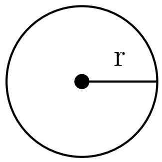
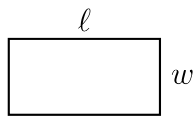
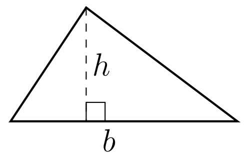
SAT Reference Sheet



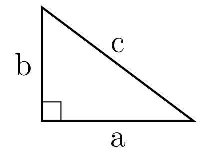
$A = \pi r^2$
$C = 2\pi r$



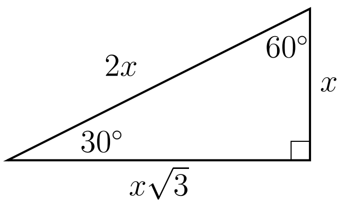
$A = \ell w$

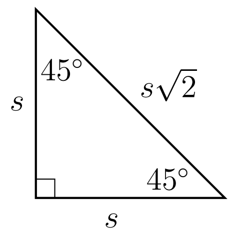


$A = 1/2bh$

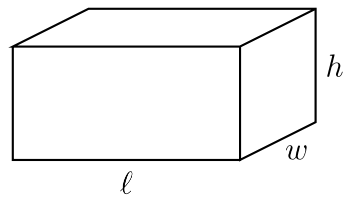


$a^2 + b^2 = c^2$

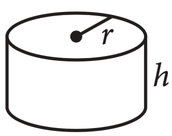




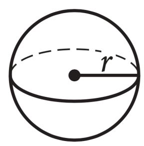
Special Right Triangles



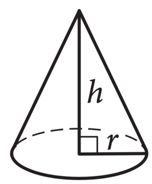
$V = \ell wh$



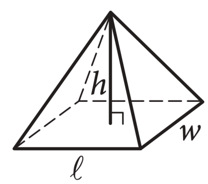
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}\ell wh$

Exponents/Roots/Logs

$2^{-2} = \frac{1}{4}$ $\quad 2^{-1} = \frac{1}{2}$ $\quad 2^0 = \frac{1}{1}$ $\quad 2^1 = \frac{2}{1}$ $\quad 2^2 = \frac{4}{1}$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$a = \sqrt[d]{b^c} \Rightarrow a = b^{c/d} \Rightarrow \sqrt[c/d]{a} = b \Rightarrow \sqrt[c]{a^d} = b$$

$i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad$ $\mathrm{x}^a \times x^b = x^{a+b}$

$(x^a)^b = x^{ab}$ $\qquad$ $\ln(a)^r = r\ln(a)$

$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ $\qquad$ $\ln(a) + \ln(b) = \ln(ab)$

$\ln(x) = log_e(x)$ $\qquad$ $\log_a b = c \Rightarrow a^c = b$

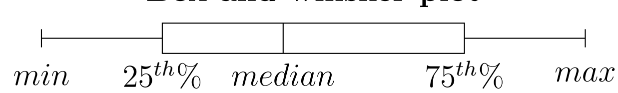
$\log_x(x) = 1$ $\qquad$ $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

Statistics

Mean - average
Median - middle value in a sorted list
Mode - most frequent
Range - difference between smallest and largest

Fundamental counting principle - multiply odds together to find the chances of two occurrences happening at once. Eg: the odds getting two heads is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Box and whisker plot



Linear Equations

Standard form: Ax+By=C
Slope-Intercept form: y=mx+b
Point-slope form: y-$y_1 = m(x - x_1)$

Midpoint formula: M= $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance formula: d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

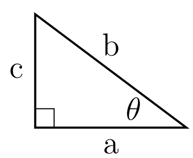
Matrices

determinants $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow ad - bc$

multiplication $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \times \begin{pmatrix} h & j \\ k & l \\ m & n \end{pmatrix} = \begin{pmatrix} (ah)+(bk)+(cm) & (aj)+(bl)+(cn) \\ (dh)+(ek)+(fm) & (dj)+(el)+(fn) \end{pmatrix}$

Trigonometry

Basic properties of triangles



$a^2 + b^2 = c^2$

$a - b < c < a + b$

$\sin(a) = \cos(90 - a)$

$\cos(a) = \sin(90 - a)$

$\sin(\theta) = \frac{a}{c}$ $\cos(\theta) = \frac{b}{c}$ $\tan(\theta) = \frac{a}{b}$

Common values of trig functions

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0

Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Simplifying trig functions

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

Trig functions with the unit circle

$\sin(\theta) = \dfrac{y}{r}$ $\cos(\theta) = \dfrac{x}{r}$ $\tan(\theta) = \dfrac{y}{x}$

$\csc(\theta) = \dfrac{r}{y}$ $\sec(\theta) = \dfrac{r}{x}$ $\cot(\theta) = \dfrac{x}{y}$

$$x^2 + y^2 = r^2$$

Reciprocal Identities

$\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 1 - 2\sin^2\theta$ $\cos(2\theta) = 2\cos^2\theta - 1$

Heron's Formula

$$\sqrt{s\cdot(s-a)(s-b)(s-c)}$$
$$s = \frac{a+b+c}{2}$$

Calculating a triangle's area

$$\text{area} = \frac{1}{2}ab\sin(C^\circ)$$

Sector Arc Length

$s = r\theta$

where θ is in radians

Sector Area

$$A = \frac{1}{2}\theta r^2$$

Calculus

Limits

If for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $\left|f(x) - L\right| < \epsilon$.

Power Rule

$$\frac{d}{dx}[ax^n] = (n \times a)x^{n-1}$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x) \times g(x)}$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \times g'(x)$$

Exponential Rule

$$\frac{d}{dx}[x^{a+b}] = x^{a+b} \times ln(x)(a+b)'$$

Log Rule

$$\frac{d}{dx}[log_a x] = \frac{1}{x \times ln(a)} \times x'$$

Basic Derivatives

$\frac{d}{dx}(\sin x) = \cos x$ $\quad \frac{d}{dx}(-\sin x) = -\cos x$ $\quad \frac{d}{dx}(\cos x) = -\sin x$ $\quad \frac{d}{dx}(-\cos x) = \sin x$

$\frac{d}{dx}(\sinh x) = \cosh x$ $\quad \frac{d}{dx}(-\sinh x) = -\cosh x$ $\quad \frac{d}{dx}(\cosh x) = \sinh x$ $\quad \frac{d}{dx}(-\cosh x) = -\sinh x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}(\cot x) = -\csc^2 x$ $\quad \frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad \frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$ $\quad$ f'(x) $= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\frac{d}{dx}(e^{2x}) = 2e^x$ $\quad \frac{d}{dx}(a^x) = a^x \ln a$ $\quad \frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \frac{d}{dx}(\log_b x) = \frac{1}{x \ln(b)}$

Position, velocity and acceleration

Position

unit ⇒ x(t)

ex. meters ⇒ x(t)

Velocity

$\frac{unit}{time}$ ⇒ x'(t)

ex. $\frac{meters}{second}$ ⇒ v(t)

Acceleration

$\frac{unit}{time^2}$ ⇒ x"(t)

ex. $\frac{meters}{second^2}$ ⇒ a(t)

Rolle's Theorem

If $f(x)$ is continuous on $[a, b]$, differentiable on (a, b) and $f(a) = f(b) = 0$, then there exists a value for c on the interval (a, b) such that $f'(c) = 0$.

Intermediate Value Theorem

If $f(x)$ is continuous on $[a, b]$ and c falls between $f(a)$ and $f(b)$, then there is at least one value of x in which $f(x) = c$ on the interval (a, b).

Mean Value Theorem

If $f(x)$ is continuous on $[a, b]$, differentiable on (a, b) and $a < c < b$, then there exists a value for c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Extreme Value Theorem

If $f(x)$ is continuous on $[a, b]$, then there exists both a minimum and a maximum on the interval $[a, b]$.

Rewriting Riemann Sums

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(x_x)\Delta x = \int_a^b f(x)dx$$

Trapezoidal Sums

$$\int_a^b f(x)dx \approx \frac{1}{2}\frac{b-a}{n}[y_0 + 2y_1 + 2y_2 ... 2y_{n-1} + 2y_{n-1} + y_n]$$

Right Riemann Sums

$$\frac{b-a}{n}[f(x_1) + f(x_2) ... f(x_n)]$$

Left Riemann Sums

$$\frac{b-a}{n}[f(x_0) + f(x_1) ... f(x_{n-1})]$$

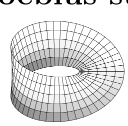
Midpoint Riemann Sums

$$\frac{b-a}{n}[f(x_{1/2}) + f(x_{3/2}) ... f(x_{n-1/2})]$$

l'Hopital's Rule

If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Moebius strip



$f(x)$	+	-	+m	-m	rel min	rel max
$f'(x)$			+	-	+m	-m
$f''(x)$					+	-

Calculus (continuted)

FTC 1

If $f(x)$ is continuous on $[a, b]$ then $g'(x) = f(b)$, where b is the upper limit of the integral.

FTC 2

If $f'(x)$ is continuous on $[a, b]$ then $\int_a^b f'(x)dx = f(b) - f(a)$.

Theorem of Fermat

If f has a local extrema at x and $f'(x)$ exists then $f'(x) = 0$.

FTC 2

If $f'(x)$ is continuous on $[a, b]$ then $\int_a^b f'(x)dx = f(b) - f(a)$.

Definition of a definite integral

Given a function $f(x)$ on $[a, b]$, subdivide $[a, b]$ into n subintervals of equal length Δx, with numbers $a = x_0$ and $b = x_n$. In each subinterval $[x_i - 1, x_i]$, pick a sample point x_i^*.

Average function value

$\frac{1}{b-a} \int_a^b f(x)dx$

Newton's Method

$$x_(n + 1) = x_n - \frac{f(x_n)}{f'(x_n)}$$

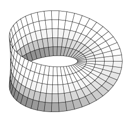
Midpoint Riemann Sums

$\frac{b-a}{n}[f(x_{1/2}) + f(x_{3/2})...f(x_{n-1/2})]$

l'Hopital's Rule

If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Moebius strip



$f(x)$	+	-	+m	-m	rel min	rel max
$f'(x)$			+	-	+m	-m
$f''(x)$					+	-

Constants

$\pi \approx 3.14159$ $e \approx 2.71828$ $\gamma \approx 0.57721$ $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61803$ $\hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -.61803$

Waves

Sine and cosine

$$y = A\sin(Bx + C) + D$$

period: $\dfrac{2\pi}{B}$ *amplitude:* $|A|$

domain: $(-\infty, \infty)$ *range:* $[D - |A|, D + |A|]$

phase shift: $Bx + C = 0$, solve for x

horizontal line of rest: D

Cosecant and secant

$$y = A\csc(Bx + C) + D$$

period: $\dfrac{2\pi}{B}$ *amplitude: none*

domain: $\csc \neq 0$ *range:* $(-\infty, D - |A|] \cup [D + |A|, \infty)$

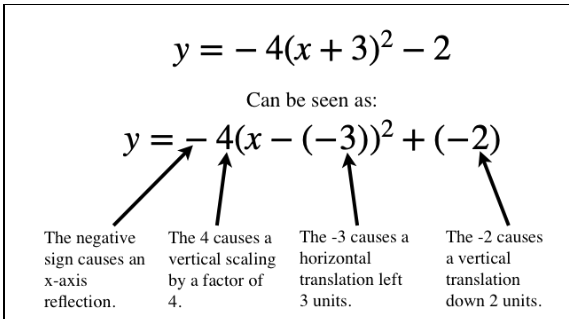
phase shift: $Bx + C = 0$, solve for x

horizontal line of rest: D

Parabolas

Standard form: $ax^2 + bx + c = 0$
Vertex form: $y = a(x - h)^2 + k$

Discriminant: From standard form, $b^2 - 4ac$
No real solutions when discriminant < 0
One real solution when discriminant $= 0$
Two real solutions when discriminant > 0



Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Units of Measurement

meter, m	kilogram, kg
second, s	ampere, A
kelvin, K	mole, mol
mole, mol	hertz, Hz
newton, N	pascal, Pa
joule, J	watt, W
coulomb, C	volt, V
ohm, Ω	henry, H
farad, F	tesla, T

Unit Prefixes

Prefix	Symbol	Factor
giga	G	10^9
mega	M	10^6
kilo	k	10^3
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 41 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1