CMSC 303 Introduction to Theory of Computation, VCU
Spring 2015, Assignment 5
Due: Tuesday, March 28, 2015 in class

Total marks: 38 marks + 4 marks bonus for typing your solutions in LaTeX.

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [6 marks] This question asks you to examine the formal definitions of a TM and related concepts closely. Based on these definitions, answer the following.
   (a) A *configuration* of a Turing Machine (TM) consists of three things. What are these three things?
   (b) Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
   (c) Can the tape alphabet Γ be the same as the input alphabet Σ?
   (d) Can a Turing machine's head *ever* be in the same location in two successive steps?
   (e) Can a TM contain just a single state?
   (f) What is the difference between a decidable language and a Turing-recognizable language?

2. [8 marks] This question gets you to practice describing TM's at a semi-low level. Give an implementation-level description of a TM that decides the language

$$L = \{x \mid x \text{ contains twice as many 0s as 1s}\}.$$

   By *implementation-level description*, we mean a description similar to Example 3.11 in the text (i.e. describe how the machine's head would move around, whether the head might mark certain tape cells, etc. . . . Please do *not* draw a full state diagram (for your sake and for ours)).

3. [9 marks] This question investigates a variant of our standard TM model from class. Our standard model included a tape which was infinite in one direction only. Consider now a TM whose tape is infinite in *both* directions (i.e. you can move left or right infinitely many spaces on the tape). We call this a TM with *doubly infinite tape*.
   (a) [3 marks] Show that a TM with doubly infinite tape can simulate a standard TM.
   (b) [5 marks] Show that a standard TM can simulate a TM with doubly infinite tape.
   (c) [1 mark] What does this imply about the sets of languages recognized by both models?

4. [10 marks] This question studies closure properties of the decidable and Turing-recognizable languages.
   (a) [5 marks] Show that the set of decidable languages is closed under concatenation.
   (b) [5 marks] Show that the set of Turing-recognizable languages is closed under concatenation. (Hint: This is trickier than part (a) because if a (deterministic) Turing machine decides to split an input string x as $x = yz$ and check if $y \in L_1$ and $z \in L_2$, i.e. to check if $x \in L_1 \circ L_2$, then running the *recognizer* for L_1 on y (say) may result in an infinite loop if $y \notin L_1$.)

5. [5 marks] This question allows you to explore variants of the computational models we've defined in class. Let a k-PDA be a pushdown automaton that has k stacks. In this sense, a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs. Show that 2-PDAs are more powerful than 1-PDAs. (Hint: Recall from A4 that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.)