

PROBLEMS OF COMPUTING LYAPUNOV EXPONENTS IN POWER ELECTRONICS

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ABSTRACT

Ideal power converters are governed by differential equations with discontinuous right-hand sides. General-purpose algorithms to compute their Lyapunov exponents assume smooth equations, so are not directly applicable to power electronics. We review the methods and their problems, focusing on the largest Lyapunov exponent, which dominates system behavior. The variational approach can incorporate an algorithm by Müller (1995) to cope with the discontinuities found in power electronics. As an example, the method is applied to verify chaos in a buck dc-dc converter.

1. INTRODUCTION

Power electronic systems have been shown to exhibit a wide variety of nonlinear phenomena [1], [2]. In dc-dc converters, this behavior has been studied by means of iterated mappings [3].

Lyapunov exponents are the best indicators to categorize the different classes of nonlinear phenomena. A positive Lyapunov exponent confirms chaos. If the system equations are known, algorithms [4] can readily be applied to compute the largest or all of the exponents. Alternatively, the method of embedded dimensions may be applied to time series resulting from simulations and experiments to estimate their exponents [5], [6]. These approaches work satisfactorily for smooth systems, i.e. those where the vector field is continuously differentiable. However, the methods may fail for systems with discontinuities. Unfortunately, there is very little literature on this issue.

Because power electronic systems are characterized by switching discontinuities and their mappings are generally difficult to derive, we require a practical and efficient algorithm to compute the Lyapunov exponents from the state equations. Recently, Müller [7] proposed a general algorithm to compute the Lyapunov exponents for dynamic systems with discontinuities, and it is applicable to power converters.

In this paper we first review some problems of implementing existing methods in systems with discontinuities. We then study the feasibility of Müller's algorithm for power electronics and apply it to verify chaotic behavior in a well known buck dc-dc converter. Finally, we present our results with discussions.

2. LYAPUNOV EXPONENTS

Broadly speaking, Lyapunov exponents (LEs) measure the exponential convergence or divergence of neighboring orbits of a dynamical system. An n th-order system has n LEs; these may be viewed as extending the notion of eigenvalues to nonlinear systems. Associated with each LE is a characteristic direction in phase space, along which the expansion or contraction of perturbations takes place.

The largest Lyapunov exponent (LLE), i.e. the most positive, is of paramount interest because it dominates the system's response to perturbations. This paper focuses on computation of the LLE, but much of the material is applicable to the other LEs as well.

If a dynamical system can be described by a mapping, the LEs can be computed from the Jacobian matrix evaluated for a long sequence. However, this approach is not easily applied to power electronic converters. The difficulty lies in obtaining the mapping, which is usually done by taking a Poincaré section, or a stroboscopic section for driven systems. Because of the switching action, the original converter is piecewise-linear in time, and this leads to discontinuities in the mapping. If the converter is considered to be piecewise linear, analytical solutions can be obtained for each topological state of the converter, but numerical computation is still needed to locate the switching instants, because transcendental equations must be solved [3]. A further drawback of the mapping approach is that power electronics engineers are generally unfamiliar with discrete-time systems. (The modeling and control literature shows a marked preference for continuous-time averaged models.)

In the alternative continuous-time approach, we are concerned with the convergence or divergence of neighboring trajectories as they evolve with time. Essentially this means tracking the evolution of a perturbation. Continuous-time systems are familiar to all power electronics practitioners, and time-domain simulation of power converters is an accepted technique. Therefore we concentrate on the continuous-time approach.

3. COMPUTATION OF THE LLE

Consider an n -dimensional, autonomous, continuous-time system governed by

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}), \quad \mathbf{f}: \mathbb{R}^n \rightarrow \mathbb{R}^n \quad (1)$$

where $\mathbf{f}(\mathbf{x})$ is the vector field. The Jacobian matrix of the vector field is $\mathbf{F}(\mathbf{x}) = D_{\mathbf{x}}\mathbf{f}(\mathbf{x})$. The LEs are given by [4]

$$\lambda_i = \lim_{t \rightarrow \infty} \left(\frac{1}{t} \ln \text{eigenval}_i \exp \mathbf{F}(\mathbf{x}) t \right), \quad i = 1 \cdots n \quad (2)$$

i.e., there are n LEs, each found from an eigenvalue of the state transition matrix. The LEs have dimensions of s^{-1} in SI units.

Because the LLE dominates over all others, it is sufficient to define it directly in terms of the evolution of an infinitesimal perturbation:

$$\lambda_{\max} = \lim_{\|\delta\mathbf{x}(0)\| \rightarrow 0} \lim_{t \rightarrow \infty} \left(\frac{1}{t} \ln \frac{\|\delta\mathbf{x}(t)\|}{\|\delta\mathbf{x}(0)\|} \right) \quad (3)$$

where $\delta \mathbf{x}(0)$ is an initial small perturbation from $\mathbf{x}(0)$ and $\|\cdot\|$ denotes a vector norm. There are at least three methods for computing the LLE: the time-series method, the difference method, and the variational method.

3.1 Time-Series Method

Time series typically arise from experimental measurements, and the method of embedded dimensions has been developed to estimate the associated LEs [5], [6]. This approach is valuable when the underlying model is unknown and all that is available is experimental data. The method has not been proven valid for discontinuous systems. Although it can be applied to simulation output, it is inefficient because it assumes little about the system. In simulation an accurate system model exists, which is used to produce the output, so it is sensible to utilize that information.

3.2 Difference Method

In the difference (or direct) method, two trajectories are computed by numerical integration:

$$\begin{aligned} &\mathbf{x}(t) \text{ with initial value } \mathbf{x}(0) \\ &\mathbf{y}(t) \text{ with initial value } \mathbf{y}(0) = \mathbf{x}(0) + \delta \mathbf{x}(0) \end{aligned}$$

The difference $\delta \mathbf{x}(t) = \mathbf{y}(t) - \mathbf{x}(t)$ expands and contracts with t . After a sufficiently long time T (many times around the attractor), the LLE is computed from

$$\lambda_{\max} \approx \frac{1}{T} \ln \frac{\|\mathbf{y}(T) - \mathbf{x}(T)\|}{\|\mathbf{y}(0) - \mathbf{x}(0)\|} \quad (4)$$

In practice this method is unusable without refinement. If the LLE is positive, as in a chaotic system, the initial perturbation grows until \mathbf{x} and \mathbf{y} are on opposite sides of the attractor, at which point they cannot diverge further. The calculated LLE is more negative than the true value. This saturation effect results from the use of a non-infinitesimal perturbation. Once $\delta \mathbf{x}$ becomes comparable to the dimensions of the attractor it can no longer be considered small. Although $\delta \mathbf{x}(0)$ should be infinitesimal, it cannot be chosen arbitrarily small in practice, because of limited machine precision.

If instead the LLE is negative, $\delta \mathbf{x}$ shrinks until eventually underflow can occur: $\mathbf{x}(T) \equiv \mathbf{y}(T)$, to machine precision. Then a run-time error occurs when attempting to evaluate the logarithm of zero in (4). Moreover, truncation and rounding errors cause significant numerical noise if $\delta \mathbf{x}$ is very small.

To avoid these effects, it is necessary to renormalize the perturbation regularly. This means choosing renormalization times $\{t_r\}$ at which to scale $\delta \mathbf{x}(t_r)$ back to its original size $\|\delta \mathbf{x}(0)\|$, multiplying it by a scaling factor s_r . Ideally this is done many times, then the LLE can be computed from the product of the scaling factors, $\lambda_{\max} \approx 1 / \prod_r s_r$.

If one is interested in all LEs, not just the largest, the more complex procedure of reorthonormalization is needed [4].

The difference method raises many difficult practical questions. What size should $\delta \mathbf{x}(0)$ be? How should the renormalization times $\{t_r\}$ be chosen? How large or small should $\delta \mathbf{x}$ be allowed to grow before it is renormalized? How accurate is the result?

3.3 Variational Method

To overcome these problems, the variational method [4] is used. Applying Taylor's theorem to (1) for an infinitesimal perturbation $\delta \mathbf{x}$,

$$\frac{d}{dt}(\mathbf{x} + \delta \mathbf{x}) = \mathbf{f}(\mathbf{x}) + \mathbf{F}(\mathbf{x}) \delta \mathbf{x} \quad (5)$$

Subtracting (1) we have the variational equation

$$\frac{d}{dt} \delta \mathbf{x} = \mathbf{F}(\mathbf{x}) \delta \mathbf{x} \quad (6)$$

Essentially this linearizes $\mathbf{f}(\mathbf{x})$ at each point along a reference trajectory $\mathbf{x}(t)$, describing the dynamics in an infinitesimal neighborhood of the point by a linear system. An infinitesimal perturbation $\delta \mathbf{x}$ would evolve according to (6). Because it is linear, $\delta \mathbf{x}(0)$ can be chosen to have any convenient non-zero value. After a sufficiently long time T the LLE is calculated from

$$\lambda_{\max} \approx \frac{1}{T} \ln \frac{\|\delta \mathbf{x}(T)\|}{\|\delta \mathbf{x}(0)\|} \quad (7)$$

Renormalization will probably still be necessary to prevent underflow and overflow, but the choice of renormalization times $\{t_r\}$ is non-critical. For instance we could renormalize whenever $\|\delta \mathbf{x}(t)\|$ leaves the interval $[10^{-3}, 10^3]$ or $[10^{-6}, 10^6]$ without affecting the result (assuming adequate machine precision).

Even when $\mathbf{F}(\mathbf{x})$ can be found analytically from $\mathbf{f}(\mathbf{x})$, \mathbf{x} must be obtained numerically. The simplest way is to solve (6) simultaneously with (1), forming a $2n$ -dimensional augmented system:

$$\frac{d}{dt} \begin{bmatrix} \mathbf{x} \\ \delta \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}) \\ \mathbf{F}(\mathbf{x}) \delta \mathbf{x} \end{bmatrix} \quad (8)$$

The variational method is an improvement upon the difference method for two reasons. First, it deals with infinitesimal perturbations, which is how the Lyapunov exponent is defined, so it is more accurate. (For instance, on a limit cycle the LLE is zero, but a perturbation from it shrinks: the limit cycle is an attractor. Thus the difference method calculates the LLE of limit cycles as slightly negative.) Second, the variational method avoids the saturation problem. In what follows, we adopt the variational method.

4. PROBLEMS

We now examine some problems affecting the variational method for finding the LLE.

4.1 Effect of Numerical Integration

The augmented system (8) is solved by numerical integration, and the integration method can influence the results. Since we are particularly concerned with the critical case of a zero LLE, which delimits chaos, it makes sense to employ an integration method that preserves zero LEs. A basic requirement is that the integrator should be stable. For the test equation $dx/dt = \lambda x$, $\lambda \in \mathbb{C}$, integrated with time step h , the stability region would ideally consist of the left half of the $h\lambda$ -plane. Without going into details, one suitable method for non-stiff systems is the forward-Euler integrator with a very small h ; another is the popular explicit fourth-order Runge-Kutta method. The backward-Euler integrator with a very small h is a suitable implicit method for

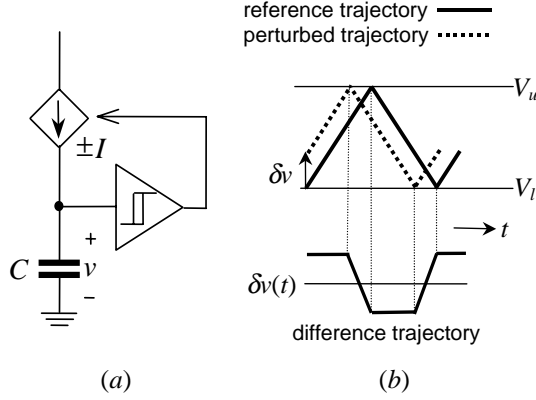


Fig. 1: (a) Triangle wave generator; (b) propagation of perturbation through switching events.

stiff systems, as is the trapezoidal method, provided they are iterated to convergence.

4.2 Discontinuities

Power converters are a special case in that the action of their idealized switches and diodes inherently introduces discontinuities into $\mathbf{f}(\mathbf{x})$, and the effects must be addressed.

Our first observation is that numerical integration must not use a fixed time step, or a large local truncation error is committed when the step spans a switching instant. The proper way to handle switching events is to detect them, locate the switching instant t_s as precisely as possible, integrate up to t_s , apply the appropriate new initial conditions at t_s , then integrate onwards.

The second observation is that if $\mathbf{f}(\mathbf{x})$ contains step discontinuities, $\mathbf{F}(\mathbf{x})$ contains impulses (Dirac delta functions). In extreme cases (e.g. a capacitance discharged via a switch), $\mathbf{F}(\mathbf{x})$ will be even more pathological. The above strategy for handling switching instants is insufficient to integrate the variational equation accurately. A simple example illustrates this.

Fig. 1(a) shows a triangle-wave generator, comprising a controlled current source $\pm I$, a capacitor C , and a Schmitt trigger (hysteretic comparator) with lower and upper thresholds V_L and V_U respectively. The Schmitt trigger output $S(v) \in \{0, 1\}$ determines the direction of I depending on the history of the capacitor voltage v . Thus v is governed by

$$\frac{dv}{dt} = f(v) = \begin{cases} I/C & \text{if } S(v) = 0 \\ -I/C & \text{if } S(v) = 1 \end{cases} \quad (9)$$

and the trajectory $v(t)$ cycles between V_L and V_U . Here $\mathbf{F}(\mathbf{x})$ is simply df/dv , which is zero everywhere apart from the switching points $v = V_L$ and $v = V_U$, where it is $\pm \infty$. Now consider a perturbation δv . As can be seen from Fig. 1(b), the perturbed trajectory is simply offset along the time axis. The difference $\delta v(t)$ between the perturbed (dashed) and unperturbed (solid) trajectory alternates between a positive and negative value at each discontinuity. Although shown for a sizeable perturbation, the principle clearly holds for an infinitesimal perturbation too, resulting in a square wave.

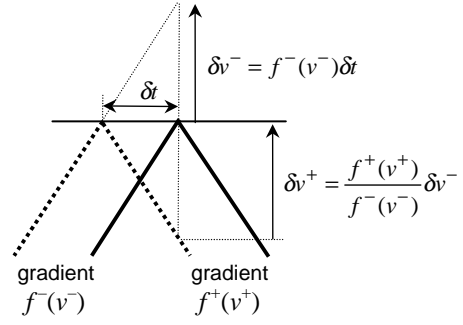


Fig. 2: Discontinuity handling for the triangle wave generator.

Since (9) is piecewise constant, the variational equation formed ignoring the discontinuities would be $d(\delta v)/dt = 0$, whose solution is a constant. This is qualitatively incorrect. One cannot simply avoid the discontinuities, one must integrate through them.

If we are unwilling to entertain numerical integration of impulses, it is necessary to correct the perturbation to its true value following each switching event. Bearing in mind that the variational method deals with infinitesimal quantities, the correction must be applied just after the switching instant t_s . Using superscripts $-$ and $+$ to indicate values just before and after a switching event respectively, the perturbation δv^- at t_s^- needs to be corrected to δv^+ at t_s^+ . This can be done using the intermediate value δt shown in Fig. 2. By geometrical arguments, $\delta v^-/f^-(v^-) = \delta t = \delta v^+/f^+(v^+)$. Therefore

$$\delta v^+ = \frac{f^+(v^+)}{f^-(v^-)} \delta v^- \quad (10)$$

Since $f^+(v^+) = -f^-(v^-)$ for every switching event, the adjustment produces a square wave, which is correct.

5. MÜLLER'S ALGORITHM

The above argument was recently extended by Müller, who proposed a general method to compute the Lyapunov exponents of dynamical systems where $\mathbf{f}(\mathbf{x})$ contains various types of discontinuities [7]. His algorithm for dealing with the propagation of perturbations through a discontinuity may be summarized as follows.

Consider a piecewise-smooth system of the form (1). Close to a switching instant t_s the augmented system (8) is defined as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{x} \\ \delta \mathbf{x} \end{bmatrix} = \begin{cases} \mathbf{f}^-(\mathbf{x}), & t < t_s \\ \mathbf{f}^+(\mathbf{x}), & t \geq t_s \end{cases} \quad (11)$$

$$\mathbf{F}(\mathbf{x}) \delta \mathbf{x}$$

The switching instant t_s occurs when an indicator function $\mathbf{h}(\mathbf{x}) = \mathbf{0}$. Let its Jacobian be $\mathbf{H}(\mathbf{x}) = D_{\mathbf{x}}\mathbf{h}(\mathbf{x})$. At switching, the state

vector is transformed by an impact function, $\mathbf{x}^+ = \mathbf{g}(\mathbf{x}^-)$. Let its Jacobian be $\mathbf{G}(\mathbf{x}) = D_{\mathbf{x}}\mathbf{g}(\mathbf{x})$. The correction of $\delta\mathbf{x}^-$ to $\delta\mathbf{x}^+$ during numerical integration of (11) can be stated as follows.

Müller's algorithm:

1. Detect the occurrence of a switching event by monitoring the sign of $\mathbf{h}(\mathbf{x})$ and/or a local truncation error estimate and/or switch drive functions.
2. Use a numerical root-finding algorithm to locate the precise instant t_s at which $\mathbf{h}(\mathbf{x}) = \mathbf{0}$.
3. Find the values of the state vector (\mathbf{x}^-) and the perturbation vector ($\delta\mathbf{x}^-$) at t_s^- .
4. Solve the following equation for δt :

$$\mathbf{H}(\mathbf{x}^-)[\delta\mathbf{x}^- + \mathbf{f}^-(\mathbf{x}^-)\delta t] = \mathbf{0} \quad (12)$$

5. Obtain $\delta\mathbf{x}^+$ from

$$\delta\mathbf{x}^+ = \mathbf{G}(\mathbf{x}^-)\delta\mathbf{x}^- + [\mathbf{G}(\mathbf{x}^-)\mathbf{f}^-(\mathbf{x}^-) - \mathbf{f}^+(\mathbf{x}^+)]\delta t \quad (13)$$

Originally developed for impacting mechanical systems, Müller's algorithm is very general and can cope with the discontinuities found in power electronics. This includes certain undesirable situations that could nevertheless occur in practice. For instance, \mathbf{x} is usually continuous across switching instants, accounted for by defining $\mathbf{g}(\mathbf{x}) = \mathbf{x}$. But if a capacitor is shunted by a switch, the voltage goes to zero when the switch closes. In Müller's algorithm the corresponding entry of $\mathbf{g}(\mathbf{x})$ is zero.

6. EXAMPLE: BUCK CONVERTER

To illustrate the application of Müller's algorithm, we treat the much-studied voltage-mode controlled PWM buck converter shown in Fig. 3 [3]. Operation in CCM is assumed. The system is autonomous because a ramp waveform generator is included. Note that this circuit includes a switch across a capacitor. The ramp voltage v_{ramp} rises from V_l to V_u at a rate $dv_{ramp}/dt = I/C_1$. When v_{ramp} reaches V_u , switch S_1 closes momentarily and v_{ramp} falls instantly to V_l . The period of the ramp is $T_s = C_1(V_u - V_l)$.

Defining $\mathbf{x} = [v \ i \ v_{ramp}]^T$, the system equation (1) is

$$\frac{d}{dt} \begin{bmatrix} v \\ i \\ v_{ramp} \end{bmatrix} = \begin{bmatrix} (i - v/R)/C \\ (SV_l - v)/L \\ I/C_1 \end{bmatrix} \quad (14)$$

Let $v_{con} = A(v - V_{ref})$. In (14), $S = 1$ if $v_{con} < v_{ramp}$ or $S = 0$ otherwise. Differentiating the RHS of (14), the variational equation (6) is found as

$$\frac{d}{dt} \begin{bmatrix} \delta v \\ \delta i \\ \delta v_{ramp} \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C & 0 \\ -1/L & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v \\ \delta i \\ \delta v_{ramp} \end{bmatrix} \quad (15)$$

In this circuit there are two indicator functions signaling the occurrence of a switching event:

$$\left. \begin{aligned} h_1(\mathbf{x}) &= v_{ramp} - V_u \\ h_2(\mathbf{x}) &= v_{ramp} - v_{con} \end{aligned} \right\} \quad (16)$$

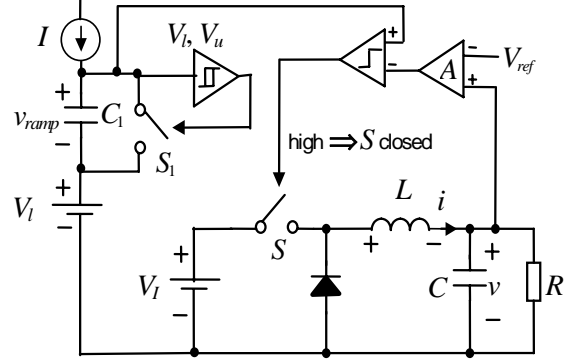


Fig. 3: Buck dc-dc converter with voltage-mode feedback control.

6.1 Indicator Function h_1

First consider $h_1(\mathbf{x}) = 0$, which indicates when the ramp reaches its upper limit. Switch S_1 closes, discharging C_1 and ending a ramp cycle. State variables v and i are continuous across the switching event but v_{ramp} is reset to V_l . Hence $\mathbf{g}_1(\mathbf{x}) = [v \ i \ V_l]^T$. The Jacobians of $\mathbf{g}_1(\mathbf{x})$ and $h_1(\mathbf{x})$ are both constant matrices in this example:

$$\mathbf{G}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{H}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (17)$$

It is straightforward to solve (12) for δt :

$$\delta t = -\delta v_{ramp} C_1 / I \quad (18)$$

There are three possibilities for switch S , depending on v_{con} :

- (i) if $v_{con} \leq V_l$ S remains closed;
- (ii) if $v_{con} \in (V_l, V_u)$ S makes a closed-to-open transition;
- (iii) if $v_{con} \geq V_u$ S remains open.

For cases (i) and (iii) there is no topological change, and applying (13) gives

$$\begin{bmatrix} \delta v^+ \\ \delta i^+ \\ \delta v_{ramp}^+ \end{bmatrix} = \begin{bmatrix} \delta v^- \\ \delta i^- \\ -I/C_1 \end{bmatrix} \delta t \quad (19)$$

But for case (ii), the closed-to-open transition means (13) now gives

$$\begin{bmatrix} \delta v^+ \\ \delta i^+ \\ \delta v_{ramp}^+ \end{bmatrix} = \begin{bmatrix} \delta v^- \\ \delta i^- + V_l/L \\ -I/C_1 \end{bmatrix} \delta t \quad (20)$$

6.2 Indicator Function h_2

Next consider $h_2(\mathbf{x}) = 0$, which indicates when the control and ramp voltages intersect. All three state variables are continuous across the switching event, so $\mathbf{g}_2(\mathbf{x}) = \mathbf{x} = [v \ i \ v_{ramp}]^T$. The Jacobian matrices of $\mathbf{g}_2(\mathbf{x})$ and $h_2(\mathbf{x}) = v_{ramp} - A(v - V_{ref})$ are:

$$\mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} -A & 0 & 1 \end{bmatrix} \quad (21)$$

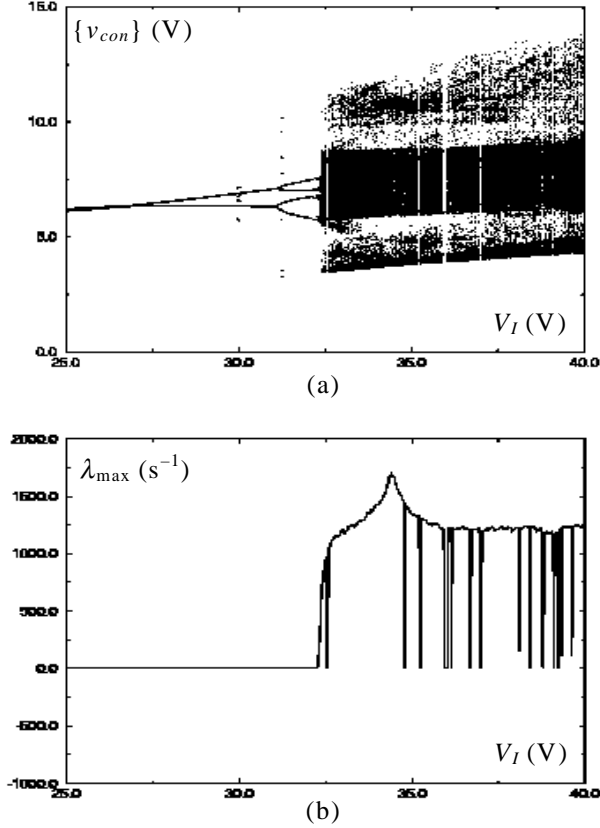


Fig. 4: (a) Bifurcation diagram for the buck converter: $\{v_{con}\}$ plotted against V_I ; (b) largest Lyapunov exponent λ_{max} plotted against V_I , computed by the variational method incorporating Muller's algorithm for discontinuities.

Now δt can be found from (12) as

$$\delta t = \frac{\delta v_{ramp}^- - A \delta v^-}{A(i^- - v^- / R) / C - I / C_1} \quad (22)$$

Finally, we find $\delta \mathbf{x}^+$ from (13) as

$$\begin{bmatrix} \delta v^+ \\ \delta i^+ \\ \delta v_{ramp}^+ \end{bmatrix} = \begin{bmatrix} \delta v^- \\ \delta i^- \pm V_I / L \\ -I / C_1 \end{bmatrix} \delta t \quad (23)$$

where the positive sign is taken for a closed-to-open transition of S and the negative sign for an open-to-closed transition.

6.3 Indicator Functions h_1 and h_2

In the special case where $v_{con} = v_{ramp} = V_u$, $h_1(\mathbf{x})$ and $h_2(\mathbf{x})$ become zero simultaneously. This case is already covered by the equality relation in case (iii) of subsection 6.1.

6.4 Results

We investigated the buck converter using the following parameter values [3]: $L = 20\text{mH}$, $C = 47\mu\text{F}$, $R = 22\Omega$, $A = 8.2$, $V_{ref} = 11.3\text{V}$, $V_I = 3.8\text{V}$, $V_u = 8.2\text{V}$, with $C_1 = 90.9\text{nF}$, $I = 1\text{mA}$ to give a ramp period of $400\mu\text{s}$. We used $V_I \in [25\text{V}, 40\text{V}]$ as the bifurcation parameter. The integration was performed by a fourth-order

Runge-Kutta routine with a time step $h = 2.5\mu\text{s}$, the switching instants being located by repeated bisection of the interval.

Fig. 4(a) shows a bifurcation diagram ($\{v_{con}\}$ sampled at $h_1(\mathbf{x}) = 0$) and Fig. 4(b) shows the computed LLE. It can be seen that periodic behavior (of various periods) is correctly indicated by a zero LLE, whilst chaotic regions have a positive LLE. Although the bifurcation diagram seems to show periodic windows at $V_I = 37.00\text{V}$, 38.10V and 39.32V , the LLE indicates otherwise. Closer investigation confirmed that the behavior is indeed chaotic here.

As a check, we modified our program to compute the LLE without Müller's algorithm, and got $\lambda_{max} \approx -0.276\text{ s}^{-1}$ for all values of V_I . This qualitatively incorrect result demonstrates that Müller's algorithm is not merely a refinement: it is a necessity for computing LEs of power converters.

The magnitude of the LLE is interesting. In the chaotic region it is about 1250 s^{-1} ; its reciprocal may be identified with a characteristic time of $1/1250 = 800\mu\text{s}$. This is just two ramp cycles, revealing that nearby trajectories diverge extremely rapidly.

7. CONCLUSION

In smooth continuous-time systems, computation of the largest Lyapunov exponent is straightforward. In idealized power converters the switching discontinuities cause problems. If they are not treated properly, the variational method gives invalid results. Müller's algorithm can be employed to ensure accurate propagation of infinitesimal perturbations through the switching events. We have used this method successfully to calculate the largest Lyapunov exponent of an idealized buck converter. Unlike mapping-based approaches, this continuous-time approach should appeal more to power electronics engineers, who are already familiar with continuous-time simulation.

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