# CMSC 303 Introduction to Theory of Computing, VCU
# Spring 2017, Assignment 3
# Due: Thursday, February 23, 2017 at start of class

Total marks: $66$ marks $+$ $4$ marks bonus $+$ $8$ bonus marks for LaTeX

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [12 marks] This question develops your ability to devise regular expressions, given an explicit definition of a language. For each of the following languages, prove they are regular by giving a regular expression which describes them. Justify your answers.
   (a) $L = \{x \mid x \text{ begins with a } 0 \text{ and ends with a } 1\}$.
   (b) $L = \{x \mid x \text{ contains at least four } 0\text{'s}\}$.
   (c) $L = \{1, 11, \epsilon\}$.
   (d) $L = \{x \mid \text{the length of } x \text{ is at most } 3\}$.
   (e) $L = \{x \mid x \text{ doesn't contain the substring } 110\}$.
   (f) $L = \{x \mid |x| > 0, \text{ i.e. } x \text{ is non-empty}\}$.
2. [20 marks] This question tests your understanding of how to translate a regular expression into a finite automaton. Using the construction of Lemma 1.55, construct NFAs recognizing the languages described by the following regular expressions.
   (a) [5 marks] $R = \emptyset^*$.
   (b) [15 marks] $R = (0 \cup 1)^* 111 (0 \cup 1)^*$.
3. [15 marks] This question tests your understanding of how to translate a finite automaton into a regular expression. Consider DFA $M = (Q, \Sigma, \delta, q, F)$ such that $Q = \{q_1, q_2, q_3\}$, $q = q_1$, $F = \{q_1, q_3\}$, and $\delta$ is given by:

   | $\delta$ | 0 | 1 |
   |---|---|---|
   | $q_1$ | $q_2$ | $q_2$ |
   | $q_2$ | $q_2$ | $q_3$ |
   | $q_3$ | $q_1$ | $q_2$ |

   Draw the state diagram for $M$, and then apply the construction of Lemma 1.60 to obtain a regular expression describing $L(M)$.
4. [15 marks] This question allows you to practice proving a language is non-regular via the Pumping Lemma. Using the Pumping Lemma (Theorem 1.70), give formal proofs that the following languages are *not* regular:
   (a) $L = \left\{www \mid w \in \{0,1\}^*\right\}$.
   (b) $L = \{1^n 0^m 1^n \mid m, n \geq 0\}$.
   (c) $L = \left\{x \mid x \in \{0,1\}^* \text{ is not a palindrome}\right\}$. Recall a palindrome is a string that looks the same forwards and backwards. Examples of palindromes are "madam" and "racecar".

5. [4 marks + 4 marks bonus] This question reveals important subtleties of the Pumping Lemma. Let $B = \{0^k x 0^k \mid k \geq 1 \text{ and } x \in \Sigma^*\}$.

   (a) [4 marks] Consider the following argument, which claims to prove that $B$ is not regular.

   Assume $B_2$ is regular, and let $p$ be the pumping length. Consider string $s = 0^p 1 0^p \in B_2$, and decompose it as $s = xyz$ with $x = \epsilon$, $y = 0^p$, $z = 10^p$. Then, pumping $s$ down by setting $i = 0$ yields string $s' = xy^i z = xy^0 z = 10^p \notin B_2$. Hence, by the Pumping Lemma, we have a contradiction. We conclude that $B_2$ is not regular.

   The question is: What is wrong with this proof?

   (b) [4 marks, bonus] Prove that, in fact, $B$ is regular.