CHAPTER

Continuous Change

7

Discrete change is change that is limited to a countable, usually finite, set of timepoints. We represent discrete change in the event calculus using effect axioms. In a number of commonsense domains ranging from the physical to the mental, we find continuous change. A function $f(t)$ is *continuous at a point* t_0 if and only if for every $\epsilon > 0$, there is a $\delta > 0$ such that $|t - t_0| < \delta$ implies $|f(t) - f(t_0)| < \epsilon$. A function is *continuous on an interval* if and only if it is continuous at all points of the interval. Examples of continuous change include the change in the height of a falling object, the location of a projectile, the water level of a filling bathtub, the volume of a balloon in the process of inflation, the frequency of a siren, the hunger level of an animal, and the anger level of a person. In the discrete event calculus, time is limited to the integers; there we speak of gradual change. In this chapter, we discuss the representation of continuous change and gradual change in the event calculus. We retrace the development of *Trajectory*. We discuss the use of trajectory and antitrajectory axioms.

7.1 TRAJECTORY AXIOMS

How should we represent continuous change? Let us consider the changing height of a falling object without air resistance.

7.1.1 EXAMPLE: FALLING OBJECT

An object $O1$ is at rest at height H prior to timepoint T_1, starts to fall at timepoint T_1, and hits the ground at timepoint $T_2 = T_1 + \sqrt{2H/G}$, where G is the acceleration due to gravity (9.8 m/s^2):

$$t \le T_1 \Rightarrow \mathit{HoldsAt}(\mathit{Height}(O1, H), t) \tag{7.1}$$

$$t > T_1 \wedge t \le T_2 \Rightarrow \mathit{HoldsAt}(\mathit{Height}(O1, H - \frac{1}{2}G(t - T_1)^2), t) \tag{7.2}$$

$$t > T_2 \Rightarrow \mathit{HoldsAt}(\mathit{Height}(O1, 0), t) \tag{7.3}$$

We have a state constraint that says that an object has a unique height at any given time:

$$\begin{aligned}&\mathit{HoldsAt}(\mathit{Height}(o, h_1), t) \wedge \mathit{HoldsAt}(\mathit{Height}(o, h_2), t) \Rightarrow \\ &h_1 = h_2\end{aligned} \tag{7.4}$$

The height of $O1$ is released from the commonsense law of inertia:

$$ReleasedAt(Height(O1,h),t) \tag{7.5}$$

From this axiomatization, we can determine the height of $O1$ at any timepoint. For instance, if $(T_1+2) \le T_2$, then the height of $O1$ at timepoint T_1+2 is $H-2G$.

Proposition 7.1. *If* $(T_1+2) \le T_2$, $\Psi = (7.4)$, *and* $\Gamma = (7.1) \wedge (7.2) \wedge (7.3) \wedge (7.5)$, *then* $\Psi \wedge \Gamma \vdash HoldsAt(Height(O1,H-2G),T_1+2)$.

Proof. From $(T_1+2) > T_1$, $(T_1+2) \le T_2$, and (7.2), we have $HoldsAt(Height(O1, H-\frac{1}{2}G((T_1+2)-T_1)^2),T_1+2)$ or $HoldsAt(Height(O1,H-2G),T_1+2)$. ■

Although this axiomatization describes a falling object, it fails to capture our commonsense knowledge about falling objects. Namely, when an object is dropped, it starts to fall; when an object hits the ground, it stops falling. Therefore, we integrate events into our representation of continuous change. One event will set in motion an episode of continuous change, whereas another event will stop the episode. We use fluents to represent episodes of continuous change.

7.1.2 EXAMPLE: FALLING OBJECT WITH EVENTS

We model a falling object using events. We have an episode of no change (the object at rest above the ground), followed by an episode of continuous change (falling), followed by another episode of no change (the object hits the ground).

We use the following axiomatization. If an agent drops an object, then the object will be falling, and the height of the object will be released from the commonsense law of inertia:

$$Initiates(Drop(a,o),Falling(o),t) \tag{7.6}$$

$$Releases(Drop(a,o),Height(o,h),t) \tag{7.7}$$

In EC and DEC, events are instantaneous. Notice that we are using the fluent *Falling* here as a way of representing an event with duration or a process. (We present a modification of EC for events with duration in Appendix C.)

We describe the motion of the object from the moment it is dropped until it stops falling as follows:

$$\begin{gathered} HoldsAt(Height(o,h),t_1) \wedge \\ Happens(Drop(a,o),t_1) \wedge 0 < t_2 \wedge \\ \neg StoppedIn(t_1,Falling(o),t_1+t_2) \Rightarrow \\ HoldsAt(Height(o,h-\frac{1}{2}Gt_2^2),t_1+t_2) \end{gathered} \tag{7.8}$$

If an object hits the ground, then the object will no longer be falling:

$$Terminates(HitGround(o),Falling(o),t) \tag{7.9}$$

A falling object hits the ground when its height becomes zero:

$$\begin{gathered} HoldsAt(Falling(o),t) \wedge \\ HoldsAt(Height(o,0),t) \Rightarrow \\ Happens(HitGround(o),t) \end{gathered} \tag{7.10}$$

If an object hits the ground and the height of the object is h, then the height of the object will no longer be released from the commonsense law of inertia and the height of the object will be h:

$$\begin{aligned}&\mathit{HoldsAt}(\mathit{Height}(o,h),t) \Rightarrow\\&\mathit{Initiates}(\mathit{HitGround}(o),\mathit{Height}(o,h),t)\end{aligned} \tag{7.11}$$

An object has a unique height:

$$\begin{aligned}&\mathit{HoldsAt}(\mathit{Height}(o,h_1),t) \wedge \mathit{HoldsAt}(\mathit{Height}(o,h_2),t) \Rightarrow\\&h_1 = h_2\end{aligned} \tag{7.12}$$

At timepoint 0, Nathan drops an apple whose height is $G/2$:

$$\neg \mathit{HoldsAt}(\mathit{Falling}(\mathit{Apple}),0) \tag{7.13}$$

$$\mathit{HoldsAt}(\mathit{Height}(\mathit{Apple},G/2),0) \tag{7.14}$$

$$\mathit{Happens}(\mathit{Drop}(\mathit{Nathan},\mathit{Apple}),0) \tag{7.15}$$

We can show that the apple will hit the ground at timepoint 1 and that its height at timepoint 2 will be zero.

Proposition 7.2. *Let* $\Sigma = (7.6) \wedge (7.7) \wedge (7.9) \wedge (7.11)$, $\Delta = (7.10) \wedge (7.15)$, $\Omega = U[\mathit{Drop}, \mathit{HitGround}] \wedge U[\mathit{Falling}, \mathit{Height}]$, $\Psi = (7.12)$, $\Pi = (7.8)$, *and* $\Gamma = (7.13) \wedge (7.14)$. *Then we have*

$$\begin{aligned}&\mathit{CIRC}[\Sigma;\mathit{Initiates},\mathit{Terminates},\mathit{Releases}] \wedge \mathit{CIRC}[\Delta;\mathit{Happens}] \wedge\\&\Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \mathrm{EC}\\&\vdash \mathit{HoldsAt}(\mathit{Height}(\mathit{Apple},0),1) \wedge\\&\mathit{Happens}(\mathit{HitGround}(\mathit{Apple}),1) \wedge\\&\mathit{HoldsAt}(\mathit{Height}(\mathit{Apple},0),2)\end{aligned}$$

Proof. From $\mathit{CIRC}[\Sigma;\mathit{Initiates},\mathit{Terminates},\mathit{Releases}]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned}&\mathit{Initiates}(e,f,t) \Leftrightarrow\\&\exists a,o\,(e = \mathit{Drop}(a,o) \wedge f = \mathit{Falling}(o)) \vee\\&\exists o,h\,(e = \mathit{HitGround}(o) \wedge\\&f = \mathit{Height}(o,h) \wedge\\&\mathit{HoldsAt}(\mathit{Height}(o,h),t))\end{aligned} \tag{7.16}$$

$$\begin{aligned}&\mathit{Terminates}(e,f,t) \Leftrightarrow\\&\exists o\,(e = \mathit{HitGround}(o) \wedge f = \mathit{Falling}(o))\end{aligned} \tag{7.17}$$

$$\begin{aligned}&\mathit{Releases}(e,f,t) \Leftrightarrow\\&\exists a,o,h\,(e = \mathit{Drop}(a,o) \wedge f = \mathit{Height}(o,h))\end{aligned} \tag{7.18}$$

From $\mathit{CIRC}[\Delta;\mathit{Happens}]$ and Theorem 2.1, we have

$$\begin{aligned}&Happens(e,t) \Leftrightarrow \\ &\exists o\,(e = HitGround(o) \wedge \\ &HoldsAt(Falling(o),t) \wedge \\ &HoldsAt(Height(o,0),t)) \vee \\ &(e = Drop(Nathan,Apple) \wedge t = 0)\end{aligned} \tag{7.19}$$

We can show

$$\neg\exists t\,(0 < t < 1 \wedge Happens(HitGround(Apple),t)) \tag{7.20}$$

To see this, suppose, to the contrary, that

$$\exists t\,(0 < t < 1 \wedge Happens(HitGround(Apple),t))$$

Let $Happens(HitGround(Apple),\tau)$ be the first such event. That is, we have

$$0 < \tau < 1 \tag{7.21}$$

$$Happens(HitGround(Apple),\tau) \tag{7.22}$$

$$\neg\exists\tau'\,(0 < \tau' < \tau \wedge Happens(HitGround(Apple),\tau')) \tag{7.23}$$

From (7.22) and (7.19), we have

$$HoldsAt(Height(Apple,0),\tau) \tag{7.24}$$

From (7.17), (7.23), and EC3, we have $\neg StoppedIn(0, Falling(Apple), \tau)$. From this, (7.14), (7.15) (which follows from (7.19)), $0 < \tau$ (which follows from (7.21)), and (7.8), we have

$$HoldsAt(Height(Apple,\frac{1}{2}G(1-\tau^2)),\tau)$$

From this, (7.21), and (7.12), we have $\neg HoldsAt(Height(Apple, 0), \tau)$, which contradicts (7.24).

From (7.17), (7.20), and EC3, we have

$$\neg StoppedIn(0,Falling(Apple),1) \tag{7.25}$$

From this, (7.14), (7.15) (which follows from (7.19)), $0 < 1$, and (7.8), we have

$$HoldsAt(Height(Apple,0),1) \tag{7.26}$$

From (7.18), we have

$$\neg ReleasedIn(0,Falling(Apple),1)$$

From this, (7.15) (which follows from (7.19)), (7.6) (which follows from (7.16)), $0 < 1$, (7.25), and EC14, we have $HoldsAt(Falling(Apple), 1)$. From this, (7.26), and (7.10) (which follows from (7.19)), we have

$$Happens(HitGround(Apple),1) \tag{7.27}$$

From (7.17) and EC3, we have

$$\neg StoppedIn(1,Height(Apple,0),2) \tag{7.28}$$

From (7.18), (7.19), and EC13, we have $\neg ReleasedIn(1, Height(Apple, 0), 2)$. From this, (7.27), (7.26), (7.11) (which follows from (7.16)), $1 < 2$, (7.28), and EC14, we have $HoldsAt(Height(Apple, 0), 2)$. ■

7.1.3 INTRODUCTION OF *TRAJECTORY* PREDICATE

We introduce the predicate $Trajectory(\beta_1, \tau_1, \beta_2, \tau_2)$, which allows us to express (7.8) more compactly.

Definition 7.1. If γ is a condition, β_1 and β_2 are fluent terms, and τ_1 and τ_2 are timepoint terms, then

$$\gamma \Rightarrow Trajectory(\beta_1, \tau_1, \beta_2, \tau_2)$$

is a ***trajectory axiom***. This represents that, if γ is true, fluent β_1 is initiated by an event that occurs at timepoint τ_1, and $\tau_2 > 0$, then fluent β_2 will be true at timepoint $\tau_1 + \tau_2$.

Thus, instead of (7.8) we may simply write:

$$\begin{aligned} &HoldsAt(Height(o, h), t_1) \Rightarrow \\ &Trajectory(Falling(o), t_1, Height(o, h - \frac{1}{2}Gt_2^2), t_2) \end{aligned} \tag{7.29}$$

Using the *Trajectory* predicate, we show that the apple hits the ground at timepoint 1 and that it has zero height at timepoint 2.

Proposition 7.3. *Let* $\Sigma = (7.6) \wedge (7.7) \wedge (7.9) \wedge (7.11)$, $\Delta = (7.10) \wedge (7.15)$, $\Omega = U[Drop, HitGround] \wedge U[Falling, Height]$, $\Psi = (7.12)$, $\Pi = (7.29)$, *and* $\Gamma = (7.13) \wedge (7.14)$. *Then we have*

$$\begin{aligned} &CIRC[\Sigma; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \\ &\Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \mathrm{EC} \\ &\vdash HoldsAt(Height(Apple, 0), 1) \wedge \\ &Happens(HitGround(Apple), 1) \wedge \\ &HoldsAt(Height(Apple, 0), 2) \end{aligned}$$

Proof. The proof is identical to that of Proposition 7.2, except that (7.8) is replaced by the conjunction of (7.29), (7.6) (which follows from (7.16)), and EC5. ■

7.2 ANTITRAJECTORY AXIOMS

The *AntiTrajectory* predicate is analogous to the *Trajectory* predicate, except that it is brought into play when a fluent is terminated rather than initiated by an occurring event.

Definition 7.2. If γ is a condition, β_1 and β_2 are fluent terms, and τ_1 and τ_2 are timepoint terms, then

$$\gamma \Rightarrow AntiTrajectory(\beta_1, \tau_1, \beta_2, \tau_2)$$

is an ***antitrajectory axiom***. This represents that if γ is true, fluent β_1 is terminated by an event that occurs at timepoint τ_1, and $\tau_2 > 0$, then fluent β_2 will be true at timepoint $\tau_1 + \tau_2$.

7.2.1 EXAMPLE: HOT AIR BALLOON

This example and the trajectory and antitrajectory axioms are due to Rob Miller and Murray Shanahan. We have two effect axioms. If an agent turns on the heater of a hot air balloon, then the heater will be on:

$$Initiates(TurnOnHeater(a,b), HeaterOn(b), t) \tag{7.30}$$

If an agent turns off the heater of a hot air balloon, then the heater will no longer be on:

$$Terminates(TurnOffHeater(a,b), HeaterOn(b), t) \tag{7.31}$$

We have a state constraint:

$$\begin{aligned} &HoldsAt(Height(b,h_1),t) \wedge \\ &HoldsAt(Height(b,h_2),t) \Rightarrow \\ &h_1 = h_2 \end{aligned} \tag{7.32}$$

We have the following trajectory and antitrajectory axioms:

$$\begin{aligned} &HoldsAt(Height(b,h),t_1) \Rightarrow \\ &Trajectory(HeaterOn(b),t_1,Height(b,h+V\cdot t_2),t_2) \end{aligned} \tag{7.33}$$

$$\begin{aligned} &HoldsAt(Height(b,h),t_1) \Rightarrow \\ &AntiTrajectory(HeaterOn(b),t_1,Height(b,h-V\cdot t_2),t_2) \end{aligned} \tag{7.34}$$

where V is the velocity of the balloon.

We have the following observations and narrative. The balloon starts out at height 0:

$$HoldsAt(Height(Balloon,0),0) \tag{7.35}$$

The height of a balloon is always released from the commonsense law of inertia:

$$ReleasedAt(Height(b,h),t) \tag{7.36}$$

At timepoint 0 the heater is turned on, and at timepoint 2 the heater is turned off:

$$Happens(TurnOnHeater(Nathan,Balloon),0) \tag{7.37}$$
$$Happens(TurnOffHeater(Nathan,Balloon),2) \tag{7.38}$$

We can show that the height of the balloon is $2V$ at timepoint 2 and is V at timepoint 3.

Proposition 7.4. *Let* $\Sigma = (7.30) \wedge (7.31)$, $\Delta = (7.37) \wedge (7.38)$, $\Omega = U[TurnOnHeater, TurnOffHeater] \wedge U[HeaterOn, Height]$, $\Psi = (7.32)$, $\Pi = (7.33) \wedge (7.34)$, *and* $\Gamma = (7.35) \wedge (7.36)$. *Then we have*

$$\begin{aligned} &CIRC[\Sigma; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \\ &\qquad \Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \mathrm{EC} \end{aligned}$$

$$\vdash HoldsAt(Height(Balloon, 2V), 2) \wedge HoldsAt(Height(Balloon, V), 3)$$

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$Initiates(e, f, t) \Leftrightarrow \exists a, b\, (e = TurnOnHeater(a, b) \wedge HeaterOn(b)) \tag{7.39}$$

$$Terminates(e, f, t) \Leftrightarrow \exists a, b\, (e = TurnOffHeater(a, b) \wedge HeaterOn(b)) \tag{7.40}$$

$$\neg Releases(e, f, t) \tag{7.41}$$

From $CIRC[\Delta; Happens]$ and Theorem 2.1, we have

$$Happens(e, t) \Leftrightarrow (e = TurnOnHeater(Nathan, Balloon) \wedge t = 0) \wedge (e = TurnOffHeater(Nathan, Balloon) \wedge t = 2) \tag{7.42}$$

From (7.42) and EC3, we have $\neg StoppedIn(0, HeaterOn(Balloon), 2)$. From this, (7.37) (which follows from (7.42)), (7.30) (which follows from (7.39)), $0 < 2$, (7.35), (7.33), and EC5, we have

$$HoldsAt(Height(Balloon, 2V), 2) \tag{7.43}$$

From (7.42) and EC4, we have $\neg StartedIn(2, HeaterOff(Balloon), 3)$. From this, (7.38) (which follows from (7.42)), (7.31) (which follows from (7.40)), $0 < 1$, (7.43), (7.34), and EC6, we have $HoldsAt(Height(Balloon, V), 3)$. ■

7.3 USING *ANTITRAJECTORY* INSTEAD OF *RELEASES*

We may sometimes use *AntiTrajectory* to replace *Releases*.

7.3.1 EXAMPLE: FALLING OBJECT WITH *ANTITRAJECTORY*

Consider again the example of the falling object. First, we state that the height of an object is always released from the commonsense law of inertia:

$$ReleasedAt(Height(o, h), t) \tag{7.44}$$

Then, we replace (7.7) and (7.11) with

$$HoldsAt(Height(o, h), t_1) \Rightarrow AntiTrajectory(Falling(o), t_1, Height(o, h), t_2) \tag{7.45}$$

That is, when an object stops falling, its height stays constant.

We can show that the apple hits the ground at timepoint 1, and has a height of zero at timepoint 2.

Proposition 7.5. *Let* $\Sigma = (7.6) \land (7.9)$, $\Delta = (7.10) \land (7.15)$, $\Omega = U[Drop, HitGround] \land U[Falling, Height]$, $\Psi = (7.12)$, $\Pi = (7.29) \land (7.45)$, *and* $\Gamma = (7.13) \land (7.14) \land (7.44)$. *Then we have*

$$\begin{aligned}&CIRC[\Sigma; Initiates, Terminates, Releases] \land CIRC[\Delta; Happens] \land\\ &\Omega \land \Psi \land \Pi \land \Gamma \land \mathrm{EC}\\ &\vdash HoldsAt(Height(Apple, 0), 1) \land\\ &Happens(HitGround(Apple), 1) \land\\ &HoldsAt(Height(Apple, 0), 2)\end{aligned}$$

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned}&Initiates(e, f, t) \Leftrightarrow\\ &\exists a, o\, (e = Drop(a, o) \land f = Falling(o))\end{aligned} \tag{7.46}$$

$$\begin{aligned}&Terminates(e, f, t) \Leftrightarrow\\ &\exists o\, (e = HitGround(o) \land f = Falling(o))\end{aligned} \tag{7.47}$$

$$\neg Releases(e, f, t) \tag{7.48}$$

From $CIRC[\Delta; Happens]$ and Theorem 2.1, we have

$$\begin{aligned}&Happens(e, t) \Leftrightarrow\\ &\exists o\, (e = HitGround(o) \land\\ &HoldsAt(Falling(o), t) \land\\ &HoldsAt(Height(o, 0), t)) \lor\\ &(e = Drop(Nathan, Apple) \land t = 0)\end{aligned} \tag{7.49}$$

We can show

$$\neg \exists t\, (0 < t < 1 \land Happens(HitGround(Apple), t)) \tag{7.50}$$

Suppose for contradiction that

$$\exists t\, (0 < t < 1 \land Happens(HitGround(Apple), t))$$

Let $Happens(HitGround(Apple), \tau)$ be the first such event. Thus we have

$$0 < \tau < 1 \tag{7.51}$$

$$Happens(HitGround(Apple), \tau). \tag{7.52}$$

$$\neg \exists \tau'\, (0 < \tau' < \tau \land Happens(HitGround(Apple), \tau')) \tag{7.53}$$

From (7.52) and (7.49), we have

$$HoldsAt(Height(Apple, 0), \tau) \tag{7.54}$$

From (7.47), (7.53), and EC3, we have $\neg StoppedIn(0, Falling(Apple), \tau)$. From this, (7.15) (which follows from (7.49)), (7.6) (which follows from (7.46)), $0 < \tau$ (which follows from 7.51), (7.14), (7.29), and EC5, we have

$$HoldsAt(Height(Apple, \frac{1}{2}G(1 - \tau^2)), \tau)$$

From this, (7.51), and (7.12), we have $\neg HoldsAt(Height(Apple, 0), \tau)$, which contradicts (7.54).

From (7.47), (7.50), and EC3, we have

$$\neg StoppedIn(0, Falling(Apple), 1) \tag{7.55}$$

From this, (7.15) (which follows from (7.49)), (7.6) (which follows from (7.46)), $0 < 1$, (7.14), (7.29), and EC5, we have

$$HoldsAt(Height(Apple, 0), 1) \tag{7.56}$$

From (7.48), we have

$$\neg ReleasedIn(0, Falling(Apple), 1)$$

From this, (7.55), (7.15) (which follows from (7.49)), (7.6) (which follows from (7.46)), $0 < 1$, and EC14, we have $HoldsAt(Falling(Apple), 1)$. From this, (7.56), and (7.10) (which follows from (7.49)), we have

$$Happens(HitGround(Apple), 1) \tag{7.57}$$

From (7.46), (7.49), and EC4, we have $\neg StartedIn(1, Falling(Apple), 2)$. From this, (7.57), (7.9) (which follows from (7.47)), $0 < 1$, (7.56), (7.45), and EC6, we have $HoldsAt(Height(Apple, 0), 2)$. ■

BIBLIOGRAPHIC NOTES

Hendrix (1973) extends the language of STRIPS (Fikes & Nilsson, 1971) to deal with “continuous, gradual change” (Hendrix, 1973, p. 146). A process such as filling a bucket is subject to “continuance conditions” (p. 161), or conditions necessary for the process to continue. Filling a bucket has the continuance conditions that the tap must be turned on, the bucket must be facing up, and the amount of water in the bucket must be less than its capacity. When any such condition is violated, the filling process is interrupted. Proposals for integrating continuous change into the situation calculus have been made by Gelfond, Lifschitz, and Rabinov (1991), Pinto (1994), and R. Miller and Shanahan (1994). See the discussion in the Bibliographic notes of Chapter 16. Proposals have been made for other formalisms for reasoning about action and change (Herrmann & Thielscher, 1996; McDermott, 1982; Sandewall, 1989a; Van Belleghem, Denecker, & De Schreye, 1994). The area of qualitative reasoning, discussed in Section 17.1, deals with continuous change. Shanahan (1995a) introduced the classical logic event calculus, which included the *Trajectory* predicate (p. 268). This predicate was previously introduced by Shanahan (1990) into a simplified version of the original event calculus (Kowalski & Sergot, 1986).

The example of a falling object has been used by McCarthy (1963), McCarthy and Hayes (1969, p. 479), and Shanahan (1990, p. 598). The method for showing $\neg StoppedIn(0, Falling(Apple), 1)$ in the proofs of Propositions 7.2, 7.3, 7.5, 10.2 is taken from Shanahan (1997b, pp. 328-329; 2004, p. 161). Reiter (2001, pp. 149-150) discusses the use of fluents to represent processes or actions with duration. The *AntiTrajectory* predicate was introduced by R. Miller and Shanahan (1999, pp. 13-15; 2002, pp. 466-470). The hot air balloon example is from R. Miller and Shanahan (1999, p. 14; 2002, p. 468). Shanahan (1990) shows how multiple flows into a vessel can be handled. Van Belleghem, Denecker, and De Schreye (1994) show how to handle multiple simultaneous (and possibly continuously changing) influences on a quantity. R. Miller and Shanahan (1996) treat continuously changing parameters in the event calculus using differential equations. We have discussed trajectories in the context of continuous change. But as R. Miller and Shanahan (2002, p. 468) note, trajectories are not required to be continuous. They give the example of using a trajectory axiom to model a blinking light.

EXERCISES

7.1 Formalize an expanding balloon.

7.2 Formalize the following. A person's level of hunger gradually increases. When a person's hunger level rises above a certain threshold, the person starts to eat. When a person finishes eating, the person's hunger level is lowered and starts to increase again.

7.3 Extend the formalization of the falling object in Section 7.1.2 so that (1) an object P is initially resting on a high object Q; (2) when P is pushed, P is no longer on Q and P starts falling; and (3) after P hits the ground, P rests on the ground.

7.4 Extend the formalization of the hot air balloon in Section 7.2.1 to deal with what happens when the balloon hits the ground.

7.5 Formalize a puck bouncing off a wall in the game of air hockey. Assume there is no friction. Assume there is a horizontal wall at $y = 0$ and a puck. If the y coordinate of a moving puck is 0, then the puck will bounce off the wall.

7.6 Extend the formalization of air hockey so that an agent may hit the puck with a mallet.

7.7 Formalize a rocket with constant thrust. When the thrust is on, it moves upward with an acceleration of $T - G$, where T is the acceleration due to the thrust. When the thrust is off, it falls with an acceleration of G.

7.8 Formalize snow falling from the sky. When it is snowing, snowflakes fall. Model the falling of individual snowflakes.