The Lorenz system:

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = \gamma x - y - xz \\ \dot{z} = xy - bz \end{cases}$$

Jacobian:

$$\mathbf{J} = \begin{pmatrix} -\sigma & \sigma & 0 \\ \gamma - z & -1 & -x \\ y & x & -b \end{pmatrix}$$

1 Parameter set 1

Table 1: The complete set of Lyapunov exponent for the Lorenz system with $\sigma = 10, b = 8/3, \gamma = 28$. The column of values are given by Liao 2008.

λ_1	0.91
λ_2	0
λ_3	-144.47

表 1 $a = 10, b = 8/3, r = 28$ 时 Lyapunov 指数比较

Table 1 The comparison of Lyapunov Exponents with $a = 10, b = 8/3, r = 28$

计算方法	λ_1	λ_2	λ_3	λ_{max}
定义法	\	\	\	1. 367 1
Wolf 法	\	\	\	0. 022 9
正交法	0. 856 4	− 0. 001 1	− 14. 518 5	0. 856 4
小数据量法	\	\	\	0. 024

Figure 1: Calculating LE with different methods

2 Parameter set 2

3 Parameter set 3

为验证上述理论分析的正确性，下面以 Lorenz 混沌和静摩擦 Duffing 振子为例进行仿真研究。

Lorenz 混沌的状态方程

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = \gamma x - y - xz \\ \dot{z} = xy - bz \end{cases} \tag{11}$$

当参数 $\sigma = 10$, $\gamma = 28$, $b = 8/3$ 时，系统呈现混沌行为[7]，且最大 Lyapunov 指数为 0.906。以系统

Figure 2:

Table 2: The complete set of Lyapunov exponent for the Lorenz system with $\sigma = 16, b = 4.0, \gamma = 40$. The column of values are given by Shimada 1979.

λ_1	1.37
λ_2	0
λ_3	-22.37

值相符合.

表 3 Lorenz 系统最大 Lyapunov 指数与嵌入维的关系

嵌入维数 m	2	3	4	5	6	7
λ	1.49184	1.5335	1.51945	1.52051	1.514	1.41107

表 4 Lorenz 系统最大 Lyapunov 指数与嵌入维的关系($m=3$)

时间延迟 k	7	11	15	18	21
λ	1.48935	1.5335	1.46001	1.4615	1.43038

Figure 3: LLE for the Lorenz system with parameter $\sigma = 16, b = 4.0, \gamma = 45.92$

Table 3: The complete set of Lyapunov exponent for the Lorenz system with $\sigma = 16, b = 4.0, \gamma = 45.92$. The second column is given by A. Wolf 1980,while the third column is given by Kehui Sun 2004.

λ_1	2.16	1.506
λ_2	0.00	0.001
λ_3	-32.4	-22.505