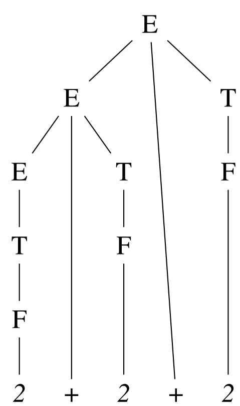
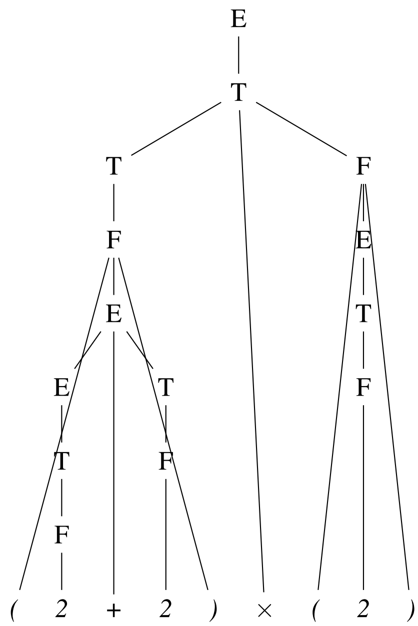
CMSC 303 Introduction to Theory of Computation, VCU
Assignment: 4
Name: Steven Hernandez

1. (a)

(b)



(c) 

2. (a)

   (b)

3. (a)

$$\begin{aligned} S &\rightarrow 0 \mid 1 \mid 0T0 \mid 1T1 \\ T &\rightarrow \epsilon \mid 1T \mid 0T \end{aligned}$$

Trivially 0 and 1 match. $0T0$ $1T1$ ensure that the first and last symbol are the same before moving past S into T. T simply allows you to add any symbols $\in \Sigma_{epsilon}$ recursively within the string obtained above.

(b)

$$\begin{aligned} S\_0 &\rightarrow 0 \mid 1 \mid 0S\_\{odd\} \mid 1S\_\{odd\} \\ S\_\{odd\} &\rightarrow \epsilon \mid 0S\_\{even\} \mid 1S\_\{even\} \\ S\_\{even\} &\rightarrow 0 \mid 1 \mid 0S\_\{odd\} \mid 1S\_\{odd\} \end{aligned}$$

Think of the labels such that S_{odd} means we have an odd length currently, thus we can only add $epsilon$ of one symbol, which then means we now have an even number of symbols (thus the S_{even}).

(c)

$$S \rightarrow \epsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$$

Unlike a, the only variable is S, this is because each time we recurse, we want to ensure whatever the sub-string contains, it always begins and ends with the same symbol, thus maintaining the palindrome.

(d)

$$S \rightarrow S$$

The grammar continues recursively forever. Never reaching only terminals, thus never reaching an accept state.

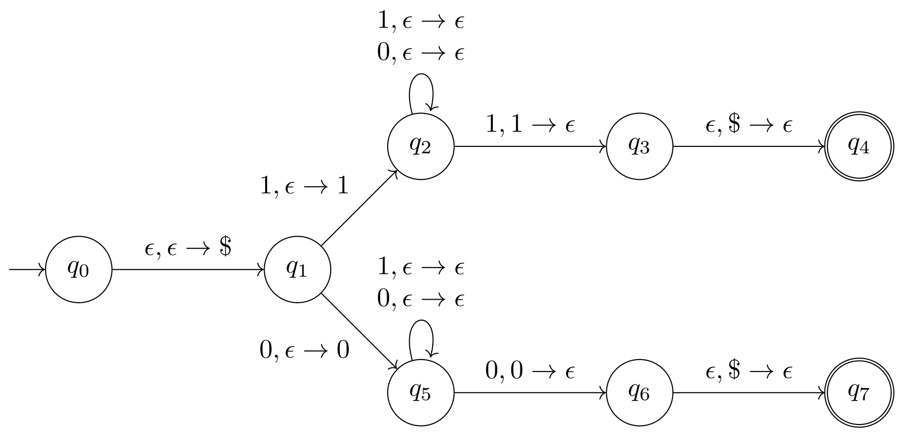
(e)

$$\begin{aligned} S &\rightarrow X\$C\$X \mid C\$X \mid X\$C \mid C \\ C &\rightarrow 1C1 \mid 0C0 \mid \$ \mid \$X\$ \\ X &\rightarrow \$X \mid 1X \mid 0X \mid \epsilon \end{aligned}$$

The idea for this grammar is that C always builds a palindrome. Note how from C, we either recursively wrap C with 0 or 1. After which, we can leave \$ in the center.

From the first step, if there are any symbols to the left or right of C, we delimite it with a \$. This keeps the grammar matching the language.

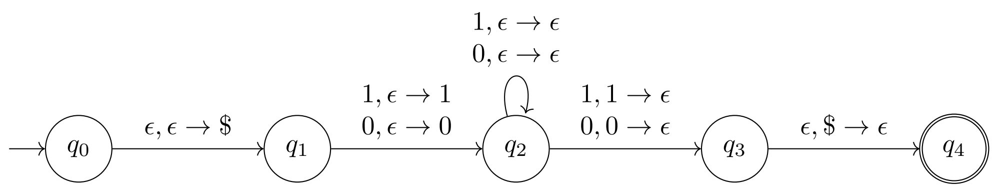
This produces the language when $i = n$ and $j = n + 1$, however does not account for $i = n$ and $j = n + t$ where $t > 1$. So, notice $C \to \$X\$$. This allows the palindrome we were building to have non-palindrome-like items in the middle of this palindrome. Notice though, that these symbols are delimited by \$ so that we keep separate the palindrome from earlier.

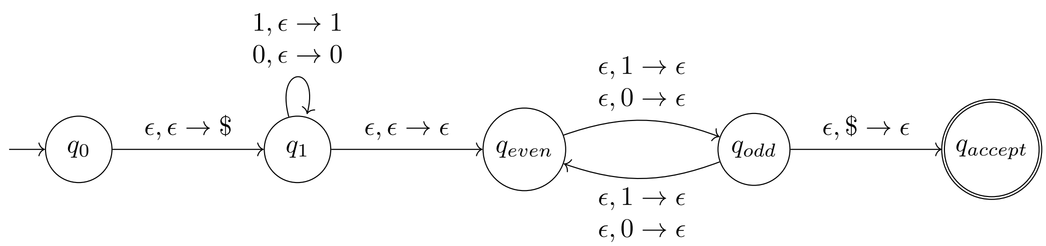


4. (a)

Notice however, this solution does not require a stack. The language could be modeled easily by an NFA with two branches as seen above.

Instead, a PDA where the stack is required, can be modeled as such:



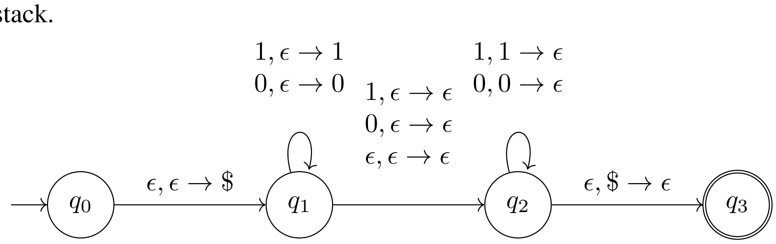


(b)

The idea here is we begin by placeing \$ on the stack. We then loop through the entire string in state q_1 and place each symbol in the stack. From this state, notice the only transitions are $\epsilon, x \to \epsilon$ where $x \in \Sigma_\epsilon$. This means we must have read all input here in q_1. Otherwise we the PDA will not have read all the input, and thus would fail. This also means we only care about the contents of the stack.

We loop back and forth between q_{even} and q_{odd} after each element $x \in \Sigma$ is popped off the stack. This continues until the stack only contains \$. At this point, if we are in q_{odd}, this means we have popped an odd number of symbols off the stack, thus the string contained an odd number of symbols.

As a note, it seems it would be relatively simple to build this logic as a normal NFA, without the use of a stack.



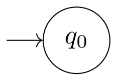
(c)

Begin by adding $\$$ to the stack. Then the PDA reads some number of symbols onto the stack on q_1. The transition from q_1 to q_2 has a few options. We begin by observing $\epsilon, \epsilon \rightarrow \epsilon$.

After this transition, we match each following symbol of the string to what has already been placed onto the stack. This matches palindromes because the stack is last in first out.

Notice however, this only matches palindromes that are an even number of characters long. Odd length palindromes would have the center symbol placed on the stack, but would not match with the symbols on the right hand side.

The other options in the transition from q_1 to q_2 allows a single symbol to be read and removed. The symbol does not effect the stack at all. This allows odd length palindromes.

(d) 

There don't need to be any transitions, nor any accept states because nothing should ever be accepted.

5.