Computational Neuroscience Homework 4

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1 Scheme of simulation

Suppose we implement and simulate the network with a discrete number of neurons: say $N = 50$ neurons labeled as $\theta_i = i\pi/N - \pi/2, i = 1, 2, ..., N$, then we have a ODE system with N variables:

$$\tau \frac{dr(\theta_i)}{dt} = -r(\theta_i) + F(I(\theta_i)). \tag{1}$$

with:

$$\begin{aligned} I(\theta_i) &= h(\theta_i) + \int_{\pi/2}^{\pi/2} \frac{d\theta'}{\pi}(J_0 + J_2 \cos(2(\theta_i - \theta')))r(\theta') \\ &= h(\theta_i) + \sum_{j=1}^{N} \frac{\pi}{N\pi}(J_0 + J_2 \cos(2(\theta_i - \theta_j)))r(\theta_j) \\ &= h(\theta_i) + \frac{J_0}{N}\sum_{j=1}^{N} r(\theta_j) + \frac{J_2}{N}\sum_{j=1}^{N}\cos(2(\theta_i - \theta_j))r(\theta_j) \end{aligned} \tag{2}$$

We then transform the continuous calculation into discretized version. Suppose $\mathbf{r} = (r(\theta_1), r(\theta_2), ..., r(\theta_N))^T$, $\mathbf{h} = (h(\theta_1), h(\theta_2), ..., h(\theta_N))^T$, and:

$$\mathbf{C} = \begin{pmatrix} 1 & \cos\left(\frac{2\pi}{N}\right) & \cdots & \cos\left(\frac{2(N-1)\pi}{N}\right) \\ \cos\left(\frac{2\pi}{N}\right) & 1 & \cdots & \cos\left(\frac{2(N-2)\pi}{N}\right) \\ \vdots & \vdots & & \vdots \\ \cos\left(\frac{2(N-1)\pi}{N}\right) & \cos\left(\frac{2(N-2)\pi}{N}\right) & \cdots & 1 \end{pmatrix}, \tag{3}$$

then we could reexpress the system as:

$$\tau \frac{d\mathbf{r}}{dt} = -\mathbf{r} + F\left(h + \frac{J_0}{N}\mathbf{1}\cdot\mathbf{r} + \frac{J_2}{N}\mathbf{C}\mathbf{r}\right) \tag{4}$$

2 Experiments

2.1 Part 1

Here we discuss the relationship between the steady state of the network with the tuning curve of a single neuron. The network is invariant with the translation of θ_{cue} as the thalamic input. If $r(\theta)$ is the steady state with respect to $\theta_{\text{cue}} = 0$, then $r(\theta - \theta_t)$ is the steady state with respect to $\theta_{\text{cue}} = \theta_t$. Also, $r(\theta)$ is symmetric function on θ. Thus, $r(\theta_t)$ is not only the steady state for the neuron with preferred angle θ_t with $\theta_{\text{cue}} = 0$ but also the response of the neuron with preferred angle 0 with $\theta_{\text{cue}} = \theta_t$, which is the tuning curve of the single neuron with preferred angle 0. Thus, the profile of the steady state is both the population activities and the turning curve of a single neuron. Figure 2 describes the tuning curve under different contrast values in the `semilogy` graph, where Figure 1 is plotted in normal scale. The network shows the orientation selectivity focus on $\theta = 0$ but spread broadly. When increasing the contrast value, the amplitude of the firing rate will be multiplied by a constant number. The constant is the same for neurons with different preferred angles, and the four curves in Figure 2 are roughly paralleled. Also, the steady state is invariant with initial values (i.e. different amplitude or initial tuned orientation), depending only on the parameters of thalamic inputs.

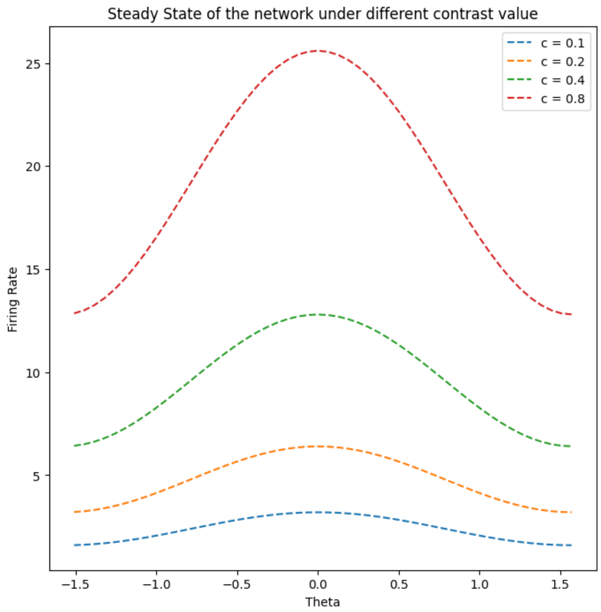

Figure 1: The steady state of the network with different contrast values (starting from $a = 2$ and $\theta_0 = \pi/4$).

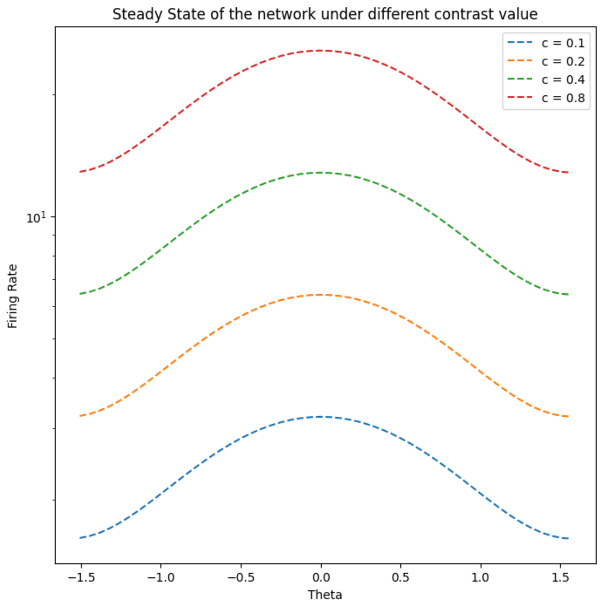

Figure 2: The steady state of the network with different contrast values (starting from $a = 2$ and $\theta_0 = \pi/4$). The y-axis is plotted in a log scale.

2.2 Part 2

The initial condition and the thalamic input are the same as Part 2.1, but the settings for J_0 and J_2 are different, meaning that the recurrent excitation input is larger and the interneuron inhibition is larger. Figure 3 describes the steady state of the network. The network shows the bell-shaped pattern for orientation selectivity, meaning that the tuning curve is focused on $\theta = 0$ but only concentrated in a narrow range of θ, and the outside regions are 0. Also, increasing the contrast value would increase the amplitude of the steady state. Similar to Part 2.1, the steady state is invariant with initial values. Furthermore, under this parameter setting, it would take much longer time for the model to appraoch the steady state if the initial condition is not tuned at θ_{cue}. It would only take $\sim$ 130ms in Part 2.1, but take $\sim$ 2200ms for the model to converge in this part, regardless which initial condition of amplitude a.

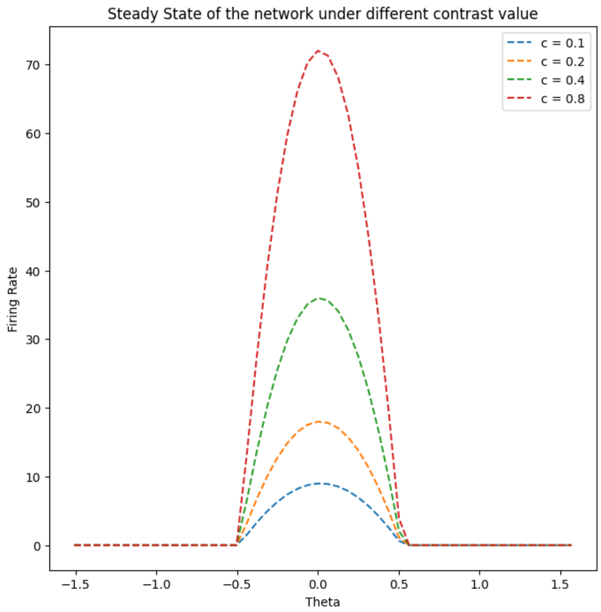

Figure 3: The steady state of the network with different contrast values (starting from $a = 2$ and $\theta_0 = \pi/4$).

2.3 Part 3

Here we would reproduce the results in Part 2.1 and 2.2 by adding a random distribution of the external input $h(\theta_i)$. Specifically:

$$h(\theta_i) = Ac(1 - \epsilon + \epsilon\cos(2(\theta - \theta_{\text{cue}}))) + \sigma_h \eta_i \tag{5}$$

where the noise term is taken from a Gaussian distribution of zero mean and standard deviation σ_h. Because of the added noise, we could not explicitly define the system's steady state. An alternative approach is to evolve the system for a long time ($\sim$ 10000ms) and take the average of the last 1000ms.

To reproduce the Part 2.1 result, we fix $\epsilon_h = 10$ and change the contrast values, as shown in Figure 4. When the contrast value is small (like $c = 0.1$), the orientation selectivity would sharply decrease (becoming flat) and the firing rate slightly increases. When c is large, the effect of changes is less distinct. To examine the changes at different noise level, we fix the contrast value $c = 0.1$ and vary the noise level. Figure 5 shows the results. When σ_h becomes larger, the firing rate of the network becomes larger and the orientation selectivity decreases. That phenomenon is reasonable, since more and more firing would be induced by the fluctuation (no selectivity) rather than the recurrent network input when c is large.

To reproduce the Part 2.2 result, similarly first we fix $\sigma_h = 10$ and observe how the orientation selectivity changes under the different contrast values. The result is shown in Figure 6. When the contrast value c gradually decreases, the tuning width of the tuning curve becomes larger than the no-noise case. Also, Figure 7 shows the result when we fix the contrast value $c = 0.1$ and vary the noise level. When σ_h becomes larger, the tuning width of the curve, as the interval of θ which could evoke firing, becomes larger. When the noise level is large enough ($\sigma_h \approx 10$), the peak of firing rate would become larger. That phenomenon is also reasonable: when we increase the noise level, the neurons receiving are not preferred-orientation stimulus can also fire by the fluctuation, which induces a broader tuning width. From the previous results, we could deduce that the contrast value would decrease the influence of the noise.

The results are qualitatively consistent with Dayan and Abbott's book. The results from Part 2.1, namely Figure 4 and 5, are similar to Figure 7.8. The results from Part 2.2, namely Figure 6 and 7, are similar to Figure 7.9.

2.4 Part 4

Figure 8 describes the network activity with different J_2 when J_0 is fixed and $\epsilon = 0$. When J_2 is small, like 1 or 1.5, the neurons at all positions have the same firing rate; thus, the network shows no orientation selectivity. When J_2 is large, like 2.5 or 4.5, the network displays a bell-shaped activity pattern, where only the neuron within a range can fire. The larger J_2 is, the narrower the tuning width is.

Also, we notice that when the system starts with tuned orientation θ_0, the network will end up with orientation selectivity focused on θ_0 as long as J_2 is large enough.

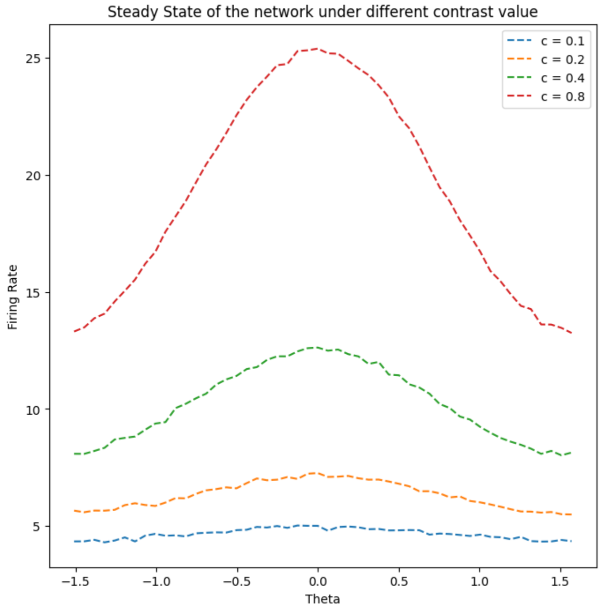

Figure 4: The steady state of the network with different contrast values and with noise $\sigma_h = 10$. Reproduced based on Part 2.1.

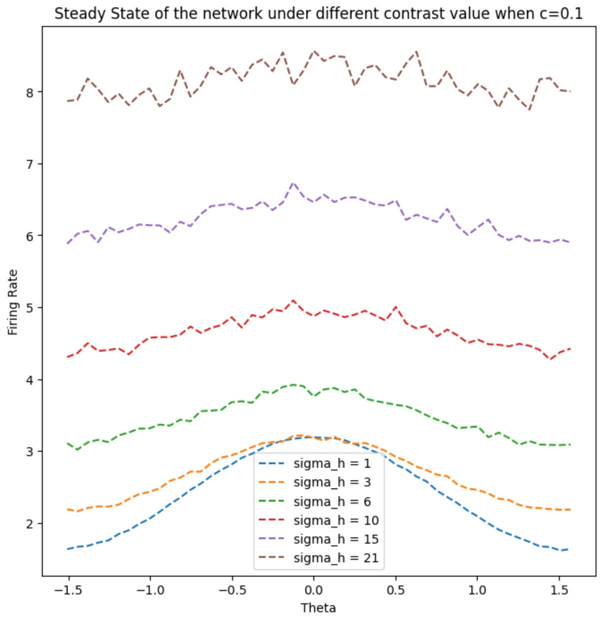

Figure 5: The steady state of the network with different noise levels. The contrast value is set as $c = 0.1$. Reproduced based on Part 2.1.

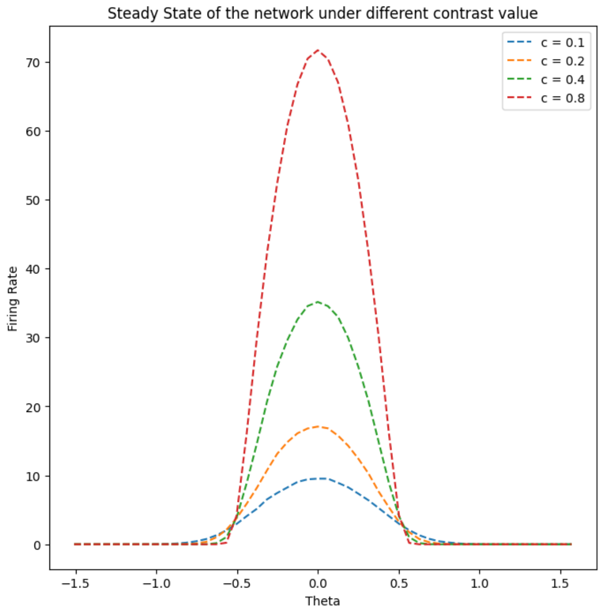

Figure 6: The steady state of the network with different contrast values and with noise $\sigma_h = 10$. Reproduced based on Part 2.2.

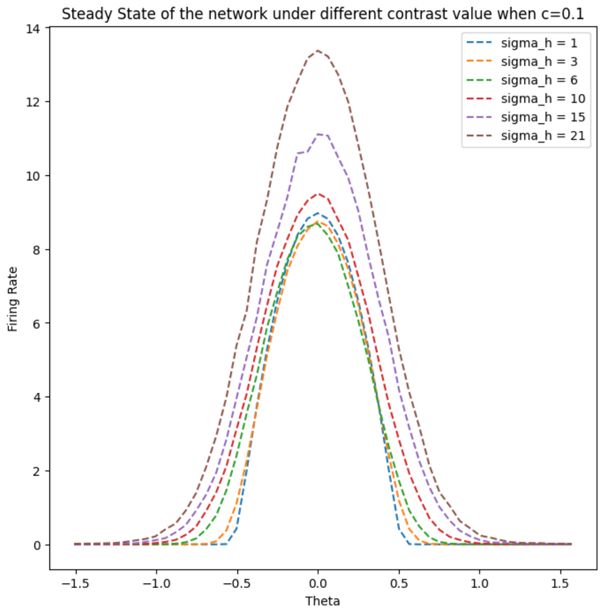

Figure 7: The steady state of the network with different noise levels. The contrast value is set as $c = 0.1$. Reproduced based on Part 2.2.

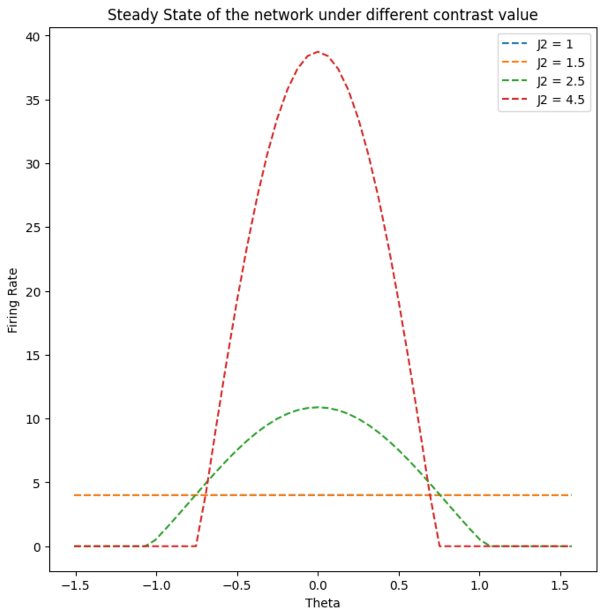

Figure 8: The steady state of the network with different J_2, as the strength of recurrent excitation input. Here $c = 0.2, \epsilon = 0, a = 2$.