CMSC 303 Introduction to Theory of Computation, VCU
Assignment: 1
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1 Exercises

1. (a) $A \not\subseteq B$
   (b) $A \subseteq B$
   (c) $\{x, y, z\}$
   (d) $\{x, y\}$
   (e) $\{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$
   (f) $\{\emptyset, \{x\}, \{y\}, \{x, y\}\}$
2. $A \times B$ would simply have $a \times b$ elements because each element in A will have to pair with each element of B.
   Let A have a elements, B have b elements and $n \in A$.
   From here we know that for just the one element n, we will have $S = \{(n, B_0), (n, B_1), \ldots, (n, B_b)\}$ such that $S \subseteq A \times B$ and where $|S| = b$.
   Because we know this will happen for all elements in A, $|A \times B| = ab$.
3. Let C be a set such that $|C|$ equals to some integer c.
   For each element in C, there are two options per element when building the sets contained within the power set; either the element appears or doesn't appear in the set.
   Thus there would be 2^c elements(sets) within C.
4. (a) $f(2) = 7$
   (b) $domain = X$
   $codomain = \{6, 7, 8, 9, 10\}$
   (c) $g(2, 10) = 6$
   (d) $domain = X \times Y$
   $codomain = Y$
   (e) $g(4, f(4)) = 8$
5. (a) $hasClassTogether$
   (b) $notXOR$ (where items $\in \{0, 1\}$)
   (c) $canSeeFaceStraightOnWithoutMirror$ (or similar relective surface)

2 Problems

1. The induction step is the problem. By removing the horse in sentence 2, the author assumes that H_1 is the same as the set assumed to be true in the first sentence. This however is not the case, the author can not make the assumption that H_1 nor H_2 contain horses of the same color from the information given.

2. **Claim:**

$$\sum_{m=0}^{n} m = \frac{n(n+1)}{2}$$

Proof. **Base Case:** Prove for $n = 0$.

$$\sum_{m=0}^{n} m = 0.$$

$$\frac{0(0+1)}{2} = \frac{0}{2} = 0.$$

$$0 = 0.$$

Induction Hypothesis: For $k \geq 1$, we assume to be true:

$$\sum_{m=0}^{k} m = \frac{k(k+1)}{2}$$

Induction Step: From the induction hypothesis, we want to prove that

$$\sum_{m=0}^{k+1} m = \frac{(k+1)(k+1+1)}{2}$$

is also true as a result.

Note:

$$\sum_{m=0}^{k+1} m = \sum_{m=0}^{k} m + (k+1)$$

$$\begin{aligned}
\sum_{m=0}^{k+1} m &= \frac{(k+1)(k+1+1)}{2} \\
&= \frac{(k+1)(k+2)}{2} \\
&= \frac{k(k+1) + 2(k+1)}{2} \\
&= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} && \text{(by distributive property)} \\
&= \frac{k(k+1)}{2} + (k+1) && \text{(dividing by 2)} \\
&= \frac{k(k+1)}{2} + (k+1).
\end{aligned}$$

□

Thus proving the claim.