Computational Neuroscience Homework 6

Zihan Zhang (Steven)

November 21, 2022

0.1 Background & Introduction

In this homework, we would consider a population rate model of a recurrent neural circuit [1, 2]. The model has two excitatory neural assemblies. Their mutual interactions are effectively inhibitory through a shared pool of inhibitory neurons (not explicitly present in the reduced two-variable rate model). r_1 and r_2 be their respective population-firing rates, and the total synaptic input current I_i and the resulting firing rates r_i of the neural population i obey the following input-output relationship ($F - I$ curve):

$$r_i = F(I_i) = \frac{aI_i - b}{1 - \exp(-d(aI_i - b))}. \tag{1}$$

where $a = 270Hz/nA, b = 108Hz, d = 0.154s$. It captures the current-frequency function of a leaky integrate-and-fire neuron. Also, assume the synaptic drive variables s_1 and s_2 obey:

$$\begin{aligned} \frac{ds_1}{dt} &= F(I_1)\gamma(1 - s_1) - s_1/\tau_s \\ \frac{ds_2}{dt} &= F(I_2)\gamma(1 - s_2) - s_2/\tau_s \end{aligned} \tag{2}$$

where $\gamma = 0.641$. Also:

$$\begin{aligned} I_1 &= g_E s_1 - g_I s_2 + I_{b_1} + g_{ext}\mu_1 \\ I_2 &= g_E s_2 - g_I s_1 + I_{b_2} + g_{ext}\mu_2 \end{aligned} \tag{3}$$

The synaptic time constant $\tau_s = 100ms$. The synaptic coupling strengths $g_E = 0.2609nA, g_I = 0.0497nA, g_{ext} = 0.00052nA$. Stimulus-selective inputs to populations 1 and 2 are governed by unitless parameters μ_1 and μ_2 respectively. I_b is the background input which has a mean I_0 and a noise component described by an Ornstein-Uhlenbeck process:

$$\begin{aligned} \tau_0 \frac{dI_{b_1}}{dt} &= -(I_{b_1} - I_0) + \eta_1(t)\sqrt{\tau_0 \sigma^2} \\ \tau_0 \frac{dI_{b_2}}{dt} &= -(I_{b_2} - I_0) + \eta_2(t)\sqrt{\tau_0 \sigma^2} \end{aligned} \tag{4}$$

where $I_0 = 0.3255nA$, filter time constant $\tau_0 = 2ms$, and noise amplitude $\sigma = 0.02nA$. $\eta(t)$ is a Gaussian white noise with zero mean and unit standard deviation.

0.2 Question 1

With a zero initial condition, Figure 1 shows the traces of the synaptic drives, which gradually converge to the resting state around 0.12. To determine the firing rate at the resting state, we solve the steady state of ODE of Equation (2) and have $r \approx 2.1273Hz$. Another way of doing this is truncating the final period of r curves (if stable) and calculating the mean, which would generate similar results.

After the system has settled in the resting state, we add a stimulus ($\mu_1 = 35, \mu_2 = 0$) for a brief period ($1s - 1.5s$), followed by a delayed memory period (Figure 2 left). Without stimulus, the system will end with the resting state $r_1 \approx r_2 \approx 0.5 - 1Hz$. A transient stimulus will induce a delayed memory period with $r_1 \approx 25Hz, r_2 \approx 0Hz$, and the persistent activity state is attained (oscillates at a high frequency). Figure 2 right shows the result using the symmetric condition where the stimulus is applied to the second population. There exists another memory state $r_1 \approx 0Hz$, $r_2 \approx 25Hz$, and it can also be switched back to the resting state.

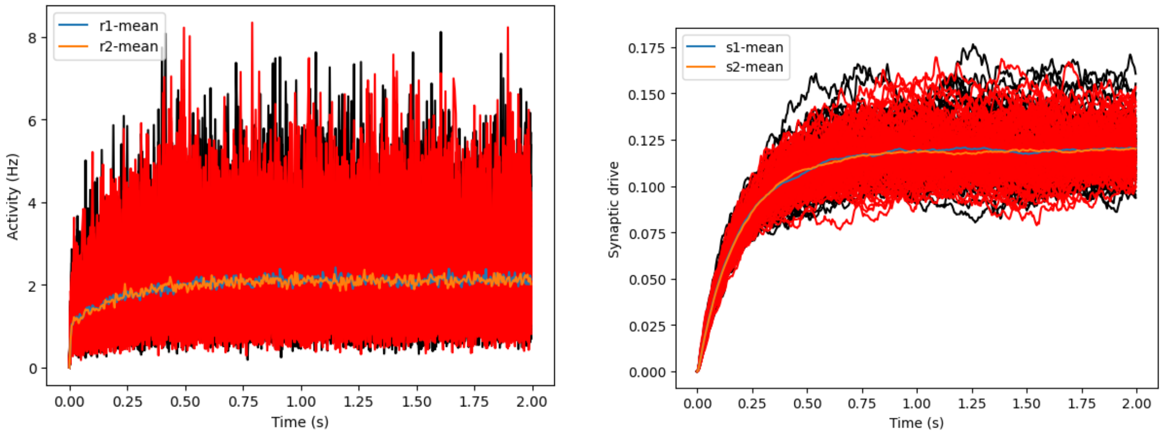

Figure 1: Run the model simulation for $2s$ without stimulus ($\mu_1 = \mu_2 = 0$) with the initial condition $s_1 = s_2 = 0$. Shows the curves for r, s, and their respective means among 100 independent trails.

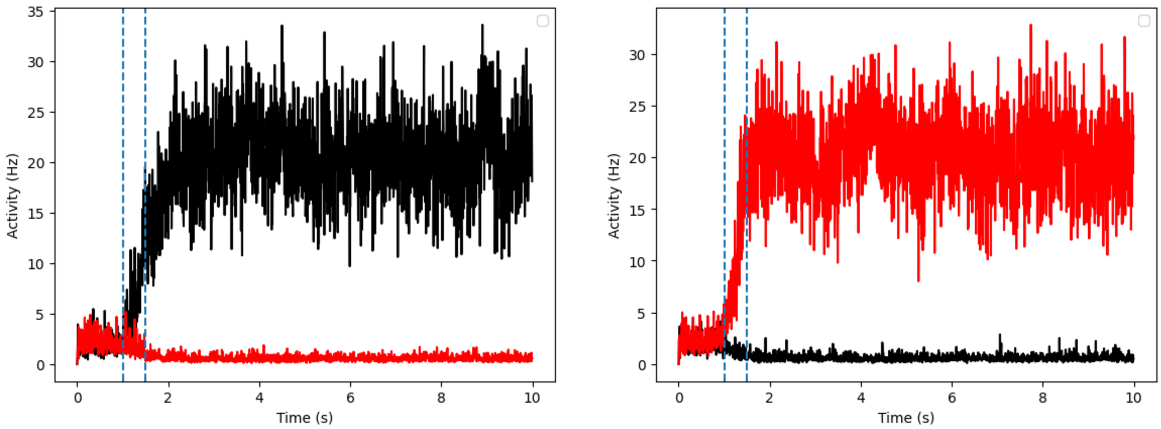

Figure 2: Add stimulus through $\mu_1 = 25$ (left) and $\mu_2 = 25$ (right) during $1 - 1.5s$, followed by a delay memory period. The results are symmetric.

After the delay period, we apply a second transient input ($\mu_1 = -60, \mu_2 = 0$) for a brief time ($5.0s - 5.5s$), which would lead the system back to the resting state. Figure 3 shows the results. Also, we compute the mean over all the trails (to get a smooth trajectory) and combine the results of the two conditions to get a full description of the trajectory of s_1 against s_2 and r_1 against r_2. Figure 4 shows the phase planes. Roughly we can see three steady states, corresponding to the resting state and two memory states, consistent with our numerical experiments.

Repeat the previous process by firstly briefly adding a cue stimulus and then a distractor stimulus during the delay. Figure 5 shows the results. The general tendency (resting, memory, resting) does not drastically change, except the firing rate of the first population slightly decreases because of the distraction.

We incrementally decrease the value of g_E and show that persistent activity disappears when the recurrent excitation is below a critical value. Experimentally speaking, the threshold value is approximately 0.2539. The result could be more accurate using a finer step size. Figure 6 shows the decreasing trend.

0.3 Question 2

We implement the coin-tossing simulations with $\mu_1 = \mu_2 = \mu_0 = 30$. In a decision-making simulation, both μ_1 and μ_2 are presented for a time interval from $t_1 = 0.5s$ to $t_2 = 1.5s, T = t_2 - t_1 = 1s$. The decision choice is determined according to which of the two active attractors wins the competition. Figure 7 shows that either of the two populations can dominate and win the competition. Also, we did observe a $50 - 50$ decision outcome when n is large. Specifically, when $n = 500$, r_1 wins for 52.6% of the trails and r_2 wins for 48.4% respectively.

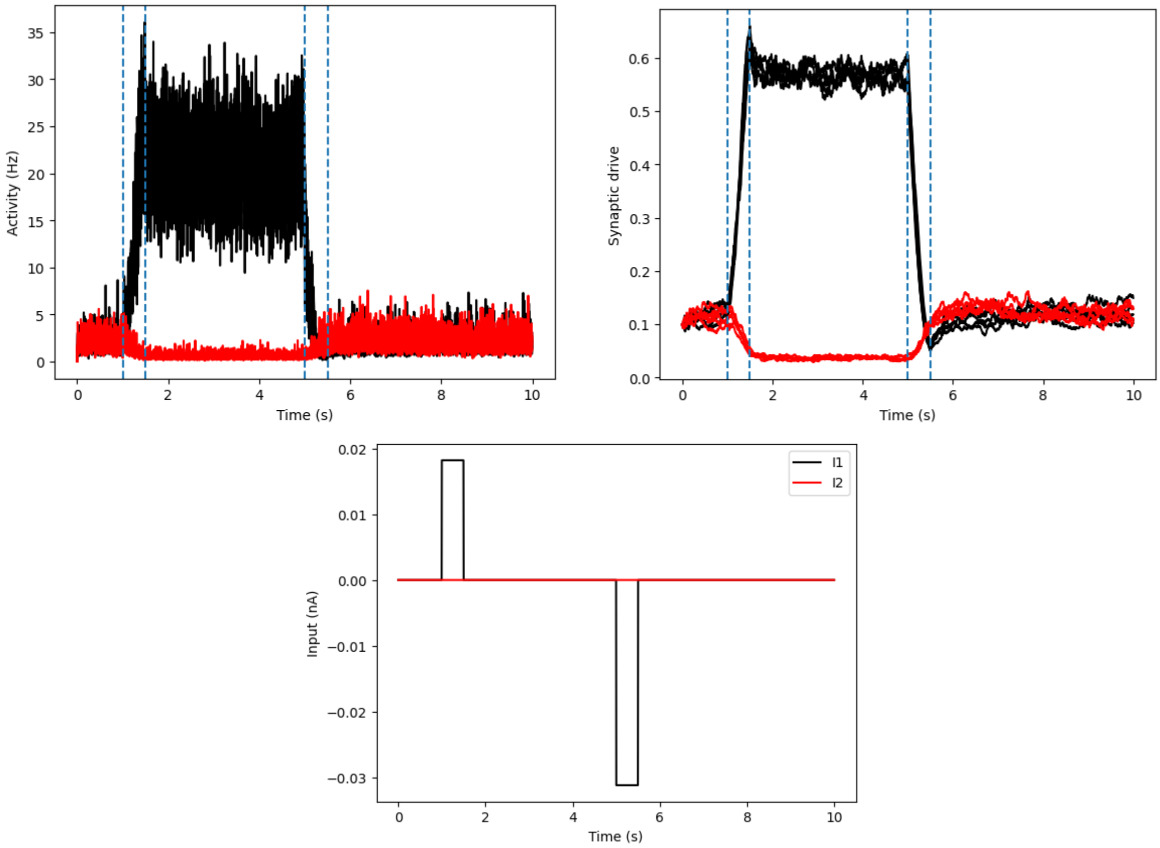

Figure 3: Continue from Figure 2. Add a second impulse during $5s$ to $5.5s$ to switch the system from a memory state back to the resting state. Shows the curves for r, s, and I for 5 independent trails.

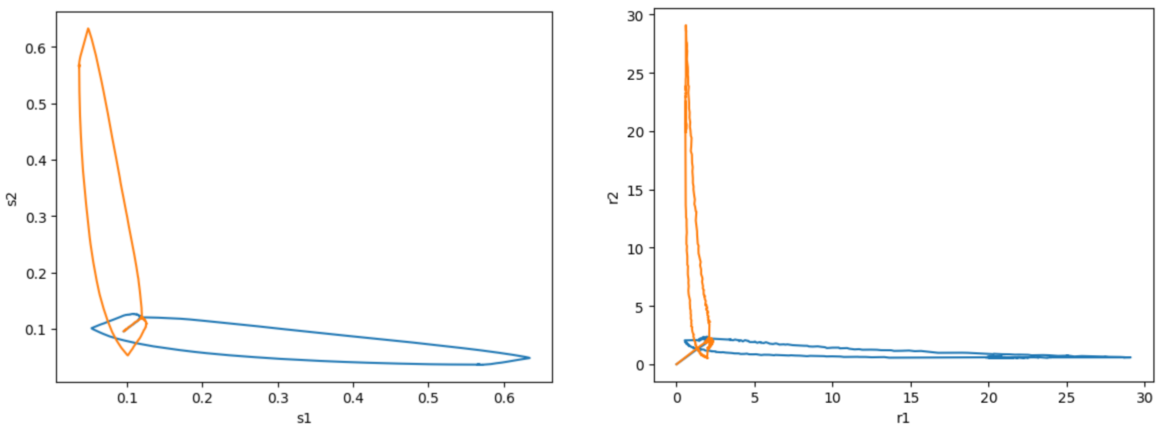

Figure 4: Phase planes of s_1 against s_2 (left) and r_1 against r_2 (right).

0.4 Question 3

We investigate the situation with stimulus-specific stimuli. With $\mu_1 = \mu_0(1 + c'), \mu_2 = \mu_2(1 - c')$ for positive c' as the coherence level (CH), we know μ_1 is more preferred. Thus, we would investigate how CH influences the performance of the decision-making by comparing the percentage of correct decisions as a function of $\log(c')$, namely the psychometric function. Figure 8 shows the result. When CH is low, the performance improves with its increase. When CH is high, the system can do perfect decisions and the percentage of the correct decisions converge to 100%.

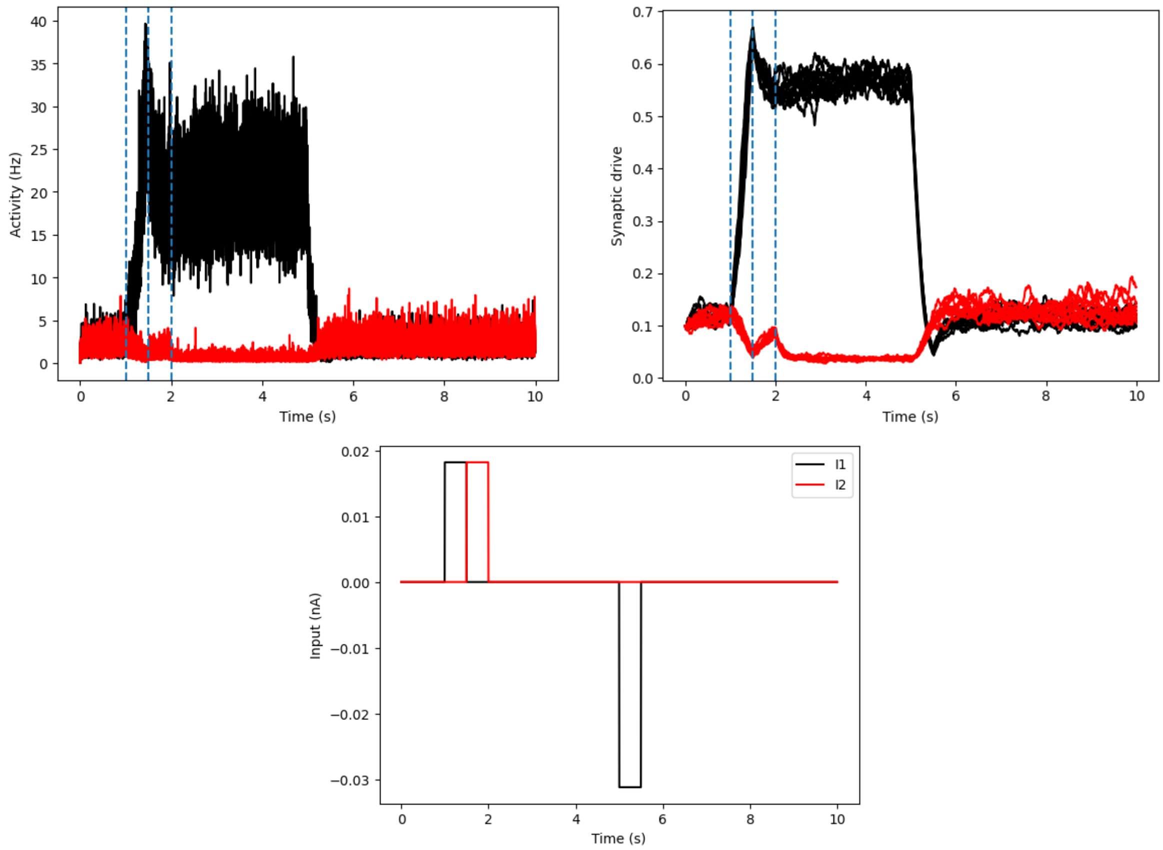

Figure 5: Continue from Figure 3. Add distractor stimulus ($\mu_1 = 0, \mu_2 = 35$) at the beginning of the delay period for 10 independent trials.

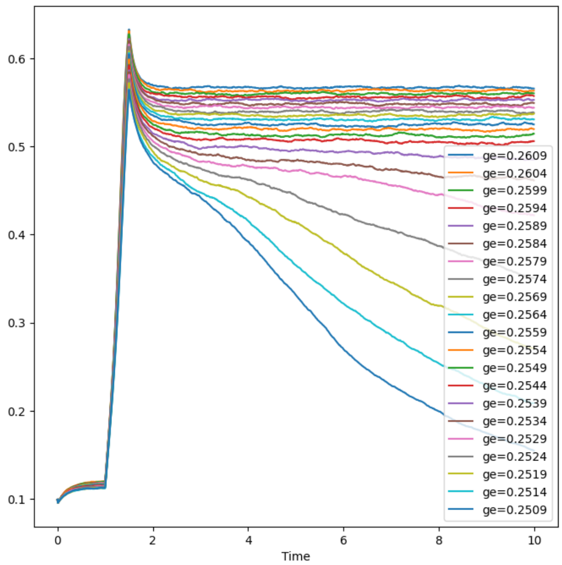

Figure 6: Decrease the value of g_E incrementally and the persistent activity disappears when $g_E < 0.2539$ roughly.

0.5 Question 4

We perform the reaction time task with the same model. With the firing threshold set as $\theta = 15Hz$, the decision is made whenever one of the two neural populations reaches the threshold first. Figure 9 shows the sample time course of firing rates for different coherence levels. For small coherence level (CH), both populations can get over the firing threshold. When CH is large, only the population which receives the high input will fire over the threshold, which is consistent with the observation in the decision-making task. Figure 10 shows all the statisti-

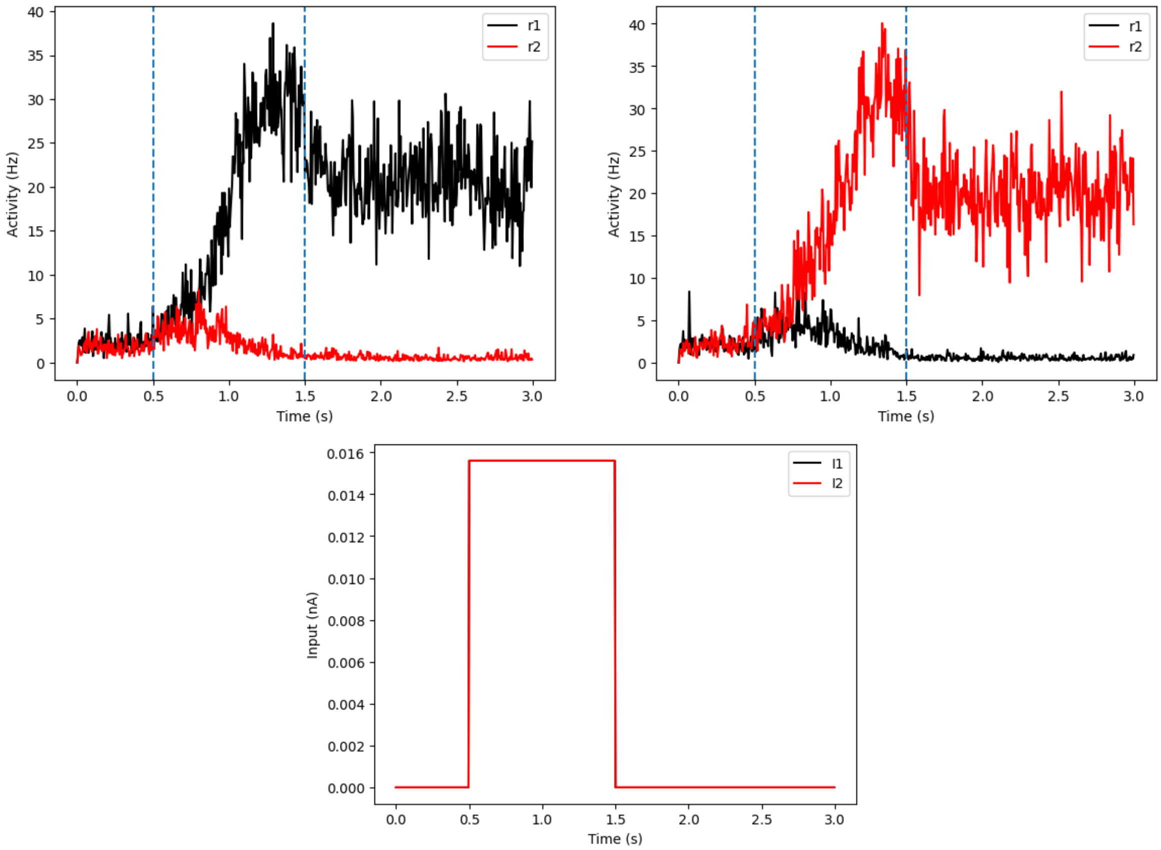

Figure 7: The typical time course of coin-tossing simulation. Either of the two population is able to dominate and win the competition.

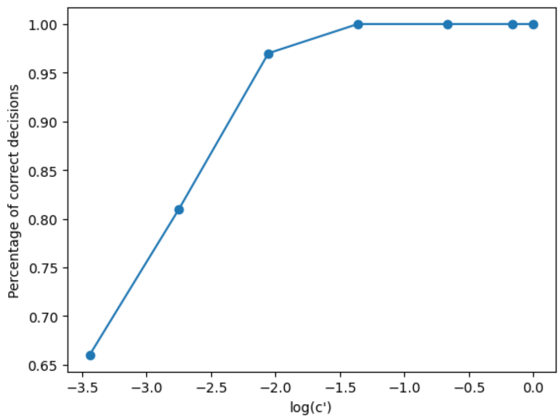

Figure 8: The percentage of correct decisions as a function of $\log(c')$.

cal results. The mean and standard deviation of reaction time both decrease with the increasing coherence level. Roughly speaking, the standard deviation of the reaction time is proportional to its mean, which validates Weber's law.

We also explore how the decision performance depends on the time integration of sensory information by computing the psychometric curve with varying duration of stimulus presentation. We fix $c' = 0.128$ and vary the duration T of stimulus from $0.1s$ to $1s$. Figure 11 shows the results. The percentage of correct decisions increases (better performance of decision making) with increasing duration of stimulus time. Besides, when the duration of the stimulus is very short ($0.1s$), the percentage of correct decisions is around 50%, which indicates that the system does not get much from the specific stimulus (just like in a coin-tossing situation).

References

[1] Xiao-Jing Wang. "Probabilistic Decision Making by Slow Reverberation in Cortical Circuits". In: *Neuron* 36.5 (2002), pp. 955–968. ISSN: 0896-6273. DOI: https://doi.org/10.1016/S0896-6273(02)01092-9. URL: https://www.sciencedirect.com/science/article/pii/S0896627302010929.

[2] KongFatt Wong-Lin and Xiao-Jing Wang. "A Recurrent Network Mechanism of Time Integration in Perceptual Decisions". In: *The Journal of Neuroscience* 26 (2006), pp. 1314–1328.

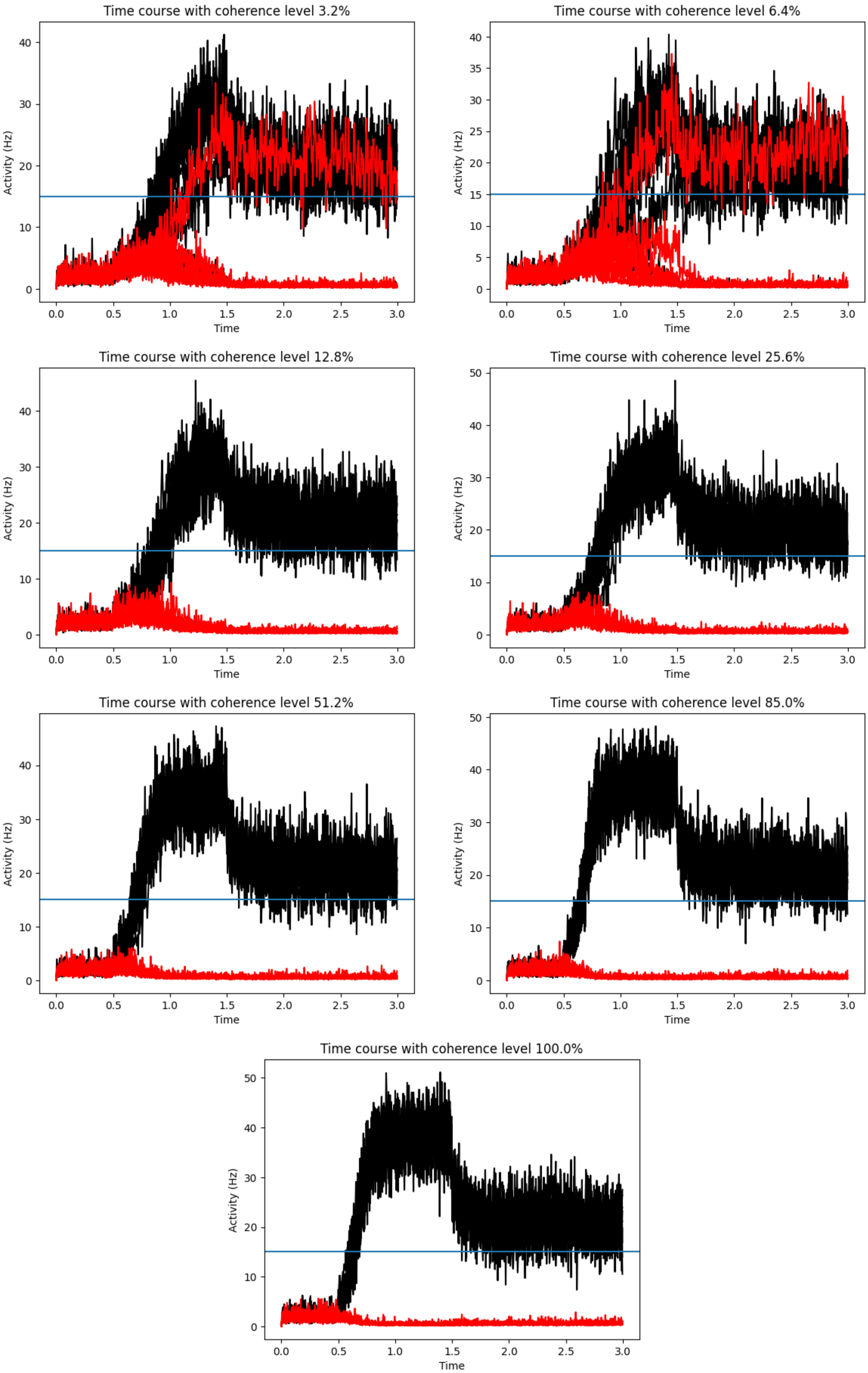

Figure 9: The time course of reaction time task for different coherence levels.

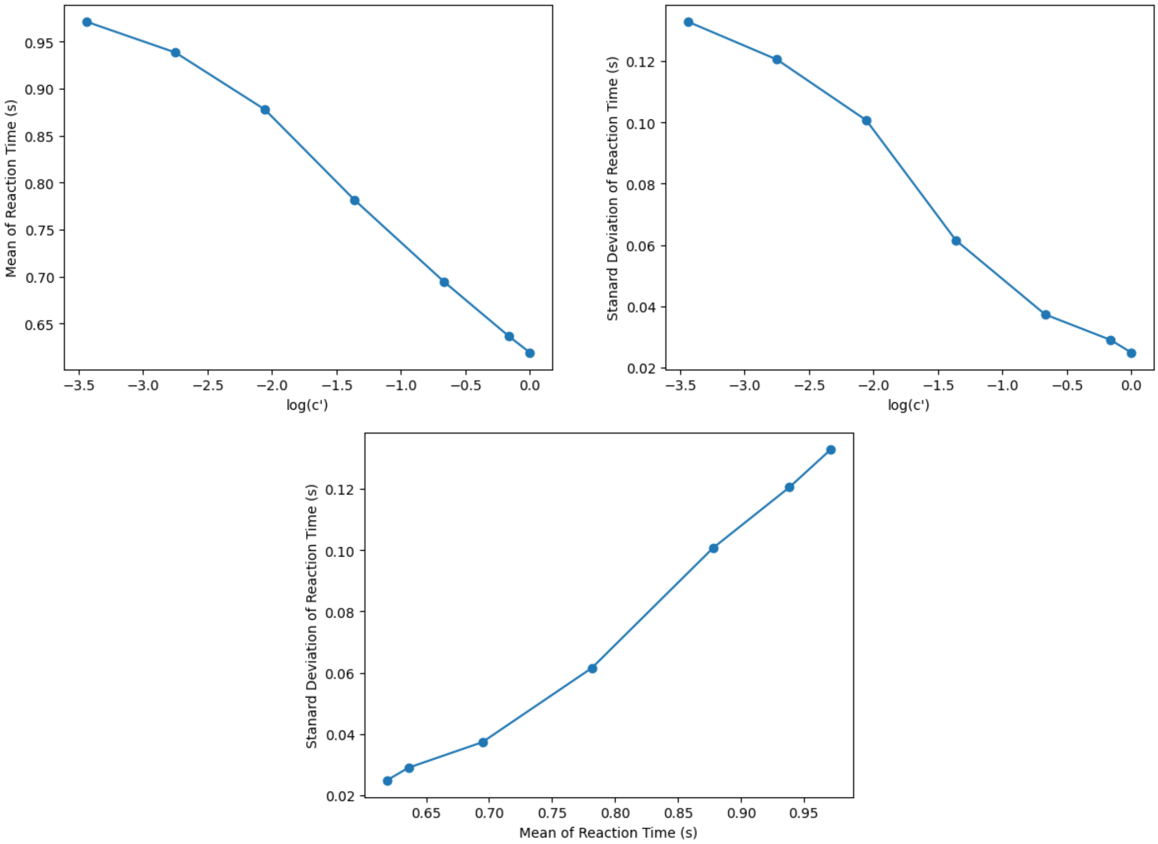

Figure 10: The trial-averaged reaction time and its standard deviation versus the coherence level.

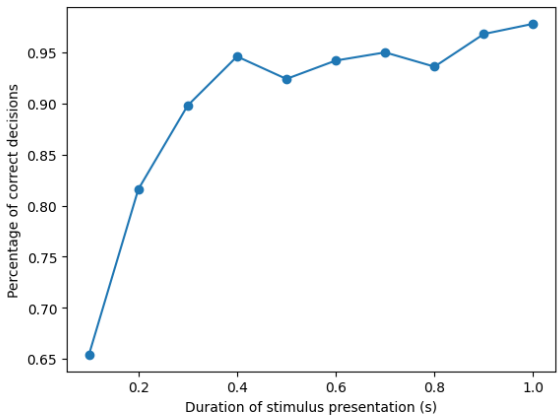

Figure 11: The percentage of correct decisions versus stimulus time.