Steven Maharaj 695281 Assignment 2, Question 3 MAST90125: Bayesian Statistical Learning

Due: Friday 20 September 2019

There are places in this assignment where R code will be required. Therefore set the random seed so assignment is reproducible.

```r
set.seed(695281)   #Please change random seed to your student id number.
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library(ggplot2)
library(tidyr)
library(TruncatedNormal)
```

```r
rtn <- function(n,b,a,mu,Sigma){
  u <- runif(n)
  g <- pnorm((b-mu)/Sigma) - pnorm((a-mu)/Sigma)
  x <-qnorm((g) * u + pnorm((a-mu)/Sigma))*Sigma + mu
}
```

PART C

We mplement a Gibbs sampler to fit the same mixed model, but now with a probit link.

Assumsing,

- $p(\boldsymbol{\beta}) \propto 1$
- $p(\boldsymbol{u}) = \mathcal{N}(\boldsymbol{0}, \sigma_u^2 \boldsymbol{I})$
- $p(\tau_u) = Ga(\alpha_u, \gamma_u)$

It can be shown that we have the folling conditional posteriors

$$p\left(\tau_u | \cdot\right) = \text{Ga}\left(\alpha_u + q/2, \gamma_u + \mathbf{u}'\mathbf{u}/2\right)$$

$$p\left(\left(\begin{array}{c} \beta \\ u \end{array}\right)\Big|\cdot\right) = \mathcal{N}\left(\left(\begin{array}{cc} X'X & X'Z \\ Z'X & Z'Z + \tau_u\boldsymbol{I}^{-1} \end{array}\right)^{-1}\left(\begin{array}{c} X'z \\ Z'z \end{array}\right), \left(\begin{array}{cc} X'X & X'Z \\ Z'X & Z'Z + \tau_u\boldsymbol{I}^{-1} \end{array}\right)^{-1}\right)$$