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Math 133

Linear Algebra Notes

Study Sheets

Part One:

- **Linear Equation:**
  Equation in the form $a_1x_1 + a_2x_2 + \ldots a_nx_n = b$

- **Solution:**
  Parallel lines - No Solution - Inconsistent
  Same line - Infinite Solutions - Consistent
  Intersection - One Solution - Consistent
  This concept extends to 3D plans or even higher dimensions
  Number of variables = dimension of space

- **Matrix Notation:**

$$7x + 5y - 3z = 16$$

$$3x - 5y + 2z = -8$$

$$5x + 3y - 7z = 0$$

Coefficient matrix

$$\begin{bmatrix} 7 & 5 & -3 \\ 3 & -5 & 2 \\ 5 & 3 & -7 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 7 & 5 & -3 & 16 \\ 3 & -5 & 2 & -8 \\ 5 & 3 & -7 & 0 \end{bmatrix}$$

m equations and n variables leads to a m x n matrix.

- **Elementary Row Operations**

  - Row Swap) Exchange any two rows.
  - (Scalar Multiplication) Multiply any row by a constant.
  - (Row Sum) Add a multiple of one row to another row

- **Reduced Row-Echelon Form**
  Example:

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$$

  - Any row with non-zero entries has a leading one
  - All entries in a column with a leading one are zero
  - Each leading one has another up and to the left

  Elimination is complete and we can look at the Rank of the matrix

$$\boxed{\text{Rank} = \text{number of leading ones}}$$

  - Rank = Number of columns in coefficient matrix → at most one solution
  - Rank < Number of columns in coefficient matrix → Infinite or No solutions

- **Types of Matrices**

$$\begin{gathered} \text{m} = \text{number of rows} \\ \text{n} = \text{number of columns} \\ \text{m} = \text{n results in a square matrix} \end{gathered}$$

  The identity matrix I is an RREF square matrix with any size.
  Example:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - An upper triangular matrix is one where all entries below the diagonal are 0.
  - A lower triangular matrix one where all entries above the diagonal are 0.
  - **Vector:** Matrix of one column
  - **Row Vector:** Matrix of one row

  Linear equations can also be represented as vectors.

$$3x + y = 7$$

$$x + 2y = 4$$

$$x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

  This is the vector form of the linear system. This shows that scalars can be factored in and out of Matrices.

- **Matrix Addition**
  Two Matrices must have the same dimension in order to add them.
  Example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 3 & 1 \\ 5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 4 \\ 9 & 8 & 5 \end{bmatrix}$$

  Reminder that this is a 2 x 3 matrix and is the sum of the first two.

- **Matrix Subtraction**
  Works in the exact same way as addition.
  *Note**
  Addition is commutative- Subtraction is not

- **Matrix Multiplication**
  In order for two matrices (n x m and q x p) to be multiplied.
  m = q MUST be true.
  Example:

$$A \times B$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5 * 1 + 2 * 7 & 6 * 1 + 8 * 2 \\ 5 * 3 + 4 * 7 & 6 * 3 + 8 * 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

  Note = Multiplication is not commutative*

- **The Determinant**

1.1 ***A*** *Find this this this and that!*