432 Class 06 Slides

thomase love. github. io/432

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Moving Forward

• Logistic Regression Models and the smart3_sh data

Setup

```
library(here); library(magrittr)
library(janitor); library(knitr)
library(patchwork); library(broom)
library(equatiomatic); library(simputation)
library(naniar)
library(rsample); library(yardstick)
library(tidyverse)

theme_set(theme_bw())
```

smart3 Variables, by Type

Variable	Туре	Description
landline	Binary (1/0)	survey conducted by landline? (vs. cell)
healthplan	Binary $(1/0)$	subject has health insurance?
age_imp	Quantitative	age (imputed from groups - see Notes)
fruit_day	Quantitative	mean servings of fruit / day
drinks_wk	Quantitative	mean alcoholic drinks / week
bmi	Quantitative	body-mass index (in kg/m^2)
physhealth	Count (0-30)	of last 30 days, # in poor physical health
dm_status	Categorical	diabetes status (4 levels, we'll collapse to 2)
activity	Categorical	physical activity level (4 levels, we'll re-level)
smoker	Categorical	smoking status (4 levels, we'll collapse to 3)
genhealth	Categorical	self-reported overall health (5 levels)

The smart3 data (built last time)

smart3 sh <- readRDS(here("data", "smart3 sh.Rds"))</pre>

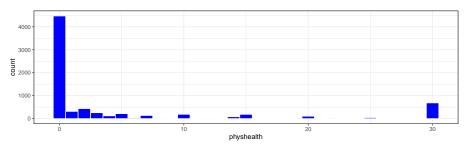
tibble [7,412 x 28] (S3: tbl_df/tbl/data.frame)

```
$ SEQNO
             : chr [1:7412] "2017000001" "2017000002" "2017
$ mmsa : chr [1:7412] "Cincinnati" "Cincinnati" "Cinc
$ mmsa wt : num [1:7412] 670 407 356 203 194 ...
$ landline : int [1:7412] 1 1 1 1 1 1 1 1 1 ...
$ age imp : num [1:7412] 36 41 55 61 57 24 65 53 51 42
$ healthplan : chr [1:7412] "1" "1" "1" "1" ...
$ dm status : Factor w/ 2 levels "Yes", "No": 2 2 2 2 2 2 2
$ fruit_day : num [1:7412] 1.43 1 3 0.5 0.72 ...
$ drinks wk : num [1:7412] 4.67 0 0 0 0.23 1.87 0 0 0.23 (
$ activity : Factor w/ 4 levels "Highly_Active",..: 2 2 3
$ smoker
             : Factor w/ 3 levels "Current", "Former", ...: 3
```

str(smart3 sh)

: int [1:7412] 0 0 2 0 2 0 0 30 2 30 ...

Days (in last 30) of poor physical health



```
smart3_sh %$% tabyl(physhealth > 0)
```

```
physhealth > 0 n percent
FALSE 4472 0.6033459
TRUE 2940 0.3966541
```

Create day6 data: predicting Pr(physhealth > 0)?

```
day6 <- smart3 sh %>%
   mutate(sick = as.numeric(physhealth > 0),
         id = as.character(
             as.numeric(SEQNO)-2017000000)) %>%
   select(id, sick, age = age imp, dm status, smoker,
         bmi, physhealth)
slice(day6, 17:19) # show rows 17-19
# A tibble: 3 x 7
 id sick age dm status smoker bmi physhealth
 <chr> <dbl> <dbl> <fct> <fct> <dbl> <int>
          0 72 No Former 31.4
1 17
                                              0
2 18 1 82 No Never 27.6
                                              5
```

3 19

0 62 Yes Current 27.5

Before fitting models, let's split our sample

What does strata = smoker do?

Impact of strata = smoker in split

```
d6 train %>% tabyl(smoker)
 smoker n percent
Current 933 0.1798728
 Former 1443 0.2781955
  Never 2811 0.5419318
d6_test %>% tabyl(smoker)
 smoker
           n
               percent
Current 400 0.1797753
```

Former 619 0.2782022 Never 1206 0.5420225

The logistic regression model

$$logit(event) = log\left(\frac{Pr(event)}{1 - Pr(event)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k$$

$$odds(event) = \frac{Pr(event)}{1 - Pr(event)}$$

$$Pr(event) = \frac{odds(event)}{odds(event) + 1}$$

$$Pr(event) = \frac{exp(logit(event))}{1 + exp(logit(event))}$$

Model 1: predict sick from smoker and age

Fit the model with and without an interaction term?

- Can we use the models to make predictions?
- 2 How should we interpret the model coefficients?
- Oan we compare the models based on in-sample performance?
- 4 How can we assess predictions using our test sample?

Model 1

$$\log \left[\frac{P(\widehat{\mathsf{sick}} = 1)}{1 - P(\widehat{\mathsf{sick}} = 1)} \right] = -0.322 + 0.005(\mathsf{age})$$

$$-0.318(\mathsf{smoker}_{\mathsf{Former}}) - 0.51(\mathsf{smoker}_{\mathsf{Never}})$$

Likelihood Ratio Tests: Model 1

```
anova(mod1, test = "LRT")
Analysis of Deviance Table
Model: binomial, link: logit
Response: sick
Terms added sequentially (first to last)
      Df Deviance Resid. Df Resid. Dev Pr(>Chi)
NULL
                      5186 6964.6
age 1 7.422 5185 6957.2 0.006442 **
smoker 2 45.045 5183 6912.1 1.654e-10 ***
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model 1

term	estimate	std.error	conf.low	conf.high	p.value
(Intercept)	-0.322	0.105	-0.494	-0.150	0.002
age	0.005	0.002	0.002	0.007	0.003
smokerFormer	-0.318	0.086	-0.460	-0.176	0.000
smokerNever	-0.510	0.077	-0.636	-0.384	0.000

Model 1 Predictions for subjects A-F

• logit(sick) = -0.322 + 0.005 age - 0.318 Former - 0.510 Never

ID	age	smoker	logit(sick)	odds(sick)	Pr(sick)
Α	33	Current	-0.157	0.8547	0.461
В	33	Former	-0.475	0.6219	0.383
C	33	Never	-0.667	0.5132	0.339
D	55	Current	-0.047	0.9541	0.488
Ε	55	Former	-0.365	0.6942	0.410
F	55	Never	-0.557	0.5729	0.364

Sample Calculation (for E):

•
$$logit(sick) = -0.322 + 0.005 (55) - 0.318 (1) - 0.510 (0) = -0.365$$

- odds(sick) = $\exp(-0.365) = 0.6942$
- Prob(sick) = 0.6942 / (1 + 0.6942) = 0.410

Model 1 (coefficients exponentiated)

term	estimate	lo90	hi90
(Intercept)	0.725	0.610	0.861
age	1.005	1.002	1.007
smokerFormer	0.728	0.631	0.839
smokerNever	0.601	0.529	0.681

 So what can we conclude about, for instance, the effect of Never smoking (as compared to Current smoking)?

Model 1 (coefficients exponentiated)

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- So what can we conclude about, for instance, the effect of Never smoking (as compared to Current smoking)?
- Suppose Chloe and Nancy are the same age, where Nancy never smoked and Chloe is a current smoker.

aat:maata	I ₀ 00	hi90
estimate	1090	1190
0.725	0.610	0.861
1.005	1.002	1.007
0.728	0.631	0.839
0.601	0.529	0.681
	1.005 0.728	0.725

 Chloe and Nancy are the same age; Nancy never smoked and Chloe smokes currently. What can we conclude about the relative odds for Nancy of a sick day as compared to Chloe?

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- Chloe and Nancy are the same age; Nancy never smoked and Chloe smokes currently. What can we conclude about the relative odds for Nancy of a sick day as compared to Chloe?
- \bullet Nancy's odds of at least one sick day in the past 30 are 60.1% of Chloe's odds.

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- Nancy's odds of at least one sick day in the past 30 are 60.1% of Chloe's odds.
- 90% CI for this odds ratio is (0.529, 0.681).

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- Chloe and Nancy are the same age; Nancy never smoked and Chloe smokes currently. What can we conclude about the relative odds for Nancy of a sick day as compared to Chloe?
- Nancy's odds of at least one sick day in the past 30 are 60.1% of Chloe's odds.
- 90% CI for this odds ratio is (0.529, 0.681).
- Chloe's odds of a sick day are (1/0.601 = 1.664) times those of Nancy.

Does this match the predictions we made?

- Suppose both Chloe and Nancy are 33 years old.
- We saw that Nancy's odds(sick) should be 0.601 times Chloe's odds(sick).

ID	age	smoker	logit(sick)	odds(sick)	Pr(sick)
		Current	-0.157	0.8547	0.461
Nancy	33	Never	-0.667	0.5132	0.339

- \bullet and we have 0.5132 / 0.8547 = 0.600 for the ratio of Nancy's odds to Chloe's odds
- ullet and Chloe's odds are $0.8547\ /\ 0.5132 = 1.665$ times those of Nancy.
- These discrepancies are just due to rounding error in my table.

Model 1 results from glance

- We'll have some additional measures of fit quality, in time.
- Deviance = -2(log Likelihood)

nobs	df.null	null.deviance	deviance	df.residual
5187	5186	6964.6	6912.1	5183

```
glance(mod1) %>%
    select(nobs, logLik, AIC, BIC) %>% kable(dig = 1)
```

nobs	logLik	AIC	BIC
5187	-3456.1	6920.1	6946.4

Model 2

$$\log \left[\frac{P(\widehat{\mathsf{sick}} = 1)}{1 - P(\widehat{\mathsf{sick}} = 1)} \right] = -0.188$$

$$+ 0.002(\mathsf{age})$$

$$- 0.563(\mathsf{smoker}_\mathsf{Former})$$

$$- 0.642(\mathsf{smoker}_\mathsf{Never})$$

$$+ 0.004(\mathsf{age} \times \mathsf{smoker}_\mathsf{Former})$$

$$+ 0.003(\mathsf{age} \times \mathsf{smoker}_\mathsf{Never})$$

Likelihood Ratio Tests: Model 2

```
anova(mod2, test = "LRT")
```

Analysis of Deviance Table

Model: binomial, link: logit

Response: sick

Terms added sequentially (first to last)

```
Df Deviance Resid. Df Resid. Dev Pr(>Chi)

NULL 5186 6964.6
age 1 7.422 5185 6957.2 0.006442 **

smoker 2 45.045 5183 6912.1 1.654e-10 ***
age:smoker 2 0.733 5181 6911.4 0.693211
```

Signif. codes:

Model 2

term	estimate	std.error	conf.low	conf.high	p.value
(Intercept)	-0.188	0.214	-0.542	0.164	0.380
age	0.002	0.004	-0.005	0.009	0.608
smokerFormer	-0.563	0.300	-1.057	-0.070	0.060
smokerNever	-0.642	0.245	-1.046	-0.238	0.009
age:smokerFormer	0.004	0.005	-0.004	0.013	0.392
age:smokerNever	0.003	0.004	-0.005	0.010	0.564

• logit(sick) = -0.188 + 0.002 age - 0.563 Former - 0.642
Never + 0.004 age*Former + 0.003 age*Never

Model 2 predictions for subjects A-F

• logit(sick) = -0.188 + 0.002 age - 0.563 Former - 0.642
Never + 0.004 age*Former + 0.003 age*Never

ID	age	smoker	logit(sick)	odds(sick)	Pr(sick)
Α	33	Current	-0.122	0.8851	0.470
В	33	Former	-0.553	0.5752	0.365
C	33	Never	-0.665	0.5143	0.340
D	55	Current	-0.078	0.9250	0.481
Ε	55	Former	-0.421	0.6564	0.396
F	55	Never	-0.555	0.5741	0.365

- Subject E: logit(sick) = -0.188 + 0.002 (55) 0.563 (1) 0.642 (0) + 0.004(55)(1) + 0.003(55)(0) = -0.421
- odds(sick) = $\exp(-0.421) = 0.6564$ so
- Prob(sick) = 0.6564 / (1 + 0.6564) = 0.396 for subject E.

Model 2 (coefficients exponentiated)

term	estimate	lo90	hi90
(Intercept)	0.828	0.582	1.178
age	1.002	0.995	1.009
smokerFormer	0.570	0.348	0.932
smokerNever	0.526	0.351	0.788
age:smokerFormer	1.004	0.996	1.013
age:smokerNever	1.003	0.995	1.010

term	estimate	lo90	hi90
(Intercept)	0.828	0.582	1.178
age	1.002	0.995	1.009
smokerFormer	0.570	0.348	0.932
smokerNever	0.526	0.351	0.788
age:smokerFormer	1.004	0.996	1.013
age:smokerNever	1.003	0.995	1.010

 Chloe and Nancy are the same age; Nancy never smoked and Chloe smokes currently. What can we conclude about the relative odds for Nancy of a sick day as compared to Chloe?

term	estimate	lo90	hi90
(Intercept)	0.828	0.582	1.178
age	1.002	0.995	1.009
smokerFormer	0.570	0.348	0.932
smokerNever	0.526	0.351	0.788
age:smokerFormer	1.004	0.996	1.013
age:smokerNever	1.003	0.995	1.010

- Chloe and Nancy are the same age; Nancy never smoked and Chloe smokes currently. What can we conclude about the relative odds for Nancy of a sick day as compared to Chloe?
- We cannot conclude anything unless we know what age Chloe and Nancy are, since the effect of smoking depends on age.

If Chloe (current smoker) and Nancy (never smoker) are each 33, then . . .

ID	age	smoker	logit(sick)	odds(sick)	Pr(sick)
Chloe	33	Current	-0.122	0.8851	0.470
Nancy	33	Never	-0.665	0.5143	0.340

- Chloe's odds of being sick are 0.8851/0.5143 = 1.72 times that of Nancy, if they are each 33 years old.
- If Chloe and Nancy are each 55, then from the table below, Chloe's odds are $0.9250\ /\ 0.5741=1.61$ times Nancy's odds of being sick.

ID	age	smoker	logit(sick)	odds(sick)	Pr(sick)
D	55	Current	-0.078	0.9250	0.481
F	55	Never	-0.555	0.5741	0.365

Comparing Model 1 to Model 2 with AIC and BIC

nobs	AIC	BIC	mod
5187	6920.1	6946.4	m1 (no int.)
5187	6923.4	6962.7	m2 (interaction)

• Which model looks like it performs better in the training sample?

Comparison with Mallows' C_p statistic?

• Same as what we got from glance for AIC in this case.

5181 6911.4 2 0.73284 6923.4

5183 6912.1

6920.1

Can we compare the models with a Test?

```
anova(mod1, mod2, test = "LRT")
Analysis of Deviance Table

Model 1: sick ~ age + smoker
Model 2: sick ~ age * smoker
   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
```

5181 6911.4 2 0.73284 0.6932

Could also consider:

• Rao's efficient score test (test = "Rao")

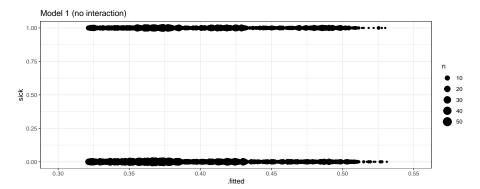
5183 6912.1

• Pearson's chi-square test (test = "Chisq")

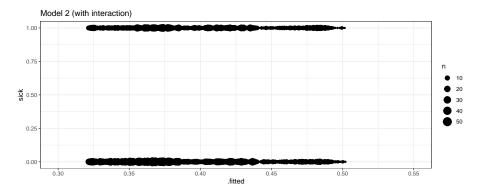
Let's get predicted probabilities in training sample

```
m1 aug <- augment(mod1, type.predict = "response")</pre>
m2_aug <- augment(mod2, type.predict = "response")</pre>
The predicted probabilities are in the .fitted column.
m1 aug %>% select(age, smoker, sick, .fitted) %>% slice(1)
# A tibble: 1 \times 4
    age smoker sick .fitted
  <dbl> <fct> <dbl> <dbl>
1 57 Current 1 0.486
m2_aug %>% select(age, smoker, sick, .fitted) %>% slice(1)
# A tibble: 1 \times 4
    age smoker sick .fitted
  <dbl> <fct> <dbl> <dbl>
1 57 Current 1 0.482
```

Observed (sick status) vs. Model 1 fitted Pr(sick)



Observed (sick status) vs. Model 2 fitted Pr(sick)



Making Classification Decisions

- Our outcome is sick, where sick = 1 if physact > 0, otherwise sick = 0.
- We can establish a classification rule based on our model's predicted probabilities of sick = 1.
- 0.5 is a natural (but not inevitable) cut point.
 - ullet if .fitted is below 0.50, we'll predict sick =0
 - if .fitted is 0.50 or larger, we'll predict sick = 1.

```
m1_aug %$% table(.fitted >= 0.50, sick)
```

```
sick
0 1
FALSE 3058 2005
TRUE 75 49
```

Standard Epidemiological Format

Confusion matrix for Model mod1 in the training sample.

```
sick_obs
sick_pred TRUE FALSE
TRUE 49 75
FALSE 2005 3058
```

Terminology associated with the Confusion Matrix

confuse_m1

```
sick_obs
sick_pred TRUE FALSE
TRUE 49 75
FALSE 2005 3058
```

- Total Observations = 49 + 75 + 2005 + 3058 = 5187
- Correct Predictions = 49 + 3058 = 3107, or 59.9% accuracy
- Incorrect Predictions = 75 + 2005 = 2080 (40.1%)
- Observed TRUE = 49 + 2005 = 2054, or 39.6% prevalence
- Predicted TRUE = 49 + 75 = 124, or 2.4% detection prevalence

Other Summaries from a Confusion Matrix

confuse_m1

```
sick_obs
sick_pred TRUE FALSE
TRUE 49 75
FALSE 2005 3058
```

- Sensitivity = 49 / (49 + 2005) = 2.4% (also called Recall)
 - ullet if the subject actually was sick, our model predicts that 2.4% of the time
- Specificity = 3058 / (3058 + 75) = 97.6%
 - if the subject was actually not sick, our model predicts that 97.6% of the time
- ullet Positive Predictive Value (or Precision) = 49 / (49 + 75) = 39.5%
 - ullet our predictions of sick were correct 39.5% of the time
- Negative Predictive Value = 3058 / (3058 + 2005) = 60.4%
 - our predictions of "not sick" were correct 60.4% of the time

Confusion Matrix for mod2 (training sample)

We can obtain a similar confusion matrix for model mod2 using the same (arbitrary) cutoff of .fitted >= 0.5 to indicate sick.

```
confuse_m1
```

```
sick_obs
sick_pred TRUE FALSE
TRUE 49 75
FALSE 2005 3058
```

 $confuse_m2$

```
sick_obs
sick_pred TRUE FALSE
TRUE 2 3
FALSE 2052 3130
```

Which of these confusion matrices looks better?

Get confusion matrix more easily?

0 2679 1642 1 454 412

Accuracy and Kappa Results for mod_1

```
metrics(data = m1_aug, truth = obs, estimate = pred) %>%
   kable(digits = 6)
```

.metric	.estimator	.estimate
accuracy	binary	0.595913
kap	binary	0.061834

- Kappa = a correlation statistic from -1 to +1, with complete agreement +1 and complete disagreement -1.
- Kappa measures the inter-rater reliability of our predicted and true classifications.

Confusion Matrix for mod_2 with 0.45 cutoff

Truth

Prediction 0 1 0 2582 1561 1 551 493

- 493 + 2582 = 3075 accurate predictions (59.3% accuracy)
- Sensitivity = 493 / (493 + 1561) = 24.0%
 - for the people who were actually sick, we made correct predictions 24% of the time
- Specificity = 2582 / (2582 + 551) = 82.4%
 - for the people who weren't actually sick, we made correct predictions 82.4% of the time with this decision rule and model mod 2.

Holdout Sample?

metrics for test sample, models 1 and 2

```
bind cols(
   metrics(data = mod1_aug_test,
           truth = obs, estimate = pred) %>%
       select(.metric, mod1 = .estimate),
   metrics(data = mod2 aug test,
           truth = obs, estimate = pred) %>%
       select(mod2 = .estimate)
# A tibble: 2 x 3
  .metric mod1 mod2
 <chr> <dbl> <dbl>
```

1 accuracy 0.609 0.604 2 kap 0.0938 0.0979

What's Next?

Expanding our options with tidymodels and the Harrell-verse. . .

- Fitting linear and logistic regression models in new ways
- Evaluating the success of our models in new ways
- Incorporating imputation approaches more seamlessly

Please don't forget to submit your Project A proposal by Monday at 9 PM.