Differentiation

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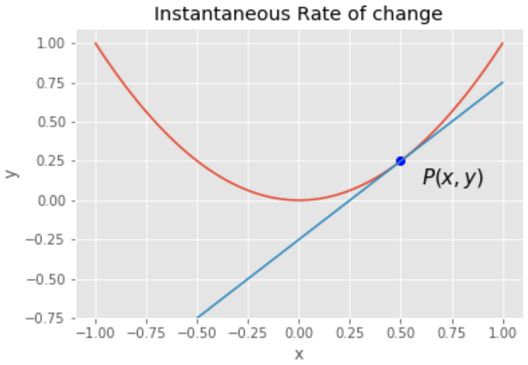
What is Differentiation

Differentiation is a tool used to find the instantaneous rate of change.
For example you are on the train. Then we freeze time. We can use differentiation to find the instantaneous rate of change (speed).

What does differentiation find?

Differentiation is a tool used to find gradient for a tangent line. That is differentiation finds the gradient of the blue line below.



Notation

- You use **Differentiation** find the **derivative** of a function.
- Use the $\frac{d}{dx}()$ to denote the derivative of something. E.g. $\frac{d}{dx}(f(x))$ translates to "the derivative with respect to x of f of x"

So you take a derivative of a function with respect to the function argument.

Notation

Other notations

- $\frac{d}{dx}(f(x))$
- $\frac{df(x)}{dx}$
- if $y = f(x)$ then we could say $\frac{dy}{dx}$
- We have the short hand $\frac{d}{dx}(f(x)) = f'(x)$

How to differentiate

Derivatives come from limit. Using limits we can find the derivative for most functions.
In practice you look up the derivative of a function.

How to differentiate

Derivatives come from limit. Using limits we can find derivatives for most functions.

In practice you look up the derivative of a function. In the final exam you are given a list of derivatives

Derivative formula sheet

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$

Figure: https://www.vcaa.vic.edu.au/Documents/exams/mathematics/mathmethods-formula-w.pdf

Examples

- $\frac{d}{dx}(x^3) = 3x^2$
- $\frac{d}{dx}(e^{2x}) = 2e^{2x}$
- $\frac{d}{dx}(sin(3x)) = 3cos(3x)$
- $\frac{d}{dx}(3cos(3x)) = -9sin(3x)$

Rules

- if $g(x) = kf(x)$ where k is a constant then $g'(x) = kf'(x)$
- if $g(x) = f(x) + h(x)$ then $g'(x) = f'(x) + h'(x)$

Advanced rules (Year 12)

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

- **Product rule** if $g(x)=f(x)h(x)$ then $g'(x)=f'(x)h(x)+h'(x)f(x)$
- **Quotient rule** if $g(x)=\frac{f(x)}{h(x)}$ then $g'(x)=\frac{f'(x)h(x)-h'(x)f(x)}{(h(x))^2}$
- **Chain rule** if $g(x)=f(h(x))$ then $g'(x)=h'(x)f'(h(x))$

Example

$$\frac{d}{dx}(2x + x^3)$$
$$= \frac{d}{dx}(2x) + \frac{d}{dx}(x^3)$$
$$= 2 + 3x^2$$

Example 1

b. Let $g(x) = (2 - x^3)^3$.

Evaluate $g'(1)$. 2 marks

https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM1-w.pdf

Example 1 Answer

$$
\begin{aligned}
g'(x) &= (-3x^2)3(2-x^3)^2 \\
g'(1) &= (-3)3(1) \\
&= -9
\end{aligned}
$$

Example 2 Answer

Let $y = x \log_e(3x)$.

a. Find $\frac{dy}{dx}$. 2 marks

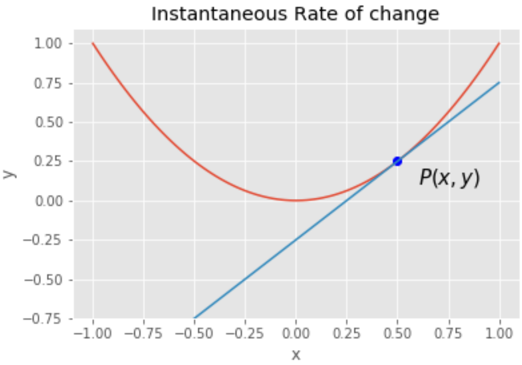
https://www.vcaa.vic.edu.au/Documents/exams/mathematics/2017/2017MM1-w.pdf

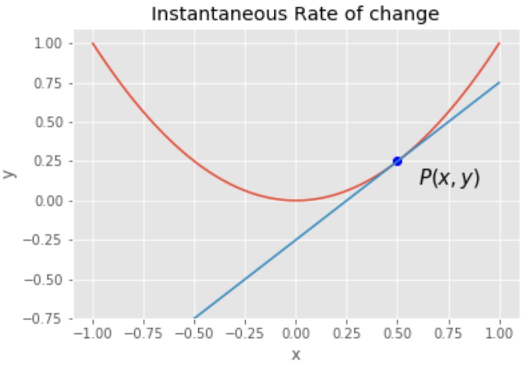
Example 2 Answer

$$\frac{dy}{dx} = log_e(3x) + x\frac{3}{3x}$$
$$\frac{dy}{dx} = log_e(3x) + 1$$

Differentiation and tangent lines

Recall that differentiation is a tool used to find the instantaneous rate of change. suppose the red curve is $f(x)$ then gradient of the blue line is $f'(x)$ evaluated at point P





The blue line has the form

$$y = mx + c$$

we have that

$$m = f'(x)$$

Example

Question Let $f(x) = x^2$ and $P = (1, 1)$. Find the equation of the tangent line on $f(x)$ at point P.
Answer: $f'(x) = 2x$ then evaluate this at $x = 1$

$$f'(1) = 2$$

so the equation of the blue line is $y = 2x + c$. We just need to find c by substituting (1,1)

$$\begin{aligned} 1 &= 2(1) + c \\ c &= -1 \end{aligned}$$

so equation of the blue line is

$$y = 2x - 1$$