Homework #3 (NEURL-GA 3042, Fall 2022)

Due date: Sunday October 16

A synaptic current $I_{syn} = g_{syn} s(V - V_{syn})$, where V is the postsynaptic membrane potential, E_{syn} is the reversal potential of the synaptic current ($V_E = 0$ mV for AMPA and NMDA receptor mediated excitatory currents, $V_I = -70$ mV for GABA$_A$ receptor mediated inhibitory current). The gating variable s represents the fraction of open synaptic channels, and obeys a simple first-order kinetics:

$$\frac{ds}{dt} = \alpha_s \sum_j \delta(t - t_j)(1 - s) - s/\tau_s \tag{1}$$

where $\sum_j \delta(t - t_j)$ is a presynaptic spike train. Let us fix $\alpha_s = 1$; $\tau_{AMPA} = 2$ ms, $\tau_{NMDA} = 50$ ms and $\tau_{GABA_A} = 8$ ms.

(1) For AMPA receptor mediated excitation with a short time constant τ_{AMPA}, $s(t)$ typically decays back to zero between input spikes as long as the presynaptic firing rate r is within a physiological range, say < 100 Hz. The interspike interval is typically $1/r$, longer than 10 ms, and we recall that an exponential decay $\sim \exp(-t/\tau)$ becomes close to 0 within 1% when $t = 5\tau$. In that case, we can assume $(1 - s) \simeq 1$ and Eq. (1) becomes

$$\frac{ds}{dt} = \alpha_s \sum_j \delta(t - t_j) - s/\tau_s \tag{2}$$

(a) Simulate Eq. (2) in response to a periodic train at two frequencies $r = 10$ and 50 Hz, for each of the two firing rates calculate the average of s,

$$< s >= \frac{1}{T} \int_0^T s(t)dt \tag{3}$$

with a long time window T.

(b) You can also calculate $< s >$ analytically, compare the analytical result with the numerical simulation result, and discuss if they agree with each other.

(2) A simplification of Eq. (1) is to replace $\sum_j \delta(t - t_j)$ by the firing rate r, thus the dynamical equation becomes

$$\frac{ds}{dt} = \alpha_s r(1-s) - s/\tau_s \tag{4}$$

(a) For a given presynaptic firing rate r, what is the time constant for the s dynamics? With $r = 20$ Hz, what is the value of this time constant for $AMPA$ and $NMDA$ receptor mediated excitation, and $GABA_A$ receptor mediated inhibition, respectively?

(b) Show that the steady state $s_{ss} = \alpha_s \tau_s r/(1+\alpha_s \tau_s r)$. Assume $\alpha_s = 1$, plot this expression as function of r between 0 and 100 Hz, for the three different types of synaptic current. Discuss each of the three curves: does it look linear? If not, when does it start to plateau? What is the physiological implication of such a saturation?