In this assignment you will implement a particle system for simulating the flocking behavior of birds based on *Boids* [Reynolds(1987)]. A short introduction from the author is available online [1]. We provide a simple starter code for basic GUI functionality and particle drawing, but you are welcome to update this code in any way you feel is convenient for you.

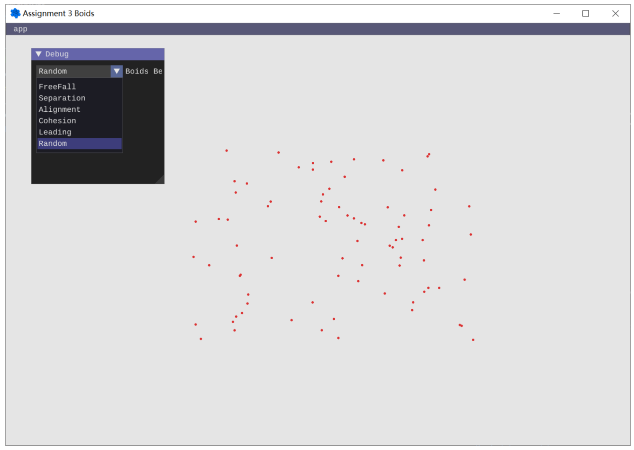


Figure 1: Starting interface.

## 1 Basic Time Integration 25%

*Boids* are modeled as orientation-less particles that have constant mass $m_i$ and time-varying position $\mathbf{x}_i$(t). In the continuous setting, the flock evolves according to the coupled system of first order ODEs

$$\dot{\mathbf{x}}(t) = \mathbf{v}(t) \tag{1}$$
$$\dot{\mathbf{v}}(t) = \mathbf{M}^{-1}\mathbf{f}(t) \tag{2}$$
$$\tag{3}$$

where $\mathbf{x}(t)$ and $\mathbf{v}(t)$ are positions and velocities at time $t$, $\mathbf{M}$ is the diagonal mass matrix, and $\mathbf{f}$ are forces acting on the particles.

To simulate the motion of the flock, the above ODE must be discretized in time using numerical integration. Implement the explicit Euler update rule for this purpose, which is given as

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h\mathbf{v}_n \ , \tag{4}$$
$$\mathbf{v}_{n+1} = \mathbf{v}_n + h\mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_n) \ , \tag{5}$$
$$\tag{6}$$

where $h$ is the step size. To test your implementation, initialize each Boid to have random (but reasonable) position and velocity. Add a constant acceleration in the $y$-direction to emulate gravity.

## 2 Advanced Time Integration 25%

Implement two additional time integration schemes: symplectic Euler and explicit midpoint.

The update rules for symplectic Euler are

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h\mathbf{v}_n \ , \tag{7}$$
$$\mathbf{v}_{n+1} = \mathbf{v}_n + h\mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_{n+1}) \ , \tag{8}$$
$$\tag{9}$$

i.e., positions are updated first, after which velocities are updated using the new positions.

[1] `https://cs.stanford.edu/people/eroberts/courses/soco/projects/2008-09/modeling-natural-systems/boids.html`

The update rules for explicit midpoint are

$$\mathbf{x}_{n+1/2} = \mathbf{x}_n + \frac{h}{2}\mathbf{v}_n \; , \tag{10}$$

$$\mathbf{v}_{n+1/2} = \mathbf{v}_n + \frac{h}{2}\mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_n) \; , \tag{11}$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h\mathbf{v}_{n+1/2} \; , \tag{12}$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + h\mathbf{M}^{-1}\mathbf{f}(\mathbf{x}_{n+1/2}) \; , \tag{13}$$

$$\tag{14}$$

i.e., a first step is taken to compute positions and velocities at the middle of interval, after which end-of-step positions and velocities are computed using mid-interval velocities and accelerations.

To test the performance of the three integration schemes, define a force function that enforces each Boid to maintain its initial distance $\bar{d}_i = |\mathbf{x}_i(0)|$ from the origin. Choose random initial positions $(\bar{x}_i, \bar{y}_i)^T$ and set the initial velocities to $(-\bar{y}_i, \bar{x}_i)^T$ such as to encourage circular motion in the plane.

For each of the three integration schemes, experiment with different step sizes. Report your findings in the ReadMe file.

## 3 Flocking 25%

**Cohesion 5%** In a real-world flock, birds have the tendency to stay close to their neighbors. To model this important behavior, define an attraction force that makes a given bird move towards the average position of neighboring birds. You can use a fixed radius or add a sliding bar to control it.

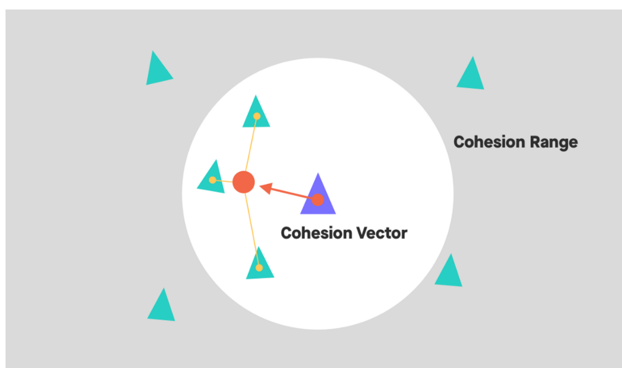


Figure 2: Cohesion.

**Alignment 5%** In addition to moving toward the average position of neighboring birds, each bird now also wants to match the average direction of the others. Using the same averaging principle as in the previous task, define and implement a force-based formulation to model this velocity alignment behavior.

**Separation 5%** The third ingredient required for realistic flocking behavior is separation: when birds come too close to each other, they separate such as to avoid collision. Add repulsive forces that avoid overcrowding.

**Collision Avoidance 10%** Implement a collision avoidance strategy where birds should steer around a given obstacle such as a square or a circle.

To showcase your implementation, create a video that illustrates each of the components in this task.

## 4 Collaborative and Adversarial Behaviors 25%

Now that you have implemented the basic flocking behavior, this task will explore higher-order coordination as well as collaborative and adversarial behaviors.

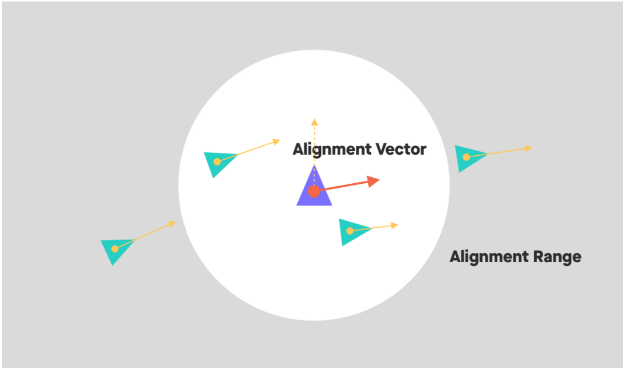


Figure 3: Alignment.

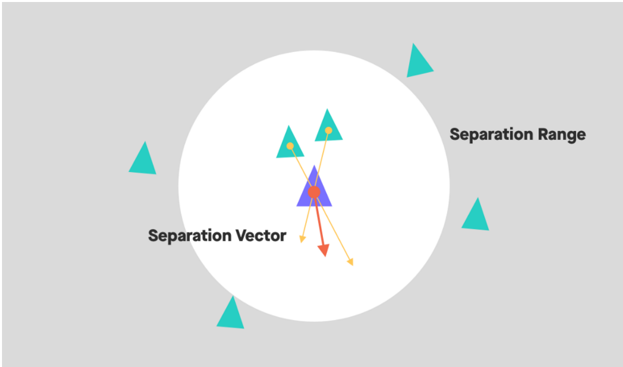


Figure 4: Separation.

**Leader Following 10%** Combining the previous forcing functions, implement a *follow the leader* mechanism in which one bird is designated as the leader whose motion the remaining birds should follow. To this end, follower birds should move towards the leader and try to head in the same direction. Meanwhile, they should also maintain a sufficient distance to each other.

You are free to set a bound on the maximum distance from which a given bird can observe the leader or any other constraints you deem reasonable. Should you choose to do that, please mention it in the ReadME file. The leading bird should be sufficiently different (in color/size/etc.) from the followers and should be controlled by the mouse.

**Collaborative and Adversarial Behaviors 15%** Divide an initial set of Boids into two groups and implement the following rules:

- if two Boids from the same group are sufficiently close, a third one is created.
- if three Boids from the same group are close to a Boid from the other group, the latter one is removed from the system.

Using the symplectic Euler method with random initial positions and velocities, observe the evolution of the populations and concisely report your findings in the ReadMe file.

Implement *control strategies* such that Boids will actively try to increase their relative population size. While there are no specific requirements on the exact strategy, the control strategy should only produce forces/accelerations and not change velocity and position directly. Test your strategy by first using it only for one group, then for both groups simultaneously. Concisely report your findings in the ReadMe file.

## 5 Submission

Please upload a video (or include a link) demonstrating the behaviors you have implemented. For any functionality you put extra effort in, please include a description in the ReadMe file.

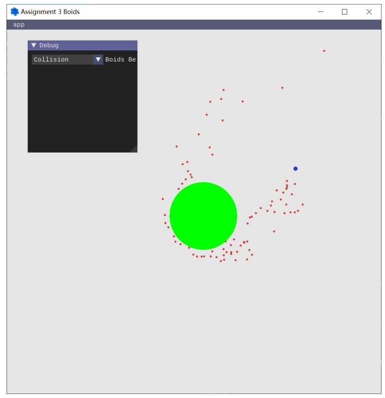


Figure 5: Sphere collision avoidance example.

Figure 2-4 credits. [2]

## References

[Reynolds(1987)] Reynolds, C.W., 1987. Flocks, herds and schools: A distributed behavioral model, in: Proceedings of the 14th annual conference on Computer graphics and interactive techniques, pp. 25–34.

[2] `https://medium.com/swlh/boids-a-simple-way-to-simulate-how-birds-flock-in-processing-69057930c229`