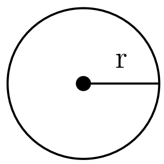
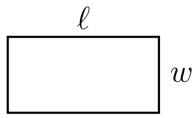
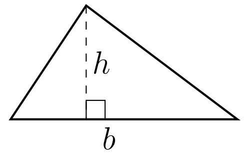
## SAT Reference Sheet



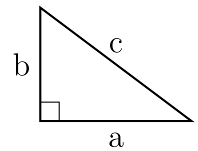

$A = \pi r^2$
$C = 2\pi r$



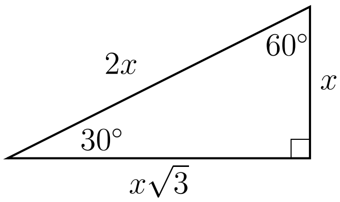

$A = \ell w$

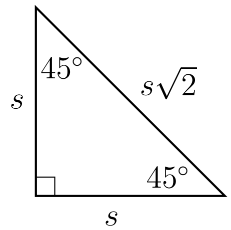



$A = 1/2bh$

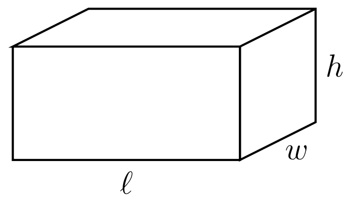



$a^2 + b^2 = c^2$

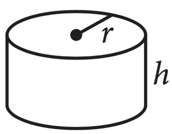


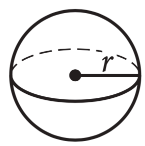

Special Right Triangles



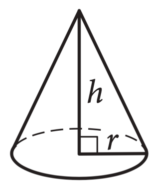

$V = \ell w h$



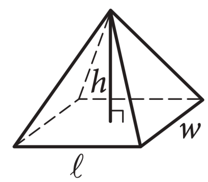

$V = \pi r^2 h$




$V = \frac{4}{3}\pi r^3$




$V = \frac{1}{3}\pi r^2 h$




$V = \frac{1}{3}\ell w h$

## Exponents/Roots/Logs

$2^{-2} = \frac{1}{4}$ $\quad 2^{-1} = \frac{1}{2}$ $\quad 2^0 = \frac{1}{1}$ $\quad 2^1 = \frac{2}{1}$ $\quad 2^2 = \frac{4}{1}$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$a = \sqrt[d]{b^c} \Rightarrow a = b^{c/d} \Rightarrow \sqrt[c/d]{a} = b \Rightarrow \sqrt[c]{a^d} = b$$

$i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad$ $\mathrm{x}^a \times x^b = x^{a+b}$

$(x^a)^b = x^{ab}$ $\qquad$ $\ln(a)^r = r\ln(a)$

$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ $\qquad$ $\ln(a) + \ln(b) = \ln(ab)$

$\ln(x) = log_e(x)$ $\qquad$ $\log_a b = c \Rightarrow a^c = b$

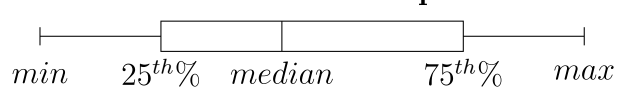
$\log_x(x) = 1$ $\qquad$ $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

## Statistics

*Mean* - average
*Median* - middle value in a sorted list
*Mode* - most frequent
*Range* - difference between smallest and largest

*Fundamental counting principle* - multiply odds together to find the chances of two occurrences happening at once. Eg: the odds getting two heads is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

### Box and whisker plot




## Linear Equations

*Standard form:* Ax+By=C
*Slope-Intercept form:* y=mx+b
*Point-slope form:* y-$y_1 = m(x - x_1)$

*Midpoint formula:* M= $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
*Distance formula:* d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

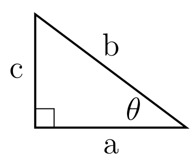
## Matrices

**determinants** $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow ad - bc$

**multiplication** $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \times \begin{pmatrix} h & j \\ k & l \\ m & n \end{pmatrix} = \begin{pmatrix} (ah)+(bk)+(cm) & (aj)+(bl)+(cn) \\ (dh)+(ek)+(fm) & (dj)+(el)+(fn) \end{pmatrix}$

## Trigonometry

### Basic properties of triangles




$a^2 + b^2 = c^2$

$a - b < c < a + b$

$\sin(a) = \cos(90 - a)$

$\cos(a) = \sin(90 - a)$

$\sin(\theta) = \frac{a}{c}$ $\cos(\theta) = \frac{b}{c}$ $\tan(\theta) = \frac{a}{b}$

### Common values of trig functions

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | 0 |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-1$ |
| $\tan\theta$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | $\infty$ | 0 |

### Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

### Law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

### Simplifying trig functions

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

### Trig functions with the unit circle

$\sin(\theta) = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$ $\tan(\theta) = \frac{y}{x}$

$\csc(\theta) = \frac{r}{y}$ $\sec(\theta) = \frac{r}{x}$ $\cot(\theta) = \frac{x}{y}$

$$x^2 + y^2 = r^2$$

### Reciprocal Identities

$\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

### Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

### Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

### Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 1 - 2\sin^2\theta$ $\cos(2\theta) = 2\cos^2\theta - 1$

### Heron's Formula

$$\sqrt{s\cdot(s-a)(s-b)(s-c)}$$
$$s = \frac{a+b+c}{2}$$

### Calculating a triangle's area

$$\text{area} = \frac{1}{2}ab\sin(C^\circ)$$

### Sector Arc Length

$s = r\theta$

where $\theta$ is in radians

### Sector Area

$$A = \frac{1}{2}\theta r^2$$

## Calculus

### Power Rule

$$\frac{d}{dx}[ax^n] = (n \times a)x^{n-1}$$

### Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x) \times g(x)}$$

### Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

### Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \times g'(x)$$

### Exponential Rule

$$\frac{d}{dx}[x^{a+b}] = x^{a+b} \times ln(x)(a+b)'$$

### Log Rule

$$\frac{d}{dx}[log_a x] = \frac{1}{x \times ln(a)} \times (x)'$$

### Basic Derivatives

$\frac{d}{dx}(\sin x) = \cos x$   $\frac{d}{dx}(-\sin x) = -\cos x$   $\frac{d}{dx}(\cos x) = -\sin x$   $\frac{d}{dx}(-\cos x) = \sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$   $\frac{d}{dx}(\cot x) = -\csc^2 x$   $\frac{d}{dx}(\sec x) = \sec x \tan x$   $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sin^{-1} \frac{u}{a}) = \frac{1}{\sqrt{a^2-u^2}}\frac{du}{dx}$   $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$   $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$   $\frac{d}{dx}(a^{u(x)}) = a^{u(x)} \ln a \frac{du}{dx}$

### Position, velocity and acceleration

*Position*

unit ⇒ x(t)

ex. meters ⇒ x(t)

*Velocity*

$\frac{unit}{time}$ ⇒ x’(t)

ex. $\frac{meters}{second}$ ⇒ v(t)

*Acceleration*

$\frac{unit}{time^2}$ ⇒ x”(t)

ex. $\frac{meters}{second^2}$ ⇒ a(t)

### Rolle’s Theorem

If $f(x)$ is continuous on $[a, b]$, differentiable on $(a, b)$ and $f(a) = f(b) = 0$, then there exists a value for $c$ on the interval $(a, b)$ such that $f'(c) = 0$.

### Intermediate Value Theorem

If $f(x)$ is continuous on $[a, b]$ and $c$ falls between $f(a)$ and $f(b)$, then there is at least one value of $x$ in which $f(x) = c$ on the interval $(a, b)$.

### Mean Value Theorem

If $f(x)$ is continuous on $[a, b]$, differentiable on $(a, b)$ and $a < c < b$, then there exists a value for $c$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

### Extreme Value Theorem

If $f(x)$ is continuous on $[a, b]$, then there exists both a minimum and a maximum on the interval $[a, b]$.

### Rewriting Riemann Sums

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_x)\Delta x = \int_a^b f(x)dx$$

### Trapezoidal Sums

$$\int_a^b f(x)dx \approx \frac{1}{2}\frac{b-a}{n}[y_0 + 2y_1 + 2y_2 ... 2y_{n-1} + 2y_{n-1} + y_n]$$

### Right Riemann Sums

$$\frac{b-a}{n}[f(x_1) + f(x_2)...f(x_n)]$$

### Left Riemann Sums

$$\frac{b-a}{n}[f(x_0) + f(x_1)...f(x_{n-1})]$$

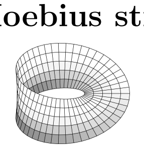
### Midpoint Riemann Sums

$$\frac{b-a}{n}[f(x_{1/2}) + f(x_{3/2})...f(x_{n-1/2})]$$

### l’Hopital’s Rule

If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

### Moebius strip



| $f(x)$ | + | - | +m | -m | rel min | rel max |
|---|---|---|---|---|---|---|
| $f'(x)$ | | | + | - | +m | -m |
| $f''(x)$ | | | | | + | - |

## Constants

$\pi \approx 3.14159$ $e \approx 2.71828$ $\gamma \approx 0.57721$ $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61803$ $\hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -.61803$

## Waves

### Sine and cosine

$$y = A\sin(Bx + C) + D$$

*period:* $\dfrac{2\pi}{B}$ *amplitude:* $|A|$

*domain:* $(-\infty, \infty)$ *range:* $[D - |A|, D + |A|]$

*phase shift:* $Bx + C = 0$, solve for $x$

*horizontal line of rest:* $D$

### Cosecant and secant

$$y = A\csc(Bx + C) + D$$

*period:* $\dfrac{2\pi}{B}$ *amplitude: none*

*domain:* $\csc \neq 0$ *range:* $(-\infty, D - |A|] \cup [D + |A|, \infty)$

*phase shift:* $Bx + C = 0$, solve for $x$

*horizontal line of rest:* $D$

## Parabolas

*Standard form*: $ax^2 + bx + c = 0$
*Vertex form*: $y = a(x - h)^2 + k$

*Discriminant*: From standard form, $b^2 - 4ac$
No real solutions when discriminant $< 0$
One real solution when discriminant $= 0$
Two real solutions when discriminant $> 0$

$$y = -4(x+3)^2 - 2$$

Can be seen as:

$$y = -4(x - (-3))^2 + (-2)$$

The negative sign causes an x-axis reflection.

The 4 causes a vertical scaling by a factor of 4.

The -3 causes a horizontal translation left 3 units.

The -2 causes a vertical translation down 2 units.

**Quadratic Formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## Units of Measurement

| | |
|---|---|
| meter, m | kilogram, kg |
| second, s | ampere, A |
| kelvin, K | mole, mol |
| mole, mol | hertz, Hz |
| newton, N | pascal, Pa |
| joule, J | watt, W |
| coulomb, C | volt, V |
| ohm, $\Omega$ | henry, H |
| farad, F | tesla, T |

## Unit Prefixes

| Prefix | Symbol | Factor |
|---|---|---|
| *giga* | $G$ | $10^9$ |
| *mega* | $M$ | $10^6$ |
| *kilo* | $k$ | $10^3$ |
| *centi* | $c$ | $10^{-2}$ |
| *milli* | $m$ | $10^{-3}$ |
| *micro* | $\mu$ | $10^{-6}$ |
| *nano* | $n$ | $10^{-9}$ |
| *pico* | $p$ | $10^{-12}$ |

## Pascal’s Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 41 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1