Rates of change

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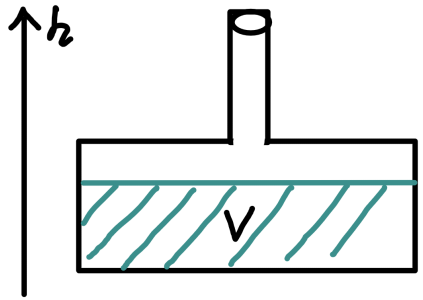
March 30, 2020

Rate of change

If we have an independent variable x and a dependent variable y one could explore how quickly y changes as x changes. This is called the rate of change. For example the gradient is constant rate of change.

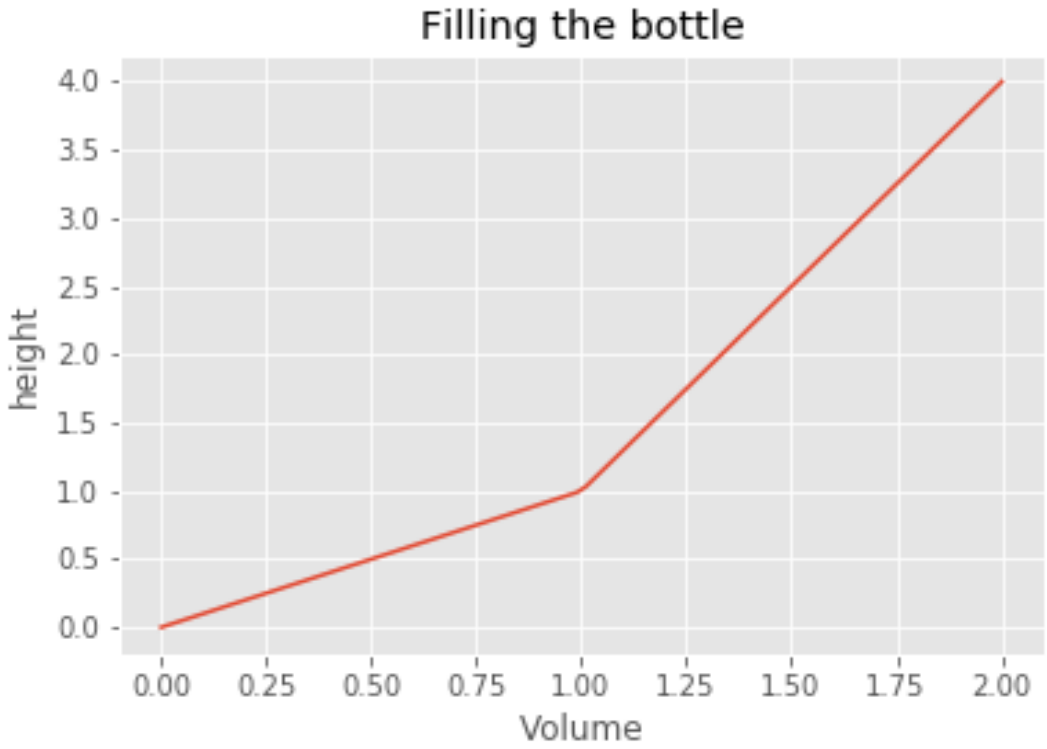
Filling a bottle

Let's try to understand what a non constant rate of change is. Consider a bottle of the following shape.



h is the height of the water from the bottom of the bottle. V is the volume of water in the bottle.

The relationship between height and volume is shown below



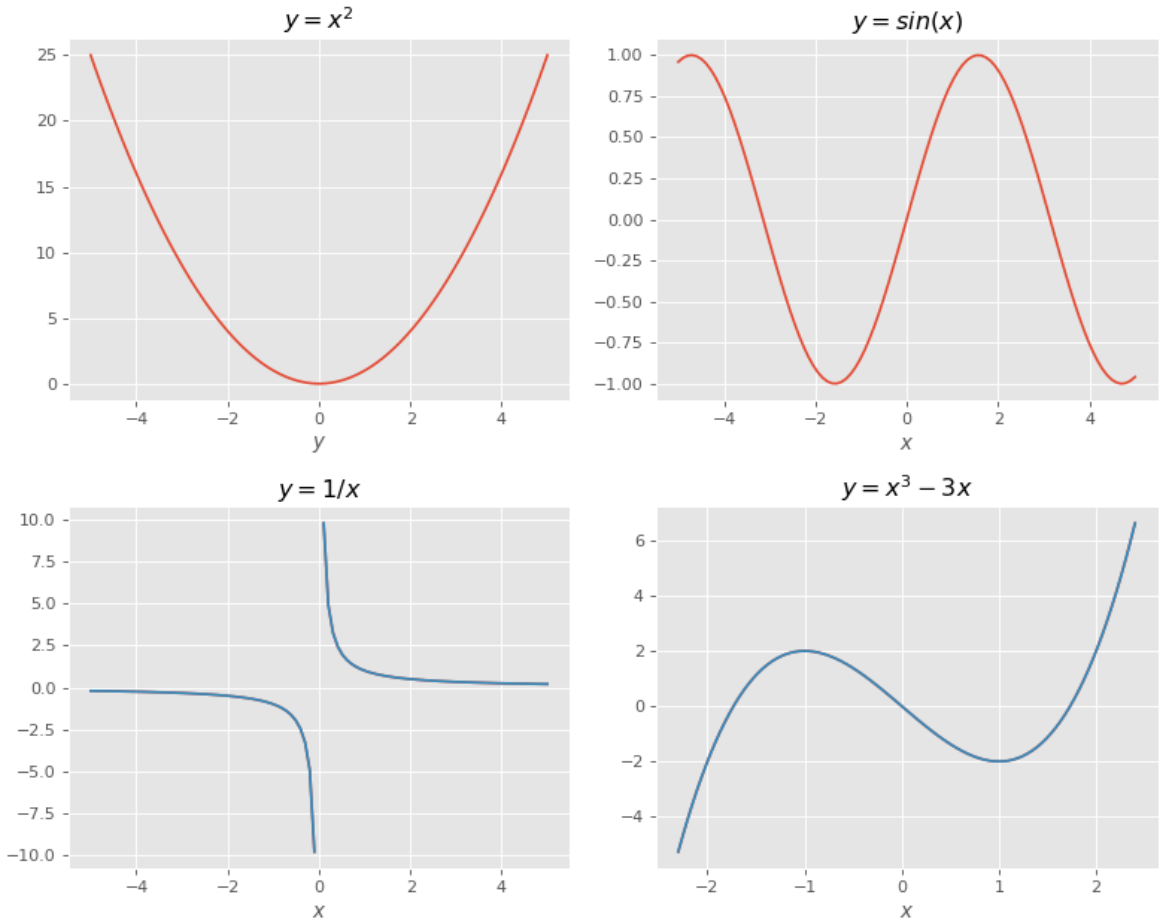
The height increases rapidly once the body is filled.

Positive and negative

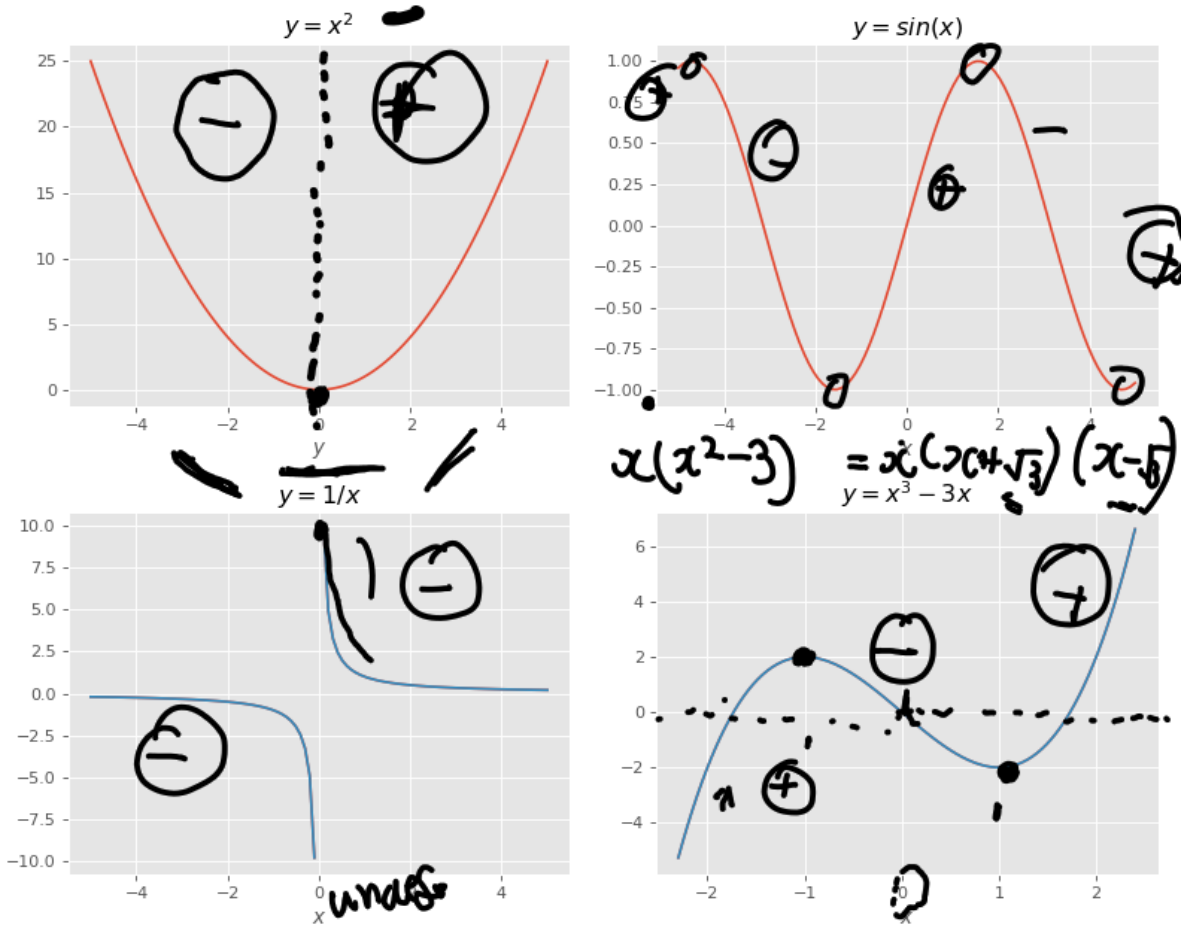
From left to Right

- if you are move up the graph you have a positive rates of change. $\nearrow$
- if you are move down the graph you have a negative rate of change. $\searrow$

Where is the rate of change positive and where is the rate of change negative for following plots.



Answers



Car speed example

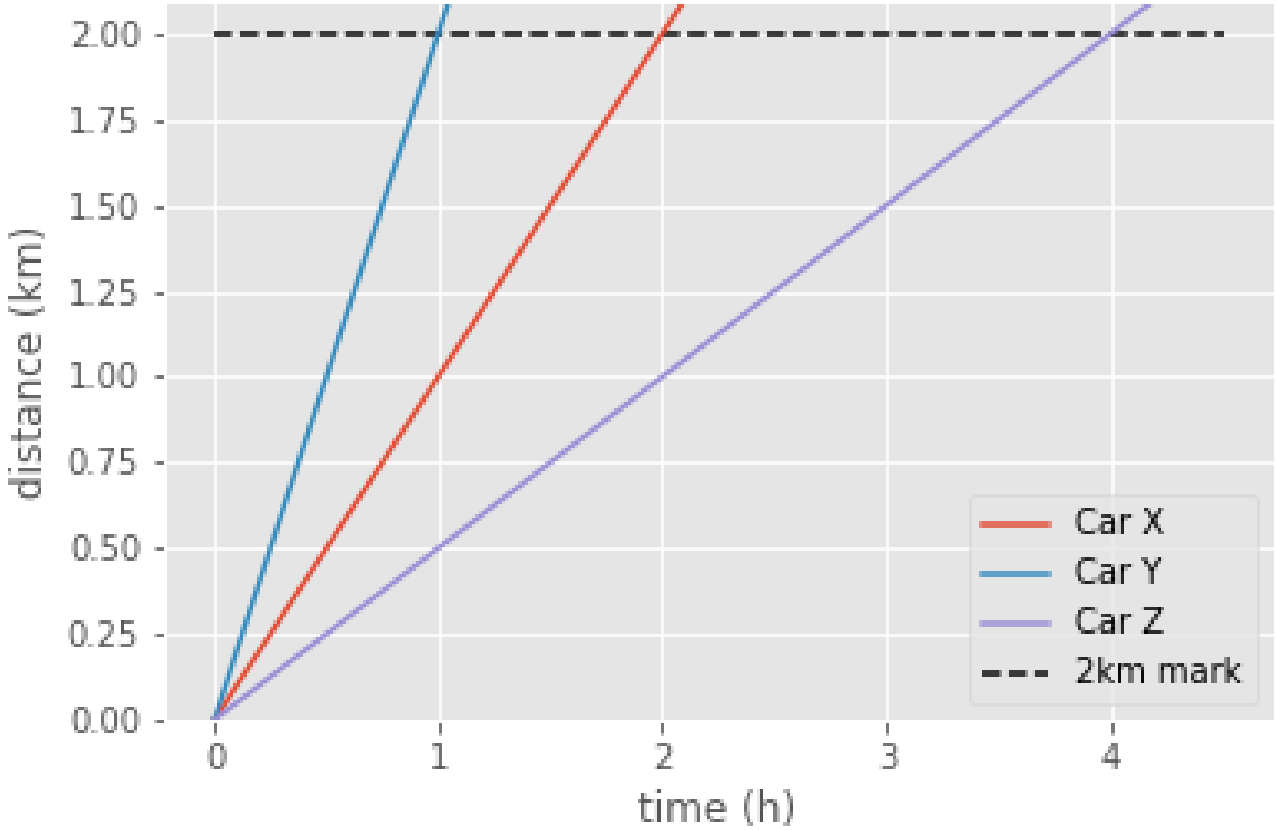
Three cars are driven over a 2-km straight track.

- The speed of Car Y is twice that of Car X
- the speed of Car Z is half that of Car X.

Illustrate this on a graph. Assume CAR X travels at 1km/h

Car speed example

Answers

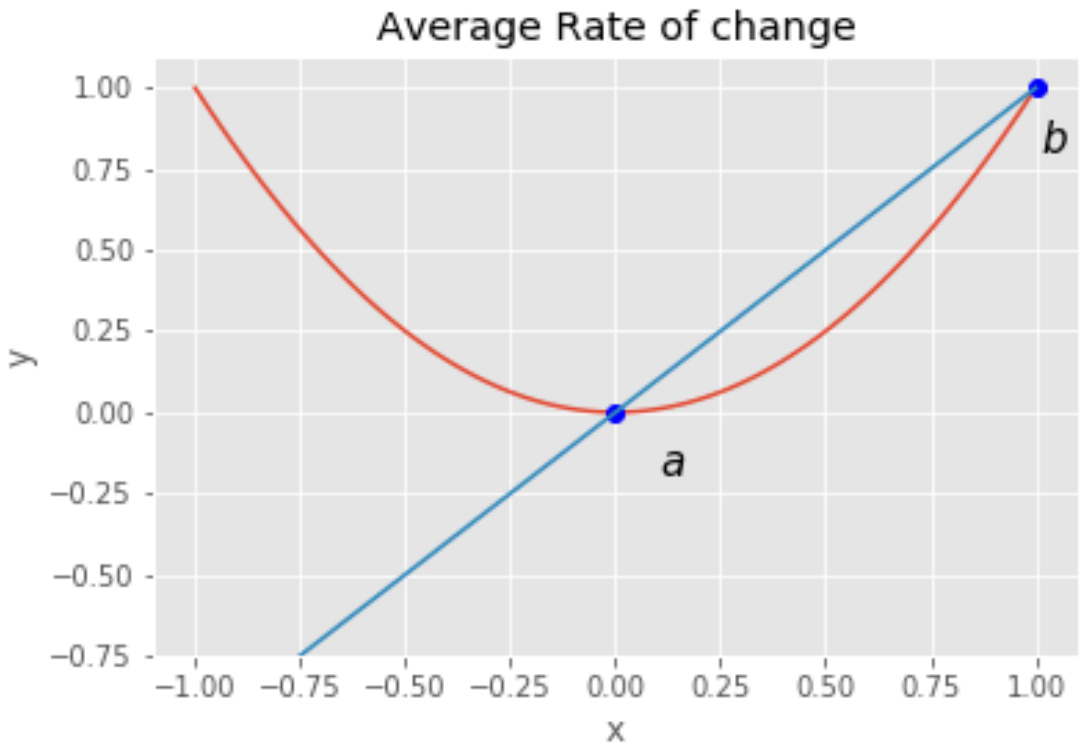


Average rate of change

We define the average rate of change from point a to be b as

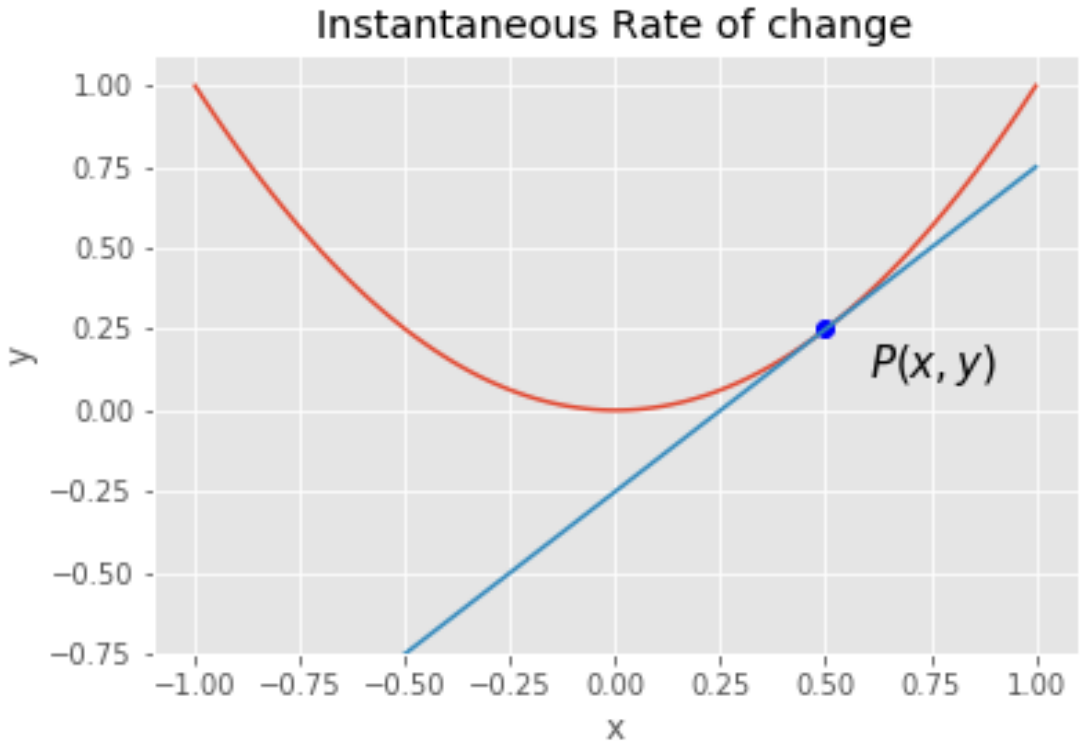
$$\frac{f(b) - f(a)}{b - a}$$

This is the gradient between two points a and b.



Instantaneous rate of change

We can also define the instantaneous rate of change.
Essentially, this is the gradient of the tangent line to a curve at some $P(x, y)$



The differentiation topic is all about finding the Instantaneous rate of change.