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Math 240

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Discrete Structures Notes

Sets

Defining Sets

- Set: Collection of objects of the same nature (Bag analogy)
  Ex: $A = \{1, 2, 6\}$

- Empty Set: Set that contains nothing
  $\emptyset = \{\}$

- x is an element of a set A **belongs**
  $x \in A$

- If all elements of A are in B
  $A \subseteq B$

- $\mathbb{N}$ The natural numbers
  $\{1, 2, 3, 4, \ldots\}$
  **Note: Zero is not natural*

- $\mathbb{Z}$ The Integers
  $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

- Using the `...` from the previous example is called `Definition by Extension`

- Two Sets are Equal if:
  $A \subseteq B \wedge B \subseteq A$
  $A = B$

- Repetitions and order does not matter in sets.

- Sets can also be defined by `Comprehension`
  Ex:
  $A = \{x \in S | x \text{ satisfies a property}\}$
  — means `such that`

- Set of Even Numbers
  $E = \{x \in \mathbb{Z} | n = 2k \text{ for some } k \in \mathbb{Z}\}$

- Set of Odd Numbers
  $O = \{x \in \mathbb{Z} | n = 2k + 1 \text{ for some } k \in \mathbb{Z}\}$

- Set of Rational Numbers
  $\mathbb{Q} = \{\frac{a}{b} | a \in \mathbb{Z},\, b \in \mathbb{N},\, GCD(a, b) = 1\}$

- Cardinality of a set is the `number of elements`
  $A = \{1, 2, 3\}, |A| = 3$
  $|\{\{1, 2\}\}| = 1$

Russel's Set Paradox

$$R = \{x \text{ is a set} | x \notin x\}$$

Ex: If $x = \{1, 2, x\}$ then $x \in x$ so $x \notin R$

The Question is $R \in R$?

1. If $R \in R$ then $R \notin R$
2. If $R \notin R$ then $R \in R$

One way to stop the paradox is to forbid recursive sets.

Operations on Sets

- We are always assuming a universe u. Which is deduced from context. A set that contains all sets which are discussed as subsets.

- **Union:**
  $A \cup B = \{e \in u \mid e \in A$ **or** $e \in B\}$

- **Intersection:**
  $A \cap B = \{e \in u \mid e \in A$ **and** $e \in B\}$

- **Complement:**
  $\bar{A} = \{e \in u \mid e \notin A\}$

- **Set Difference:**
  $A \setminus B = A - B$
  $\{x \in u \mid x \in A, x \notin B\}$

- **Symmetric Difference:**
  $A \oplus B = \{x \in u \mid x \in A \text{ or } x \in B\}$

- Two Identities:
  $A \oplus B = (A \setminus B) \cup (B \setminus A)$
  $A \setminus B = A \cap \bar{B}$

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Laws of Boolean Algebra

- Identity Laws:
  $A \cap u = A$, $A \cup \emptyset = A$
- Domination Laws:
  $A \cup u = u$, $A \cap \emptyset = \emptyset$
- Idempotent Laws:
  $A \cup A = A$, $A \cap A = A$
- Double Complementary Law:
  $\bar{\bar{A}} = A$
- Commutative Laws:
  $A \cup B = B \cup A$, $A \cap B = B \cap A$
- Associative laws:
  $A \cup (B \cup C) = (A \cup B) \cup C$
  $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Law:
  Applies for both union and intersection
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup B)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- De Morgan's Laws
  $(A \cup B)' = \bar{A} \cap \bar{B}$
  $(A \cap B)' = \bar{A} \cup \bar{B}$
- Absorption Laws
  $A \cup (A \cap B) = A$
  $A \cap (A \cup B) = A$
- Set Product
  $A \times B = \{(a, b) \in R^2 \mid a \in A, b \in B\}$
  $|A \times B| = |A| \times |B|$
- Power Set
  $P(A) =$ set of all subsets of A
  $= \{x \in u \mid x \subseteq A\}$
  $|P(A)| = 2^{|A|}$
  $P(\{1, 2, 3, \} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\})$

  Each element can be a Yes or No if it shows up in a given combination yes or no is a set of 2 which means it is 2^n combinations because you multiply 2 possiblilies n times.