Homework #6 (NEURL-GA 3042, Fall 2022)

A firing-rate model of working memory and decision-making
Due date: Sunday November 20

Consider a population rate model of a recurrent neural circuit, described in Wang (2002) and Wong and Wang (2006). The code in Python can be found at https://github.com/xjwanglab/book/tree/master/wong2006.

The model has two excitatory neural assemblies. Their mutual interactions are effectively inhibitory through a shared pool of inhibitory neurons (not explicitly present in the reduced two-variable rate model). Let r_1 and r_2 be their respective population-firing rates, and the total synaptic input current I_i and the resulting firing rate r_i of the neural population i obey the following input-output relationship ($F-I$ curve):

$$r_i = F(I_i) = \frac{aI_i - b}{1 - \exp(-d(aI_i - b))} \tag{1}$$

which captures the current-frequency function of a leaky integrate-and-fire neuron. The parameter values are $a = 270$ Hz/nA, $b = 108$ Hz, $d = 0.154$ sec.

Assume that the 'synaptic drive variables' s_1 and s_2 obey

$$\frac{ds_1}{dt} = F(I_1)\gamma(1 - s_1) - s_1/\tau_s \tag{2}$$

$$\frac{ds_2}{dt} = F(I_2)\gamma(1 - s_2) - s_2/\tau_s \tag{3}$$

where $\gamma = 0.641$. $I_1 = g_E s_1 - g_I s_2 + I_{b1} + g_{ext}\mu_1$, $I_2 = g_E s_2 - g_I s_1 + I_{b2} + g_{ext}\mu_2$. The synaptic time constant $\tau_s = 100$ ms. The synaptic coupling strengths $g_E = 0.2609$ nA, $g_I = 0.0497$ nA and $g_{ext} = 0.00052$ nA. Stimulus-selective inputs to populations 1 and 2 are governed by unitless parameters μ_1 and μ_2, respectively. I_b is the background input which has a mean (I_0) and a noise component described by an Ornstein-Uhlenbeck process:

$$\tau_0 \frac{dI_{b1}}{dt} = -(I_{b1} - I_0) + \eta_1(t)\sqrt{\tau_0 \sigma^2} \tag{4}$$

$$\tau_0 \frac{dI_{b2}}{dt} = -(I_{b2} - I_0) + \eta_2(t)\sqrt{\tau_0 \sigma^2} \tag{5}$$

where $I_0 = 0.3255$ nA, filter time constant $\tau_0 = 2$ ms, and noise amplitude $\sigma = 0.02$ nA; $\eta(t)$ is a Gaussian white-noise with zero mean and unit standard deviation.

Caution: $F(I)$ is given in Hz, but in the s-equations, it should be divided by 1000 so that it has the unit of 1/msec.

(1) Consider first a delayed response task.

(a) First, run the model simulation for 500-1000 msec without stimulus ($\mu_1 = \mu_2 = 0$). With the initial condition $s_1 = s_2 = 0$, show that the system is at a resting state, and determine the firing rate at that state.

(b) After the system has settled in the resting state, show a stimulus ($\mu_1 = 35$, $\mu_2 = 0$) for a brief period of time (say 300 msec), followed by a delay 'memory period' (say for 3 sec). Do you see a 'persistent activity' state, if so with what r_1 and r_2 values in Hz? Repeat the simulation with $\mu_1 = 0$, $\mu_2 = 35$, do you observe another memory state and what are the corresponding r_1 and r_2 in Hz?

Find a way (using a second transient input at the end of the delay period) to switch the system from a memory state back to the resting state.

(c) Display time courses of $s_1(t)$ and $s_2(t)$, as well as firing rates $r_1(t) = F(I_1)$ and $r_2(t) = F(I_2)$. Also, plot s_1 against s_2, or r_1 against r_2 that traces a trajectory (each point corresponds to a moment in time) in the two-dimensional 'state space'. Describe what you observe.

(d) Repeat (b) but now show stimulus 1 as a cue ($\mu_1 = 35$, $\mu_2 = 0$) briefly first, and during the delay show stimulus 2 as a distractor ($\mu_1 = 0$, $\mu_2 = 35$), what do you see?

(e) Decrease the value of g_E incrementally (by steps of 0.01 nA), and show that persistent activity disappears when the recurrent excitation is below a critical value. What is this critical level of recurrent excitation?

(2) 'Coin-tossing' simulations with $\mu_1 = \mu_2 = \mu_0 = 30$ ($c' = 0$). In a decision-making simulation, both μ_1 and μ_2 are presented for a time interval, say from $t_1 = 500$ ms to $t_2 = 1500$ ms ($T = t_2 - t_1 = 1$ sec), and your total simulation time should be much longer (say 3 sec). The decision choice is determined according to which of the two active attractors wins the competition.

In different (n) trials (each with a different seed for the random number generator, but always with the same initial condition $s_1 = s_2 = 0.1$), what do you observe? Do you see 50-50 decision outcome if n is large, say n=100-500?

(3) Stimulus-specific stimuli are given by μ_1 and μ_2. The 'coherence level' is

defined as $c' = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$. For example, if $\mu_1 = 0.84$ and $\mu_2 = 0.8$, then $c' = 0.0244$ or 2.44%. Repeat (2) with several $c' = 0.032, 0.064, 0.128, 0.256, 0.512, 0.85, 1.0$ (for example, with $\mu_1 = \mu_0(1 + c')$ and $\mu_2 = \mu_0(1 - c')$). Plot the 'psychometric function', namely the percentage of correct decisions (choice=1 is correct if $\mu_1 > \mu_2$) as a function of $\log(c')$.

(4) Reaction time task. Set a firing threshold $\theta = 15$ Hz. In any trial, the decision is made whenever one of the two neural populations reaches this threshold first. Run simulations over many trials for each c' as in (3).

(a) Show sample time courses of firing rates for different coherence levels.

(b) Plot the psychometric function, namely the trial-averaged reaction time as a function of $\log(c')$.

(c) Calculate the standard deviation of the reaction time, as a function of c'. Is the RT standard deviation proportional to its mean?

(d) Bonus: explore how the decision performance depends on time integration of sensory information, by computing the psychometric curve with varing durations of stimulus presentation ($T = 100, 300, 500, 800$ ms).

References

[1] Wong K-F and Wang X-J (2006) A recurrent network mechanism for time integration in perceptual decisions. J. Neurosci 26, 1314-1328.

[2] Wang X-J (2002) Probabilistic decision making by slow reverberation in neocortical circuits. Neuron 36, 955-968.

[3] Roitman JD and Shadlen MN (2002) Response of neurons in the lateral intraparietal area during a combined visual discrimination reaction time task. J Neurosci 22, 9475-9489.

[4] Gold, JI and Shadlen MN (2007). The neural basis of decision making. Annu. Rev. Neurosci. 30, 535-574.

[5] Wang X-J (2008) Decision making in recurrent neural circuits. Neuron 60, 215-234.