Introduction of Gamma Oscillation by Synaptic Inhibition through Wang-Buzsaki Model (1996)

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Special Topic: Modeling and Simulation in Science, Engineering, and Economics

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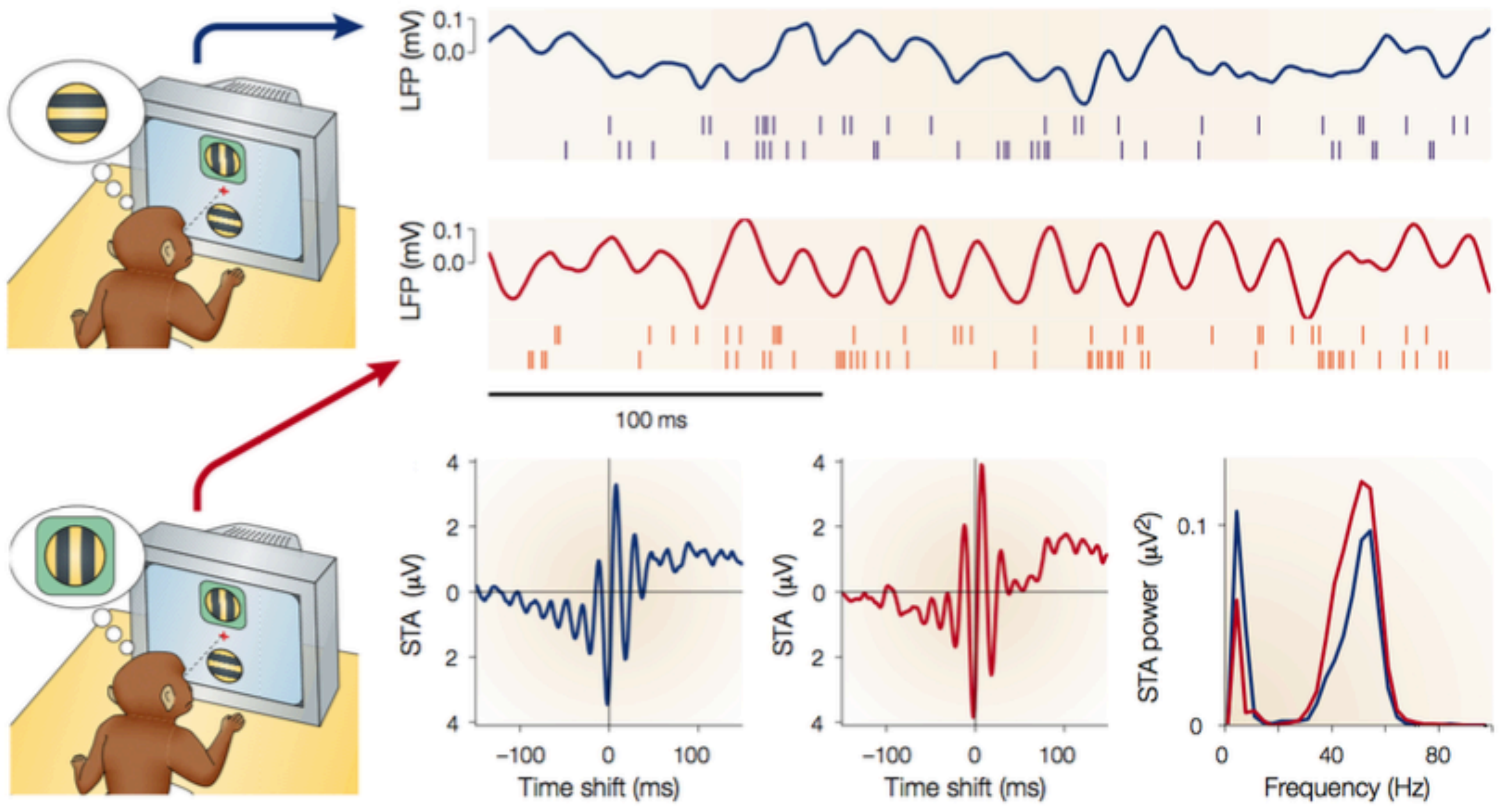
Gamma Oscillation

- Fast neuronal oscillations (30-60 Hz) observed in spiking activity of neurons of the neocortex depend on stimulus.
- Under certain behavioral condition, the synchronous rhythm is <u>confined</u> to selective neural populations rather than spatially uniform across the cortex.



Selective attention induces changes in synchrony in the visual cortex
[Fries et al. (2001), Wang (2010)]
Red (blue): attention directed inside (outside) the RF

LFP displays enhanced gamma oscillations for preferred spatial location:

- Synchronization in time with the population rhythm
- Changes of firing rates of sensory neurons

Synchronization of Neural Oscillators

- Brain rhythms as an emerging phenomenon in a network of oscillatory neurons.
- Whereas Gamma oscillations can be synchronized over a range of spatial scales, rhythmogenesis arises locally from microcircuit mechanisms [Fisahn et al. 1998, Atallah and Scanziani 2009].
    - Recurrent excitation (E-E); Mutual inhibition (I-I); Feedback Inhibition (E-I-E).

Synchronizing coupled neurons only when synaptic excitation is very fast

WB Model (Type I single neuron and $GABA_A$ receptor for synaptic coupling)

HH-type & Modified Integrate-and-Fire Model with nonlinear sodium and potassium conductances

Model Neuron & Synapse

$$c_{\mathrm{m}}\frac{dV}{dt} = -I_{\mathrm{Na}} - I_{\mathrm{K}} - I_{\mathrm{L}} - I_{\mathrm{syn}} + I_{\mathrm{app}}$$

- Spike-generating Na^{+} and K^{+} voltage-dependent ion currents + transient sodium current.
  - Shift of (in)activation curves and impact m_{∞} & h_{∞} and n_{∞} comparing to Hodgkin–Huxley.
- Gating variables are described in first-order kinetics (e.g. $\frac{dh}{dt} = \phi\left(\alpha_{\mathrm{h}}(1-h) - \beta_{\mathrm{h}}h\right)$).

1. Action potential is followed by a brief afterhyperpolarization
   - Relatively small maximal conductance g_{K} and fast gating process of I_{K}.
2. Interneurons have ability to fire repetitive spikes at high frequencies
   - Fast kinetics of I_{Na} inactivation & I_{K} activation and relatively high threshold of I_{K}.
   - Sensitive to input heterogeneities at smaller I_{app} values.

Fraction of open synaptic ion channels.

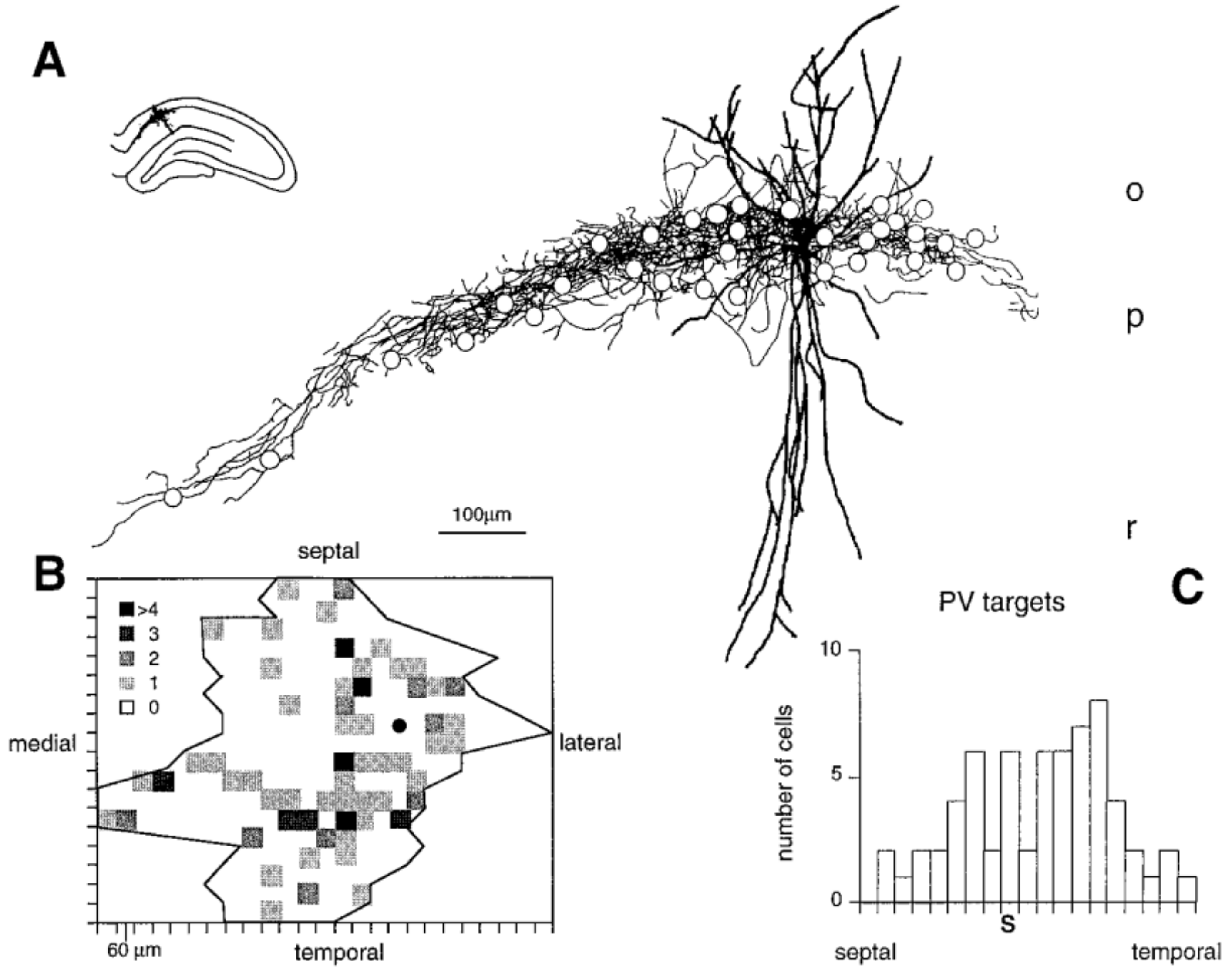
$$I_{\mathrm{syn}} = g_{\mathrm{syn}}s(V - E_{\mathrm{syn}}),\ \frac{ds}{dt} = \alpha F(V_{\mathrm{pre}})(1-s) - \beta s\ .$$

Normalized concentration of the postsynaptic transmitter-receptor complex

Modeling Network

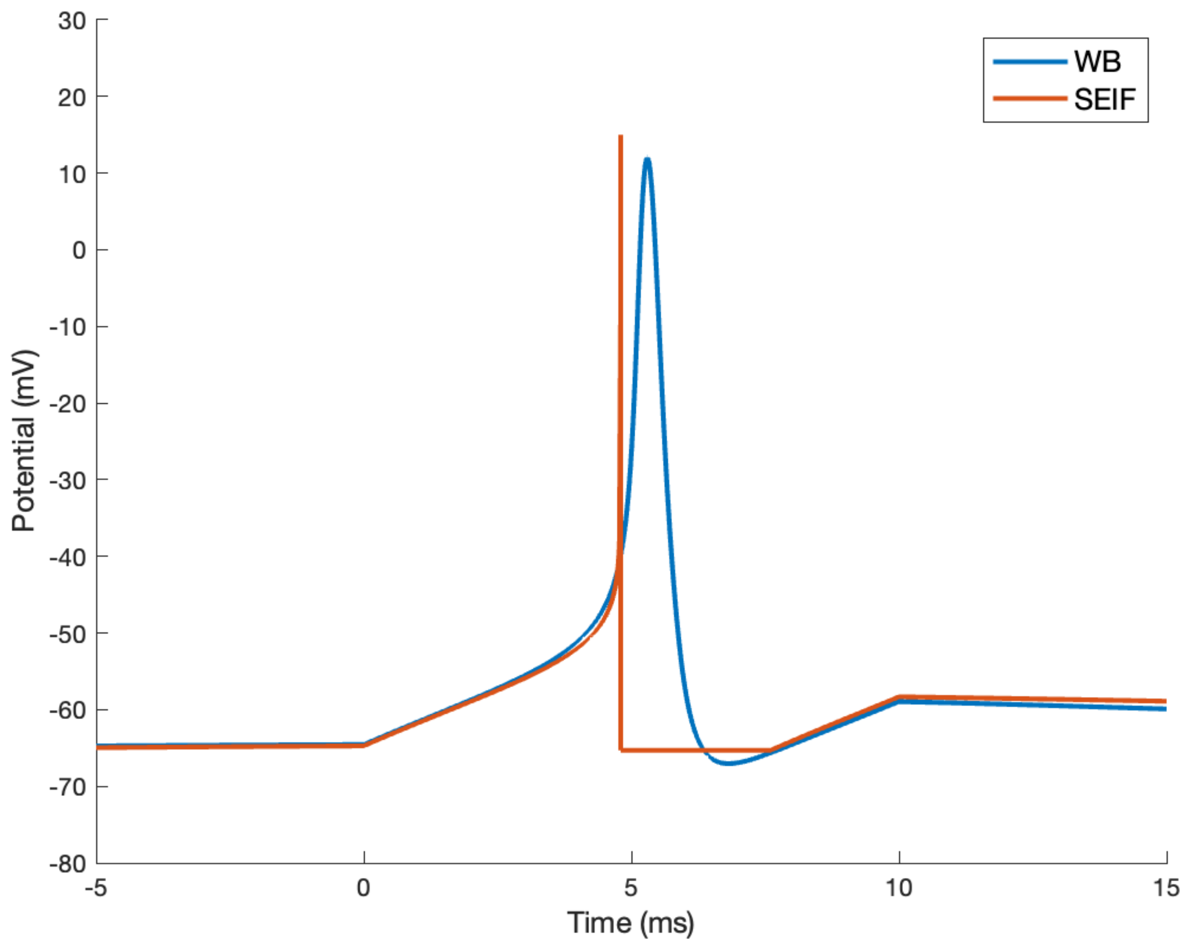
- Random network connectivity M_{syn} & Fully coupled (all-to-all) connectivity
  - Estimate from CA1 double staining of PV interneuron in hippocampus [Sik et al., 1995].
  - Axonal arborization of an intracellularly labeled basket cell was largely confined in the striatum pyramidale.
- Heterogeneous inputs (Gaussian distribution).
- Network coherence & Population coherence
  - Cross-correlation of spike trains at zero time lag within a time bin of $\Delta t = \tau$.

$$\kappa_{ij}(\tau) = \frac{\sum_{l=1}^{K} X(l)Y(l)}{\sqrt{\sum_{l=1}^{K} X(l) \sum_{l=1}^{K} Y(l)}}$$

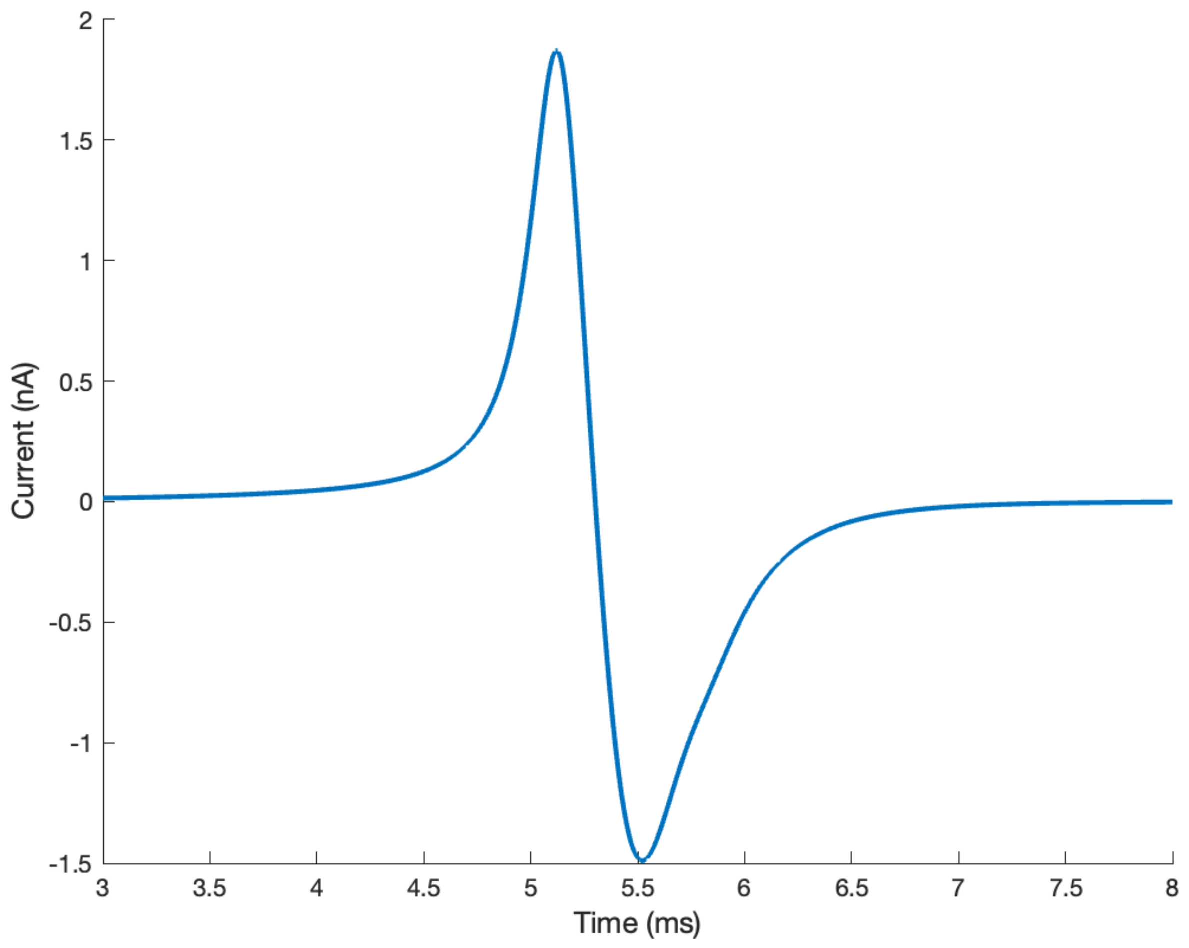


Wang and Buzsaki [1996]

Response Properties of Single-Compartment Model

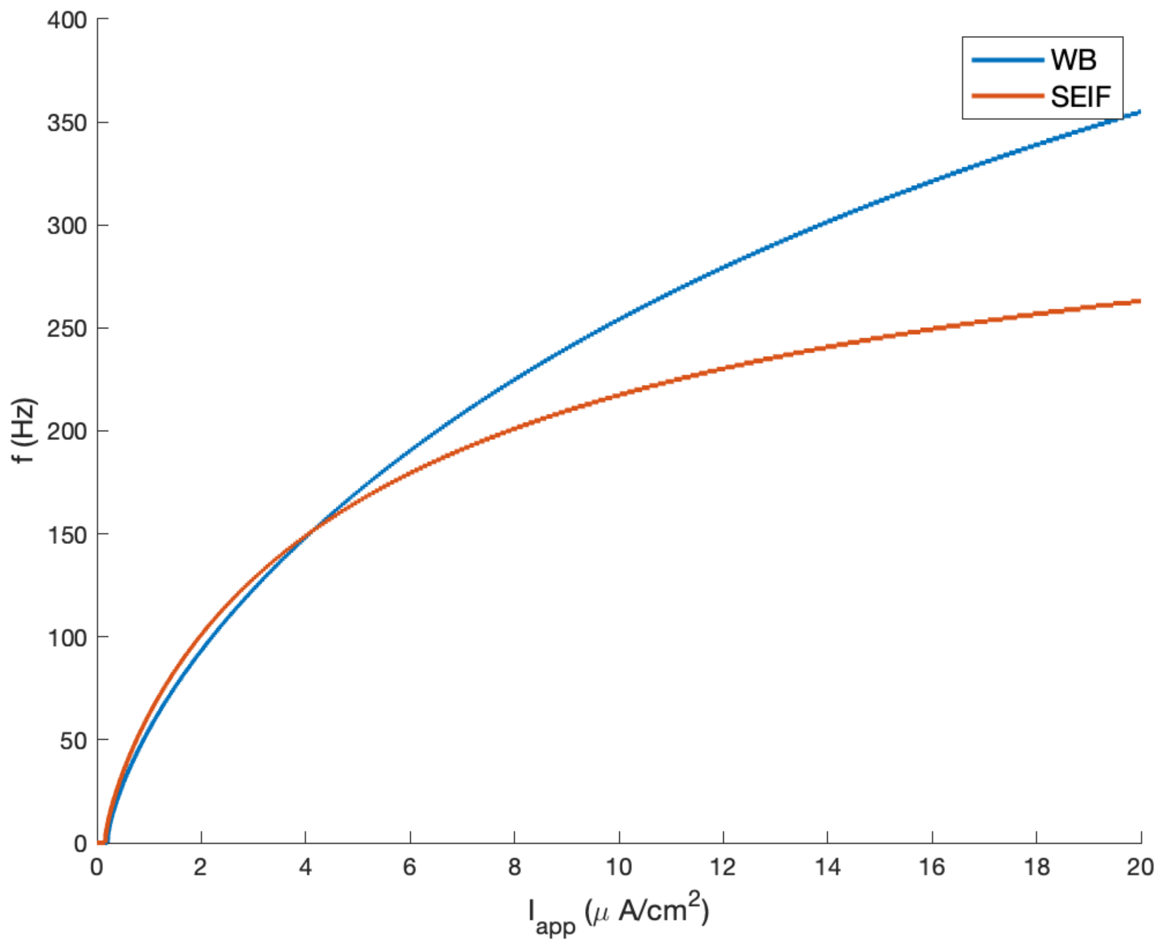


Spike responses driven by a step current input.

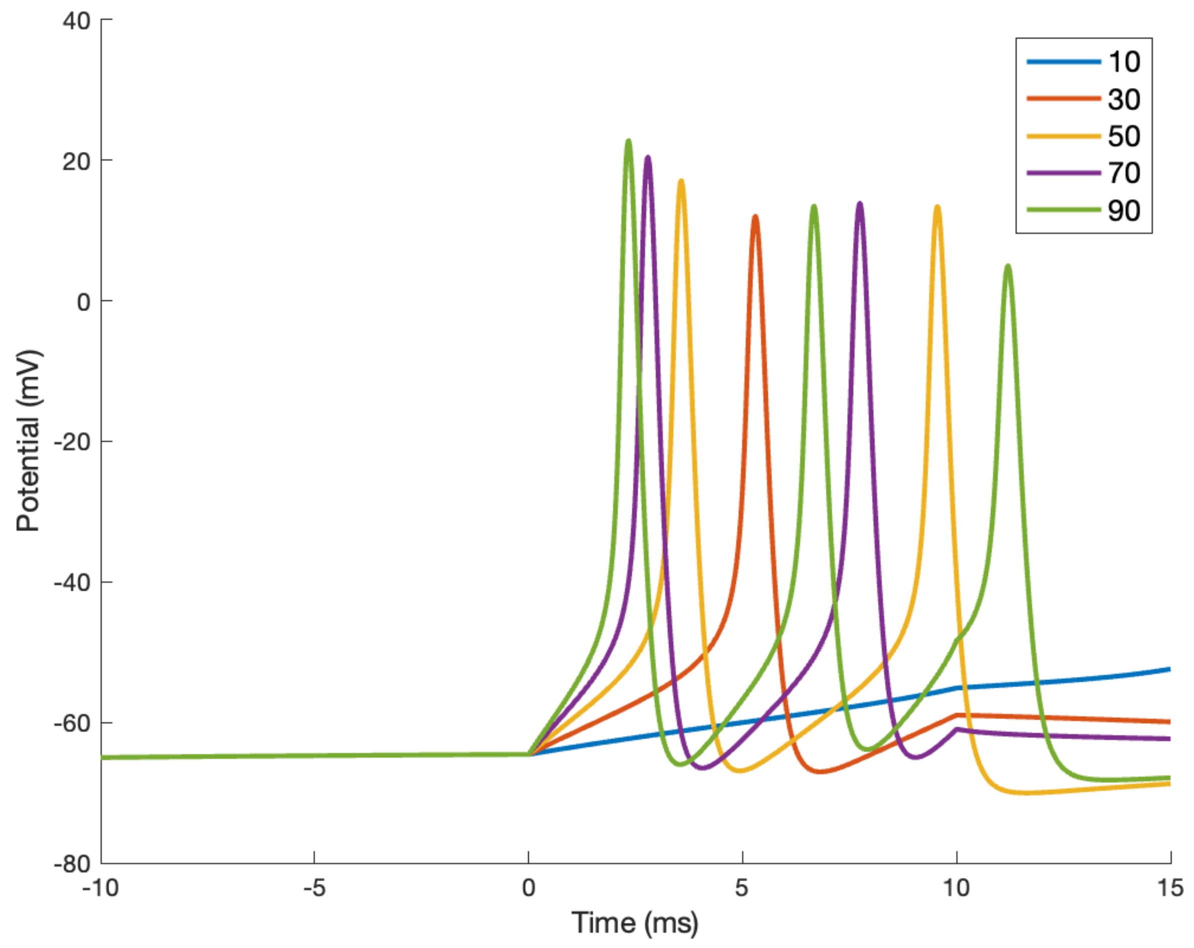


Membrane current.

Well-interpolated by exp(·)



$f - I$ curve.



Potentials under various I_{app}.

Comparison with (Standard)Exponential Integrate-and-Fire Model [Fourcaud-Trocme et al. (2003)]

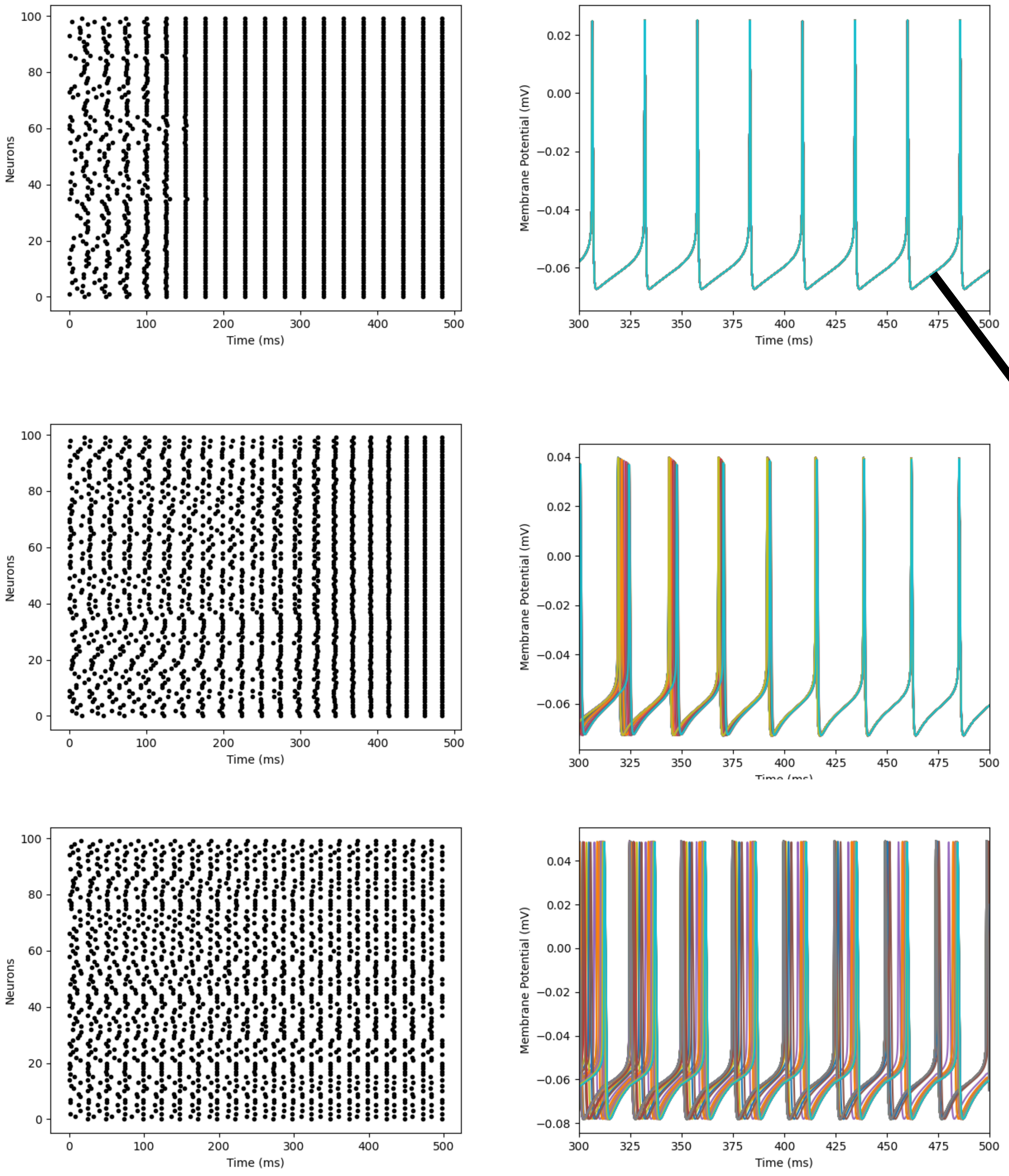
- Positive feedback underlying action potential.

$$c_{\mathrm{m}}\frac{dV}{dt} = -g_{\mathrm{L}}\left(V - E_{\mathrm{L}}\right) + \Delta\exp\left(\left(V - V_{\mathrm{th}}\right)/\Delta\right) + I_{\mathrm{app}}$$

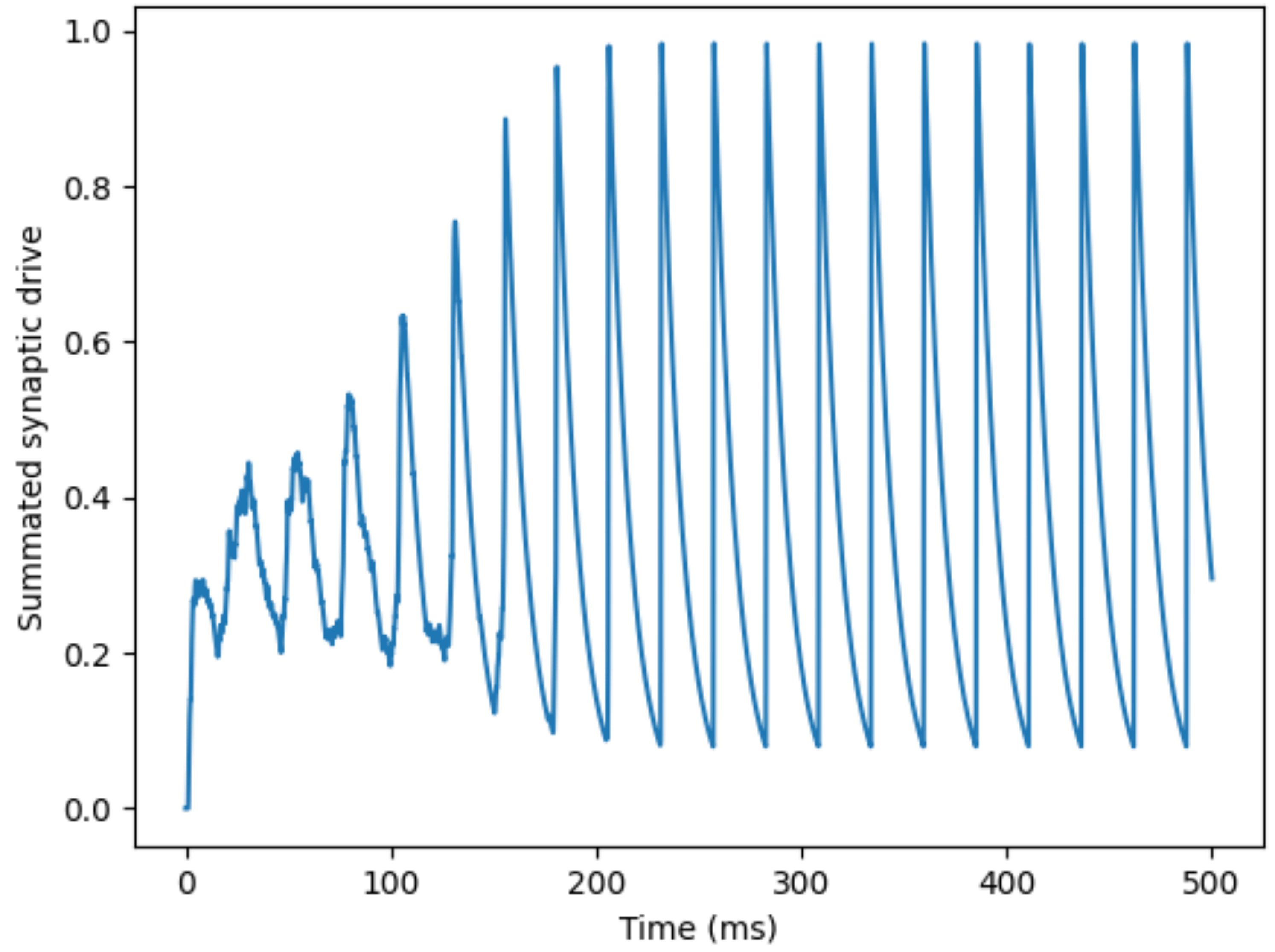
$$r = \beta\sqrt{I_{\mathrm{app}} - I_{\mathrm{c}}}$$

Synchronization by $GABA_A$ synapses

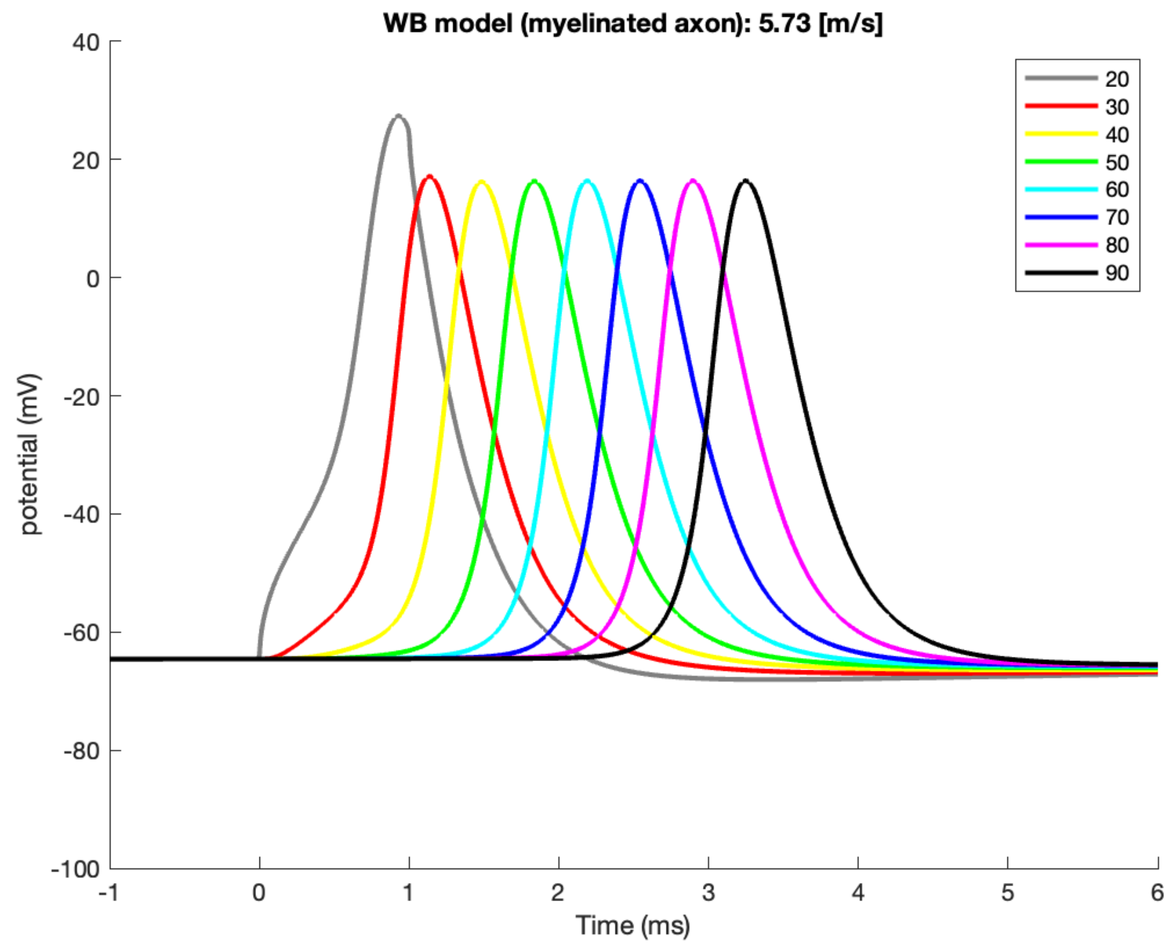
I_{app} increases
ϕ decreases

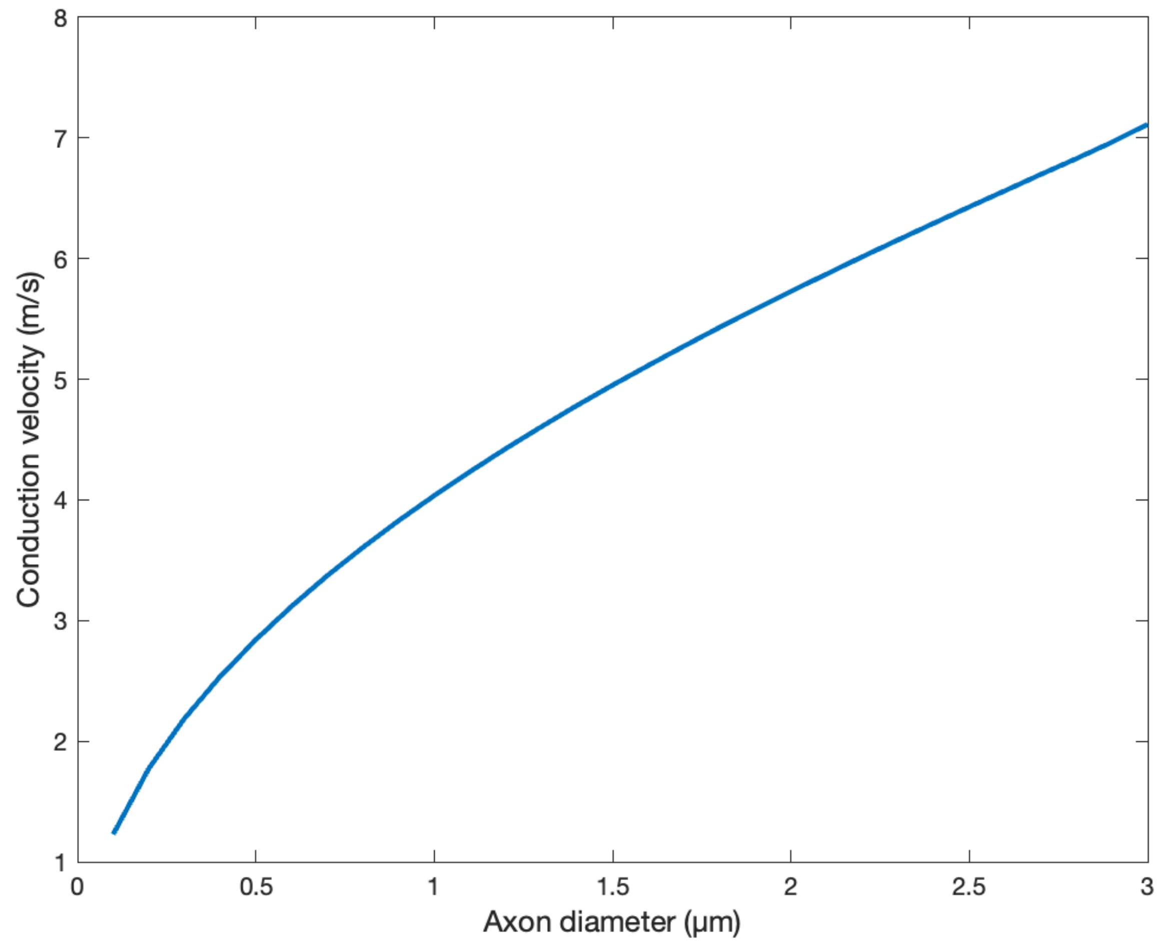


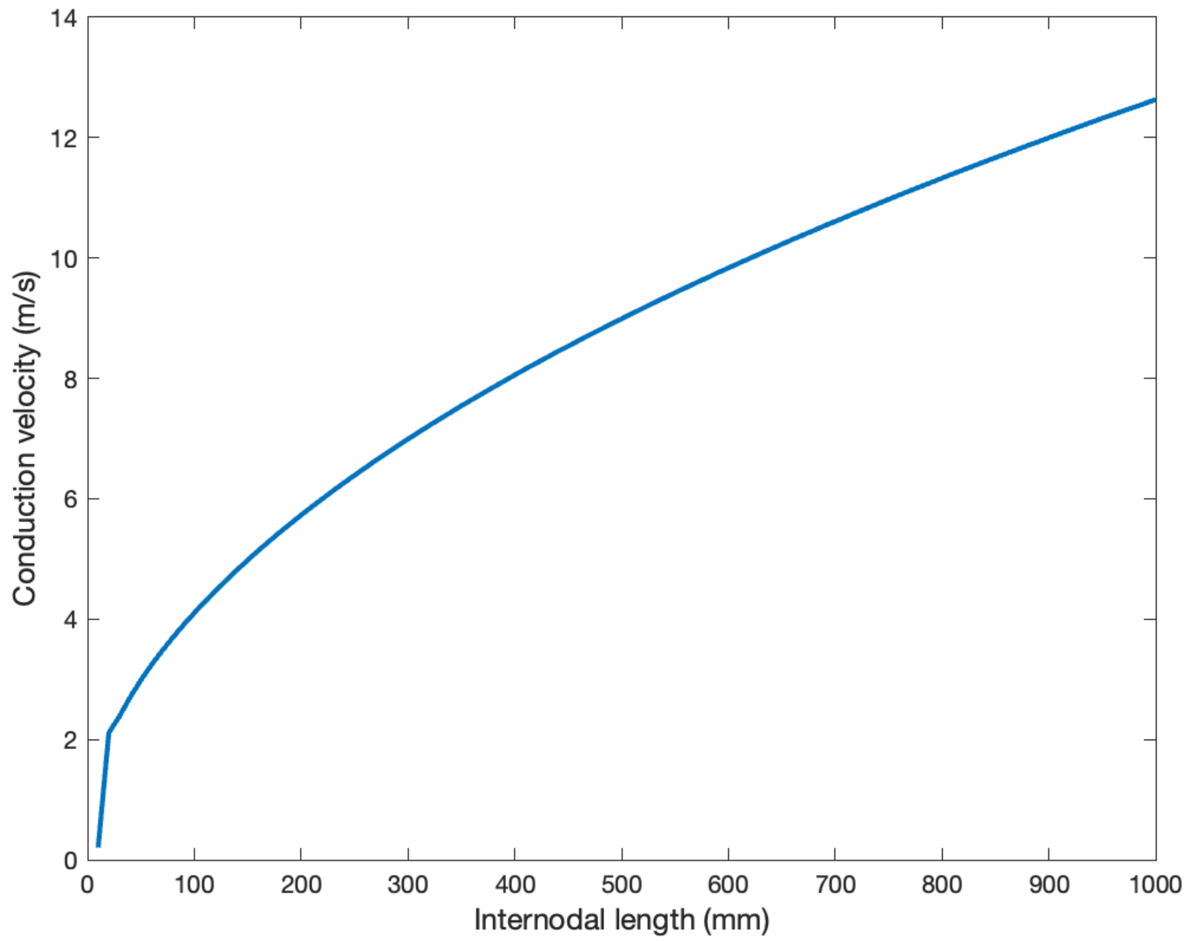
- Rastergrams & Membrane potentials of cells ($N = 100$) under various I_{app} and ϕ to preserve a similar oscillation frequency.
- With smaller ϕ values, I_K activation & I_{Na} inactivation is slower and the AHP amplitude is more negative.
- When $V_{AHP} < E_{syn}$, the full synchrony is lost and switches into 2 clusters alternating in time.



Application of WB Model: Myelinated Axons







- Series of excitable units interconnected with an axial resistance (with negligible capacitance and transmembrane conductance) [Moore et al., 1978].
- Each nodal compartment is described in WB model.
- 1D non-branching axon and stimulus current is injected intracellularly.

  Axonal current: $I_{\text{axon}^j} = g^{j-1}_{\text{axon}}(V_{j-1} - V_j) + g^{j}_{\text{axon}}(V_{j+1} - V_j)$

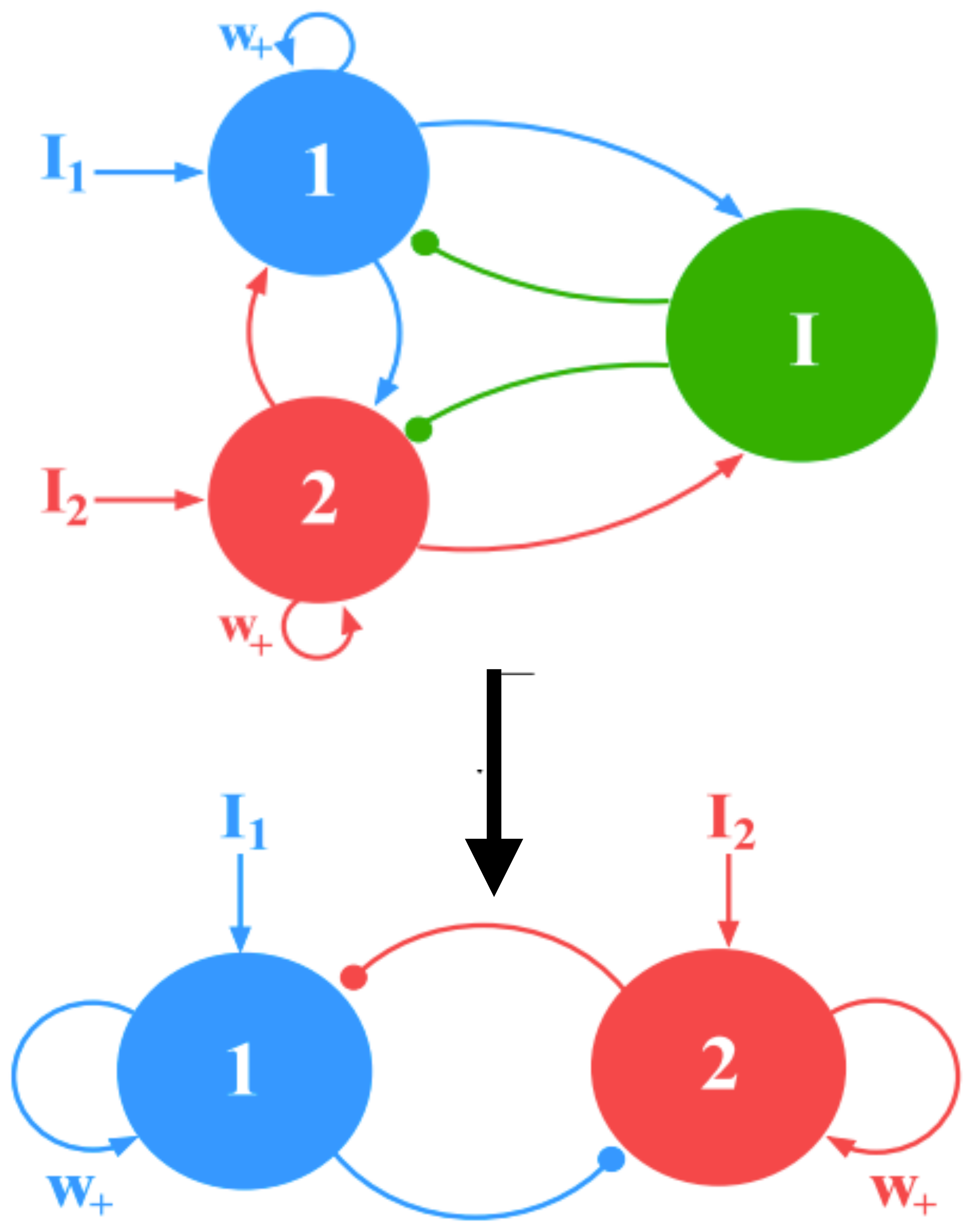
Preliminary Approach
Population Rate Model of Recurrent Neural Circuit

- Two excitatory neural assemblies
  - Strong recurrent excitation between neurons with similar stimulus selectivity.
  - Effective mutual inhibition — competition.

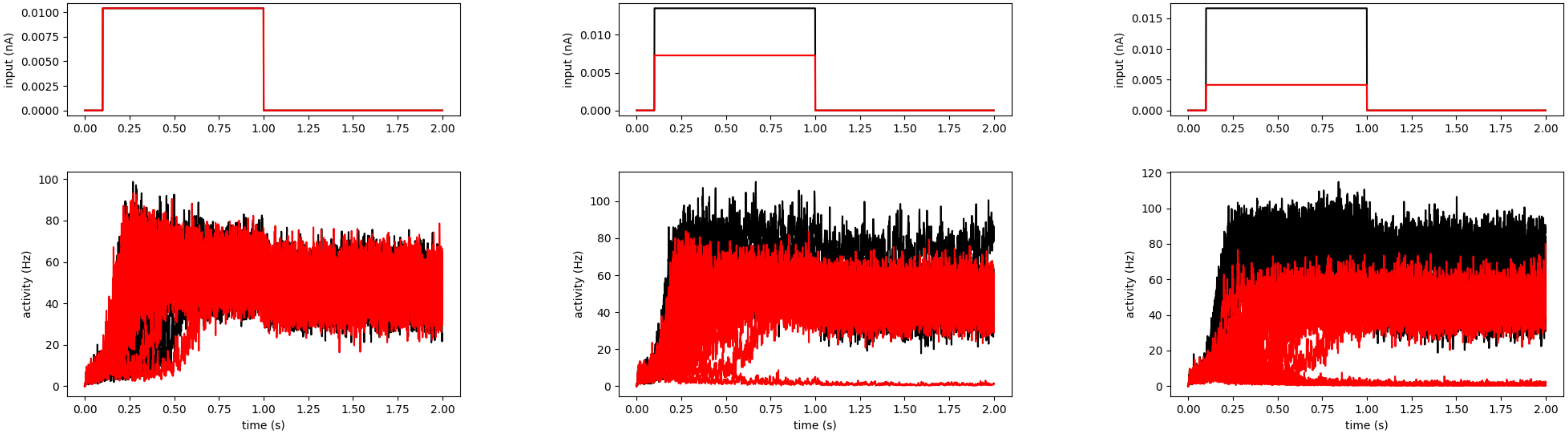
$$\frac{ds_1}{dt} = F\left(I_1\right)\gamma\left(1 - s_1\right) - s_1/\tau_s,\ r_i = F(I_i)$$

$$I_1 = g_E s_1 - g_I s_2 + I_{b1} + g_{ext}\mu_1$$

$$\tau_0 \frac{dI_{b1}}{dt} = -\left(I_{b1} - I_0\right) + \eta_1(t)\sqrt{\tau_0\sigma^2}\ \text{(Ornstein-Uhlenbeck)}$$



Wong and Wang (2006)



Different formulations of background inputs; $n = 20$.

Summary

- Reproduce the behaviors and dynamics of neurons and synapses described in Wang-Buzsaki 1996 model.
- Regenerate the synchronization by $GABA_A$ synapses in the neuronal network.
- Use WB model to simulate the myelinated axons and characterize the action potential propagation.
- Adapt Wong-Wang 2006 model to describe two excitatory WB neural assemblies and their mutual inhibitory interactions.