Mathematics

Study Notes

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Abstract

This document pertains to mathematical concepts which I have studied. It includes my notes, work, and solutions when dealing with various problems. The source code for this document can be found on my *GitHub* at any time.

Contents

1 Definitions

1.1 Binomial Theorems

Definition 1

$$(a+b)^2 = a^2 + 2ab + b^2 \tag{1}$$

$$(a-b)^2 = a^2 - 2ab + b^2 \tag{2}$$

$$(a+b)(a-b) = a^2 - b^2 \tag{3}$$

For higher exponentiations:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \tag{4}$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \tag{5}$$

$$(-a-b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3 \tag{6}$$

Binomial Formula:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \tag{7}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \tag{8}$$

1.2 Fractions

Definition 2

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \tag{9}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b} \tag{10}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \tag{11}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd} \tag{12}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd} \tag{13}$$

Inverse:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \tag{14}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot b}{d \cdot c} \tag{15}$$

$$\frac{a \cdot \frac{b}{c}}{\frac{d}{e}} = \frac{\frac{a \cdot c + b}{c}}{\frac{d}{c}} = \frac{a \cdot c + b}{c} \cdot \frac{c}{d} = \frac{(a \cdot c + b) \cdot c}{c \cdot d} \tag{16}$$

$$\frac{ab}{c} = \frac{a}{c} \cdot b \tag{17}$$

$$\frac{a}{b} = \frac{1}{b} \cdot a \tag{18}$$

$$\frac{a \div b}{c} = \frac{a}{c} \div b \tag{19}$$

1.3 Parentheses Rules

Definition 3

$$+\,(a+b) = a+b \tag{20}$$

$$+\,(-a-b) = -a-b \tag{21}$$

$$-\,(a-b) = -a+b \tag{22}$$

$$-\,(-a+b) = +a-b \tag{23}$$

$$-\,(a+b) = -a-b \tag{24}$$

Associative properties:

$$(a+b)+c = a+(b+c) \tag{25}$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) \tag{26}$$

Distributive properties:

$$a \cdot (b+c) = (a \cdot b) + (a \cdot c) \tag{27}$$

$$(a+b) \cdot c = (a \cdot c) + (b \cdot c) \tag{28}$$

Commutative properties:

$$a + b = b + a \tag{29}$$

$$a \cdot b = b \cdot a \tag{30}$$

1.4 Multiply with -1

Definition 4 *Mathematical operators may be swapped by multiplying with -1, because the result does not change.*

$$a + b = c \quad \leftrightarrow \quad -1 \cdot (-a - b) = c \tag{31}$$

Example:

$$(a - b)^2 = (b - a)^2 \tag{32}$$

1.5 Square Roots

Definition 5

$$\sqrt[1]{a} = a \tag{33}$$

$$\sqrt[2]{a} = \sqrt{a} \tag{34}$$

$$\sqrt{a^2} = a \tag{35}$$

$$\left(\sqrt{a}\right)^2 = a \tag{36}$$

$$\frac{1}{\sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \frac{1}{n} \tag{37}$$

$$\sqrt{n} \cdot \sqrt{n} = n \tag{38}$$

Addition:

$$a \sqrt[n]{x} + b \sqrt[n]{x} = (a + b) \sqrt[n]{x} \tag{39}$$

Subtraction:

$$a \sqrt[n]{x} - b \sqrt[n]{x} = (a - b) \sqrt[n]{x} \tag{40}$$

Multiplication:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b} \tag{41}$$

Division:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \tag{42}$$

Root exponentiation:

$$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{m} \tag{43}$$

Root extraction:

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m\cdot n]{a} \tag{44}$$

Transforming roots into exponents:

$$\sqrt[n]{a} = a \cdot \frac{1}{n} \tag{45}$$

$$\sqrt{a} = a \cdot \frac{1}{2} \tag{46}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \tag{47}$$

1.6 Exponentiation

Definition 6

$$x^n \cdot x^b = x^{n+b} \tag{48}$$

$$x^n \div x^b = \frac{x^n}{x^b} = x^{n-b} \tag{49}$$

$$\left(x^a\right)^b = x^{a\cdot b} \tag{50}$$

$$a^n \cdot a^b = (a \cdot b)^n \tag{51}$$

$$a^n \div b^n = \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \tag{52}$$

$$x^0 = 1 \tag{53}$$

$$x^1 = x \tag{54}$$

$$x^{-n} = \frac{1}{x^n} \tag{55}$$

$$\frac{1}{x} = x^{-1} \tag{56}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x} \tag{57}$$

Disclaimer (for 56): If n is even, then x must be > 0!

$$x^{\frac{m}{n}} = \sqrt[n]{m} \tag{58}$$

$$x^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{x^m}} \tag{59}$$

Addition:

$$ax^n + bx^n = (a+b)x^n \tag{60}$$

Subtraction:

$$ax^n - bx^n = (a-b)x^n \tag{61}$$

Transform a single root into a exponent:

$$\sqrt{a} = (a)^{\frac{1}{2}} \cdots \sqrt[3]{a} = (a)^{\frac{1}{3}} \cdots \tag{62}$$

2 Calculus

2.1 Simplifying Algebraic Terms

Exercise 1:

$$
\begin{aligned}
[12x + 5x \cdot 2 - (10x - 8x)] + 18x \div 3 & \\
& \quad \textit{Apply parentheses rule (22)} \\
= [12x + 5x \cdot 2 - 10x + 8x] + 18x \div 3 & \\
& \quad \textit{Substract } -10x + 8 \\
= [12x + 5x \cdot 2 - 2x] + 18x \div 3 & \\
& \quad \textit{Multiply } 5x \cdot 2 \\
= 20x + 18x \div 3 & \\
= 20x + 6x & \\
= 26x &
\end{aligned}
$$

Exercise 2:

$$
\begin{aligned}
x - ((x - 4) - (14 + 2x)) + 1 & \\
& \quad \textit{Apply parentheses rule (24)} \\
\Leftrightarrow x - (x - 4 - 14 - 2x) + 1 & \\
& \quad \textit{(22)} \\
\Leftrightarrow x - x + 4 + 14 + 2x + 1 & \\
\Leftrightarrow 19 + 2x & \\
& \quad \textit{(29)} \\
\Leftrightarrow 2x + 19 &
\end{aligned}
$$

2.2 Square Root Equations

Exercise 1:

$$
\begin{aligned}
2x + \sqrt{x^2+9} &= 4x + 3 && \big)\ -2x \\
\sqrt{x^2+9} &= 2x + 3 && \big)\ (\ )^2 \\
x^2 + 9 &= (2x+3)^2 && \big)\ \textit{Apply first binomial rule (1)} \\
x^2 + 9 &= 4x^2 + 2x \cdot 2 \cdot 3 + 9 && \big)\ \textit{Summarize} \\
x^2 + 9 &= 4x^2 + 12x + 9 && \big)\ -x^2 - 9 \\
0 &= 3x^2 + 12x
\end{aligned}
$$

$$0 = \underbrace{x}_{x_1}\underbrace{(3x+12)}_{x_2}$$

$$\Rightarrow x_1 = 0 \quad | \quad \text{Verify for } x_1\text{:} \quad 2 \cdot 0 + \sqrt{0^2+9} = 3$$

$$\Rightarrow x_2 = 3x + 12 = 0 \quad \Leftrightarrow \quad x_2 = -4 \quad | \quad \text{Verify for } x_2\text{:} \quad 4 \cdot (-4) + 3 = -13$$

$$\Rightarrow 3 = -13$$

Q.E.D

Exercise 2:

$$
\begin{aligned}
\sqrt{x+2} + \sqrt{x-1} &= 3 && \big)\ (\ )^2 \\
(\sqrt{x+2} + \sqrt{x-1})^2 &= 3^2 && \big)\ \textit{Apply first binomial rule (1)} \\
\sqrt{x+2}^2 + 2 \cdot \sqrt{x+2} \cdot \sqrt{x-1} + \sqrt{x-1}^2 &= 9 && \big)\ \textit{Square, and apply rule (35)} \\
x + 2 + 2\sqrt{(x+2)\cdot(x-1)} + x - 1 &= 9 && \big)\ x + 2 + x - 1 \\
2x + 1 + 2\sqrt{(x+2)\cdot(x-1)} &= 9 && \big)\ -2x - 1 \\
2\sqrt{(x+2)\cdot(x-1)} &= -2x + 8 && \big)\ \div 2 \\
\sqrt{(x+2)\cdot(x-1)} &= -x + 4 && \big)\ \textit{Apply third binomial rule (3)} \\
\sqrt{x^2 - 1x + 2x - 2} &= -x + 4 && \big)\ (\ )^2 \\
x^2 - 1x + 2x - 2 &= (-x+4)^2 && \big)\ \textit{No binomial rule applies for } (-x+4)^2 \\
x^2 + 1x - 2 &= x^2 - 2 \cdot 4x + 16 && \big)\ -x^2 - 1x + 2 \\
0 &= -9x + 18 && \big)\ +9x \\
9x &= 18 && \big)\ \div 9 \\
x &= 2
\end{aligned}
$$

Exercise 3:

$$
\begin{aligned}
\sqrt{-3x-1-\sqrt{4x+5}} &= 1 && (\ )^2 \\
-3x-1-\sqrt{4x+5} &= 1 && +3x+1 \\
-\sqrt{4x+5} &= 3x+2 && (\ )^2 \\
4x+5 &= (3x+2)^2 && \text{Apply first binomial rule (1)} \\
4x+5 &= 9x^2+2\cdot 3x\cdot 2+4 && \\
4x+5 &= 9x^2+12x+4 && -4x-5 \\
0 &= 9x^2+8x-1 && \div 9 \\
0 &= x^2+\frac{8}{9}\cdot x-\frac{1}{9} && \text{Apply } \mathbf{PQ} \text{ formula} \\
x_{1,2} &= -\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q} && \\
x_1 &\Leftrightarrow \sqrt{-\frac{11}{3}}=1 && \\
x_2 &\Leftrightarrow 1=1 && \\
&\Rightarrow L=\{-1\} &&
\end{aligned}
$$

Exercise 4:

$$
\begin{aligned}
x &= \sqrt{x+20} && x^2 \\
x^2 &= x+20 && -x-20 \\
&\Leftrightarrow x^2-x-20=0 && \\
&\Leftrightarrow (x-5)(x+4)=0 && \\
&\Rightarrow x=5 \quad \wedge x=-4 && \\
\Rightarrow \sqrt{5+20}=5 \quad | \quad &\sqrt{-4+20}=\sqrt{16}=4 && \\
&\Rightarrow L=\{5\} &&
\end{aligned}
$$

2.3 Changing the Subject of an Equation

Exercise 1:

$$
\begin{aligned}
\frac{6x+7}{9} - \frac{10x+7}{18} &= \frac{9x+5}{14} - \frac{9x-16}{20} && \text{Find lowest common denominator.} \\
\Leftrightarrow \frac{6x+7}{2\cdot 9} - \frac{10x+7}{18} &= \frac{9x+5}{14\cdot 10} - \frac{9x-16}{20\cdot 7} \\
\Leftrightarrow \frac{12+14}{18} - \frac{10x+7}{18} &= \frac{90x+50}{140} - \frac{63x-112}{140} \\
\Leftrightarrow \frac{12x+14-(10x+7)}{18} &= \frac{90x+50-(63x-112)}{140} \\
\Leftrightarrow \frac{12x+14-10x-7}{18} &= \frac{90x+50-63x+112}{140} \\
\Leftrightarrow \frac{2x+7}{18} &= \frac{27x+162}{140} && \cdot 18 \\
\Leftrightarrow 2x+7 &= \frac{(27x+162)\cdot 18}{140} && \text{Reduce fraction with 2.} \\
\Leftrightarrow 2x+7 &= \frac{(27x+162)9}{70} && \cdot 70 \\
\Leftrightarrow 140x+490 &= 9(27x+162) \\
\Leftrightarrow 140x+490 &= 243x+1458 && -140x \\
\Leftrightarrow 490 &= 103x+1458 && -1458 \\
\Leftrightarrow -968 &= 103x && \text{Apply (29)} \\
\Leftrightarrow 103x &= -968 && \div 103 \\
\Leftrightarrow x &= -\frac{968}{103}
\end{aligned}
$$

Exercise 2:

$$
\begin{aligned}
\frac{3x-9}{6x-1} &= \frac{4x-16}{8x-5} && \cdot(8x-5)\cdot(6x-1)\\
\Leftrightarrow (3x-9)\cdot(8x-5) &= (4x-16)\cdot(6x-1)\\
\Leftrightarrow 24x^2-15x-72x+45 &= 24x^2-4x-96x+16 && -24x^2\\
\Leftrightarrow -15x-72x+45 &= -4x-96x+16 && -45\\
\Leftrightarrow -15x-72x &= -4x-96x-29\\
\Leftrightarrow -87x &= -100x-29 && +100x\\
\Leftrightarrow 13x &= -29 && \div 13\\
\Leftrightarrow x &= -\frac{29}{13}
\end{aligned}
$$

$$
\begin{aligned}
x^2+(x-2)^2 &= 10\\
\Leftrightarrow x^2+(x-2)(x-2) &= 10\\
\Leftrightarrow x^2+x^2-2x-2x+4 &= 10\\
\Leftrightarrow 2x^2-4x+4 &= 10 && -10\\
\Leftrightarrow 2x^2-4x-6 &= 0 && \div 2\\
\Leftrightarrow x^2-2x-3 &= 0 && \text{Apply } \textbf{PQ} \text{ formula.}\\
\Leftrightarrow x_1 = 3 \quad | \quad x_2 &= -1\\
\Rightarrow L &= \{3\}
\end{aligned}
$$

2.4 Exponential Equations

Exercise 1:

$$\frac{(2^{-4})^{-5}\cdot 2^{17}}{(2^{-3})^{-6}\cdot(2^{-4})^{3}} \qquad \text{Apply (50).}$$

$$=\frac{2^{(-4)\cdot(-5)}\cdot 2^{17}}{2^{(-3)\cdot(-6)}\cdot 2^{(-4)\cdot 3}}$$

$$=\frac{2^{20}\cdot 2^{17}}{2^{18}\cdot 2^{-12}} \qquad \text{Apply (48) and (49)}$$

$$=2^{20+17-18-(-12)}$$

$$=20^{20+17-18+12}$$

$$=20^{31}$$

Exercise 2:

$$\frac{3^{7}\cdot(3^{-2})^{3}}{3^{-4}\cdot 3^{7}}\div\frac{(3^{4})^{-3}}{(3^{-2})^{-6}} \qquad \text{Apply rule (50)}$$

$$=\frac{3^{7}\cdot 3^{(-2)\cdot 3}}{3^{-4}\cdot 3^{7}}\div\frac{3^{4\cdot(-3)}}{3^{(-2)\cdot(-6)}} \qquad \text{Apply rule (14) and (48)}$$

$$=\frac{3^{7}\cdot 3^{-6}}{3^{-4+7}}\cdot\frac{3^{12}}{3^{-12}}$$

$$=\frac{3^{7}\cdot 3^{-6}\cdot 3^{12}}{3^{3}\cdot 3^{-12}} \qquad \text{Apply (48) and (49)}$$

$$=3^{7-6+12-3-(-12)}$$

$$=3^{22}$$

Exercise 3:

$$\frac{12x^{-2}y^{3}}{8z^{2}}\cdot\frac{4y^{-2}z}{3x^{-5}}\div\frac{6z^{-3}}{2y^{-4}z} \qquad \text{Apply rule (14).}$$

$$=\frac{12x^{-2}y^{3}}{8z^{2}}\cdot\frac{4y^{-2}z}{3x^{-5}}\cdot\frac{2y^{-4}z}{6z^{-3}} \qquad \text{Reduce } \frac{12\cdot 4\cdot 2}{8\cdot 3\cdot 6} \text{ and apply (48) and (49).}$$

$$=\frac{2}{3}x^{-2-(-5)}\cdot y^{3-2-4}\cdot z^{1+1-2-(-3)}$$

$$=\frac{2}{3}x^{3}y^{-3}z^{3} \qquad \text{Apply rule (55).}$$

$$=\frac{2}{3}\frac{x^{3}z^{3}}{y^{3}}$$

Exercise 4:

$$\frac{2^4 \cdot x^5 \cdot y^7 \cdot z^8}{8 \cdot x^2 \cdot y^5 \cdot z^{10}} \div \frac{2 \cdot x^2 \cdot y^5 \cdot z^8}{16 \cdot x^4 \cdot y^3 \cdot z^5} \qquad \text{Apply rule (14).}$$

$$= \frac{2^4 \cdot x^5 \cdot y^7 \cdot z^8}{8 \cdot x^2 \cdot y^5 \cdot z^{10}} \cdot \frac{16 \cdot x^4 \cdot y^3 \cdot z^5}{2 \cdot x^2 \cdot y^5 \cdot z^8} \qquad \text{Reduce } \frac{2}{8} \cdot \frac{16}{2} \text{ and apply (48).}$$

$$= \frac{2}{1} \cdot \frac{x^{5+4} \cdot y^{7+3} \cdot z^{8+5}}{x^{2+2} \cdot y^{5+5} \cdot z^{10+8}}$$

$$= \frac{2}{1} \cdot \frac{x^9 \cdot y^{10} \cdot z^{13}}{x^4 \cdot y^{10} \cdot z^{18}} \qquad \text{Apply rule (49).}$$

$$= 2 \cdot x^5 \cdot z^{-5} \qquad \text{Apply rule (55).}$$

$$= \frac{2 \cdot x^5}{z^5}$$

Exercise 5:

$$\frac{4x^{2-m}y^{3 \cdot m}}{7z^{m-n}} \div \frac{5z^{m+n}x^{3-m}}{14y^{1-2m}} \qquad \text{Apply rule (14).}$$

$$= \frac{4x^{2-m}y^{3 \cdot m}}{7z^{m-n}} \cdot \frac{14y^{1-2m}}{5z^{m+n}x^{3-m}} \qquad \text{Reduce } \frac{4}{7} \cdot \frac{14}{5} \text{ and apply (48) and (49).}$$

$$= \frac{8}{5} \cdot \frac{x^{2-m-(3-m)} \cdot y^{3m+1-2m}}{z^{m-n+m+n}}$$

$$= \frac{8}{5} \cdot \frac{x^{-1} \cdot y^{m+1}}{z^{2m}} \qquad \text{Apply rule (55).}$$

$$= \frac{8}{5} \frac{y^{m+1}}{x \cdot z^{2m}}$$

2.5 Square Root Equations

Exercise 1:

$$\begin{aligned}
&a^{\frac{2}{5}} \cdot \sqrt[5]{a^3} && \text{Apply rule (46).}\\
&a^{\frac{2}{5}} \cdot (a^3)^{\frac{1}{5}} && \text{Apply rule (50).}\\
&= a^{\frac{2}{5}} \cdot a^{\frac{3}{5}} && \text{Apply rule (48).}\\
&= a^{\frac{2}{5}+\frac{3}{5}} && \\
&= a^1 && \text{Apply rule (54).}\\
&= a &&
\end{aligned}$$

Exercise 2:

$$\begin{aligned}
&\sqrt{a\sqrt[3]{a^2}} \div \left(a\sqrt{a^{-3}\sqrt{a^{-1}}}\right) && \text{Apply rule (47).}\\
&= \sqrt{a^1 \cdot a^{\frac{2}{3}}} \div \left(a^1 \cdot \sqrt{a^{-3} \cdot a^{-\frac{1}{2}}}\right) && \text{Apply rule (48) and (62).}\\
&= \left(a^{1+\frac{2}{3}}\right)^{\frac{1}{2}} \div \left(a^1 \cdot \left(a^{-3-\frac{1}{2}}\right)^{\frac{1}{2}}\right) && \text{Apply rule (50).}\\
&= a^{\frac{5}{3}\cdot\frac{1}{2}} \div \left(a^1 \cdot a^{-\frac{7}{2}\cdot\frac{1}{2}}\right) && \text{Simplify and apply rule (48).}\\
&= a^{\frac{5}{6}} \div a^{1-\frac{7}{4}} && \\
&= a^{\frac{5}{6}} \div a^{-\frac{3}{4}} && \text{Apply rule (49).}\\
&= a^{\frac{5}{6}-(-\frac{3}{4})} && \\
&= a^{\frac{19}{12}} &&
\end{aligned}$$

Exercise 3:

$$\begin{aligned}
&\sqrt[3]{(a^2)^{-5} \cdot \sqrt[4]{a^{16}}} && \text{Apply rule (62), (50), and (47).}\\
&= \left(a^{-10} \cdot a^{\frac{16}{4}}\right)^{\frac{1}{3}} && \\
&= \left(a^{-10} \cdot a^4\right)^{\frac{1}{3}} && \text{Apply rule (48).}\\
&= \left(a^{-10+4}\right)^{\frac{1}{3}} && \\
&= \left(a^{-6}\right)^{\frac{1}{3}} && \text{Apply rule (50) and (55).}\\
&= a^{-2} = \frac{1}{a^2} &&
\end{aligned}$$

Exercise 4:

$$\frac{1}{\sqrt[3]{a\sqrt[5]{a^{-20}}}} \qquad \text{Apply rule (62) and (47).}$$

$$= \frac{1}{\left(a^1 \cdot a^{\frac{-20}{5}}\right)^{\frac{1}{3}}} \qquad \frac{-20}{5} \text{ equals } -4.$$

$$= \frac{1}{(a^1 \cdot a^{-4})^{\frac{1}{3}}} \qquad \text{Apply rule (48).}$$

$$= \frac{1}{(a^{1-4})^{\frac{1}{3}}} = \frac{1}{(a^{-3})^{\frac{1}{3}}} \qquad \text{Apply rule (50).}$$

$$= \frac{1}{a^{-1}} = a^1 = a$$

Exercise 5:

$$\sqrt[3]{a\sqrt{a}} \div \sqrt{a^{-3}\sqrt[4]{a^6}} \qquad \text{Apply rule (62) and (47).}$$

$$= \left(a^1 \cdot a^{\frac{1}{2}}\right)^{\frac{1}{3}} \div \left(a^{-3} \cdot a^{\frac{6}{4}}\right)^{\frac{1}{2}} \qquad \text{eq-50}$$

$$= \left(a^{\frac{1}{3}} \cdot a^{\frac{1}{6}}\right) \div \left(a^{-\frac{3}{2}} \cdot a^{\frac{3}{4}}\right) \qquad \text{Apply rule (48).}$$

$$= \left(a^{\frac{1}{3}+\frac{1}{6}}\right) \div \left(a^{\frac{-3}{2}+\frac{3}{4}}\right) = a^{\frac{1}{2}} \div a^{\frac{-3}{4}} \qquad \text{Apply rule (49).}$$

$$= a^{\frac{1}{2}-\left(\frac{-3}{4}\right)} = a^{\frac{1}{2}+\frac{3}{4}}$$

$$= a^{\frac{5}{4}}$$

3 Inequalities

Coming up...

4 Ring Theory

In the near future...