CMSC303 Introduction to Theory of Computation, VCU
Spring 2017, Assignment 1

Due: Tues Jan 24, 2017 at start of class

Total marks: 26 marks + 3 bonus marks for LaTeX

NOTE: As this is a warmup assignment intended to refresh your memory on background material, it will be marked only for completeness, *not* correctness. It is your responsibility to compare your answers with the solutions (to be posted after the due date) to gauge how well you understand the concepts on this assignment.

1 Exercises

1. (6 marks) Sipser, Ex. 0.3: Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.
   (a) (1 mark) Is A a subset of B?
   (b) (1 mark) Is B a subset of A?
   (c) (1 mark) What is $A \cup B$?
   (d) (1 mark) What is $A \cap B$?
   (e) (1 mark) What is $A \times B$?
   (f) (1 mark) What is the power set of B?

2. (2 marks) Sipser, Ex. 0.4: If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

3. (2 marks) Sipser, Ex. 0.5: If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

4. (7 marks) Sipser, Ex. 0.6: Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f : X \mapsto Y$ and the binary function $g : X \times Y \mapsto Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

   (a) (1 mark) What is the value of $f(2)$?
   (b) (2 marks) What are the domain and co-domain of f?
   (c) (1 mark) What is the value of $g(2, 10)$?
   (d) (2 marks) What are the domain and co-domain of g?
   (e) (1 mark) What is the value of $g(4, f(4))$?

5. (3 marks) Sipser, Ex. 0.7: For each part, give a relation that satisfies the condition.

(a) (1 mark) Reflexive and symmetric but not transitive.

(b) (1 mark) Reflexive and transitive but not symmetric.

(c) (1 mark) Symmetric and transitive but not reflexive.

2 Problems

1. (2 marks) Sipser, Prob. 0.12 (0.11 in 2nd edition): Find the error in the following proof that all horses are the same color.
   CLAIM: In any set of h horses, all horses are the same color.
   PROOF: By induction on h.

   Base Case: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

   Induction Step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all horses in H must be the same color, and the proof is complete.

2. (4 marks) Prove using induction that
$$\sum_{m=0}^{n} m = \frac{n(n+1)}{2}.$$