VE477
Introduction to Algorithms

Homework 4

Manuel — UM-JI (Fall 2020)

Reminders

- Write in a neat and legible handwriting or use LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb, and object)
- Be critical on your results

*Questions preceded by a * are optional. Although they can be skipped without any deduction, it is important to know and understand the results they contain.*

Ex. 1 — *Time vs. space*

The goal of this exercise is to consider the best available hardware and compare the feasibility of heavy computation in terms of (i) time and (ii) memory.

- As of June 2015 the fastest supercomputer publicly known is called NUDT Tianhe-2. Its speed is 33.86 PFLOPS and its storage is 12.4 PB[1].
- As of August 2015 one of the fastest CPU for desktop computer is the Intel Core i7-5775R Processor which has four core running at maximal frequency of 3.8 GHz[2].
- As of August 2015 the largest hard drive is almost 16TB[3].

1. How long would it take to perform 2^{64} operations on NUDT Tianhe-2? What about 2^{80} operations?
2. How many desktop computers would be necessary to perform 2^{64} operations in no more than a day. What about 2^{80} operations in no more than a month?
3. How many hard drives would be necessary to store 2^{64} bits. What about 2^{80} bits?

Ex. 2 — *Critical thinking*

Given a set S of n integers, generate a subset S' of S composed of k elements, each selected with probability k/n. Explain how to obtain S' in only one pass.

Ex. 3 — *Algorithm and complexity*

In the following triangle each entry is the sum of the three entries directly above it.

				1				
			1	1	1			
		1	2	3	2	1		
	1	3	6	7	6	3	1	
1	4	10	16	19	16	10	4	1

1. Write the pseudo-code of a simple algorithm which returns the sum on all the elements in the i-th line, when given i as input.

[1] Source: top500.org.
[2] Source: intel.com.
[3] Source: arstechnica.co.uk.

2. Determine the complexity of this algorithm, and prove its correctness.

* **Ex. 4 —** *From SAT to 3-SAT*

Rewrite the following SAT formula into a 3-SAT formula.

$$(x_1 \vee x_2 \vee \neg x_3 \vee x_4 \vee x_5 \vee \neg x_6) \wedge (\neg x_1 \vee \neg x_2 \vee x_3 \vee \neg x_4 \vee x_5 \vee x_6) \wedge (x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4 \vee x_5 \vee \neg x_6) \wedge (x_1 \vee \neg x_2).$$

Ex. 5 — *Clique problem*

* 1. Explain what the Clique problem is.

2. Prove that Clique is in $\mathcal{NP}$.

3. Given a 3-SAT formula F with k clauses, construct a graph G such that F is satisfiable if and only if G has a k-clique.

4. Conclude on the complexity class of the Clique problem.

Ex. 6 — *IND-SET problem*

* 1. What is the maximum independent set problem?

2. What is the independent set (IND-SET) decision problem?

3. Prove that IND-SET is in $\mathcal{NP}$.

4. Construct a graph G' such that "G has a k-clique" is equivalent to "G' has an independent set of size k".

5. Conclude on the complexity class of the IND-SET problem.