$$\frac{1}{\sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \frac{1}{n} \tag{37}$$

$$\sqrt{n} \cdot \sqrt{n} = n \tag{38}$$

Addition:

$$a \sqrt[n]{x} + b \sqrt[n]{x} = (a + b) \sqrt[n]{x} \tag{39}$$

Subtraction:

$$a \sqrt[n]{x} - b \sqrt[n]{x} = (a - b) \sqrt[n]{x} \tag{40}$$

Multiplication:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b} \tag{41}$$

Division:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \tag{42}$$

Root exponentiation:

$$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{m} \tag{43}$$

Root extraction:

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a} \tag{44}$$

Transforming roots into exponents:

$$\sqrt[n]{a} = a \cdot \frac{1}{n} \tag{45}$$

$$\sqrt{a} = a \cdot \frac{1}{2} \tag{46}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \tag{47}$$

## 1.6 Exponentiation

**Definition 6**

$$x^n \cdot x^b = x^{n+b} \tag{48}$$

$$x^n \div x^b = \frac{x^n}{x^b} = x^{n-b} \tag{49}$$

$$\left(x^a\right)^b = x^{a \cdot b} \tag{50}$$

$$a^n \cdot a^b = (a \cdot b)^n \tag{51}$$

$$a^n \div b^n = \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \tag{52}$$

$$x^0 = 1 \tag{53}$$

$$x^1 = x \tag{54}$$

$$x^{-n} = \frac{1}{x^n} \tag{55}$$

$$\frac{1}{x} = x^{-1} \tag{56}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x} \tag{57}$$

**Disclaimer (for 56):** If n is even, then x must be $> 0$!

$$x^{\frac{m}{n}} = \sqrt[n]{m} \tag{58}$$

$$x^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{x^m}} \tag{59}$$

Addition:

$$ax^n + bx^n = (a+b)x^n \tag{60}$$

Subtraction:

$$ax^n - bx^n = (a-b)x^n \tag{61}$$

Transform a single root into a exponent:

$$\sqrt{a} = (a)^{\frac{1}{2}} \cdots \sqrt[3]{a} = (a)^{\frac{1}{3}} \cdots \tag{62}$$

# 2 Calculus

## 2.1 Simplifying Algebraic Terms

Exercise 1:

$$
\begin{aligned}
&[12x + 5x \cdot 2 - (10x - 8x)] + 18x \div 3 \\
&= [12x + 5x \cdot 2 - 10x + 8x] + 18x \div 3 && \text{Apply parentheses rule (22)} \\
&= [12x + 5x \cdot 2 - 2x] + 18x \div 3 && \text{Substract } -10x + 8 \\
&= 20x + 18x \div 3 && \text{Multiply } 5x \cdot 2 \\
&= 20x + 6x \\
&= 26x
\end{aligned}
$$

Exercise 2:

$$
\begin{aligned}
&x - ((x - 4) - (14 + 2x)) + 1 \\
&\Leftrightarrow x - (x - 4 - 14 - 2x) + 1 && \text{Apply parentheses rule (24)} \\
&\Leftrightarrow x - x + 4 + 14 + 2x + 1 && \text{(22)} \\
&\Leftrightarrow 19 + 2x \\
&\Leftrightarrow 2x + 19 && \text{(29)}
\end{aligned}
$$

## 2.2 Square Root Equations

Exercise 1:

$$
\begin{aligned}
2x + \sqrt{x^2+9} &= 4x + 3 && \big) -2x \\
\sqrt{x^2+9} &= 2x + 3 && \big) \,(\,)^2 \\
x^2 + 9 &= (2x+3)^2 && \big) \textit{ Apply first binomial rule (1)} \\
x^2 + 9 &= 4x^2 + 2x \cdot 2 \cdot 3 + 9 && \big) \textit{ Summarize} \\
x^2 + 9 &= 4x^2 + 12x + 9 && \big) -x^2 - 9 \\
0 &= 3x^2 + 12x
\end{aligned}
$$

$$0 = \underbrace{x}_{x_1}\underbrace{(3x+12)}_{x_2}$$

$$\Rightarrow x_1 = 0 \quad | \quad \text{Verify for } x_1\text{:} \quad 2 \cdot 0 + \sqrt{0^2+9} = 3$$

$$\Rightarrow x_2 = 3x + 12 = 0 \quad \Leftrightarrow \quad x_2 = -4 \quad | \quad \text{Verify for } x_2\text{:} \quad 4 \cdot (-4) + 3 = -13$$

$$\Rightarrow 3 = -13$$

$$Q.E.D$$

Exercise 2:

$$
\begin{aligned}
\sqrt{x+2} + \sqrt{x-1} &= 3 && \big) \,(\,)^2 \\
(\sqrt{x+2} + \sqrt{x-1})^2 &= 3^2 && \big) \textit{ Apply first binomial rule (1)} \\
\sqrt{x+2}^2 + 2 \cdot \sqrt{x+2} \cdot \sqrt{x-1} + \sqrt{x-1}^2 &= 9 && \big) \textit{ Square, and apply rule (35)} \\
x + 2 + 2\sqrt{(x+2)\cdot(x-1)} + x - 1 &= 9 && \big) \, x + 2 + x - 1 \\
2x + 1 + 2\sqrt{(x+2)\cdot(x-1)} &= 9 && \big) \, -2x - 1 \\
2\sqrt{(x+2)\cdot(x-1)} &= -2x + 8 && \big) \, \div 2 \\
\sqrt{(x+2)\cdot(x-1)} &= -x + 4 && \big) \textit{ Apply third binomial rule (3)} \\
\sqrt{x^2 - 1x + 2x - 2} &= -x + 4 && \big) \,(\,)^2 \\
x^2 - 1x + 2x - 2 &= (-x+4)^2 && \big) \textit{ No binomial rule applies for } (-x+4)^2 \\
x^2 + 1x - 2 &= x^2 - 2 \cdot 4x + 16 && \big) \, -x^2 - 1x + 2 \\
0 &= -9x + 18 && \big) \, +9x \\
9x &= 18 && \big) \, \div 9 \\
x &= 2
\end{aligned}
$$

Exercise 3:

$$\begin{aligned}
\sqrt{-3x-1-\sqrt{4x+5}} &= 1 && ()^2 \\
-3x-1-\sqrt{4x+5} &= 1 && +3x+1 \\
-\sqrt{4x+5} &= 3x+2 && ()^2 \\
4x+5 &= (3x+2)^2 && \text{Apply first binomial rule (1)} \\
4x+5 &= 9x^2+2\cdot 3x\cdot 2+4 \\
4x+5 &= 9x^2+12x+4 && -4x-5 \\
0 &= 9x^2+8x-1 && \div 9 \\
0 &= x^2+\frac{8}{9}\cdot x-\frac{1}{9} && \text{Apply } \mathbf{PQ} \text{ formula} \\
x_{1,2} &= -\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q} \\
x_1 &\Leftrightarrow \sqrt{-\frac{11}{3}}=1 \\
x_2 &\Leftrightarrow 1=1 \\
&\Rightarrow L=\{-1\}
\end{aligned}$$

Exercise 4:

$$\begin{aligned}
x &= \sqrt{x+20} && x^2 \\
x^2 &= x+20 && -x-20 \\
&\Leftrightarrow x^2-x-20=0 \\
&\Leftrightarrow (x-5)(x+4)=0 \\
&\Rightarrow x=5 \quad \wedge x=-4 \\
\Rightarrow \sqrt{5+20}=5 \quad | \quad \sqrt{-4+20} &= \sqrt{16}=4 \\
&\Rightarrow L=\{5\}
\end{aligned}$$

## 2.3 Changing the Subject of an Equation

Exercise 1:

$$\frac{6x+7}{9} - \frac{10x+7}{18} = \frac{9x+5}{14} - \frac{9x-16}{20} \quad \text{Find lowest common denominator.}$$

$$\Leftrightarrow \frac{6x+7}{2\cdot 9} - \frac{10x+7}{18} = \frac{9x+5}{14\cdot 10} - \frac{9x-16}{20\cdot 7}$$

$$\Leftrightarrow \frac{12+14}{18} - \frac{10x+7}{18} = \frac{90x+50}{140} - \frac{63x-112}{140}$$

$$\Leftrightarrow \frac{12x+14-(10x+7)}{18} = \frac{90x+50-(63x-112)}{140}$$

$$\Leftrightarrow \frac{12x+14-10x-7}{18} = \frac{90x+50-63x+112}{140}$$

$$\Leftrightarrow \frac{2x+7}{18} = \frac{27x+162}{140} \quad \cdot 18$$

$$\Leftrightarrow 2x+7 = \frac{(27x+162)\cdot 18}{140} \quad \text{Reduce fraction with 2.}$$

$$\Leftrightarrow 2x+7 = \frac{(27x+162)9}{70} \quad \cdot 70$$

$$\Leftrightarrow 140x+490 = 9(27x+162)$$

$$\Leftrightarrow 140x+490 = 243x+1458 \quad -140x$$

$$\Leftrightarrow 490 = 103x+1458 \quad -1458$$

$$\Leftrightarrow -968 = 103x \quad \text{Apply (29)}$$

$$\Leftrightarrow 103x = -968 \quad \div 103$$

$$\Leftrightarrow x = -\frac{968}{103}$$

Exercise 2:

$$\frac{3x-9}{6x-1} = \frac{4x-16}{8x-5} \qquad \cdot(8x-5)\cdot(6x-1)$$

$$\Leftrightarrow (3x-9)\cdot(8x-5) = (4x-16)\cdot(6x-1)$$

$$\Leftrightarrow 24x^2 - 15x - 72x + 45 = 24x^2 - 4x - 96x + 16 \qquad -24x^2$$

$$\Leftrightarrow -15x - 72x + 45 = -4x - 96x + 16 \qquad -45$$

$$\Leftrightarrow -15x - 72x = -4x - 96x - 29$$

$$\Leftrightarrow -87x = -100x - 29 \qquad +100x$$

$$\Leftrightarrow 13x = -29 \qquad \div 13$$

$$\Leftrightarrow x = -\frac{29}{13}$$

$$x^2 + (x-2)^2 = 10$$

$$\Leftrightarrow x^2 + (x-2)(x-2) = 10$$

$$\Leftrightarrow x^2 + x^2 - 2x - 2x + 4 = 10$$

$$\Leftrightarrow 2x^2 - 4x + 4 = 10 \qquad -10$$

$$\Leftrightarrow 2x^2 - 4x - 6 = 0 \qquad \div 2$$

$$\Leftrightarrow x^2 - 2x - 3 = 0 \qquad \text{Apply } \mathbf{PQ} \text{ formula.}$$

$$\Leftrightarrow x_1 = 3 \quad | \quad x_2 = -1$$

$$\Rightarrow L = \{3\}$$

## 2.4 Exponential Equations

Exercise 1:

$$\begin{aligned}
&\frac{(2^{-4})^{-5}\cdot 2^{17}}{(2^{-3})^{-6}\cdot(2^{-4})^{3}} && \text{Apply (50).}\\
&= \frac{2^{(-4)\cdot(-5)}\cdot 2^{17}}{2^{(-3)\cdot(-6)}\cdot 2^{(-4)\cdot 3}}\\
&= \frac{2^{20}\cdot 2^{17}}{2^{18}\cdot 2^{-12}} && \text{Apply (48) and (49)}\\
&= 2^{20+17-18-(-12)}\\
&= 20^{20+17-18+12}\\
&= 20^{31}
\end{aligned}$$

Exercise 2:

$$\begin{aligned}
&\frac{3^7\cdot(3^{-2})^3}{3^{-4}\cdot 3^7}\div\frac{(3^4)^{-3}}{(3^{-2})^{-6}} && \text{Apply rule (50)}\\
&= \frac{3^7\cdot 3^{(-2)\cdot 3}}{3^{-4}\cdot 3^7}\div\frac{3^{4\cdot(-3)}}{3^{(-2)\cdot(-6)}} && \text{Apply rule (14) and (48)}\\
&= \frac{3^7\cdot 3^{-6}}{3^{-4+7}}\cdot\frac{3^{12}}{3^{-12}}\\
&= \frac{3^7\cdot 3^{-6}\cdot 3^{12}}{3^3\cdot 3^{-12}} && \text{Apply (48) and (49)}\\
&= 3^{7-6+12-3-(-12)}\\
&= 3^{22}
\end{aligned}$$

Exercise 3:

$$\begin{aligned}
&\frac{12x^{-2}y^3}{8z^2}\cdot\frac{4y^{-2}z}{3x^{-5}}\div\frac{6z^{-3}}{2y^{-4}z} && \text{Apply rule (14).}\\
&= \frac{12x^{-2}y^3}{8z^2}\cdot\frac{4y^{-2}z}{3x^{-5}}\cdot\frac{2y^{-4}z}{6z^{-3}} && \text{Reduce } \frac{12\cdot 4\cdot 2}{8\cdot 3\cdot 6} \text{ and apply (48) and (49).}\\
&= \frac{2}{3}x^{-2-(-5)}\cdot y^{3-2-4}\cdot z^{1+1-2-(-3)}\\
&= \frac{2}{3}x^3y^{-3}z^3 && \text{Apply rule (55).}\\
&= \frac{2}{3}\frac{x^3z^3}{y^3}
\end{aligned}$$

Exercise 4:

$$\begin{aligned}
&\frac{2^4 \cdot x^5 \cdot y^7 \cdot z^8}{8 \cdot x^2 \cdot y^5 \cdot z^{10}} \div \frac{2 \cdot x^2 \cdot y^5 \cdot z^8}{16 \cdot x^4 \cdot y^3 \cdot z^5} && \text{Apply rule (14).} \\
&= \frac{2^4 \cdot x^5 \cdot y^7 \cdot z^8}{8 \cdot x^2 \cdot y^5 \cdot z^{10}} \cdot \frac{16 \cdot x^4 \cdot y^3 \cdot z^5}{2 \cdot x^2 \cdot y^5 \cdot z^8} && \text{Reduce } \frac{2}{8} \cdot \frac{16}{2} \text{ and apply (48).} \\
&= \frac{2}{1} \cdot \frac{x^{5+4} \cdot y^{7+3} \cdot z^{8+5}}{x^{2+2} \cdot y^{5+5} \cdot z^{10+8}} && \\
&= \frac{2}{1} \cdot \frac{x^9 \cdot y^{10} \cdot z^{13}}{x^4 \cdot y^{10} \cdot z^{18}} && \text{Apply rule (49).} \\
&= 2 \cdot x^5 \cdot z^{-5} && \text{Apply rule (55).} \\
&= \frac{2 \cdot x^5}{z^5} &&
\end{aligned}$$

Exercise 5:

$$\begin{aligned}
&\frac{4x^{2-m}y^{3\cdot m}}{7z^{m-n}} \div \frac{5z^{m+n}x^{3-m}}{14y^{1-2m}} && \text{Apply rule (14).} \\
&= \frac{4x^{2-m}y^{3\cdot m}}{7z^{m-n}} \cdot \frac{14y^{1-2m}}{5z^{m+n}x^{3-m}} && \text{Reduce } \frac{4}{7} \cdot \frac{14}{5} \text{ and apply (48) and (49).} \\
&= \frac{8}{5} \cdot \frac{x^{2-m-(3-m)} \cdot y^{3m+1-2m}}{z^{m-n+m+n}} && \\
&= \frac{8}{5} \cdot \frac{x^{-1} \cdot y^{m+1}}{z^{2m}} && \text{Apply rule (55).} \\
&= \frac{8}{5} \frac{y^{m+1}}{x \cdot z^{2m}} &&
\end{aligned}$$

## 2.5 Square Root Equations

Exercise 1:

$$
\begin{aligned}
& a^{\frac{2}{5}} \cdot \sqrt[5]{a^3} && \text{Apply rule (46).} \\
& a^{\frac{2}{5}} \cdot \left(a^3\right)^{\frac{1}{5}} && \text{Apply rule (50).} \\
&= a^{\frac{2}{5}} \cdot a^{\frac{3}{5}} && \text{Apply rule (48).} \\
&= a^{\frac{2}{5}+\frac{3}{5}} \\
&= a^1 && \text{Apply rule (54).} \\
&= a
\end{aligned}
$$

Exercise 2:

$$
\begin{aligned}
& \sqrt{a\sqrt[3]{a^2}} \quad \div \quad \left(a\sqrt{a^{-3}\sqrt{a^{-1}}}\right) && \text{Apply rule (47).} \\
&= \sqrt{a^1 \cdot a^{\frac{2}{3}}} \quad \div \quad \left(a^1 \cdot \sqrt{a^{-3} \cdot a^{-\frac{1}{2}}}\right) && \text{Apply rule (48) and (62).} \\
&= \left(a^{1+\frac{2}{3}}\right)^{\frac{1}{2}} \quad \div \quad \left(a^1 \cdot \left(a^{-3-\frac{1}{2}}\right)^{\frac{1}{2}}\right) && \text{Apply rule (50).} \\
&= a^{\frac{5}{3}\cdot\frac{1}{2}} \quad \div \quad \left(a^1 \cdot a^{-\frac{7}{2}\cdot\frac{1}{2}}\right) && \text{Simplify and apply rule (48).} \\
&= a^{\frac{5}{6}} \div a^{1-\frac{7}{4}} \\
&= a^{\frac{5}{6}} \div a^{-\frac{3}{4}} && \text{Apply rule (49).} \\
&= a^{\frac{5}{6}-(-\frac{3}{4})} \\
&= a^{\frac{19}{12}}
\end{aligned}
$$

Exercise 3:

$$
\begin{aligned}
& \sqrt[3]{(a^2)^{-5} \cdot \sqrt[4]{a^{16}}} && \text{Apply rule (62), (50), and (47).} \\
&= \left(a^{-10} \cdot a^{\frac{16}{4}}\right)^{\frac{1}{3}} \\
&= \left(a^{-10} \cdot a^4\right)^{\frac{1}{3}} && \text{Apply rule (48).} \\
&= \left(a^{-10+4}\right)^{\frac{1}{3}} \\
&= \left(a^{-6}\right)^{\frac{1}{3}} && \text{Apply rule (50) and (55).} \\
&= a^{-2} = \frac{1}{a^2}
\end{aligned}
$$

Exercise 4:

$$\frac{1}{\sqrt[3]{a\sqrt[5]{a^{-20}}}}$$ Apply rule (62) and (47).

$$= \frac{1}{\left(a^1 \cdot a^{\frac{-20}{5}}\right)^{\frac{1}{3}}}$$ $\frac{-20}{5}$ equals $-4$.

$$= \frac{1}{\left(a^1 \cdot a^{-4}\right)^{\frac{1}{3}}}$$ Apply rule (48).

$$= \frac{1}{\left(a^{1-4}\right)^{\frac{1}{3}}} = \frac{1}{\left(a^{-3}\right)^{\frac{1}{3}}}$$ Apply rule (50).

$$= \frac{1}{a^{-1}} = a^1 = a$$

Exercise 5:

$$\sqrt[3]{a\sqrt{a}} \div \sqrt{a^{-3}\sqrt[4]{a^6}}$$ Apply rule (62) and (47).

$$= \left(a^1 \cdot a^{\frac{1}{2}}\right)^{\frac{1}{3}} \div \left(a^{-3} \cdot a^{\frac{6}{4}}\right)^{\frac{1}{2}}$$ eq-50

$$= \left(a^{\frac{1}{3}} \cdot a^{\frac{1}{6}}\right) \div \left(a^{-\frac{3}{2}} \cdot a^{\frac{3}{4}}\right)$$ Apply rule (48).

$$= \left(a^{\frac{1}{3}+\frac{1}{6}}\right) \div \left(a^{\frac{-3}{2}+\frac{3}{4}}\right) = a^{\frac{1}{2}} \div a^{\frac{-3}{4}}$$ Apply rule (49).

$$= a^{\frac{1}{2}-\left(\frac{-3}{4}\right)} = a^{\frac{1}{2}+\frac{3}{4}}$$

$$= a^{\frac{5}{4}}$$

# 3 Inequalities

Coming up...

# 4 Ring Theory

In the near future...