CMSC 303 Introduction to Theory of Computation, VCU
Assignment: 6
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1. (a) Claim: $|N| = |Z|$ where $N = \{0, 1, 2, 3, \ldots\}$ and $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
   Proof: By bijection, we create a function $f : N \mapsto Z$ such that all even numbers in N map into a some even number in Z and odd numbers in N map to negative numbers in Z.

$$f(x) = \begin{cases} -\dfrac{x+1}{2} & \text{if } x \text{ is odd} \\ \dfrac{x}{2} & \text{if } x \text{ is even} \end{cases}$$

   (b)

   Claim: $|N| \neq |B|$ where $B = \left\{x | x \in \{0,1\}^*\right\}$

   Proof: We can prove this by by contradiction.

   Suppose we only only concern ourselves with very the large (infinite) strings. We assume $\exists$ list L of all thes strings.

$$\begin{aligned} L = & 011001100\ldots \\ & 100110011\ldots \\ & 100100100\ldots \\ & 111111111\ldots \\ & \ldots\ldots\ldots \end{aligned} \tag{1}$$

   We can construct the string $x \in B$ which is not in L by taking the ith symbol of x to be the opposite symbol from the ith entry of L. Thus, contradiction.

2. (a) $A \leq B$ means that B is harder than A (or equally hard).
   (b) A mapping reduction gives us a way to handle deciding whether a problem is decidable or not from knowing that some other problem is decidable or not.
   So for example with an example of adding and multiplying. Multiplying $\leq$ adding because multiplying can be achieved by simply using adding. Thus, adding is a more powerful construct.
   With this, we can say that if A is not decidable, B is not decidable either because B is harder than A. As well as, if B is decidable, A is also decidable, because A is not as hard as B.
   (c)
   (d) i. A is decidable because B is 'harder', yet is decidable.
   ii. B is undecidable because B is 'harder' than A and A is not decidable.
   iii. It is unknown whether A is decidable or not, because while B is not decidable, it is also said to be harder than A.

3. (a)
   (b)
   (c)