条件概率：Conditional Probability
The conditional probability of an event F happening given event E => P(F|E)
F given E

Chain rule $\quad P(F|E)P(E) = P(E \cap F) = P(E|F)P(F).$

全概率公式：Total Probability
Random Variables => X   possible values => x
binomial coefficient 二项式系数/组合数

$$p(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

P to the power k

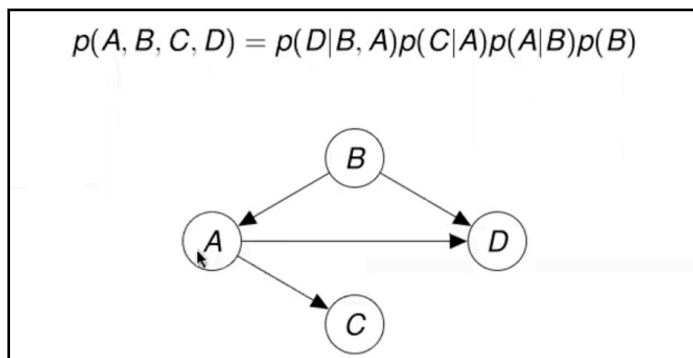$$\sum_{i=1}^{\infty}$$ (weighted) sum over xxx

Expectations 期望
Variance 方差
times 乘 multiply.   >. great than   X' = X prime
factorization 分解式子 factorized 分解 v
分子 numerator 分母 denominator

贝叶斯网络：

$p(A, B, C, D) = p(D|B, A)p(C|A)p(A|B)p(B)$

$$p(D) = \sum_A \sum_B \sum_C p(A, B, C, D),$$
$$= \sum_A \sum_B \sum_C p(D|B, A)p(C|A)p(A|B)p(B),$$
$$= \sum_A \sum_B p(D|B, A)\underbrace{\left(\sum_C p(C|A)\right)}_{=1} p(A|B)p(B),$$
$$= \sum_A \sum_B p(D|B, A)p(A|B)p(B).$$

$$p(C) = \sum_A \sum_B \sum_D p(A, B, C, D),$$
$$= \sum_A \sum_B \sum_D p(D|B, A)p(C|A)p(A|B)p(B),$$
$$= \sum_A \sum_B \underbrace{\left(\sum_D p(D|B, A)\right)}_{=1} p(C|A)p(A|B)p(B),$$
$$= \sum_A \sum_B p(C|A)p(A|B)p(B),$$

$$p(C|B) = \sum_A \sum_D p(A, C, D|B),$$
$$= \sum_A \sum_D \frac{p(A, B, C, D)}{p(B)},$$
$$= \frac{1}{\cancel{p(B)}} \sum_A \underbrace{\sum_D p(D|B, A)}_{=1} p(C|A)p(A|B)\cancel{p(B)},$$
$$= \sum_A p(C|A)p(A|B).$$

$$p(A|D) = \sum_B \sum_C \frac{p(D|B, A)p(C|A)p(A|B)p(B)}{p(D)},$$
$$= \frac{1}{p(D)} \underbrace{\sum_C p(C|A)}_{=1} \sum_B p(D|B, A) \underbrace{\underbrace{p(A|B)p(B)}_{=p(A,B)}}_{p(D,A,B)},$$
$$= \frac{1}{p(D)} \sum_B p(D|B, A)p(A|B)p(B),$$

Example 1

$$p(Ac, PT, T, W, Po) = p(T|PT, W, Ac)p(PT)p(W)p(Ac|W)p(Po|Ac)$$

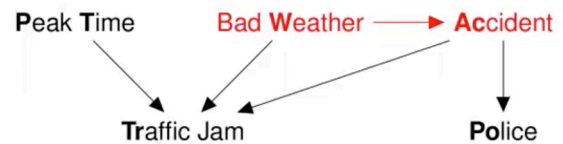**Peak Time** — **Bad Weather** → **Accident**, **Traffic Jam**, **Police**

$$p(PT = 1) = 0.17, p(W = 1) = 0.3,$$
$$p(Po = 1|Ac = 1) = 0.9, p(Po = 1|Ac = 0) = 0,$$
$$p(Ac = 1|W = 1) = 0.1, p(Ac = 1|W = 0) = 0.01$$

Tr	PT	W	$p(Tr|PT, W, Ac = 0)$
0	0	0	0.5
0	0	1	0.8
0	1	0	0.6
0	1	1	0.6
1	0	0	0.05
1	0	1	0.2
1	1	0	0.4
1	1	1	0.9

Question: Compute $p(W = 1|Ac = 1)$.

Peak Time — **Bad Weather** → **Accident**, **Traffic Jam**, **Police**

Solution: From Bayes' Theorem:

$$p(W = 1|Ac = 1) = \frac{p(Ac = 1|W = 1)p(W = 1)}{p(Ac = 1)}$$

Since we computed $P(Ac = 1)$ previously:

$$p(W = 1|Ac = 1) = \frac{0.1 \times 0.3}{0.037}$$

Question: Compute $p(Ac = 1)$.

Peak Time — **Bad Weather** → **Accident**, **Traffic Jam**, **Police**

Solution:

$$p(Ac = 0) = p(Ac = 0|W = 0)p(W = 0) + p(Ac = 0|W = 1)p(W = 1)$$
$$= (1 - 0.01) \times (1 - 0.3) + (1 - 0.1) \times 0.3,$$
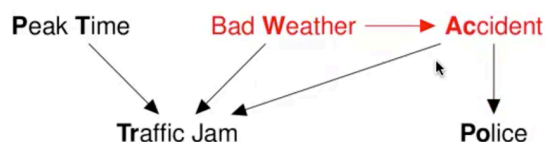$$= 0.963,$$
$$p(Ac = 1) = 1 - P(Ac = 0) = \underline{0.037}.$$

Question: Compute $p(Po = 1)$.

Peak Time — **Bad Weather** → **Accident**, **Traffic Jam**, **Police**
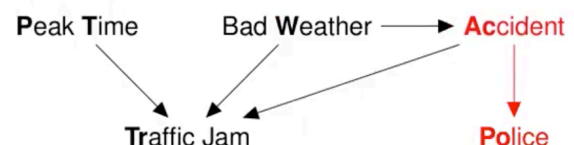
Solution:

$$p(Po = 1) = p(Po = 1|Ac = 0)p(Ac = 0) + p(Po = 1|Ac = 1)p(Ac = 1),$$
$$= 0 \times 0.963 + 0.9 \times 0.037,$$
$$= \underline{0.0333}$$

Example 2

$$\sum_A P(A|B,C) = 1$$

且其他P没有依赖A才可消掉

求和公式的字母不能有题目给定的字母

根据需要变化的字母来定！！

Peak Time   **Bad Weather** ⟶ **Ac**cident

Traffic Jam   **Police**

$p(PT = 1) = 0.17, p(W = 1) = 0.3$

Tr	PT	W	$p(Tr|PT, W, Ac = 0)$
1	0	0	0.05
1	0	1	0.2
1	1	0	0.4
1	1	1	0.9

$$p(Tr = 1 | Ac = 0) = \sum_W \sum_{PT} p(Tr = 1 | PT, W, Ac = 0)p(PT)p(W),$$
$$= 0.05(1 - 0.17)(1 - 0.3)$$
$$+ 0.2(1 - 0.17)(0.3)$$
$$+ 0.4(0.17)(1 - 0.3)$$
$$+ 0.9(0.17)(0.3)$$