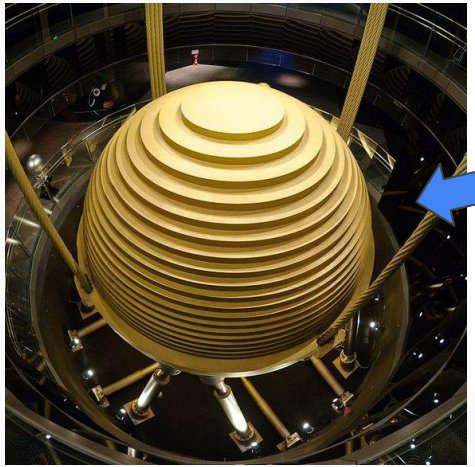
Modeling a Tuned Mass Damper

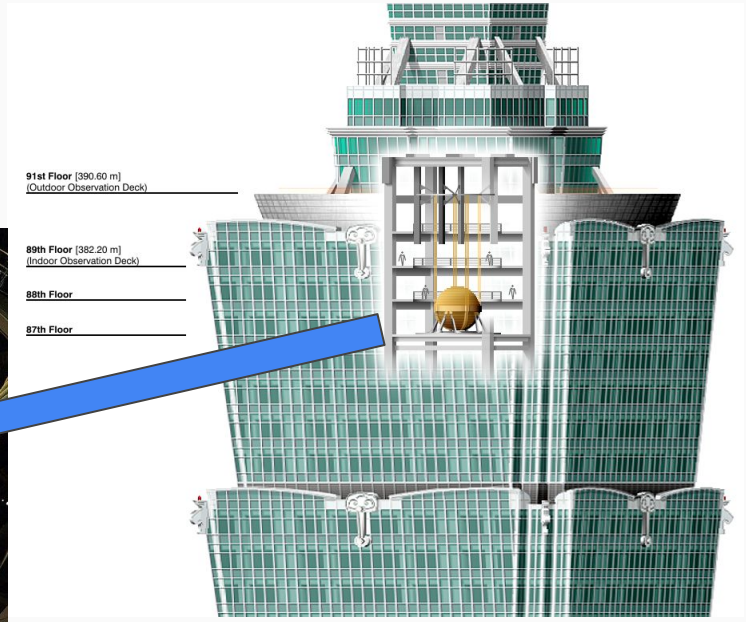
Course: PHY 480, Section 1

By: Steven Vancamp and Patrick Tutt

Background and Motivation

- External forces (wind, earthquakes) can induce oscillations in man-made structures like bridges and skyscrapers
- These oscillations can be dangerous to occupants and building integrity
- Tuned Mass dampers (TMDs) can be used within structures to counteract these effects



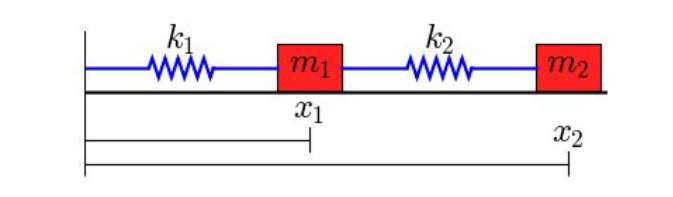


TMD at Taipei 101

Project Goals

- Can we model a coupled mass-spring system?
- Do we observe damped oscillations of the larger mass?
- Can we optimize the physical parameters of the system (spring constant and mass of damper)?
- Can we use a coupled mass-spring system as a stepping stone to more complicated models?
- Can we optimize in this case?

Simple Model: Coupled Mass Spring System

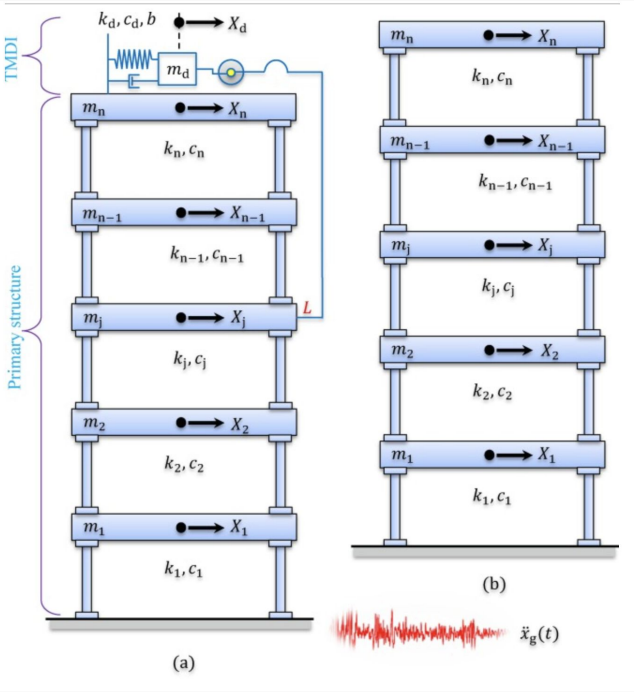


$$m_1\ddot{x_1} + b_1\dot{x_1} + k_1(x_1 - L_1) - k_2(x_2 - x_1 - L_2) = 0$$

$$m_2\ddot{x_2} + b_2\dot{x_2} + k_2(x_2 - x_1 - L_2) = 0$$

- A 2nd order D.E. can describe this case
- Decoupling the system with a series of 1st order D.E. allows for easy solving with numerical methods
- Parameters m_2 and k_2 can then be optimized

TMD: Multiple story building



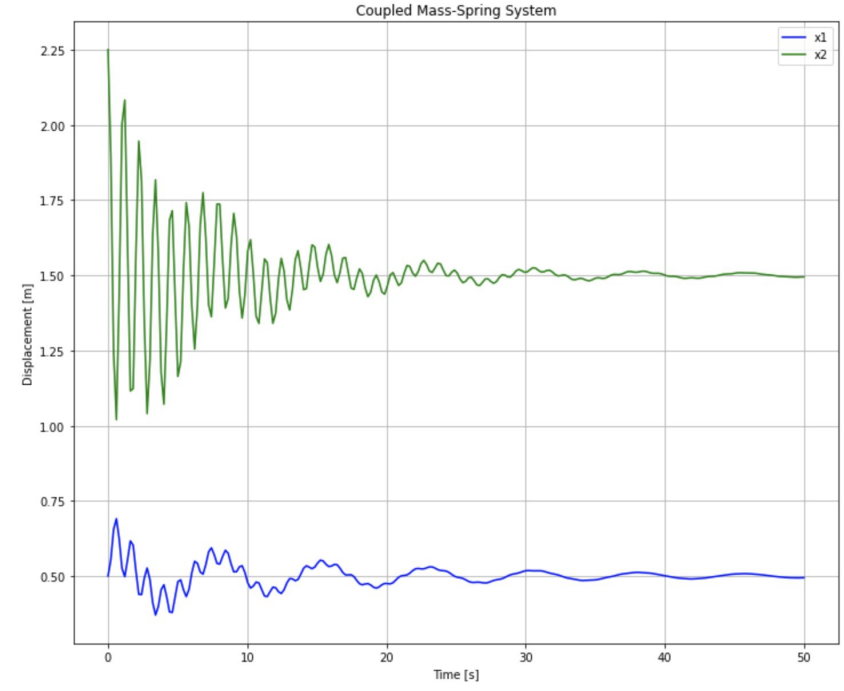
$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{m}\ddot{x}_g(t)$$

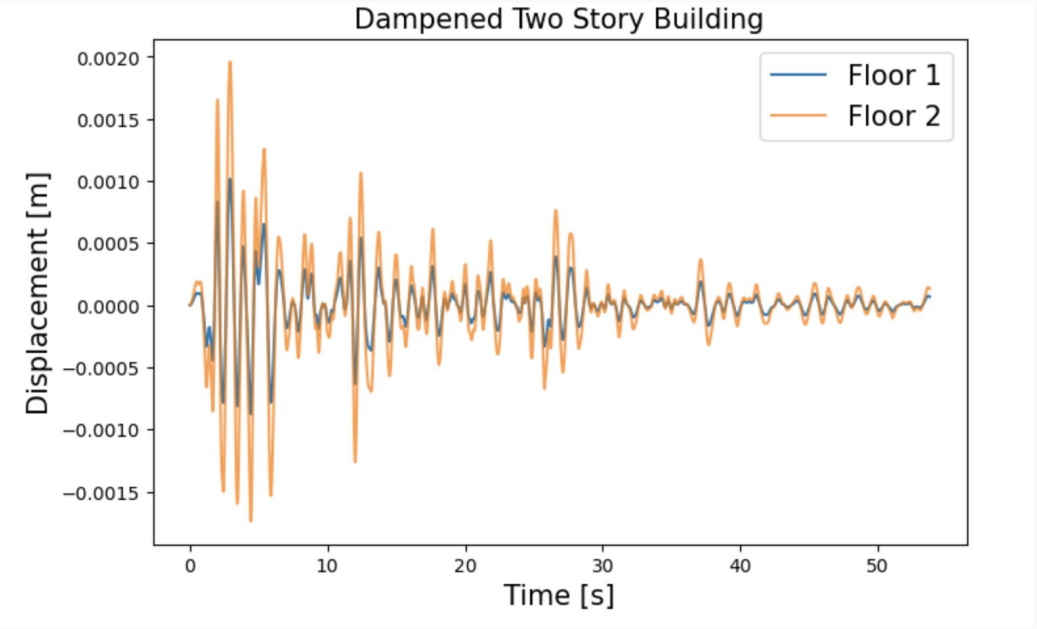
- A more complicated 2nd order D.E. with matrices describes this case
- Matrices determine coupling between each floor

$$\mathbf{K} = \begin{bmatrix} (k_1+k_2) & -k_2 & & & & \\ -k_2 & (k_2+k_3) & -k_3 & & 0 & \\ & -k_3 & . & . & & \\ & & . & . & -k_n & \\ & 0 & & -k_n & (k_n+k_d) & -k_d \\ & & & & -k_d & k_d \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} (c_1+c_2) & -c_2 & & & & \\ -c_2 & (c_2+c_3) & -c_3 & & 0 & \\ & -c_3 & . & . & & \\ & & . & . & -c_n & \\ & 0 & & -c_n & (c_n+c_d) & -c_d \\ & & & & -c_d & c_d \end{bmatrix}$$

Results


Coupled Mass-Spring System
x1
x2
Displacement [m]
Time [s]
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0
10
20
30
40
50


Dampened Two Story Building
Floor 1
Floor 2
Displacement [m]
Time [s]
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
0
10
20
30
40
50

Conclusion and Next Steps

Accomplishments:

- Dampened oscillations demonstrated in coupled mass-spring system and 2-story building model

Next Steps:

- Optimization of TMD parameters
  - Markov Chain Monte Carlo (MCMC)