Homework #4 (NEURL-GA 3042, Fall 2022)

Due date: Tuesday October 25

Consider a network of neurons, described by the following equation (cf. Ben-Yisha, Bar-Or and Sompolinsky, Proc. Natl. Acad. Sci. (USA) 92: 3844-3848 (1995)),

$$\tau \frac{dr(\theta)}{dt} = -r(\theta) + F(I(\theta)),$$

where $\tau = 10$ ms, $r(\theta)$ is the firing rate of a neuron with preferred orientation θ between $-\pi/2$ and $+\pi/2$. $I(\theta)$ is the input it receives, and $F(I)$ is the input-output relationship (f$-$I curve). We will use a simple threshold linear $F(I) = [I]_+$, which is zero if $I < 0$, and I if $I > 0$. The input is the sum of an external (thalamic) input $h(\theta)$ and a recurrent input,

$$I(\theta) = h(\theta) + \int_{-\pi/2}^{\pi/2} \frac{d\theta'}{\pi}(J_0 + J_2 \cos(2(\theta - \theta')))r(\theta').$$

and $h(\theta) = Ac(1 - \epsilon + \epsilon \cos(2(\theta - \theta_{cue})))$ where θ_{cue} is the peak of thalamic input, say $\theta_{cue} = 0$ (at the center of the network). The parameter ϵ expresses the tuning of the thalamic input, c is the contrast.

Implement and simulate this network with a discrete number of neurons, say $N = 50$ (you can also try different values of N). Neurons are now labeled as $\theta_i = (\pi/N)i - \pi/2$, $i = 1, 2, ..., N$. Start with a slightly tuned initial condition, say $r(\theta_i, t = 0) = a\cos(2(\theta_i - \theta_0))$ with $a = 2$ or 5 Hz, $\theta_0 = 0$ or $\pi/4$.

(1) Use $J_0 = -0.5$, $J_2 = 1$, $A = 40$ Hz and $\epsilon = 0.1$. For each of several contrast values $c = 0.1(10\%), 0.2(20\%), 0.4(40\%)$ and $0.8(80\%)$, plot the activity profile in the steady state, $r(\theta_i)$ as function of θ_i. Relate the activity profile in the network to the tuning curve of a single neuron, and discuss its dependence on the contrast.

(2) Repeat (1) with $J_0 = -7.3$, $J_2 = 11$, $A = 40$ Hz and $\epsilon = 0.1$. Discuss the different contrast dependence that you observe.

(3) As in (1-2), but now use a random distribution of $h(\theta_i)$. For each cell i, $h(\theta_i) = Ac(1 - \epsilon + \epsilon\cos(2\theta_i)) + \sigma_h \eta_i$ where the noise term is taken from a Gaussian distribution of zero mean and standard deviation σ_h (say $\sigma_h = 3$, but try several values). Can you reproduce qualitatively figures 7.8 and 7.9 in Dayan and Abbott's book?

(4) Let $\epsilon = 0$ (the input is completely untuned). Explore the parameter space (J_0, J_2). For example, fix $J_0 = -1$, and vary $J_2 = 1, 1.5, 2.5, 4.5$, Describe what you observe.