

Discrete Structures Notes

Sets

Defining Sets

- Set: Collection of objects of the same nature (Bag analogy)
Ex: $A = \{1, 2, 6\}$
- Empty Set: Set that contains nothing
 $\emptyset = \{\}$
- x is an element of a set A **belongs**
 $x \in A$
- If all elements of A are in B
 $A \subseteq B$
- \mathbb{N} The natural numbers
 $\{1, 2, 3, 4, \dots\}$
**Note: Zero is not natural*
- \mathbb{Z} The Integers
 $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- Using the \dots from the previous example is called **Definition by Extension**
- Two Sets are Equal if:
 $A \subseteq B \wedge B \subseteq A$
 $A = B$
- Repetitions and order does not matter in sets.
- Sets can also be defined by **Comprehension**
Ex:
 $A = \{x \in S | x \text{ satisfies a property}\}$
— means **such that**
- Set of Even Numbers
 $E = \{x \in \mathbb{Z} | n = 2k \text{ for some } k \in \mathbb{Z}\}$
- Set of Odd Numbers
 $O = \{x \in \mathbb{Z} | n = 2k + 1 \text{ for some } k \in \mathbb{Z}\}$

- Set of Rational Numbers
 $\mathbb{Q} = \{\frac{a}{b} | a \in \mathbb{Z}, b \in \mathbb{N}, GCD(a, b) = 1\}$
- Cardinality of a set is the **number of elements**
 $A = \{1, 2, 3\}, |A| = 3$
 $|\{\{1, 2\}\}| = 1$

Russel's Set Paradox

$$R = \{x \text{ is a set} | x \notin x\}$$

Ex: If $x = \{1, 2, x\}$ then $x \in x$ so $x \notin R$

The Question is $R \in R$?

1. If $R \in R$ then $R \notin R$
2. If $R \notin R$ then $R \in R$

One way to stop the paradox is to forbid recursive sets.

Operations on Sets

- We are always assuming a universe u . Which is deduced from context. A set that contains all sets which are discussed as subsets.
- **Union:**
 $A \cup B = \{e \in u | e \in A \text{ or } e \in B\}$
- **Intersection:**
 $A \cap B = \{e \in u | e \in A \text{ and } e \in B\}$
- **Complement:**
 $\bar{A} = \{e \in u | e \notin A\}$
- **Set Difference:**
 $A \setminus B = A - B$
 $\{x \in u | x \in A, x \notin B\}$
- **Symmetric Difference:**
 $A \oplus B = \{x \in u | x \in A \text{ or } x \in B\}$
- Two Identities:
 $A \oplus B = (A \setminus B) \cup (B \setminus A)$
 $A \setminus B = A \cap \bar{B}$

Laws of Boolean Algebra

- Identity Laws:
 $A \cap u = A, A \cup \emptyset = A$
- Domination Laws:
 $A \cup u = u, A \cap \emptyset = \emptyset$
- Idempotent Laws:
 $A \cup A = A, A \cap A = A$
- Double Complementary Law:
 $\bar{\bar{A}} = A$
- Commutative Laws:
 $A \cup B = B \cup A, A \cap B = B \cap A$
- Associative laws:
 $A \cup (B \cup C) = (A \cup B) \cup C$
 $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Law:
 Applies for both union and intersection
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- De Morgan's Laws
 $(A \cup B)' = \bar{A} \cap \bar{B}$
 $(A \cap B)' = \bar{A} \cup \bar{B}$
- Absorption Laws
 $A \cup (A \cap B) = A$
 $A \cap (A \cup B) = A$
- Set Product
 $A \times B = \{(a, b) \in R^2 \mid a \in A, b \in B\}$
 $|A \times B| = |A| \times |B|$
- Power Set
 $P(A) = \text{set of all subsets of } A$
 $= \{x \in u \mid x \subseteq A\}$
 $|P(A)| = 2^{|A|}$
 $P(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

Each element can be a Yes or No if it shows up in a given combination yes or no is a set of 2 which means it is 2^n combinations because you multiply 2 possibilities n times.

1.1 **A** Find this this this and that!