Homework #2 (NEURL-GA 3042, Fall 2022)

Due date: Sunday October 2

The leaky integrate-and-fire (LIF) model is based on an RC circuit equation

$$C\frac{dV_m}{dt} = -G_L(V_m - V_L) + I \tag{1}$$

if $V(t = t_{spike}) = V_{th} = -50$ mV, a spike is discharged and V is reset to $V_{reset} = -60$ mV for a refractory period of $\tau_{ref} = 2$ ms. C is the capacitance (0.5 nF), G_L is the leak conductance (0.025 μS), and V_L is the leak reversal potential (-70 mV). I is the input current (in nA). When $I = 0$, the membrane potential is in the resting state of $V_{ss} = V_L$.

(1) Implement this differential equation in a computer code in Python, and simulate numerically this model with a range of I values, from 0 to 1 nA. Plot $V_m(t)$ as function of time for a few selective values of f.

For a fixed $I = 0.55$, use several different values of the timestep for numerical integration ($dt = 5, 1, 0.1, 0.01$ ms) and discuss how the result depends on dt.

(2) What is the critical value (I_c) of I above which the model fires spikes repetitively? Calculate the firing frequency f (in number of spikes per second, or Hertz), and plot f versus I. Compare on the same graph this simulated $f - I$ curve with the analytical $f(I)$ function (see Eq. (4) of Chapter 2).

(3) Add a noise term into the input current,

$$C\frac{dV_m}{dt} = -G_L(V_m - V_L) + I + \sigma w(t) \tag{2}$$

where I is the mean current, σ is the noise level, and $w(t)$ is a white-noise ($< w >= 0, < w(t)w(t') >= \delta(t - t')$, and the probability density for w is a Gaussian, $p(w) = 1/\sqrt{2\pi}\exp(-w^2/2)$). Note that σ has the unit of nA$\sqrt{ms}$. For a given noise level (say $\sigma = 0.3$), simulate the model for a range of I values (0 to 1).

(a) Show $V_m(t)$ for a few selective I values, including an example for I just below the deterministic current threshold I_c, and another one for I above I_c.

(b) Calculate the firing rate f as the inverse of the mean interspike intervals, and plot f versus I. Superimpose it with the $f - I$ curve without noise ($\sigma = 0$) from Part (2).

(c) Calculate the coefficient of variation (CV) of the interspike intervals for different I values, and plot CV versus the mean firing rate f.

Note: Please provide a detailed write-up of your work and discuss your observations/results. Do not just give answers and simulation plots.

With noise, integrate the model using the first-order Euler method. For a differential equation $dx/dt = f(x) + \sigma w(t)$, the iteration is given by $x_{(n+1)} = x_n + f(x_n) * dt + \sigma * \sqrt{dt} * w_n$, where w_n is from a Gaussian distribution, independently chosen at each timestep.

You may need to run the simulation for a long total time T in order to average out the noise and obtain reliable estimates of f and CV. Try several time durations (say $T = 2, 5, 10, 20$ seconds) and compare the results.