Assignment 2 < THMSTE021 > JE2.5.X, JE2.6, JE2.7

July 27, 2020

Brief Remark Plots.plot() - with plotly() backend - used as default plotting function. Julia 1.3.1 used in the creation of this notebook.

0.1 Julia Exercise 2.5.1 – Visualising Sampled Sinusoid

0.1.1 Simulate a sinusoidal signal over enough time to see several cycles.

```
[1]: using Plots
plotly()
Plots.PlotlyBackend()
```

Info: For saving to png with the Plotly backend ORCA has to be installed. @ Plots /Users/steventhomi/.julia/packages/Plots/5srrj/src/backends.jl:371

[1]: Plots.PlotlyBackend()

```
[2]: f01 = 50 #Hz

startTime = -0.1;
stopTime = 0.1;

x(t) = sin(2*pi*f01*t);

plot(x, startTime, stopTime,label=false)
```

0.1.2 Generic functions - line plot, scatter plot

```
[3]: default(size=(600,300)) # Set default Plot canvas size

# default plot that joins samples with straight lines

function linePlot(k)

fnyquist = 2*f01 # Nyquist sampling rate
fs1 = k*fnyquist # Define sample rate

T01 = (1/f01)
```

```
Δt1 = 1/fs1
t1 = 0:Δt1:10*T01;

x(t1) = sin(2*pi*f01*t1); # Create array containing function to be plotted

plot(t1,x,xaxis=("time [s]"),title = "Line Plot showing the samples"

→f0=$(f01)Hz fs=$(round(fs1))Hz",label=false);
end
```

[3]: linePlot (generic function with 1 method)

- [4]: scatterPlot (generic function with 1 method)
 - 0.1.3 Plot the sampled waveforms in order to show the visual effect of sampling at:
 - 0.1.4 100x the Nyquist rate

```
[5]: linePlot(100)
```

- [6]: scatterPlot(100)
 - 0.1.5 10x the Nyquist rate

```
[7]: linePlot(10)
```

```
[8]: scatterPlot(10)
```

0.1.6 2x the Nyquist rate

```
[9]: linePlot(2)
[10]: scatterPlot(2)
     0.1.7 1.1x the Nyquist rate
[11]: linePlot(1.1)
[12]: scatterPlot(1.1)
     0.1.8 On the Nyquist rate
[13]: linePlot(1)
[14]: scatterPlot(1)
     0.1.9 0.7x the Nyquist rate
[15]: k = 0.7
      linePlot(k)
[16]: scatterPlot(k)
[17]: print("fs-f0 = ", round(f01*(k*2 - 1)));
     fs-f0 = 20.0
     0.1.10 0.55x the Nyquist rate
[18]: k = 0.55
      linePlot(k)
      scatterPlot(k)
[19]:
[20]: print("fs-f0 = ", round(f01*(k*2 - 1)));
```

fs-f0 = 5.0

The visual plots resemble a closer approximation to the actual signal as the sample interval (Δt) increases. This condition is met when: - fs is greater than 1 and approaches infinity - fs is less than 1 and approaches negative infinity

- 0.2 Julia Exercise 2.5.2 DFT / FFT Introduction
- 0.2.1 Insert a dft(x) function into Julia and compare it to the fft() function.
- 0.2.2 Generic functions dft

```
[21]: function dft(x)
         N=length(x)
         X = zeros(N)+im*zeros(N) # Complex array of O+Oim
            for n=1:N
                X[k] = X[k] + x[n]*exp(-im*2*pi*(k-1)*(n-1)/N)
         end
         return X
     end
[21]: dft (generic function with 1 method)
[22]: y = [0,1,1,0,0,0,0,0];
[23]: using FFTW;
[24]: Oshow fft(y);
    fft(y) = Complex{Float64}[2.0 + 0.0im, 0.7071067811865476 -
    1.7071067811865475im, -1.0 - 1.0im, -0.7071067811865476 + 0.2928932188134524im,
    0.0 + 0.0im, -0.7071067811865476 - 0.2928932188134524im, -1.0 + 1.0im,
    0.7071067811865476 + 1.7071067811865475im]
    dft(y) = Complex{Float64}[2.0 + 0.0im, 0.7071067811865477 -
```

[25]: Oshow dft(y);

```
-0.7071067811865477 + 0.2928932188134524im, 0.0 + 1.2246467991473532e-16im,
0.9999999999997im, 0.7071067811865469 + 1.7071067811865477im]
```

0.2.3 Plot the magnitude and phase

```
[26]: Y = fft(y);
      plot(abs.(Y), lab = "magnitude", title = "Magnitude Plot")
[27]: plot(angle.(Y), lab = "phase", title = "Phase Plot")
```

0.2.4 Insert a idft(x) function into Julia and compare it to the ifft() function.

0.2.5 Generic function - idft

```
[28]: function idft(X)
    N=length(X)
    x = zeros(N)+im*zeros(N) # Complex array of 0+0im
    for n=1:N
        for k=1:N
            x[n] = x[n] + X[k]*exp(im*2*pi*(k-1)*(n-1)/N)
        end
    end
    return (x./N)
end
[28]: idft (generic function with 1 method)
```

0.2.6 Compare the speed of the dft() and fft() functions

0.2.7 Generic function - timer

```
[31]: function timer(N)

y = randn(N);

t_dft = @elapsed dft(y);

t_fft = @elapsed fft(y);

At = t_dft - t_fft;

println("dft of length $(N) took $(t_dft) seconds");
println("fft of length $(N) took $(t_fft) seconds");

println("\nlag between functions lasted $(At) seconds");
```

```
end
```

[31]: timer (generic function with 1 method)

0.2.8 1024 Samples

```
[32]: timer(1024)
```

```
dft of length 1024 took 0.036809291 seconds fft of length 1024 took 0.000139757 seconds
```

lag between functions lasted 0.036669534000000004 seconds

0.2.9 4096 Samples

```
[33]: timer(4096)
```

```
dft of length 4096 took 0.587233114 seconds fft of length 4096 took 0.000158347 seconds
```

lag between functions lasted 0.587074767 seconds

The time lag is more noticeable with larger array sample sizes

0.2.10 Compare the largest power-of-2 size the fft() and dft() functions can compute within 1 second

```
[34]: N = 1048576
y = randn(N);

t_fft = @elapsed fft(y);

println("fft of length $(N) took $(t_fft) seconds");
```

fft of length 1048576 took 0.077071245 seconds

The algorithm is highly efficient with values of the 2-to-the-power-of-20 range

```
[35]: N = 4096
y = randn(N);

t_dft = @elapsed dft(y);

println("dft of length $(N) took $(t_dft) seconds");
```

dft of length 4096 took 0.538222898 seconds

The fft() is faster than the dft() for powers of 2 as documented above

- 0.3 Julia Exercise 2.5.3 FFT of a sine wave
- 0.3.1 Time domain: $v(t) = 4 \cos (20 t) + 2 \cos (30 t)$

```
[36]: t = -0.5:0.001:0.5
v(t) = 4*cos(20*pi*t) + 2*cos(30*pi*t)

plot(t,v,title="Time Domain",xaxis="time",label=false)
```

[37]: functionToArray (generic function with 1 method)

```
[38]: b = functionToArray(0.2,0.0001)
V = fft(b);

plot(fftshift(abs.(V)),xaxis = ("samples", (975, 1025)), title="Frequency_

→Domain",label="magnitude")
```

There is lack of close detail in the magnitude of the frequency domain.

```
[39]: plot(angle.(V),title="Frequency Domain",xaxis = ("samples", (990,⊔ →1010)),label="phase")
```

0.3.2 Applying zero padding in the time domain

```
[40]: N = length(b)
m = zeros(16*N) # Make array 16x longer.

m[1:N] = b; # Copy x into first N samples. The rest contains zeros.

Y = fft(m);
```

```
plot(fftshift(abs.(Y)),title="Frequency Domain",xaxis = ("samples", (14500,⊔ →17500)),label="magnitude")
```

Closer detail can be observed in the magnitude of the frequency domain after application of zero padding. Two Sa() functions can be seen at the locations of the two sinusoidal frequencies.

```
[41]: plot(angle.(Y),title="Frequency Domain",xaxis = ("samples", (15500, ⊔ →16500)),label="phase")
```

- 0.4 Julia Exercise 2.5.4 Effect of ADC quantization
- 0.4.1 Simulate a sinusoid voltage v = cos.(2pif0*t) that lies in the range: Amin = -0.5 to Amax = 0.5

```
[42]: f0 = 50 #Hz
A = 0.5 #Amplitude

x(t) = A*cos(2*pi*f0*t)

plot(x,-0.02,0.02,title="Voltage",label=false)
```

- [43]: functionToArray2 (generic function with 1 method)
 - 0.4.2 Quantize the signal into a power-of-2 levels

```
[44]: function signalQuantization(Nbits)

x(t) = A*cos(2*pi*f0*t)
m = functionToArray2(0.2, 0.0001);
```

```
Nlevels = 2^Nbits
          Amax = 1+0.00001 # Add a small amount to prevent problem at extreme
          Amin = -1-0.00001
          m_{quantized} = (round.((m.-Amin)/(Amax-Amin)*Nlevels.-0.5).+0.5)/_{\square}
       →Nlevels*(Amax-Amin) .+ Amin;
          return fft(m_quantized) # M_QUANTIZED
      end
[44]: signalQuantization (generic function with 1 method)
     0.4.3 Number of Bits = 2
[45]: M_QUANTIZED = signalQuantization(2)
      plot(abs.(fftshift(M_QUANTIZED)),title="Frequency Domain",label="magnitude")
[46]: plot(20*log10.(abs.(fftshift(M_QUANTIZED))),title="Frequency_"
       →Domain(dBv)",label="magnitude") # dBv scale to see wide dynamic range.
[47]: plot(abs.(ifft(M_QUANTIZED)),title="Time Domain",label="magnitude")
     0.4.4 Number of Bits = 3
[48]: M_QUANTIZED = signalQuantization(3)
      plot(abs.(fftshift(M_QUANTIZED)),title="Frequency Domain",label="magnitude")
[49]: plot(20*log10.(abs.(fftshift(M_QUANTIZED))),title="Frequency_"
       →Domain(dBv)", label="magnitude") # dBv scale to see wide dynamic range.
[50]: plot(abs.(ifft(M_QUANTIZED)),title="Time Domain",label="magnitude")
     0.4.5 Number of Bits = 4
[51]: M_QUANTIZED = signalQuantization(4)
      plot(abs.(fftshift(M_QUANTIZED)),title="Frequency Domain",label="magnitude")
[52]: plot(20*log10.(abs.(fftshift(M QUANTIZED))),title="Frequency,
       →Domain(dBv)",label="magnitude") # dBv scale to see wide dynamic range.
[53]: plot(abs.(ifft(M_QUANTIZED)),title="Time Domain",label="magnitude")
```

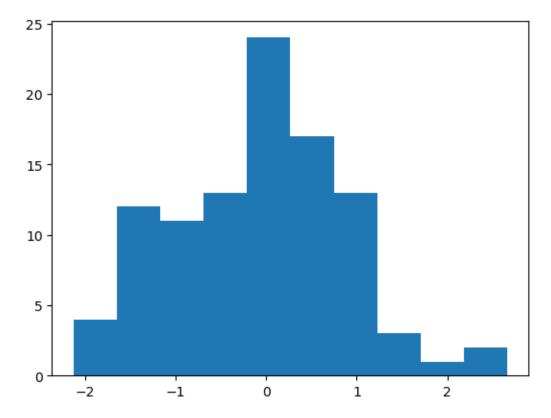
The frequency domain experiences a decrease in the magnitude of its lower frequency components with an increase in the number of bits used for quantization process. The time domain experiences a distortion as the number of bits used in the quantization process decrease.

0.5 Julia Exercise 2.5.5 – Simulating bandlimited noise

0.5.1 Noise Simulation

```
[54]: using PyPlot
```

WARNING: using PyPlot.plot in module Main conflicts with an existing identifier.



0.5.2 Time and Frequency Domains

```
[56]: plot(t,randnum,title="Time Domain",label=false) # inspect sampled time domain_ <math>\rightarrow title("Sampled noise, bandwidth B=fs/2")
```

0.5.3 Applying frequency-domain zero padding

```
[58]: pad_factor = 10
Ny = pad_factor * N;

Y = zeros(Ny)+im*zeros(Ny) # Create a complex array of zeros

k_mid = Int(N/2)

Y[1:k_mid]=X[1:k_mid]; # Insert the first half of X

Y[Ny-k_mid+1:Ny]=X[k_mid+1:N]; # Insert the 2nd half of X at the end

plot(abs.(Y),title="Zero-Padded Frequency Domain",label=false) # inspect padded_□

→ array
```

```
[59]: y = real(ifft(Y)) # Go back to time domain, discard the imaginary components

Ny = length(y);

t_new = range(0, step=Δt/pad_factor, length=Ny) # Define time axis

plot(t_new,y,title="Zero-Padded Time Domain",label=false)
```

```
[60]: plot(t_new,y,xaxis = ("first 300 samples", (0, 0.03)),title="300 Sample_u 

Close-Up",label=false) # Plot just first 300 samples
```

0.5.4 Bandlimiting Noise using an Ideal LPF

0.5.5 Create and display an ideal LPF

```
[61]:  \Delta = 2*pi/(N*\Delta t) \# Sample spacing in freq domain in rad/s 
= 0:\Delta : (N-1)*\Delta 
f = /(2*) 
B = 100 \# filter bandwidth in Hz 
rect(t) = (abs.(t).<=0.5)*1.0; \# rect function definition
```

```
H = rect(/(4**B)) + rect( ( .- 2*/Δt)/(4**B) );
plot(f,H,xaxis = ("Frequency in Hz"),title="Ideal Low Pass Filter",label=false)
```

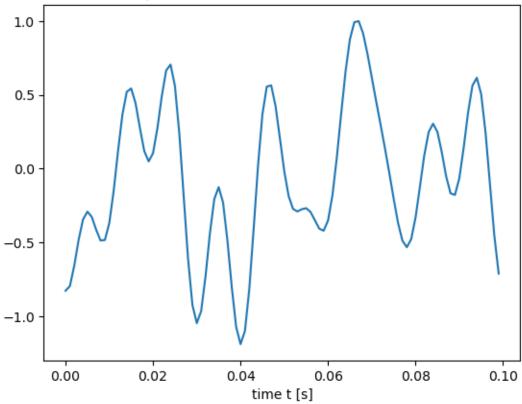
0.5.6 Apply Filter to Noise

```
[62]: X_filtered = X.*H;
plot(abs.(X_filtered),label=false)
```

```
[63]: x_filtered = ifft(X_filtered)
x_filtered = real(x_filtered)

PyPlot.plot(t,x_filtered)
title("Lowpass Filtered noise, bandwidth B=$(B)")
xlabel("time t [s]");
```





The original time waveform had more spikes (high frequency components) while the bandlimited time waveform is more smooth (absence of high frequency components).

```
[64]: using Statistics;

y = std(y)
xf = std(x_filtered)

println("The original time waveform has of: ", y);
println("\nThe bandlimited time waveform has of: ", xf);
```

The original time waveform has of: 0.09460059757910222

The bandlimited time waveform has of: 0.5091540134349454

0.6 Julia Exercise 2.5.6 – Discrete fast convolution

```
[65]: T = 1
t = 0:(T/100):4 # Define time axis

p = rect( (t.-T)/T )
q = rect( (t.-2T)/(2T) )

p1 = plot(t,p,xaxis=("time [s]"),title="Period T",label=false)
p2 = plot(t,q,xaxis=("time [s]"),title="Period 2T",label=false)

xgrid!(t)
plot(p1,p2,layout = (1, 2))
```

```
[66]: z = ifft(fft(p).*fft(q));
plot(t,real(fftshift(z)),xaxis=("time [s]"),title="Convolution",label=false)
```

0.7 Julia Exercise 3.6.1 a) – Discrete fast correlation

0.7.1 Produce an auto-correlation function: Rxx(t') = conj(X())*X()

```
[67]: T = 1;
t = 0:(T/100):4;

s = rect( (t.-T)/T )

x_auotcorr_x = ifft( conj( fft(s) ) .* fft(s) );

plot(t,fftshift(real(x_auotcorr_x)),xaxis=("time [s]"),title="Auto-Correlation_u")
→Function",label=false)
```

```
[85]: A = 1;

y = -A*rect((t.-T/2)/T) .+ A*rect((t.-2*T)/(2*T));
```

- 0.8 Julia Exercise 3.7.1 Matched Filter
- 0.8.1 Define a chirp function.
- 0.8.2 Setting: centre frequency of 10 kHz; bandwidth of 2 kHz; pulse length of 5 ms.

```
[69]: T = 0.005 # s, pulse length

f0 = 10000 # Hz, centre frequency
B = 2000 # Hz, desired bandwidth of the pulse
K = B/T # Hz/sec, chirp rate

chirp(t) = rect(t/T) * cos(2*pi*t*(f0+(0.5*K*t)));

plot(chirp,xaxis=("time", (0, 0.01)),label=false) # zoom to Os to 0.01s range
```

0.8.3 Define a physically realizable - delayed - chirp function.

```
[70]: v_tx(t) = chirp(t-T/2)

plot(v_tx,xaxis=("time", (0, 0.01)),label=false)
```

0.8.4 Sampled Time Axis

```
[71]: function nyquistSample(k)
    fnyquist = 2*f0  # Nyquist sampling rate
    fs = k*fnyquist # Define sample rate

R_max = 10  # metres
    t_max = 2*R_max/c

Δt = 1/fs
    t_new = 0:Δt:t_max;

v_tx(t) = chirp(t-T/2); # Create array containing function to be plotted

plot(t_new,v_tx,title = "Sampled Time Axis waveform",label=false)
end
```

[71]: nyquistSample (generic function with 1 method)

```
[72]: c = 343 # m/s
nyquistSample(4)
```

0.8.5 Simulated Echo from Target

```
[73]: R = 6 # metres, range of target
A = 1/(R^2); # amplitude decay

t_delay = 2*(R/c); # distance to + from target
Δt = 1/f0 # sample frequency

t = 0:Δt:(T/2+t_delay+T); # range from origin to shift+delay+period

= T/2+t_delay # cummulative lag

v_rx(t) = A*chirp(t-);

plot(t,v_rx,title="Simulated Echo from Target",label=false)
```

0.8.6 Plot time domain waveforms, showing what goes into and out of the matched filter

0.8.7 Echo from Target with Additive Noise

```
[74]: v_rnx(t) = randn()+v_rx(t); # echo + additive noise
n(t) = v_rnx(t)-v_rx(t); # noisy echo - echo

plotA = plot(t,n,title="Noise waveform",label=false)
plotB = plot(t,v_rnx,title="Noisy Echo waveform",label=false)

plot(plotA,plotB)
```

A sampling rate of 20x the nyquist rate was used to form a distorted, noisy signal.

0.8.8 Apply a matched filter which is created from the reference chirp

```
[75]: plot1 = plot(v_tx,xaxis=("time", (0, 0.025)),title = "Reference_

→Chirp",label=false) # zoom in to 0s to 0.025s

plot2 = plot(v_rnx,xaxis=("time", (0, 0.01)),title = "Noisy Echo waveform_

→x(t)",label=false) # zoom in to 0s to 0.01s

plot(plot1,plot2)
```

```
[76]: h(t) = v_tx(-t+t_{delay}); plot(h,0.025,0.05,title = "Impulse Response h(t)",label=false) # zoom in to 0. \leftrightarrow 0.025s to 0.05s
```

0.8.9 Generic Functions

[77]: xToArray (generic function with 1 method)

[78]: hToArray (generic function with 1 method)

The FFTW.fft() and FFTW.iff() functions only take in array data types as formal parameters. It is for this reason that I am forced to convert my functions of time to a closely sampled, accurate array representation.

0.8.10 Plot of output as a function of time

```
[79]: T0 = 0.04;

\[ \Delta t = 0.00005; \]
\[ \text{xarray} = \text{xToArray(T0, } \Delta t); \]
```

```
harray = hToArray(T0, Δt);

taxis = 0:Δt:T0;
yarray = ifft(fft(xarray) .* fft(harray));

plot(taxis,real(yarray),title="Matched Filter Output",label=false)
```

0.8.11 Plot the magnitude of the FFT of the pulse, and also of the matched filter.

Questions: 1. Is the bandwidth of the chirp pulse (as seen in the frequency domain) as expected? 2. What is the shape of the envelope of the output of the matched filter? 3. What happens if you increase the bandwidth? 4. What happens if you increase the length of the pulse?

Solutions: (1) The bandwith is as expected. (2) The envelope peaks at 0.035 seconds, begins at 0.030 seconds and ends at 0.040 seconds. (3) There is no change to the matched filter output. (4) The pulse energy increases, causing the matched filter to easily distinguish between the chirp and its additive noise during filtration.

0.8.12 Resolution

```
[81]: function resolutionN(start,stop)
count = 0;
  while start<stop
     start = start + Δt; # Δt was used to create xarray, and harray above
     count = count + 1;
  end
  return count # total number of elements within given range
end</pre>
```

[81]: resolutionN (generic function with 1 method)

```
[82]: start = 0.03; # derived from observation of above plot
stop = 0.04;
numberOfValues = resolutionN(start,stop);
```

```
println("The resolution in seconds is $(1/numberOfValues) seconds");
```

The resolution in seconds is 0.005 seconds

0.8.13 Estimate the Peak SNR at input and at output

```
[83]: using QuadGK # integration library, integration method in Calculus library is \rightarrow deprecated
```

```
[84]: peakInputSignalPower = v_rnx()^2; # is the delay
    peakOutputSignalPower = real(yarray[Int(0.035/Δt)])^2; # at t = 0.035
    n_in(t) = (v_rnx(t)-v_rx(t))^2;

# quadgk() returns a pair (I,E) of the estimated integral I
# and an estimated upper bound on the absolute error E

noiseAtInput = quadgk(n_in,0,0.01); # integrate the noise^2 from 0 to 0.01

SNR_input = peakInputSignalPower/noiseAtInput[1];

println("Peak SNR at input: ",SNR_input)
```

Peak SNR at input: 2.452903614741369