Steven Maharaj 695281 Assignment 2, Question 2

Due: Friday 20 September 2019

There are places in this assignment where R code will be required. Therefore set the random seed so assignment is reproducible.

```
set.seed(695281) #Please change random seed to your student id number.
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##     filter, lag
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(mvtnorm)
library(coda)
library(ggplot2)
library(tidyr)
```

PART B

For This Question we define proper priors for $\boldsymbol{\beta}, \boldsymbol{\tau}$ using the results from part a. That is using the following formula.

$$p(\theta|\boldsymbol{y}) = \frac{p(\boldsymbol{y_2}|\theta)p(\theta|\boldsymbol{y_1})}{p(\boldsymbol{y_2}|\boldsymbol{y_1})}$$

Using the posterior from the previous part we let the prior for this part be

$$p(\boldsymbol{\beta}|\tau) = \mathcal{N}(\hat{\boldsymbol{\beta}}_1, (\boldsymbol{X}_1\boldsymbol{X}_1)^{-1}/\tau)$$

$$p(\boldsymbol{\beta}|\tau) = Ga(\frac{n_1 - p}{2}, \frac{(n_1 - p)s^2}{2})$$

where the subscript 1 indicates the results are from group 1 (analysed in 2a) alone. Using the results from lecture 13, we drop the prior for τ_β, and in all other places in the joint distribution, replace τ_β and τ_e with τ.

- $\boldsymbol{K} = (\boldsymbol{X}_1\boldsymbol{X}_1)^{-1}$
- $\beta_0 = \hat{\beta}_1$
- $\alpha = \frac{n_1 - p}{2}$
- $\gamma = \frac{(n_1 - p)s^2}{2}$

Thus we get the following conditional posteriors

$$p(\tau|\mathbf{y}, \boldsymbol{\beta}, \beta_0, \mathbf{K}) = \text{Ga}\left(\alpha + \frac{n+p}{2}, \gamma + \frac{(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + (\boldsymbol{\beta} - \beta_0)' \mathbf{K}^{-1} (\boldsymbol{\beta} - \beta_0)}{2}\right)$$

$$p\left(\boldsymbol{\beta}|\mathbf{y},\boldsymbol{\beta}_0,\mathbf{K},\tau\right) == \mathcal{N}\left(\left(\mathbf{X}'\mathbf{X}+\mathbf{K}^{-1}\right)^{-1}\left(\mathbf{X}'\mathbf{y}+\mathbf{K}^{-1}\beta_0\right),\left(\mathbf{X}'\mathbf{X}+\mathbf{K}^{-1}\right)^{-1}/\tau\right)$$