VE281

Data Structures and Algorithms

Binary Search Tree Additional Operations

Learning Objectives:

- Know some additional efficient operations of binary search tree
- Know how these operations are implemented and their time complexity

Recap: Average-Case Time Complexity

	Search	**Insert**	**Remove**
Linked List	$O(n)$	$O(n)$	$O(n)$
Sorted Array	$O(\log n)$	$O(n)$	$O(n)$
Hash Table	$O(1)$	$O(1)$	$O(1)$
BST	$O(\log n)$	$O(\log n)$	$O(\log n)$

So, why we use BST, not hash table?

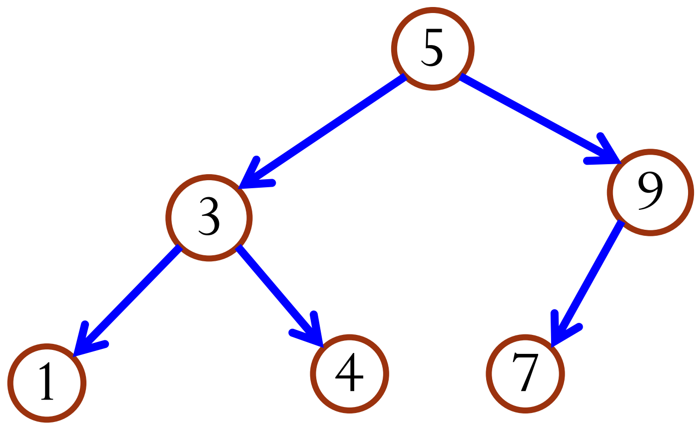
Why BST?

- Other Operations Supported by BST

Operation	Average-Case Time Complexity
Output in Sorted Order	$O(n)$
Get Min/Max	$O(\log n)$
Get Predecessor/Successor	$O(\log n)$
Rank Search	$O(\log n)$
Range Search	$O(n)$

Note: Hash table does not support efficient implementation of the above methods.

Output in Sorted Order

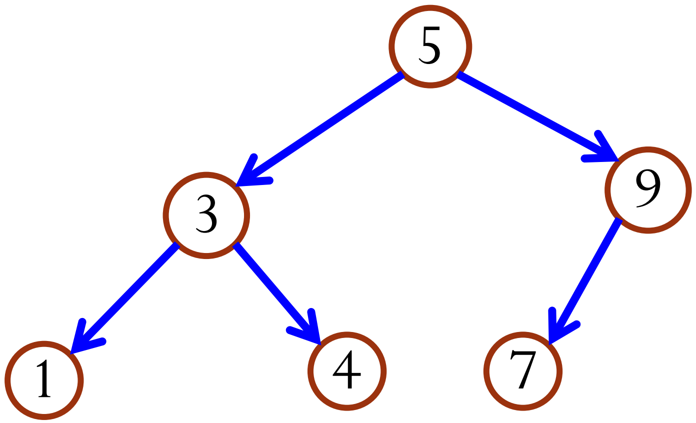


- Output: 1, 3, 4, 5, 7, 9
- **How?**
  - In-order depth-first traversal.

> – Visit the left subtree
> – Visit the node
> – Visit the right subtree

- Time complexity: $O(n)$.

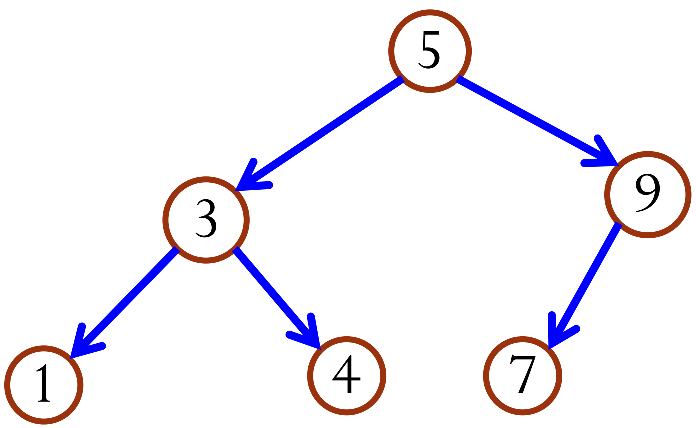
Get Min/Max



- To get **min** (**max**) key of the tree:
  - Start at root.
  - Follow **left** child pointer (**right for max**) until you cannot go anymore.
  - Return the last key found.
- Time complexity? $O(\text{height})$. On average: $O(\log n)$.

Get Predecessor/Successor

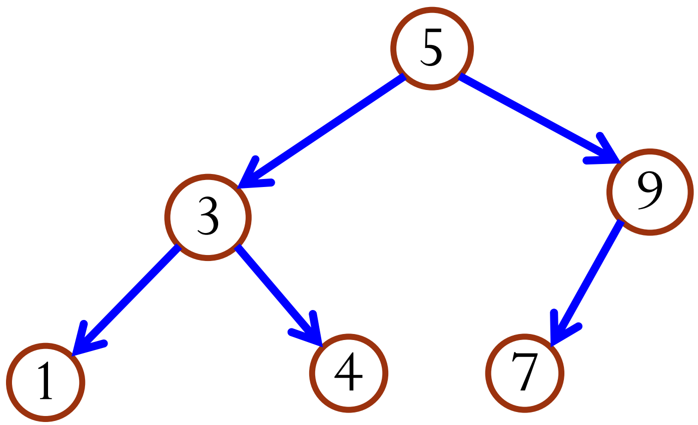
- Given **a node** in the BST, get its predecessor/successor.
  - **Predecessor**: the node with the **largest** key that is **smaller** than the current key.
  - **Successor**: the node with the **smallest** key that is **larger** than the current key.
  - **Predecessor**/**Successor** is in the sense of in-order depth-first traversal.



What's predecessor of key 5?

What's successor of key 5?

Get Predecessor of a Node



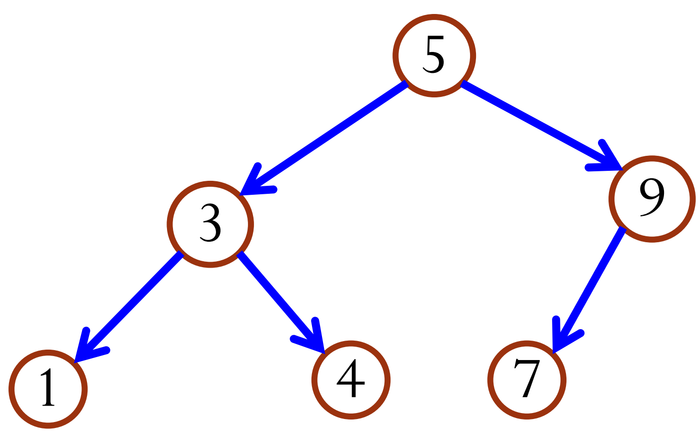
What's predecessor of key 5?

What's predecessor of key 7?

- **Easy case**: left subtree of the node is **nonempty**…
  - … return **max** key in left subtree.
- **Otherwise**: left subtree is **emtpy** …
  - … follow **parent pointers** until you get to a key less than the current key.
  - Equivalent: its first **left** ancestor.
- Time complexity? $O(\text{height})$. On average: $O(\log n)$.

Rank Search

- **Rank**: the index of the key in the **ascending order**.
  - We assume that the smallest key has rank 0.
- **Rank search**: get the key with rank k (i.e., the k-th smallest key).
  - Hash table does not support efficient rank search.
  - How to do rank search with a BST?
    - Simple solution: keep counting during an in-order depth-first traversal.



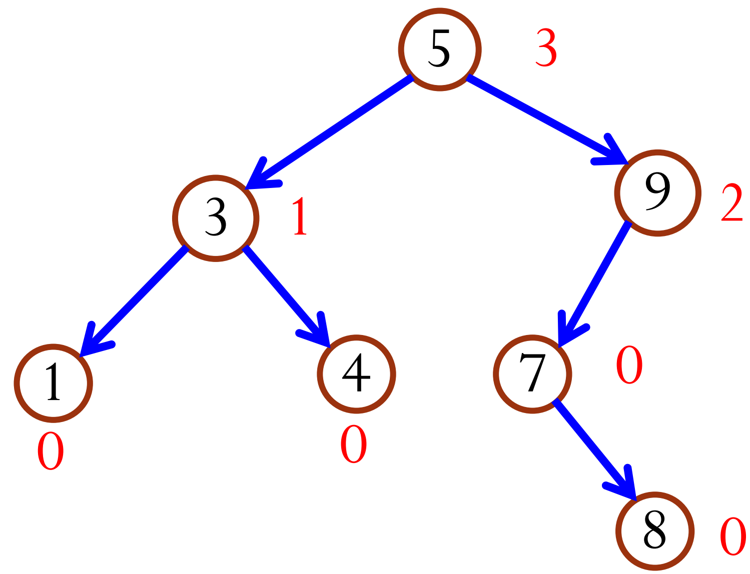
What's the average-case time complexity?

Can we do better?

BST with `leftSize`

- Each node has an additional field **`leftSize`**, indicating the number of nodes in its left subtree.

```
struct node {
   Item item;
   int leftSize;
   node *left;
   node *right;
};
```



Which Statements Are Correct?

- Suppose we modify the basic BST to implement a BST with leftsize. Select all the correct statements.

A. The search method should be updated.

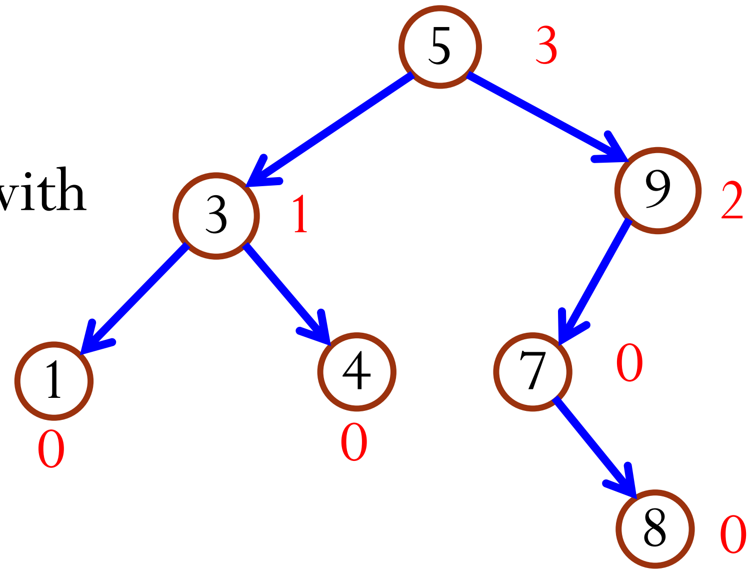
B. The insertion method should be updated, but not for the removal method.

C. The removal method should be updated, but not for the insertion method.

D. Both the insertion and removal methods should be updated.

Rank Search

- Can we increase the efficiency of rank search with a BST with **`leftSize`**?



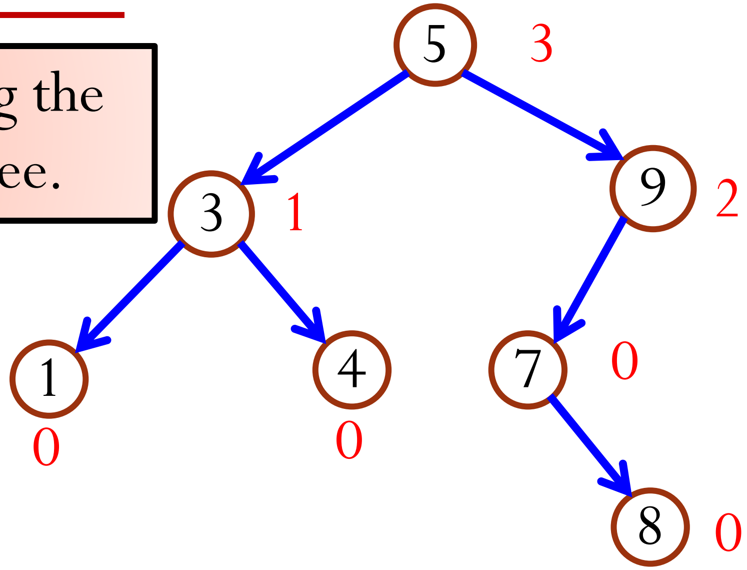
- What is the node with
  - rank = 3?
  - rank = 2?
  - rank = 5?
- Observation: **`x.leftSize`** = the rank of **`x`** in the **tree rooted** at **`x`**.
  - The rank of node 9 is 2 in the tree rooted at node 9.

Rank Search

```
node *rankSearch(node *root, int rank) {
  if(root == NULL) return NULL;
  if(rank == root->leftSize) return root;
  if(rank < root->leftSize)
    return rankSearch(root->left, rank);
  else
    return rankSearch(root->right,
      rank - 1 - root->leftSize);
}
```

The number of nodes including the current **root** and its left subtree.

What will **rankSearch(root,5)** return?



Rank Search

Example

```
node *rankSearch(node *root, int rank) {
  if(root == NULL) return NULL;
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```
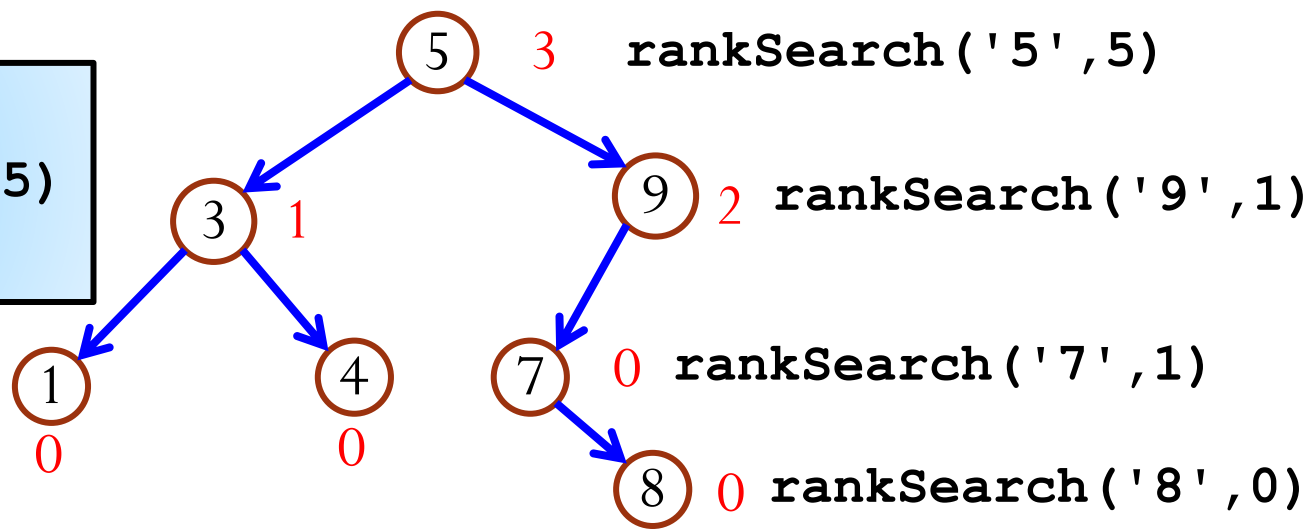

What will **`rankSearch(root,5)`** return?



Rank Search

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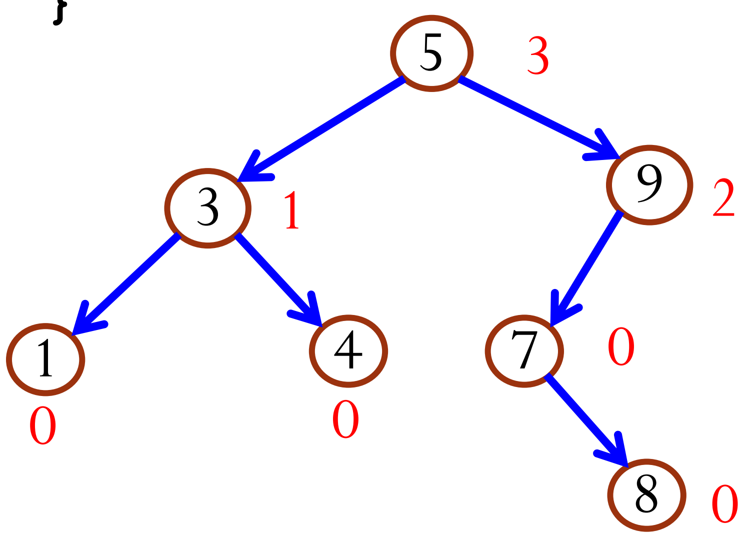
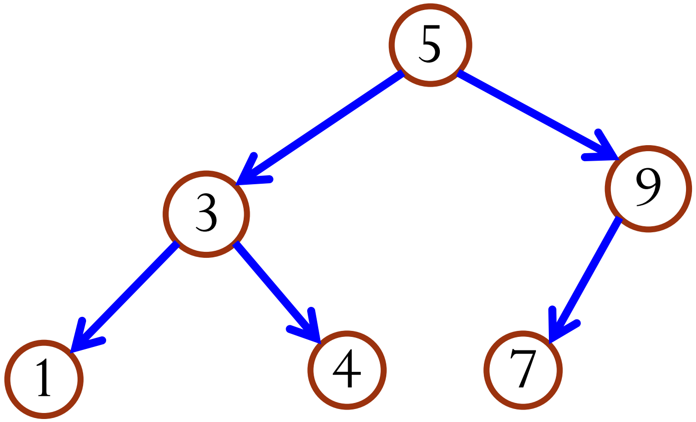


Time complexity?

$O(\text{height})$. On average: $O(\log n)$.

Range Search

- Instead of finding an exact match, find all items whose keys fall **between a range of values, inclusive**, in **sorted order**
  - E.g., between 4 and 8, inclusive.



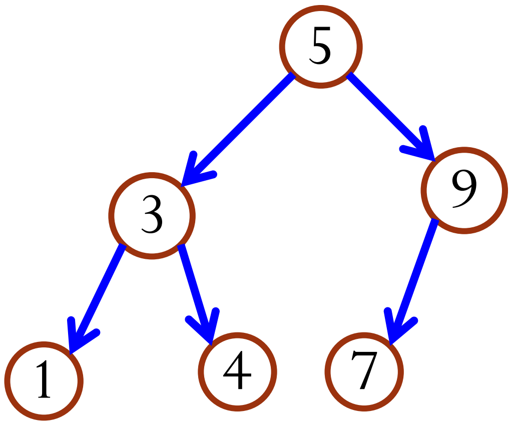
How could you implement range search?

- Example applications:
  - Buy ticket for travel between certain dates.

Range Search

Algorithm

1. Compute range of left subtree.
   - If search range covers all or part of left subtree, search left. (**recursive call**)
2. If root is in search range, add root to results.
3. Compute range of right subtree.
   - If search range covers all or part of right subtree, search right. (**recursive call**)
4. Return results.




```
void rangeSearch(node *root, Key searchRange[],
  Key treeRange[], List results)
```

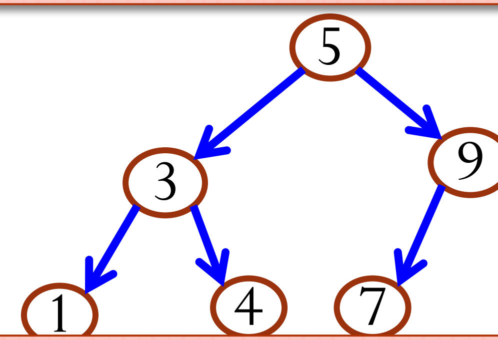
Range Search

Example

```
rangeSearch('5', [4,8], (-∞,+∞), results)
                 searchRange  treeRange
```

Call rangeSearch('3', [4,8],(-∞,5),results)

```
Does (-∞,5) overlap [4,8]?        Yes
  Does (-∞,3) overlap [4,8]?      No
  Is 3 in [4,8]?                  No
  Does (3,5) overlap [4,8]?       Yes
    Is 4 in [4,8]?                results ← 4
Is 5 in [4,8]?                    results ← 5
Does (5,+∞) overlap [4,8]?        Yes
  Does (5,9) overlap [4,8]?       Yes
    Is 7 in [4,8]?                results ← 7
  Is 9 in [4,8]?                  No
  Does (9,+∞) overlap [4,8]?      No
```



Call rangeSearch('9', [4,8],(5,+∞),results)

results:
4,5,7

Note: results are in order

Range Search
Supporting Functions

- If node is in the search range, add node to the **`results`** list.

- Compute subtree's range:
  - Replace upper bound of left subtree by node's key
    - If possible, node's key "minus one".
  - Replace lower bound of right subtree by node's key
    - If possible, node's key "plus one".

- If search range covers all or part of subtree, search subtree.
  - Recursive calls

Range Search

1. Compute range of left subtree.
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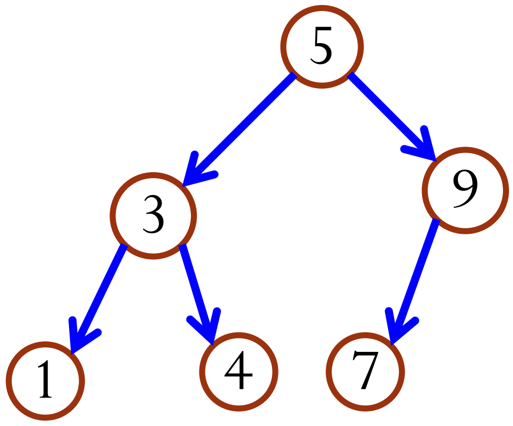
   - If search range covers all or part of right subtree, search right. (**recursive call**)
4. Return results.



Time complexity?

$O(n)$