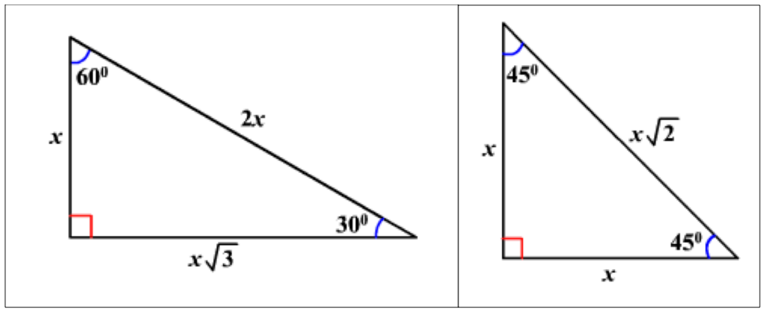
Special Right Triangles

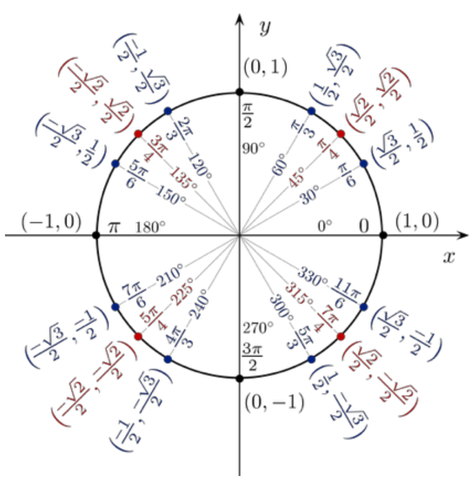


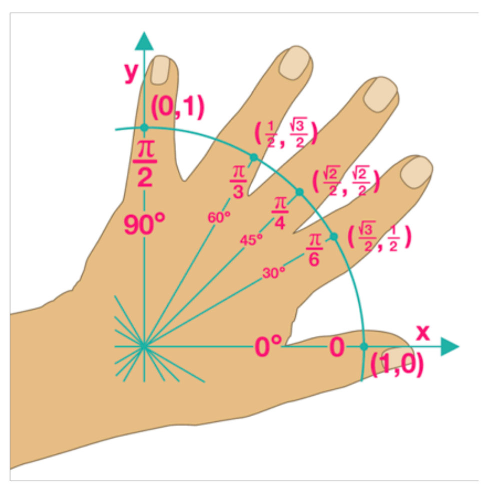
Converting between radians and degrees

$$\text{radians} = \left(\frac{\pi}{180^\circ}\right) \cdot degrees$$

$$\text{degrees} = \left(\frac{180^\circ}{\pi}\right) \cdot radians$$

The Unit Circle + hand trick





Definitions of sine, cosine, tangent and their reciprocals

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \csc\theta = \frac{hypotenuse}{opposite}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

Using sine, cosine and tangent with the unit circle

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$\mathrm{x}^2 + y^2 = r^2$$

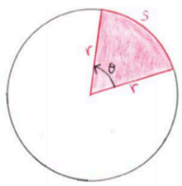
Simplifying trigonometric expressions (sine and cosine)

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Arc length, sector area, and radius formulas

Arc Length of a Sector of a Circle

For a circle of radius r, the length of the arc, s, intercepted by a central angle of θ radians is given by $s = r\theta$.



Area of a Sector of a Circle

For a circle of radius r, and central angle of θ radians, the area, A, of a sector of a circle is given by $A = \frac{1}{2}\theta r^2$.

CAUTION

The formula for the area of a sector of a circle, $A = \frac{1}{2}\theta r^2$ is only valid if the angle θ is in radians. An angle given in degrees must first be converted to radians.

Trigonometric identities

Reciprocal Identities

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

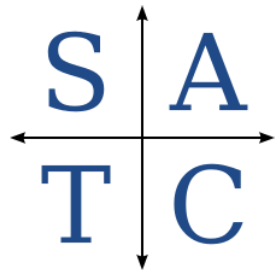
Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

All Students Take Calculus



Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1$$

Solving Generic Triangles

Heron's Formula: $\sqrt{s \cdot (s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$

Two sides and an included angle: area $= \frac{1}{2}ab\sin(C)$

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B}$$

Waves

Sine

Y=Asin$(Bx + C) + D$

Amp $\Rightarrow |A|$

Period $\Rightarrow \dfrac{2\pi}{B}$

Phase shift $\Rightarrow Bx + C = 0, solve for x$

Horizontal line of rest $\Rightarrow D$

Range $\Rightarrow [D - |A|, D + |A|]$

Domain $\Rightarrow (-\infty, \infty)$

Cosine

Y=Acos$(Bx + C) + D$

Amp $\Rightarrow |A|$

Period $\Rightarrow \dfrac{2\pi}{B}$

Phase shift $\Rightarrow Bx + C = 0, solve for x$

Horizontal line of rest $\Rightarrow D$

Range $\Rightarrow [D - |A|, D + |A|]$

Domain $\Rightarrow (-\infty, \infty)$

Secant

Y=Asec$(Bx + C) + D$

Amp $\Rightarrow None$

Period $\Rightarrow \dfrac{2\pi}{B}$

Phase shift $\Rightarrow Bx + C = 0, solve for x$

Horizontal line of rest $\Rightarrow D$

Range $\Rightarrow (-\infty, D - |A|] \cup [D + |A|, \infty)$

Domain $\Rightarrow$ cos $\neq 0$
(Happens when cosine crosses the HLR and is represented with a v.a.)

Cosecant

Y=Acsc$(Bx + C) + D$

Amp $\Rightarrow None$

Period $\Rightarrow \dfrac{2\pi}{B}$

Phase shift $\Rightarrow Bx + C = 0, solve for x$

Horizontal line of rest $\Rightarrow D$

Range $\Rightarrow (-\infty, D - |A|] \cup [D + |A|, \infty)$

Domain $\Rightarrow$ sin $\neq 0$
(Happens when sine crosses the HLR and is represented with a v.a.)