CMSC303 Introduction to Theory of Computation, VCU
Spring 2017, Assignment 2

Total marks: 29 marks + 5 bonus marks + 3 bonus marks for LaTeX

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. (6 marks) This question tests your comfort with "boundary cases" of DFA's. Draw the state diagrams of DFA's recognizing each of the following languages.
   (a) (2 marks) $L = \{\epsilon\}$ for ϵ the empty string.
   (b) (2 marks) $L = \emptyset$.
   (c) (2 marks) $L = \{0, 1\}^*$.
2. (8 marks) This question tests your ability to design DFAs and NFAs.
   (a) (2 marks) Draw the state diagram for a DFA recognizing language $L_1 = \{x \mid x \text{ contains at least two 1s}\}$.
   (b) (2 marks) Draw the state diagram for a DFA recognizing language $L_2 = \{x \mid x \text{ contains at most one } 0\}$.
   (c) (4 marks) Draw the state diagram for a NFA recognizing language $L_3 = L_1 \cup L_2$.
3. (3 marks) In this question, you will study closure of regular languages under certain operations. Specifically, show that if M is a DFA that recognizes language B, then swapping the accept and nonaccept states in M yields a new DFA M' recognizing the complement of B, $\overline{B}$. Which operation does this imply the regular languages are closed under?
4. (6 marks) This question tests your ability to prove a language is regular using the closure properties of regular languages. Given languages A and B, define the operation $\cdot$ as

   $$A \cdot B := \{x \mid x \in A \text{ and } x \text{ does not contain any string in } B \text{ as a substring.}\}.$$

   Prove that the class of regular languages is closed under the $\cdot$ operation. (Hint: Recall that by DeMorgan's law, for any sets X and Y, one has $X \cap Y = \neg(\neg X \cup \neg Y)$.)
5. (6 marks) This question tests your understanding of the equivalence between DFAs and NFAs. Consider NFA $M = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$ for δ defined as:

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\emptyset$	$\{q_1\}$	$\{q_1\}$

   Draw both the state diagrams for M and for a DFA M' equivalent to M based on the construction of Theorem 1.39 in the text (recall the latter proves that DFAs and NFAs are equivalent).
6. (Bonus, 5 marks) This question demonstrates that although DFAs and NFAs are equivalent in terms of the sets of languages they recognize, they are *provably not* equivalent in terms of efficiency (i.e. DFAs may require **many** more states to recognize a language than an NFA). Consider the language $C_k = \Sigma^* 0 \Sigma^{k-1}$ for $k \geq 1$. Convince yourself that an NFA with $k + 1$ states for recognizing C_k exists (no need to include this in your assignment answer). Now, prove that for any k, C_k cannot be recognized by a DFA with less than 2^k states.