CS 303 Introduction to Theory of Computing, VCU
Spring 2017, Assignment 4
Due: Tuesday, March 14, 2017 at start of class

Total marks: 62 marks $+$ 6 bonus marks for LaTeX

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [12 marks] This question develops a basic understanding of CFGs and parse trees. Consider the grammar G below.

$$E \rightarrow E + T \mid T \tag{1}$$
$$T \rightarrow T \times F \mid F \tag{2}$$
$$F \rightarrow (E) \mid 2 \tag{3}$$

   For each string below, give a parse tree of a derivation in G.

   (a) 2
   (b) $2 + 2 + 2$
   (c) $((2 + 2) \times (2))$

2. [10 marks] We have seen in class that the sets of both regular and context-free languages are closed under the union, concatenation, and star operations. We have also seen in A2 that the regular languages are closed under intersection and complement. In this question, you will investigate whether the latter also holds for context-free languages.

   (a) Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ to show that the class of context-free languages is not closed under intersection. You may use the fact that the language $C = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.
   (b) Using part (a) above, show now that the set of context-free languages is not closed under complement.

3. [15 marks] This question develops your ability to design CFGs. For each of the following languages, give a CFG. Assume the alphabet is $\Sigma = \{0, 1\}$. For string $x = x_1 x_2 \cdots x_n$, we define $x^R := x_n \cdots x_2 x_1$, i.e. this operation reverses the order of the symbols in x. Justify your answers briefly.

   (a) $\{x \mid x$ starts and ends with the same symbol$\}$.
   (b) $\{x \mid$ the length of x is odd$\}$.
   (c) $\left\{x \mid x = x^R\text{, that is, } x \text{ is a palindrome}\right\}$.
   (d) $\emptyset$.
   (e) For this part, set $\Sigma = \{a, b, \$\}$. The language is then

$$\left\{x_1 \$ x_2 \$ \cdots \$ x_k \mid k \geq 1\text{, each } x_i \in \{a, b\}^*\text{, and for some } i \text{ and } j\text{, } x_i = x_j^R\right\}.$$

4. [15 marks] This question develops your ability to design PDAs. For parts (a), (b), (c), and (d) of question 3 above, give state diagrams of pushdown automata. For each automata, include a brief description of the idea behind its design.

5. [10 marks] This question forces you to practice the generic construction for mapping a CFG to a PDA. Specifically, for the grammar G from question 1, use the construction from Theorem 2.20 to construct an equivalent PDA P.