

432 Class 05 Slides

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Moving Forward

- Predicting a Binary outcome
 - using a linear probability model
 - using logistic regression and `glm`
- Creating the `smart3` and `smart3_sh` data
 - A “shadow” to track what is imputed

Setup

```
library(conflicted)                                # a new idea
library(here); library(magrittr)
library(janitor); library(knitr)
library(patchwork); library(broom)
library(equatiomatic)
library(simputation); library(naniar)
library(faraway)                                  # for orings data
library(rms)
library(tidyverse)

theme_set(theme_bw())
conflict_prefer("summarize", "dplyr") # choose over Hmisc
conflict_prefer("filter", "dplyr") # choose over stats
options(dplyr.summarise.inform = FALSE)
```

A First Example: Space Shuttle O-Rings

Challenger Space Shuttle Data

The US space shuttle Challenger exploded on 1986-01-28. An investigation ensued into the reliability of the shuttle's propulsion system. The explosion was eventually traced to the failure of one of the three field joints on one of the two solid booster rockets. Each of these six field joints includes two O-rings which can fail.

The discussion among engineers and managers raised concern that the probability of failure of the O-rings depended on the temperature at launch, which was forecast to be 31 degrees F. There are strong engineering reasons based on the composition of O-rings to support the judgment that failure probability may rise monotonically as temperature drops.

We have data on 23 space shuttle flights that preceded *Challenger* on primary o-ring erosion and/or blowby and on the temperature in degrees Fahrenheit. No previous liftoff temperature was under 53 degrees F.

The “O-rings” data

```
orings1 <- faraway::orings %>%  
  tibble() %>%  
  mutate(burst = case_when( damage > 0 ~ 1,  
                             TRUE ~ 0))  
  
orings1 %>% summary()
```

| temp | damage | burst |
|---------------|----------------|----------------|
| Min. :53.00 | Min. :0.0000 | Min. :0.0000 |
| 1st Qu.:67.00 | 1st Qu.:0.0000 | 1st Qu.:0.0000 |
| Median :70.00 | Median :0.0000 | Median :0.0000 |
| Mean :69.57 | Mean :0.4783 | Mean :0.3043 |
| 3rd Qu.:75.00 | 3rd Qu.:1.0000 | 3rd Qu.:1.0000 |
| Max. :81.00 | Max. :5.0000 | Max. :1.0000 |

- damage = number of damage incidents out of 6 possible
- we set burst = 1 if damage > 0

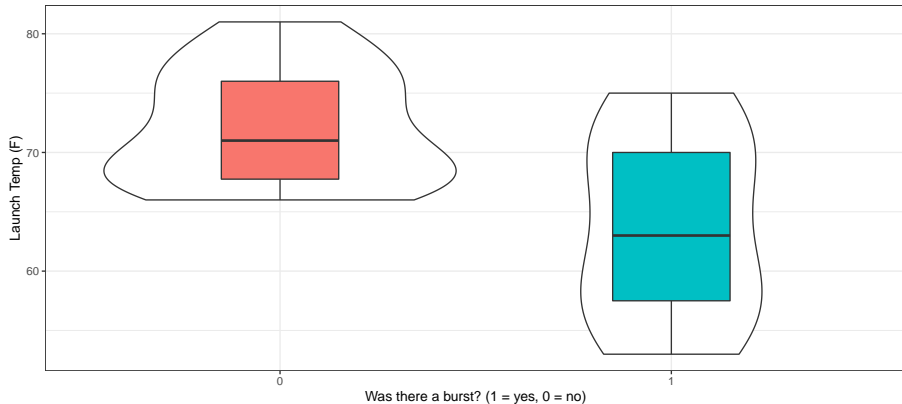
Code to plot burst and temp in our usual way...

```
ggplot(orings1, aes(x = factor(burst), y = temp)) +  
  geom_violin() +  
  geom_boxplot(aes(fill = factor(burst)), width = 0.3) +  
  guides(fill = "none") +  
  labs(title = "Are bursts more common at low temperatures?",  
        subtitle = "23 prior space shuttle launches",  
        x = "Was there a burst? (1 = yes, 0 = no)",  
        y = "Launch Temp (F)")
```

Plotted Association of burst and temp

Are bursts more common at low temperatures?

23 prior space shuttle launches



What if we want to predict $\text{Prob}(\text{burst})$ using temp?

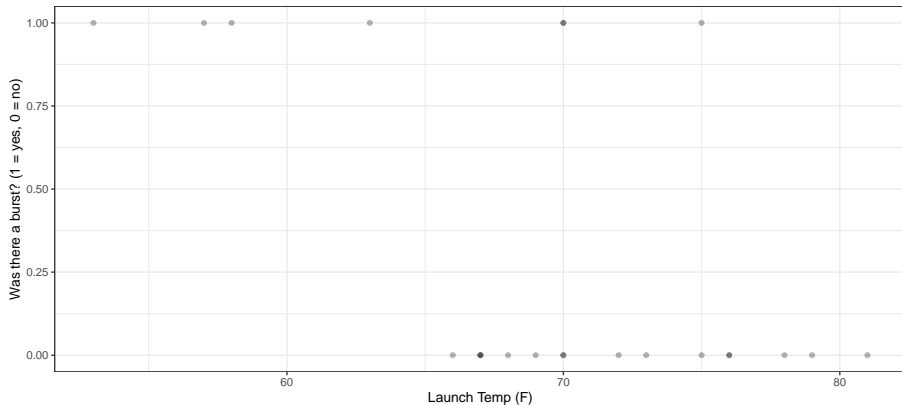
We want to treat the binary variable burst as the outcome, and temp as the predictor...

```
ggplot(orings1, aes(x = temp, y = burst)) +  
  geom_point(alpha = 0.3) +  
  labs(title = "Are bursts more common at low temperatures",  
        subtitle = "23 prior space shuttle launches",  
        y = "Was there a burst? (1 = yes, 0 = no)",  
        x = "Launch Temp (F)")
```

Plot of Prob(burst) by temperature at launch

Are bursts more common at low temperatures

23 prior space shuttle launches



Fit a linear model to predict Prob(burst)?

```
mod1 <- lm(burst ~ temp, data = orings1)
```

```
tidy(mod1, conf.int = T) %>% kable(digits = 3)
```

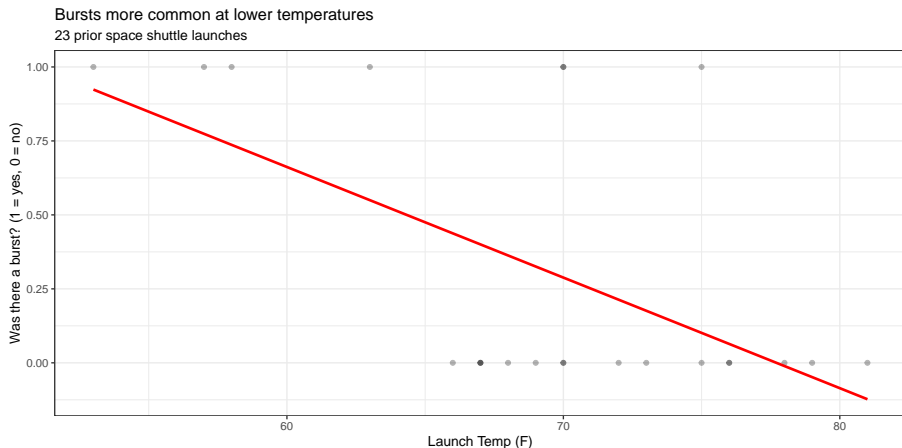
| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|-------------|----------|-----------|-----------|---------|----------|-----------|
| (Intercept) | 2.905 | 0.842 | 3.450 | 0.002 | 1.154 | 4.656 |
| temp | -0.037 | 0.012 | -3.103 | 0.005 | -0.062 | -0.012 |

- This is a **linear probability model**.

```
extract_eq(mod1, use_coefs = TRUE, coef_digits = 3)
```

$$\widehat{\text{burst}} = 2.905 - 0.037(\text{temp}) \quad (1)$$

Add linear probability model to our plot?



- It would help if we could see the individual launches. . .

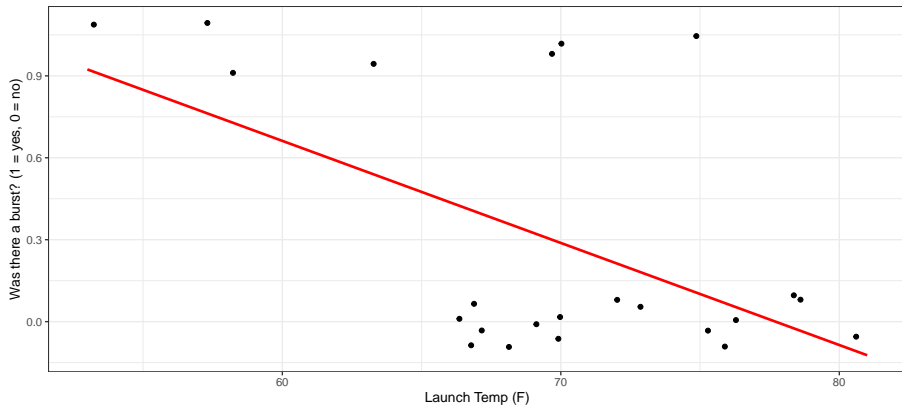
Add vertical jitter and our mod1 model?

```
ggplot(orings1, aes(x = temp, y = burst)) +  
  geom_jitter(height = 0.1) +  
  geom_smooth(method = "lm", se = F, col = "red",  
              formula = y ~ x) +  
  labs(title = "Bursts more common at lower temperatures",  
        subtitle = "23 prior space shuttle launches",  
        y = "Was there a burst? (1 = yes, 0 = no)",  
        x = "Launch Temp (F)")
```

Resulting plot with points jittered and linear model

Bursts more common at lower temperatures

23 prior space shuttle launches



- What's wrong with this picture?

Making Predictions with mod1

```
tidy(mod1, conf.int = T) %>%  
  kable(digits = c(0,5,3,3,3,3,3))
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|-------------|----------|-----------|-----------|---------|----------|-----------|
| (Intercept) | 2.90476 | 0.842 | 3.450 | 0.002 | 1.154 | 4.656 |
| temp | -0.03738 | 0.012 | -3.103 | 0.005 | -0.062 | -0.012 |

- What does mod1 predict for the probability of a burst if the temperature at launch is 70 degrees F?

$$Prob(burst) = 2.90476 - 0.03738(70) = 0.288$$

- What if the temperature was actually 60 degrees F?

Making Several Predictions with `mod1`

Let's use our linear probability model `mod1` to predict the probability of a burst at some other temperatures...

```
newtemps <- tibble(temp = c(80, 70, 60, 50, 31))
```

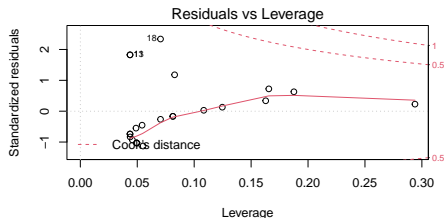
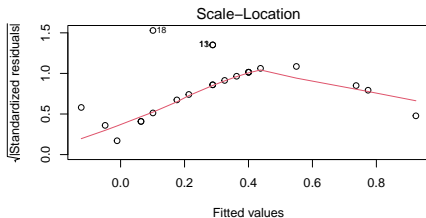
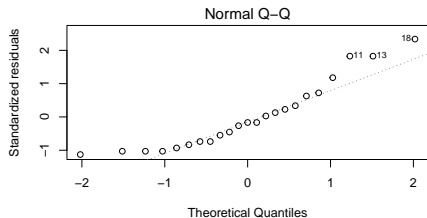
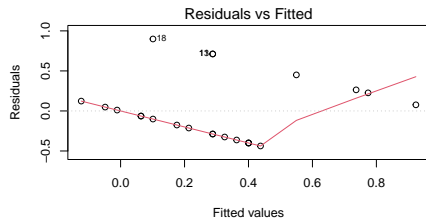
```
augment(mod1, newdata = newtemps)
```

```
# A tibble: 5 x 2
```

| | temp | .fitted |
|---|-------|---------|
| | <dbl> | <dbl> |
| 1 | 80 | -0.0857 |
| 2 | 70 | 0.288 |
| 3 | 60 | 0.662 |
| 4 | 50 | 1.04 |
| 5 | 31 | 1.75 |

- Uh, oh.

Residual Plots for mod1?



• Uh, oh.

Models to predict a Binary Outcome

Our outcome takes on two values (zero or one) and we then model the probability of a “one” response given a linear function of predictors.

Idea 1: Use a *linear probability model*

- Main problem: predicted probabilities that are less than 0 and/or greater than 1
- Also, how can we assume Normally distributed residuals when outcomes are 1 or 0?

Idea 2: Build a *non-linear* regression approach

- Most common approach: logistic regression, part of the class of *generalized* linear models

The Logit Link and Logistic Function

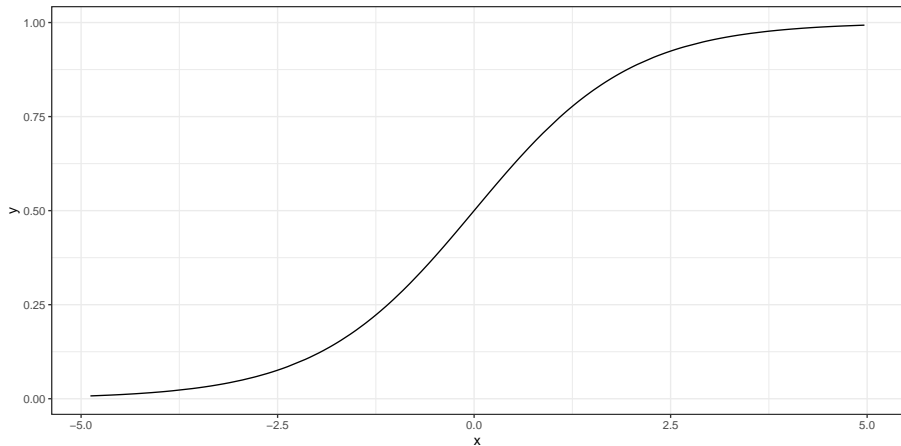
The function we use in logistic regression is called the **logit link**.

$$\text{logit}(\pi) = \log\left(\frac{\pi}{1 - \pi}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

The inverse of the logit function is called the **logistic function**. If $\text{logit}(\pi) = \eta$, then $\pi = \frac{\exp(\eta)}{1 + \exp(\eta)}$.

- The logistic function $\frac{e^x}{1 + e^x}$ takes any value x in the real numbers and returns a value between 0 and 1.

The Logistic Function $y = \frac{e^x}{1+e^x}$



The logit or log odds

We usually focus on the **logit** in statistical work, which is the inverse of the logistic function.

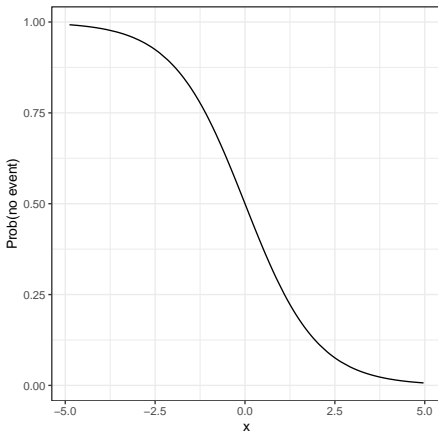
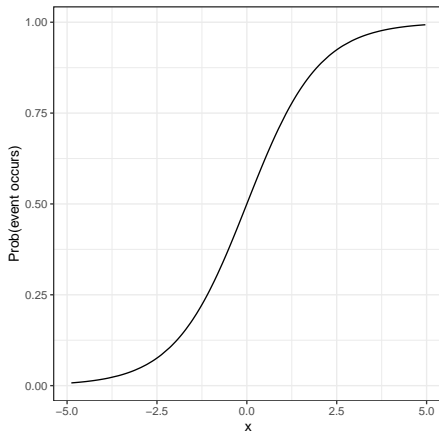
- If we have a probability $\pi < 0.5$, then $\text{logit}(\pi) < 0$.
- If our probability $\pi > 0.5$, then $\text{logit}(\pi) > 0$.
- Finally, if $\pi = 0.5$, then $\text{logit}(\pi) = 0$.

Why is this helpful?

- $\log(\text{odds}(Y = 1))$ or $\text{logit}(Y = 1)$ covers all real numbers.
- $\text{Prob}(Y = 1)$ is restricted to $[0, 1]$.

Predicting $\Pr(\text{event})$ or $\Pr(\text{no event})$

- Can we flip the story?



Returning to the prediction of Prob(burst)

We'll use the `glm` function in R, specifying a logistic regression model.

- Instead of predicting $Pr(burst)$, we're predicting $\log(odds(burst))$ or $\text{logit}(burst)$.

```
mod2 <- glm(burst ~ temp, data = orings1,  
            family = binomial(link = "logit"))  
  
tidy(mod2, conf.int = TRUE) %>%  
  select(term, estimate, std.error, conf.low, conf.high) %>%  
  kable(digits = c(0,4,3,3,3))
```

| term | estimate | std.error | conf.low | conf.high |
|-------------|----------|-----------|----------|-----------|
| (Intercept) | 15.0429 | 7.379 | 3.331 | 34.342 |
| temp | -0.2322 | 0.108 | -0.515 | -0.061 |

Our model mod2

```
extract_eq(mod2, use_coefs = TRUE, coef_digits = 4)
```

$$\log \left[\frac{P(\widehat{\text{burst}} = 1)}{1 - P(\widehat{\text{burst}} = 1)} \right] = 15.0429 - 0.2322(\text{temp}) \quad (2)$$

$$\text{logit}(\text{burst}) = \log(\text{odds}(\text{burst})) = 15.0429 - 0.2322\text{temp}$$

- For a temperature of 70 F at launch, what is the prediction?

Let's look at the results

- For a temperature of 70 F at launch, what is the prediction?

$$\log(\text{odds}(\text{burst})) = 15.0429 - 0.2322 (70) = -1.211$$

- Exponentiate to get the odds, on our way to estimating the probability.

$$\text{odds}(\text{burst}) = \exp(-1.211) = 0.2979$$

- so, we can estimate the probability by

$$Pr(\text{burst}) = \frac{0.2979}{(0.2979 + 1)} = 0.230.$$

Prediction from mod2 for temp = 60

What is the predicted probability of a burst if the temperature is 60 degrees?

- $\log(\text{odds}(\text{burst})) = 15.0429 - 0.2322 (60) = 1.1109$
- $\text{odds}(\text{burst}) = \exp(1.1109) = 3.0371$
- $\text{Pr}(\text{burst}) = 3.0371 / (3.0371 + 1) = 0.752$

Will augment do this, as well?

```
temps <- tibble(temp = c(60,70))
```

```
augment(mod2, newdata = temps, type.predict = "link")
```

```
# A tibble: 2 x 2
```

| | temp | .fitted |
|---|-------|---------|
| | <dbl> | <dbl> |
| 1 | 60 | 1.11 |
| 2 | 70 | -1.21 |

```
augment(mod2, newdata = temps, type.predict = "response")
```

```
# A tibble: 2 x 2
```

| | temp | .fitted |
|---|-------|---------|
| | <dbl> | <dbl> |
| 1 | 60 | 0.753 |
| 2 | 70 | 0.230 |

Plotting the Logistic Regression Model

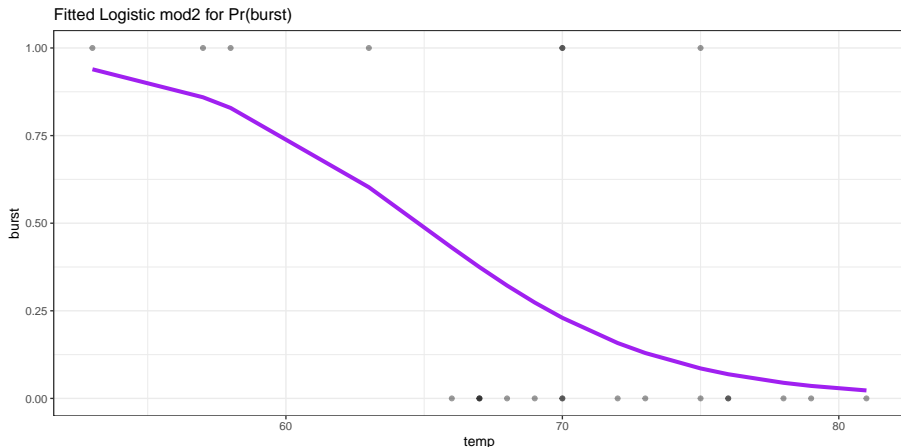
Use the `augment` function to get the fitted probabilities into the original data, then plot.

```
mod2_aug <- augment(mod2, type.predict = "response")

ggplot(mod2_aug, aes(x = temp, y = burst)) +
  geom_point(alpha = 0.4) +
  geom_line(aes(x = temp, y = .fitted),
            col = "purple", size = 1.5) +
  labs(title = "Fitted Logistic mod2 for Pr(burst)")
```

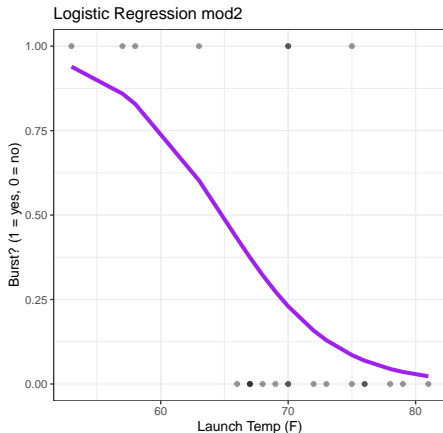
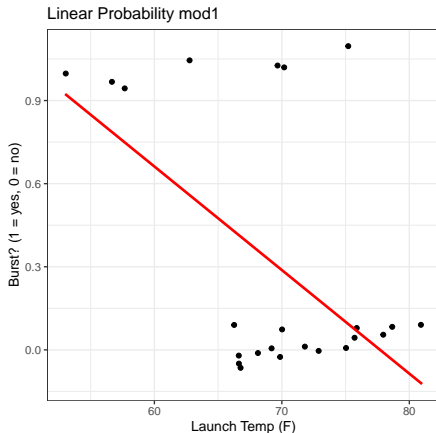
- Results on next slide

Plotting Model `m2`



Note that we're just connecting the predictions made for observed `temp` values with `geom_line`, so the appearance of the function isn't as smooth as the actual logistic regression model.

Comparing the fits of mod1 and mod2...



Could we try exponentiating the mod2 coefficients?

How can we interpret the coefficients of the model?

$$\text{logit}(\text{burst}) = \log(\text{odds}(\text{burst})) = 15.043 - 0.232\text{temp}$$

Exponentiating the coefficients is helpful. . .

```
exp(-0.232)
```

```
[1] 0.7929461
```

Suppose Launch A's temperature was one degree higher than Launch B's.

- The **odds** of Launch A having a burst are 0.793 times as large as they are for Launch B.
- Odds Ratio estimate comparing two launches whose temp differs by 1 degree is 0.793

Exponentiated and tidied slope of temp (mod2)

```
tidy(mod2, exponentiate = TRUE, conf.int = TRUE) %>%  
  filter(term == "temp") %>%  
  select(term, estimate, std.error, conf.low, conf.high) %>%  
  kable(digits = 3)
```

| term | estimate | std.error | conf.low | conf.high |
|------|----------|-----------|----------|-----------|
| temp | 0.793 | 0.108 | 0.597 | 0.941 |

- What would it mean if the Odds Ratio for temp was 1?
- How about an odds ratio that was greater than 1?

Regression on a Binary Outcome

Linear Probability Model (a linear model)

```
lm(event ~ predictor1 + predictor2 + ..., data = tibblename)
```

- $\Pr(\text{event})$ is linear in the predictors

Logistic Regression Model (generalized linear model)

```
glm(event ~ pred1 + pred2 + ..., data = tibblename,  
     family = binomial(link = "logit"))
```

- Logistic Regression forces a prediction in $(0, 1)$
- $\log(\text{odds}(\text{event}))$ is linear in the predictors

The logistic regression model

$$\text{logit}(\text{event}) = \log \left(\frac{\text{Pr}(\text{event})}{1 - \text{Pr}(\text{event})} \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

$$\text{odds}(\text{event}) = \frac{\text{Pr}(\text{event})}{1 - \text{Pr}(\text{event})}$$

$$\text{Pr}(\text{event}) = \frac{\text{odds}(\text{event})}{\text{odds}(\text{event}) + 1}$$

$$\text{Pr}(\text{event}) = \frac{\exp(\text{logit}(\text{event}))}{1 + \exp(\text{logit}(\text{event}))}$$

Building a smart3 tibble

BRFSS and SMART (Creating smart3)

```
smart3 <- read_csv(here("data/smart_ohio.csv")) %>%  
  mutate(SEQNO = as.character(SEQNO)) %>%  
  select(SEQNO, mmsa, mmsa_wt, landline,  
         age_imp, healthplan, dm_status,  
         fruit_day, drinks_wk, activity,  
         smoker, physhealth, bmi, genhealth)
```

smart3 Variables, by Type

| Variable | Type | Description |
|------------|--------------|--|
| landline | Binary (1/0) | survey conducted by landline? (vs. cell) |
| healthplan | Binary (1/0) | subject has health insurance? |
| age_imp | Quantitative | age (imputed from groups - see Notes) |
| fruit_day | Quantitative | mean servings of fruit / day |
| drinks_wk | Quantitative | mean alcoholic drinks / week |
| bmi | Quantitative | body-mass index (in kg/m ²) |
| physhealth | Count (0-30) | of last 30 days, # in poor physical health |
| dm_status | Categorical | diabetes status (4 levels, <i>we'll collapse to 2</i>) |
| activity | Categorical | physical activity level (4 levels, <i>we'll re-level</i>) |
| smoker | Categorical | smoking status (4 levels, <i>we'll collapse to 3</i>) |
| genhealth | Categorical | self-reported overall health (5 levels) |

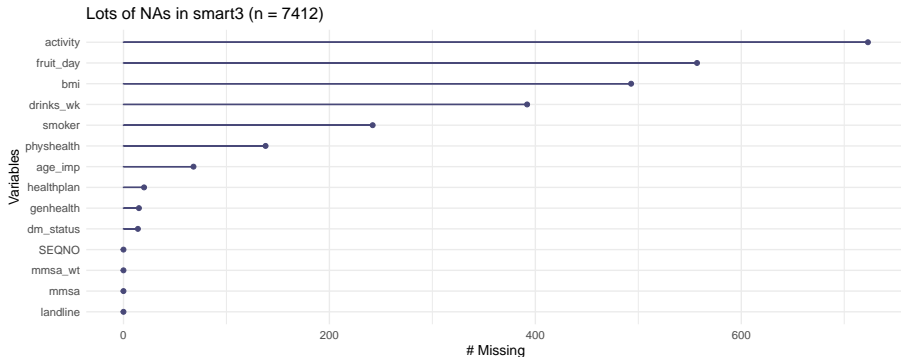
Collapsing Two Factors, Re-leveling another

```
smart3 <- smart3 %>% type.convert() %>%  
  mutate(SEQNO = as.character(SEQNO)) %>%  
  mutate(dm_status =  
    fct_collapse(factor(dm_status),  
                  Yes = "Diabetes",  
                  No = c("No-Diabetes",  
                        "Pre-Diabetes",  
                        "Pregnancy-Induced")))) %>%  
  mutate(smoker =  
    fct_collapse(factor(smoker),  
                  Current = c("Current_not_daily",  
                              "Current_daily")))) %>%  
  mutate(activity =  
    fct_relevel(factor(activity),  
                 "Highly_Active", "Active",  
                 "Insufficiently_Active",  
                 "Inactive"))
```

Visualizing Missingness in Variables

```
gg_miss_var(smart3) +  
  labs(title = "Lots of NAs in smart3 (n = 7412)")
```

Warning: It is deprecated to specify ``guide = FALSE`` to remove a guide. Please use ``guide = "none"`` instead.



Creating a “Shadow” to track what is imputed

```
smart3_sh <- smart3 %>% bind_shadow()
```


smart3_sh creates new variables, ending in _NA

```
names(smart3_sh)
```

```
[1] "SEQNO"          "mmsa"           "mmsa_wt"
[4] "landline"       "age_imp"        "healthplan"
[7] "dm_status"      "fruit_day"      "drinks_wk"
[10] "activity"       "smoker"         "physhealth"
[13] "bmi"            "genhealth"      "SEQNO_NA"
[16] "mmsa_NA"        "mmsa_wt_NA"     "landline_NA"
[19] "age_imp_NA"     "healthplan_NA"  "dm_status_NA"
[22] "fruit_day_NA"   "drinks_wk_NA"   "activity_NA"
[25] "smoker_NA"      "physhealth_NA"  "bmi_NA"
[28] "genhealth_NA"
```

What are the new variables tracking?

```
smart3_sh %>% count(smoker, smoker_NA)
```

```
# A tibble: 4 x 3
  smoker  smoker_NA      n
  <fct>   <fct>    <int>
1 Current !NA        1290
2 Former  !NA        1999
3 Never   !NA        3881
4 <NA>    NA         242
```

The fct_explicit_na warning: A pain point

My general preference is to not use `fct_explicit_na`, and if I see a warning about that, I typically suppress it from printing.

“Simple” Imputation Strategy

```
set.seed(2022432)
smart3_sh <- smart3_sh %>%
  data.frame() %>%
    impute_rhd(dm_status + smoker ~ 1) %>%
    impute_rhd(healthplan + activity ~ 1) %>%
    impute_rlm(age_imp + fruit_day + drinks_wk + bmi ~
      mmsa + landline + healthplan) %>%
    impute_knn(physhealth ~ bmi) %>%
    impute_cart(genhealth ~ activity + physhealth +
      mmsa + healthplan) %>%
  tibble()
```

Check to see that imputation worked...

Before imputation, what fraction of our cases are complete?

```
pct_complete_case(smart3)
```

```
[1] 81.08473
```

After imputation, do any of our cases have missing values?

```
pct_miss_case(smart3_sh)
```

```
[1] 0
```

Saving the smart3 and smart3_sh tibbles to .Rds

```
saveRDS(smart3, "data/smart3.Rds")
```

```
saveRDS(smart3_sh, "data/smart3_sh.Rds")
```

Using diabetes status (yes/no) to predict
whether $\text{BMI} > 30$

Create binary outcome variable

```
smart3_sh <- readRDS("data/smart3_sh.Rds") %>%  
  mutate(bmigt30 = as.numeric(bmi > 30),  
         dm_status = fct_relevel(dm_status, "No"))  
  
smart3_sh %>%  
  group_by(bmigt30) %>%  
  summarize(n = n(), mean(bmi), min(bmi), max(bmi)) %>%  
  kable(digits = 2)
```

| bmigt30 | n | mean(bmi) | min(bmi) | max(bmi) |
|---------|------|-----------|----------|----------|
| 0 | 5074 | 25.26 | 13.30 | 30.00 |
| 1 | 2338 | 35.85 | 30.01 | 75.52 |

Predicting BMI > 30 using diabetes status (a factor)

```
mod_DM <- smart3_sh %$%  
  glm(bmigt30 ~ dm_status,  
      family = binomial(link = logit))  
  
tidy(mod_DM) %>% select(term, estimate) %>%  
  kable(digits = 3)
```

| term | estimate |
|--------------|----------|
| (Intercept) | -0.949 |
| dm_statusYes | 1.048 |

Equation: $\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 (\text{dm_statusYes})$

How can we interpret this result?

Interpreting the mod_DM Equation

$$\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 (\text{dm_statusYes})$$

- Harry has diabetes.
 - His predicted $\text{logit}(\text{BMI} > 30)$ is $-0.949 + 1.048 (1) = 0.099$
- Sally does not have diabetes.
 - Her predicted $\text{logit}(\text{BMI} > 30)$ is $-0.949 + 1.048 (0) = -0.949$

Now, $\text{logit}(\text{BMI} > 30) = \log(\text{odds}(\text{BMI} > 30))$, so exponentiate to get the odds...

- Harry has predicted $\text{odds}(\text{BMI} > 30) = \exp(0.099) = 1.104$
- Sally has predicted $\text{odds}(\text{BMI} > 30) = \exp(-0.949) = 0.387$

Can we convert these odds into something more intuitive?

Converting Odds to Probabilities

- Harry has predicted $\text{odds}(\text{BMI} > 30) = \exp(0.099) = 1.104$
- Sally has predicted $\text{odds}(\text{BMI} > 30) = \exp(-0.949) = 0.387$

$$\text{odds}(\text{BMI} > 30) = \frac{\text{Pr}(\text{BMI} > 30)}{1 - \text{Pr}(\text{BMI} > 30)}$$

and

$$\text{Pr}(\text{BMI} > 30) = \frac{\text{odds}(\text{BMI} > 30)}{\text{odds}(\text{BMI} > 30) + 1}$$

- So Harry's predicted $\text{Pr}(\text{BMI} > 30) = 1.104 / 2.104 = 0.52$
- Sally's predicted $\text{Pr}(\text{BMI} > 30) = 0.387 / 1.387 = 0.28$
- odds range from 0 to ∞ , and $\log(\text{odds})$ range from $-\infty$ to ∞ .
- odds > 1 if probability > 0.5 . If odds = 1, then probability = 0.5.

What about the odds ratio?

$\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 (\text{dm_statusYes})$

- Harry, with diabetes, has $\text{odds}(\text{BMI} > 30) = 1.104$
- Sally, without diabetes, has $\text{odds}(\text{BMI} > 30) = 0.387$

Odds Ratio for $\text{BMI} > 30$ associated with having diabetes (vs. not) =

$$\frac{1.104}{0.387} = 2.85$$

- Our model estimates that a subject with diabetes has 2.85 times the odds (285% of the odds) of a subject without diabetes of having $\text{BMI} > 30$.

Can we calculate the odds ratio from the equation's coefficients?

- Yes, $\exp(1.048) = 2.85$.

Tidy with exponentiation

```
tidy(mod_DM, exponentiate = TRUE,  
      conf.int = TRUE, conf.level = 0.9) %>%  
  select(term, estimate, conf.low, conf.high) %>%  
  kable(digits = 3)
```

| term | estimate | conf.low | conf.high |
|--------------|----------|----------|-----------|
| (Intercept) | 0.387 | 0.369 | 0.405 |
| dm_statusYes | 2.851 | 2.556 | 3.181 |

- The odds ratio for BMI > 30 among subjects with diabetes as compared to those without diabetes is 2.851
- The odds of BMI > 30 are 285.1% as large (2.851 times as large) for subjects with diabetes as they are for subjects without diabetes, according to this model.
- A 90% uncertainty interval for the odds ratio estimate includes (2.556, 3.181).

Interpreting these summaries

Connecting the Odds Ratio and Log Odds Ratio to probability statements. . .

- If the probabilities were the same (for diabetes and non-diabetes subjects) of having $\text{BMI} > 30$, then the odds would also be the same, and so the odds ratio would be 1.
- If the probabilities of $\text{BMI} > 30$ were the same and thus the odds were the same, then the log odds ratio would be $\log(1) = 0$.

$\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 (\text{dm_statusYes})$

- 1 If the log odds of a coefficient (like diabetes = Yes) are negative, then what does that imply?
- 2 What if we flipped the order of the levels for diabetes so our model was about diabetes = No?

Flipping the model changes slope and intercept!

```
mod_DM_no <- smart3_sh %$%  
  glm(bmigt30 ~ (dm_status == "No"),  
      family = binomial(link = logit))  
  
tidy(mod_DM_no) %>% select(term, estimate) %>%  
  kable(digits = 3)
```

| term | estimate |
|-----------------------|----------|
| (Intercept) | 0.098 |
| dm_status == "No"TRUE | -1.048 |

Old: $\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 (\text{dm_statusYes})$

New: $\text{logit}(\text{BMI} > 30) = 0.098 - 1.048 (\text{dm_status} = \text{No})$

Predictions from the two models?

DMYes: $\text{logit}(\text{BMI} > 30) = -0.949 + 1.048 \text{ (dm_status = Yes)}$

DMNo: $\text{logit}(\text{BMI} > 30) = 0.098 - 1.048 \text{ (dm_status = No)}$

Harry lives with diabetes. Sally does not.

Using the DMYes model:

- $\text{logit}(\text{Harry's BMI} > 30) = -0.949 + 1.048 = 0.098$
- $\text{logit}(\text{Sally's BMI} > 30) = -0.949$

Using the DMNo model:

- $\text{logit}(\text{Harry's BMI} > 30) = 0.098$
- $\text{logit}(\text{Sally's BMI} > 30) = 0.098 - 1.048 = -0.949$

Comparison to the 2x2 Table Results?

```
smart3_sh %>% tabyl(bmigt30, dm_status)
```

| bmigt30 | No | Yes |
|---------|------|-----|
| 0 | 4551 | 523 |
| 1 | 1761 | 577 |

That's not quite the 2x2 table we want.

We want to switch the order of both variables

```
temp1 <- smart3_sh %>%  
  select(bmigt30, dm_status, SEQNO)  
  
temp1 <- temp1 %>%  
  mutate(bmigt30 = fct_relevel(factor(bmigt30), "1"),  
         dm_status = fct_relevel(dm_status, "Yes"))  
  
temp1 %>% tabyl(bmigt30, dm_status)
```

| | bmigt30 Yes | No |
|---|-------------|------|
| 1 | 577 | 1761 |
| 0 | 523 | 4551 |

Resulting 2x2 Table Result

```
temp1 %$% Epi::twoby2(bmigt30, dm_status)
```

2 by 2 table analysis:

Outcome : Yes

Comparing : 1 vs. 0

| | Yes | No | P(Yes) | 95% conf. interval |
|---|-----|------|--------|--------------------|
| 1 | 577 | 1761 | 0.2468 | 0.2297 0.2647 |
| 0 | 523 | 4551 | 0.1031 | 0.0950 0.1117 |

| | | 95% conf. interval |
|-----------------------------|--------|--------------------|
| Relative Risk: | 2.3943 | 2.1498 2.6666 |
| Sample Odds Ratio: | 2.8512 | 2.5024 3.2486 |
| Conditional MLE Odds Ratio: | 2.8506 | 2.4969 3.2554 |
| Probability difference: | 0.1437 | 0.1246 0.1633 |

Using smoking status (multi-categorical) to
predict diabetes (yes/no)

Can we use smoker to predict dm_status?

```
smart3_sh %>% tabyl(smoker, dm_status) %>%  
  adorn_totals() %>%  
  adorn_percentages(den = "row") %>%  
  adorn_pct_formatting() %>%  
  adorn_ns(position = "front")
```

| smoker | No | Yes |
|---------|--------------|--------------|
| Current | 1170 (87.8%) | 163 (12.2%) |
| Former | 1689 (81.9%) | 373 (18.1%) |
| Never | 3453 (86.0%) | 564 (14.0%) |
| Total | 6312 (85.2%) | 1100 (14.8%) |

Logistic Regression for dm_status by smoker

```
mod_SM <- glm(dm_status ~ smoker,  
              family = binomial(link = "logit"),  
              data = smart3_sh)  
  
tidy(mod_SM) %>% select(term, estimate) %>% kable(dig = 3)
```

| term | estimate |
|--------------|----------|
| (Intercept) | -1.971 |
| smokerFormer | 0.461 |
| smokerNever | 0.159 |

What is being fit, exactly?

```
extract_eq(mod_SM, use_coefs = TRUE, coef_digits = 3,  
           wrap = TRUE, terms_per_line = 2, label = NA)
```

$$\log \left[\frac{P(\widehat{\text{dm_status}} = \text{Yes})}{1 - P(\widehat{\text{dm_status}} = \text{Yes})} \right] = -1.971 + 0.461(\text{smoker}_{\text{Former}}) + 0.159(\text{smoker}_{\text{Never}}) \quad (3)$$

Resulting Predictions from mod_SM

$\text{logit}(\text{dm} = \text{Yes}) = -1.971 + 0.461 \text{ Former} + 0.159 \text{ Never}$

- from logit to odds via exponentiation

| Smoking Status | logit(DM = Yes) | odds(DM = Yes) |
|----------------|---------------------------|------------------------|
| Current | -1.971 | $\exp(-1.971) = 0.139$ |
| Former | $-1.971 + 0.461 = -1.510$ | $\exp(-1.510) = 0.221$ |
| Never | $-1.971 + 0.159 = -1.812$ | $\exp(-1.812) = 0.163$ |

- convert from odds to probabilities (do these match our table?)

| Smoking Status | odds(DM = Yes) | Pr(DM = Yes) |
|----------------|----------------|----------------------|
| Current | 0.139 | $.139/1.139 = 0.122$ |
| Former | 0.221 | $.221/1.221 = 0.181$ |
| Never | 0.163 | $.163/1.163 = 0.140$ |

Next Time

- Binary regression models with multiple predictors
- Assessing the quality of fit for a logistic model