CHAPTER

10 Space

Many instances of commonsense reasoning involve space. In this chapter, we present two event calculus axiomatizations of space: relational space and metric space. We describe how these axiomatizations can be used to solve some sample problems. A closely related issue that we consider is object identity: Two objects observed at different times and locations in space may or may not be the same object.

10.1 RELATIONAL SPACE

In the commonsense world, objects stand in various spatial relations to other objects. For example, a pencil is in a jar, a person is in a room, a person is holding a book, a glass is on a table, or a person is wearing a shirt. A *relational space* consists of a set of objects and a binary relation on the set. In this section we present a representation of a relational space in the event calculus. (Other representations of relational space are discussed in Sections 6.1.1, 6.3.1, and 6.4.1.)

10.1.1 BASIC REPRESENTATION

We start with a basic representation of relational space, which serves as a prototype for more elaborate representations. The representation consists of:

- an object sort, with variables o, o_1, o_2, ...
- an agent sort, which is a subsort of the object sort, with variables a, a_1, a_2, ...
- a fluent $IN(o_1, o_2)$, which represents an abstract spatial relation between object o_1 and object o_2
- an event $MOVE(a, o_1, o_2, o_3)$, which represents the action of agent a adding the abstract spatial relation between object o_1 and object o_3, and dropping the abstract spatial relation between object o_1 and object o_2

IN represents an irreflexive, antisymmetric, intransitive, and functional relation.

Axiom RS1.

$$\neg HoldsAt(IN(o, o), t)$$

Axiom RS2.

$$HoldsAt(IN(o_1, o_2), t) \Rightarrow \neg HoldsAt(IN(o_2, o_1), t)$$

Axiom RS3.

$$HoldsAt(IN(o_1, o_2), t) \wedge HoldsAt(IN(o_2, o_3), t) \Rightarrow \neg HoldsAt(IN(o_1, o_3), t)$$

Axiom RS4.

$$HoldsAt(IN(o, o_1), t) \wedge HoldsAt(IN(o, o_2), t) \Rightarrow o_1 = o_2$$

MOVE initiates and terminates *IN*.

Axiom RS5.

$$Initiates(MOVE(a, o_1, o_2, o_3), IN(o_1, o_3), t)$$

Axiom RS6.

$$Terminates(MOVE(a, o_1, o_2, o_3), IN(o_1, o_2), t)$$

10.1.2 EXTENDED REPRESENTATION

We now extend the basic representation to deal with agents and physical objects within rooms. We add the following subsorts of the object sort:

- a physical object sort, with variables p, p_1, p_2, ...
- a room sort, with variables r, r_1, r_2, ...

As shown in Table 10.1, we use *IN* to represent different spatial relationships between objects, depending on the sorts of the arguments of *IN*, and *MOVE* to represent different spatial actions, depending on the sorts of the arguments to *MOVE*. For example, $IN(p_1, p_2)$ represents that physical object p_1 is in or inside physical object p_2, whereas $IN(p, a)$ represents that agent a is holding physical object p.

Table 10.1 Meaning of *IN* and *MOVE*[a]

Fluent or Event	Meaning
$IN(p_1, p_2)$	p_1 is in p_2
$IN(a, r)$	a is in r
$IN(p, r)$	p is in r
$IN(p, a)$	a is holding p
$MOVE(a, p_1, r, p_2)$	a (in r) puts p_1 (in r) into p_2
$MOVE(a, p_1, p_2, r)$	a (in r) removes p_1 from p_2 into r
$MOVE(a, a, r_1, r_2)$	a goes from r_1 to r_2
$MOVE(a, p, r, a)$	a (in r) picks up p
$MOVE(a, p, a, r)$	a (in r) sets down p into r

[a] *a is an agent; p, p_1, and p_2 are physical objects; r, r_1, and r_2 are rooms.*

The fluent $IN(o, r)$ represents that object o is directly in room r. We introduce a fluent $INROOM(o, r)$, which represents that object o is indirectly in room r.

Axiom RS7.

$$HoldsAt(IN(o,r),t) \Rightarrow HoldsAt(INROOM(o,r),t)$$

Axiom RS8.

$$HoldsAt(IN(o_1,o_2),t) \wedge HoldsAt(INROOM(o_2,r),t) \Rightarrow \\ HoldsAt(INROOM(o_1,r),t)$$

An object can be indirectly in at most one room at a time.

Axiom RS9.

$$HoldsAt(INROOM(o,r_1),t) \wedge HoldsAt(INROOM(o,r_2),t) \Rightarrow r_1 = r_2$$

We then replace the general RS5 and RS6 with the more specific RS10 through RS19, which follow.

In order for an agent to put physical object p_1 into physical object p_2, the agent and p_1 must be directly in the same room and p_2 must be indirectly in that room.

Axiom RS10.

$$HoldsAt(IN(a,r),t) \wedge \\ HoldsAt(IN(p_1,r),t) \wedge \\ HoldsAt(INROOM(p_2,r),t) \Rightarrow \\ Initiates(MOVE(a,p_1,r,p_2),IN(p_1,p_2),t)$$

Axiom RS11.

$$HoldsAt(IN(a,r),t) \wedge \\ HoldsAt(IN(p_1,r),t) \wedge \\ HoldsAt(INROOM(p_2,r),t) \Rightarrow \\ Terminates(MOVE(a,p_1,r,p_2),IN(p_1,r),t)$$

In order for an agent to remove physical object p_1 from physical object p_2 and put p_1 in a room, the agent must be directly in that room.

Axiom RS12.

$$HoldsAt(IN(a,r),t) \Rightarrow \\ Initiates(MOVE(a,p_1,p_2,r),IN(p_1,r),t)$$

Axiom RS13.

$$HoldsAt(IN(a,r),t) \Rightarrow \\ Terminates(MOVE(a,p_1,p_2,r),IN(p_1,p_2),t)$$

In order for an agent to go from room r_1 to room r_2, the agent must be directly in r_1.

Axiom RS14.

$$HoldsAt(IN(a,r_1),t) \Rightarrow \\ Initiates(MOVE(a,a,r_1,r_2),IN(a,r_2),t)$$

Axiom RS15.

$$HoldsAt(IN(a, r_1), t) \Rightarrow$$
$$Terminates(MOVE(a, a, r_1, r_2), IN(a, r_1), t)$$

In order for an agent to pick up a physical object, the agent and the physical object must be directly in the same room.

Axiom RS16.

$$HoldsAt(IN(a, r), t) \wedge$$
$$HoldsAt(IN(p, r), t) \Rightarrow$$
$$Initiates(MOVE(a, p, r, a), IN(p, a), t)$$

Axiom RS17.

$$HoldsAt(IN(a, r), t) \wedge HoldsAt(IN(p, r), t) \Rightarrow$$
$$Terminates(MOVE(a, p, r, a), IN(p, r), t)$$

In order for an agent to set down a physical object into a room, the agent must be holding the physical object, and the agent must be directly in the room.

Axiom RS18.

$$HoldsAt(IN(p, a), t) \wedge$$
$$HoldsAt(IN(a, r), t) \Rightarrow$$
$$Initiates(MOVE(a, p, a, r), IN(p, r), t)$$

Axiom RS19.

$$HoldsAt(IN(p, a), t) \wedge$$
$$HoldsAt(IN(a, r), t) \Rightarrow$$
$$Terminates(MOVE(a, p, a, r), IN(p, a), t)$$

10.1.3 EXAMPLE: MOVING A NEWSPAPER AND A BOX

Lisa, a newspaper, and a box are in the living room. She puts the newspaper in the box, picks up the box, and walks into the kitchen:

$$HoldsAt(IN(Lisa, LivingRoom), 0) \tag{10.1}$$
$$HoldsAt(IN(Newspaper, LivingRoom), 0) \tag{10.2}$$
$$HoldsAt(IN(Box, LivingRoom), 0) \tag{10.3}$$
$$Happens(MOVE(Lisa, Newspaper, LivingRoom, Box), 0) \tag{10.4}$$
$$Happens(MOVE(Lisa, Box, LivingRoom, Lisa), 1) \tag{10.5}$$
$$Happens(MOVE(Lisa, Lisa, LivingRoom, Kitchen), 2) \tag{10.6}$$

Lisa then sets down the box and walks back into the living room:

$$Happens(MOVE(Lisa, Box, Lisa, Kitchen), 3) \tag{10.7}$$
$$Happens(MOVE(Lisa, Lisa, Kitchen, LivingRoom), 4) \tag{10.8}$$

The fluent *INROOM* is always released from the commonsense law of inertia, and *IN* is never released from this law:

$$ReleasedAt(INROOM(o_1, o_2), t) \tag{10.9}$$

$$\neg ReleasedAt(IN(o_1, o_2), t) \tag{10.10}$$

We can show that Lisa will be in the living room, but the newspaper and box will be in the kitchen.

Proposition 10.1. *Let Σ be the conjunction of RS10 through RS19. Let Δ be the conjunction of (10.4), (10.5), (10.6), (10.7), and (10.8). Let Ω be U[IN, INROOM]. Let Ψ be the conjunction of RS1 through RS4, and RS7 through RS9. Let Γ be the conjunction of (10.1), (10.2), (10.3), (10.9), and (10.10). Then we have*

$$\begin{aligned}&CIRC[\Sigma; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge\\ &\Omega \wedge \Psi \wedge \Gamma \wedge \mathrm{EC}\\ &\vdash HoldsAt(IN(Lisa, LivingRoom), 5) \wedge\\ &HoldsAt(IN(Box, Kitchen), 5) \wedge\\ &HoldsAt(INROOM(Newspaper, Kitchen), 5)\end{aligned}$$

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned}&Initiates(e, f, t) \Leftrightarrow \\ &\exists a, p_1, p_2, r\ (e = MOVE(a, p_1, r, p_2)\ \wedge\\ &f = IN(p_1, p_2) \wedge\\ &HoldsAt(IN(a, r), t) \wedge\\ &HoldsAt(IN(p_1, r), t) \wedge\\ &HoldsAt(INROOM(p_2, r), t)) \vee\\ &\exists a, p_1, p_2, r\ (e = MOVE(a, p_1, p_2, r) \wedge\\ &f = IN(p_1, r), \wedge\\ &HoldsAt(IN(a, r), t)) \vee\\ &\exists a, r_1, r_2\ (e = MOVE(a, a, r_1, r_2) \wedge\\ &f = IN(a, r_2) \wedge\\ &HoldsAt(IN(a, r_1), t)) \vee\\ &\exists a, p, r\ (e = MOVE(a, p, r, a) \wedge\\ &f = IN(p, a) \wedge\\ &HoldsAt(IN(a, r), t) \wedge\\ &HoldsAt(IN(p, r), t)) \vee\\ &\exists a, p, r\ (e = MOVE(a, p, a, r) \wedge\\ &f = IN(p, r) \wedge\\ &HoldsAt(IN(p, a), t) \wedge\\ &HoldsAt(IN(a, r), t))\end{aligned} \tag{10.11}$$

$$\begin{aligned}
&Terminates(e,f,t) \Leftrightarrow &(10.12)\\
&\exists a,p_1,p_2,r\,(e = MOVE(a,p_1,r,p_2) \wedge\\
&f = IN(p_1,r) \wedge\\
&HoldsAt(IN(a,r),t) \wedge\\
&HoldsAt(IN(p_1,r),t) \wedge\\
&HoldsAt(INROOM(p_2,r),t)) \vee\\
&\exists a,p_1,p_2,r\,(e = MOVE(a,p_1,p_2,r) \wedge\\
&f = IN(p_1,p_2) \wedge\\
&HoldsAt(IN(a,r),t)) \vee\\
&\exists a,r_1,r_2\,(e = MOVE(a,a,r_1,r_2) \wedge\\
&f = IN(a,r_1) \wedge\\
&HoldsAt(IN(a,r_1),t)) \vee\\
&\exists a,p,r\,(e = MOVE(a,p,r,a) \wedge\\
&f = IN(p,r) \wedge\\
&HoldsAt(IN(a,r),t) \wedge\\
&HoldsAt(IN(p,r),t)) \vee\\
&\exists a,p,r\,(e = MOVE(a,p,a,r) \wedge\\
&f = IN(p,a) \wedge\\
&HoldsAt(IN(p,a),t) \wedge\\
&HoldsAt(IN(a,r),t))
\end{aligned}$$

$$\neg Releases(e,f,t) \tag{10.13}$$

From $CIRC[\Delta; Happens]$ and Theorem 2.1, we have

$$\begin{aligned}
&Happens(e,t) \Leftrightarrow &(10.14)\\
&(e = MOVE(Lisa, Newspaper, LivingRoom, Box) \wedge t = 0) \vee\\
&(e = MOVE(Lisa, Box, LivingRoom, Lisa) \wedge t = 1) \vee\\
&(e = MOVE(Lisa, Lisa, LivingRoom, Kitchen) \wedge t = 2) \vee\\
&(e = MOVE(Lisa, Box, Lisa, Kitchen) \wedge t = 3) \vee\\
&(e = MOVE(Lisa, Lisa, Kitchen, LivingRoom) \wedge t = 4)
\end{aligned}$$

From (10.1), $0 < 2$, $PersistsBetween(0, IN(Lisa, LivingRoom), 2)$ (which follows from (10.10) and EC7), $\neg Clipped(0, IN(Lisa, LivingRoom), 2)$ (which follows from (10.12), (10.14), and EC1), and EC9, we have

$$HoldsAt(IN(Lisa, LivingRoom), 2) \tag{10.15}$$

From (10.6) (which follows from (10.14)), (10.15), RS14 (which follows from (10.11)), $2 < 4$, $\neg StoppedIn(2, IN(Lisa, Kitchen), 4)$ (which follows from (10.12), (10.14), and EC3), $\neg ReleasedIn(2, IN(Lisa, Kitchen), 4)$ (which follows from (10.13) and EC13), and EC14, we have $HoldsAt(IN(Lisa, Kitchen), 4)$. From this, (10.8) (which follows from (10.14)), RS14 (which follows from (10.11)), $4 < 5$, $\neg StoppedIn(4, IN(Lisa, LivingRoom), 5)$ (which follows from (10.14) and EC3), $\neg ReleasedIn(4, IN(Lisa, LivingRoom), 5)$ (which follows from (10.13) and EC13), and EC14, we have $HoldsAt(IN(Lisa, LivingRoom), 5)$.

From (10.1), $0 < 1$, $PersistsBetween(0, IN(Lisa, LivingRoom), 1)$ (which follows from (10.10) and EC7), $\neg Clipped(0, IN(Lisa, LivingRoom), 1)$ (which follows from (10.12), (10.14), and EC1), and EC9, we have

$$HoldsAt(IN(Lisa, LivingRoom), 1) \tag{10.16}$$

From (10.3), $0 < 1$, $PersistsBetween(0, IN(Box, LivingRoom), 1)$ (which follows from (10.10) and EC7), $\neg Clipped(0, IN(Box, LivingRoom), 1)$ (which follows from (10.12), (10.14), and EC1), and EC9, we have

$$HoldsAt(IN(Box, LivingRoom), 1) \tag{10.17}$$

From (10.6) (which follows from (10.14)), (10.15), RS14 (which follows from (10.11)), $2 < 3$, $\neg StoppedIn(2, IN(Lisa, Kitchen), 3)$ (which follows from (10.14) and EC3), $\neg ReleasedIn(2, IN(Lisa, Kitchen), 3)$ (which follows from (10.13) and EC13), and EC14, we have

$$HoldsAt(IN(Lisa, Kitchen), 3) \tag{10.18}$$

From (10.5) (which follows from (10.14)), (10.16), (10.17), RS16, $1 < 3$, $\neg StoppedIn(1, IN(Box, Lisa), 3)$ (which follows from (10.12), (10.14), and EC3), $\neg ReleasedIn(1, IN(Box, Lisa), 3)$ (which follows from (10.13) and EC13), and EC14, we have $HoldsAt(IN(Box, Lisa), 3)$. From this, (10.7) (which follows from (10.14)), (10.18), RS18 (which follows from (10.11)), $3 < 5$, $\neg StoppedIn(3, IN(Box, Kitchen), 5)$ (which follows from (10.12), (10.14), and EC3), $\neg ReleasedIn(3, IN(Box, Kitchen), 5)$ (which follows from (10.13) and EC13), and EC14, we have

$$HoldsAt(IN(Box, Kitchen), 5) \tag{10.19}$$

From (10.3) and RS7, we have $HoldsAt(INROOM(Box, LivingRoom), 0)$. From this, (10.4) (which follows from (10.14)), (10.1), (10.2), RS10, $0 < 5$, $\neg StoppedIn(0, IN(Newspaper, Box), 5)$ (which follows from (10.12), (10.14), and EC3), $\neg ReleasedIn(0, IN(Newspaper, Box), 5)$ (which follows from (10.13) and EC13), and EC14, we have

$$HoldsAt(IN(Newspaper, Box), 5) \tag{10.20}$$

From (10.19) and RS7, we have $HoldsAt(INROOM(Box, Kitchen), 5)$. From this, (10.20), and RS8, we have $HoldsAt(INROOM(Newspaper, Kitchen), 5)$. ■

10.2 METRIC SPACE

Definition 10.1. A ***metric space*** is a set M of objects called ***points*** and a function d (the distance function or metric) from $M \times M$ to the set of real numbers such that, for every $x, y, z \in M$,

- $d(x, y) \geq 0$
- $d(x, y) = 0$ if and only if $x = y$
- $d(x, y) = d(y, x)$
- $d(x, z) \leq d(x, y) + d(y, z)$ (known as the ***triangle inequality***)

The commonsense world involves objects located at specific points in a metric space and objects that trace specific paths over time in that space. For example, when an apple located eight feet above the ground starts to fall, it follows a particular path toward the ground. In this section, we present a representation of metric space in the event calculus. (Another representation of metric space is discussed in Section 8.2.2.)

We start with a basic representation as we did for relational space:

- an object sort, with variables o, o_1, o_2, ...
- a real number sort, with variables x, x_1, x_2, ...; y, y_1, y_2, ...; and z, z_1, z_2, ...
- a representation for points in three-dimensional space consisting of triples of real numbers $\langle x, y, z\rangle$
- a fluent $AT(o, x, y, z)$, which represents that object o is located at point $\langle x, y, z\rangle$
- a function $D(x_1, y_1, z_1, x_2, y_2, z_2)$, which represents the distance between the points $\langle x_1, y_1, z_1\rangle$ and $\langle x_2, y_2, z_2\rangle$

We may wish to require an object to be located at one point at a time via the following axiom.

Axiom MS1.

$$HoldsAt(AT(o, x_1, y_1, z_1), t) \wedge HoldsAt(AT(o, x_2, y_2, z_2), t) \Rightarrow$$
$$x_1 = x_2 \wedge y_1 = y_2 \wedge z_1 = z_2$$

We specify the *DIST* function.

Axiom MS2.

$$DIST(x_1, y_1, z_1, x_2, y_2, z_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

We can then extend this basic representation with motion and collisions.

10.2.1 EXAMPLE: TWO BASEBALLS COLLIDING

Nathan and Ryan each throw a baseball into the air. A moment later, the baseballs collide.

We treat this example using both relational and metric space. We start with some effect axioms. If an agent is holding an object and the agent throws the object into the air with a velocity vector $\langle v_x, v_y, v_z\rangle$, where the z axis represents up, then the object will be flying with that velocity vector:

$$HoldsAt(IN(o, a), t) \Rightarrow$$
$$Initiates(Throw(a, o, v_x, v_y, v_z), Flying(o, v_x, v_y, v_z), t) \tag{10.21}$$

If an agent is holding an object and the agent throws the object into the air, then the agent will no longer be holding the object:

$$HoldsAt(IN(o, a), t) \Rightarrow$$
$$Terminates(Throw(a, o, v_x, v_y, v_z), IN(o, a), t) \tag{10.22}$$

If an object is thrown into the air, then its location and velocity will no longer be subject to the commonsense law of inertia:

$$HoldsAt(IN(o,a),t) \Rightarrow \tag{10.23}$$
$$Releases(Throw(a,o,v_x,v_y,v_z),AT(o,x,y,z),t)$$

$$HoldsAt(IN(o,a),t) \Rightarrow \tag{10.24}$$
$$Releases(Throw(a,o,v_{x1},v_{y1},v_{z1}),Velocity(o,v_{x2},v_{y2},v_{z2}),t)$$

We specify the location and velocity of the flying object using the following trajectory axioms:

$$\begin{aligned} &HoldsAt(AT(o,x_0,y_0,z_0),t) \wedge \\ &x_1 = x_0 + v_x t_2 \wedge \\ &y_1 = y_0 + v_y t_2 \wedge \\ &z_1 = z_0 + v_z t_2 - \frac{1}{2} G t_2^2 \Rightarrow \\ &Trajectory(Flying(o,v_x,v_y,v_z),t_1,AT(o,x_1,y_1,z_1),t_2) \end{aligned} \tag{10.25}$$

$$\begin{aligned} &v_{z2} = v_{z1} - G t_2 \Rightarrow \\ &Trajectory(Flying(o,v_x,v_y,v_{z1}),t_1,Velocity(o,v_x,v_y,v_{z2}),t_2) \end{aligned} \tag{10.26}$$

We have state constraints that say that at any given moment an object has a unique velocity and is flying with a unique velocity:

$$\begin{aligned} &HoldsAt(Velocity(o,v_{x1},v_{y1},v_{z1}),t) \wedge \\ &HoldsAt(Velocity(o,v_{x2},v_{y2},v_{z2}),t) \Rightarrow \\ &v_{x1} = v_{x2} \wedge v_{y1} = v_{y2} \wedge v_{z1} = v_{z2} \end{aligned} \tag{10.27}$$

$$\begin{aligned} &HoldsAt(Flying(o,v_{x1},v_{y1},v_{z1}),t) \wedge \\ &HoldsAt(Flying(o,v_{x2},v_{y2},v_{z2}),t) \Rightarrow \\ &v_{x1} = v_{x2} \wedge v_{y1} = v_{y2} \wedge v_{z1} = v_{z2} \end{aligned} \tag{10.28}$$

We use a trigger axiom to represent that an elastic collision occurs when two flying objects are at the same location:

$$\begin{aligned} &o_1 \neq o_2 \wedge \\ &HoldsAt(Flying(o_1,v_{1x},v_{1y},v_{1z}),t) \wedge \\ &HoldsAt(Flying(o_2,v_{2x},v_{2y},v_{2z}),t) \wedge \\ &HoldsAt(AT(o_1,x,y,z),t) \wedge \\ &HoldsAt(AT(o_2,x,y,z),t) \Rightarrow \\ &Happens(Collide(o_1,o_2),t) \end{aligned} \tag{10.29}$$

We use effect axioms to represent that, when object o_1 elastically collides with object o_2, object o_1 assumes the velocity of o_2:

$$HoldsAt(Velocity(o_1, v_{1x}, v_{1y}, v_{1z}), t) \wedge \\ HoldsAt(Velocity(o_2, v_{2x}, v_{2y}, v_{2z}), t) \Rightarrow \\ Initiates(Collide(o_1, o_2), Flying(o_1, v_{2x}, v_{2y}, v_{2z}), t) \tag{10.30}$$

$$HoldsAt(Velocity(o_1, v_x, v_y, v_z), t) \Rightarrow \\ Terminates(Collide(o_1, o_2), Flying(o_1, v_x, v_y, v_z), t) \tag{10.31}$$

That is, two elastically colliding objects exchange their velocities. We assume the objects are points of equal mass.

Let us use the following observations and narrative. Initially, one baseball is at rest at location $\langle 0, 0, G/2 \rangle$, and another baseball is at rest at location $\langle 1, 0, G/2 \rangle$:

$$\neg ReleasedAt(f, t) \tag{10.32}$$

$$HoldsAt(AT(Baseball1, 0, 0, G/2), 0) \tag{10.33}$$

$$HoldsAt(Velocity(Baseball1, 0, 0, 0), 0) \tag{10.34}$$

$$HoldsAt(AT(Baseball2, 1, 0, G/2), 0) \tag{10.35}$$

$$HoldsAt(Velocity(Baseball2, 0, 0, 0), 0) \tag{10.36}$$

Nathan is holding the first baseball, and Ryan is holding the second baseball:

$$HoldsAt(IN(Baseball1, Nathan), 0) \tag{10.37}$$

$$HoldsAt(IN(Baseball2, Ryan), 0) \tag{10.38}$$

Nathan and Ryan throw the baseballs toward one another. Nathan throws the first baseball with velocity $\langle 1, 0, 0 \rangle$, and Ryan throws the second baseball with velocity $\langle -1, 0, 0 \rangle$:

$$Happens(Throw(Nathan, Baseball1, 1, 0, 0), 0) \tag{10.39}$$

$$Happens(Throw(Ryan, Baseball2, -1, 0, 0), 0) \tag{10.40}$$

We also have the fact that the two baseballs are not the same object:

$$Baseball1 \neq Baseball2 \tag{10.41}$$

We can then show that the baseballs will collide. In order to show this, we must assume that the two baseballs are the only objects in the metric space:

$$HoldsAt(AT(o, x, y, z), t) \Rightarrow (o = Baseball1 \vee o = Baseball2) \tag{10.42}$$

Without this assumption, either of the baseballs might first collide with some other object.

We start with a lemma that states that, if object o_1 collides with object o_2, then o_2 collides with o_1:

Lemma 10.1. *If* $\Gamma =$ (10.29), *then we have*

$$CIRC[\Gamma; Happens] \wedge Happens(Collide(o_1, o_2), t) \Rightarrow \\ Happens(Collide(o_2, o_1), t)$$

Proof. Let ω_1 and ω_2 be arbitrary objects, and τ be an arbitrary timepoint. We must show

$$CIRC[\Gamma; Happens] \wedge Happens(Collide(\omega_1, \omega_2), \tau) \Rightarrow \\ Happens(Collide(\omega_2, \omega_1), \tau)$$

Suppose the following:

$$CIRC[\Gamma; Happens] \tag{10.43}$$

$$Happens(Collide(\omega_1, \omega_2), \tau) \tag{10.44}$$

From (10.43) and Theorem 2.1, we have

$$\begin{aligned} & Happens(Collide(o_1, o_2), t) \Leftrightarrow \\ & \exists v_{1x}, v_{1y}, v_{1z}, v_{2x}, v_{2y}, v_{2z}, x, y, z \\ & (o_1 \neq o_2 \wedge \\ & HoldsAt(Flying(o_1, v_{1x}, v_{1y}, v_{1z}), t) \wedge \\ & HoldsAt(Flying(o_2, v_{2x}, v_{2y}, v_{2z}), t) \wedge \\ & HoldsAt(AT(o_1, x, y, z), t) \wedge \\ & HoldsAt(AT(o_2, x, y, z), t)) \end{aligned} \tag{10.45}$$

From this and (10.44), we have:

$$\omega_1 \neq \omega_2. \tag{10.46}$$

$$HoldsAt(Flying(\omega_1, V1X, V1Y, V1Z), \tau) \tag{10.47}$$

$$HoldsAt(Flying(\omega_2, V2X, V2Y, V2Z), \tau) \tag{10.48}$$

$$HoldsAt(AT(\omega_1, X, Y, Z), \tau) \tag{10.49}$$

$$HoldsAt(AT(\omega_2, X, Y, Z), \tau) \tag{10.50}$$

for some $V1X$, $V1Y$, $V1Z$, $V2X$, $V2Y$, $V2Z$, X, Y, and Z. From (10.46), (10.47), (10.48), (10.49), (10.50), and (10.45), we have $Happens(Collide(\omega_2, \omega_1), \tau)$, as required. ■

Now we show that the baseballs collide.

Proposition 10.2. *Let Σ be the conjunction of (10.21), (10.22), (10.23), (10.24), (10.30), and (10.31). Let Δ be the conjunction of (10.29), (10.39), and (10.40). Let Ω be $U[IN, AT, Flying, Velocity] \wedge U[Throw, Collide] \wedge$ (10.41). Let Ψ be the conjunction of RS1 through RS4, MS1, (10.27), (10.28), and (10.42). Let Π be the conjunction of (10.25) and (10.26). Let Γ be the conjunction of (10.32), (10.33), (10.34), (10.35), (10.36), (10.37), and (10.38). Then we have*

$$CIRC[\Sigma; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \\ \Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \mathrm{EC} \\ \vdash Happens(Collide(Baseball1, Baseball2), 0.5)$$

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned} & Initiates(e, f, t) \Leftrightarrow \\ & \exists a, o, v_x, v_y, v_z\ (e = Throw(a, o, v_x, v_y, v_z) \wedge \\ & f = Flying(o, v_x, v_y, v_z) \wedge \end{aligned} \tag{10.51}$$

$$
\begin{aligned}
&HoldsAt(IN(o,a),t)) \vee \\
&\exists o_1, o_2, v_{1x}, v_{1y}, v_{1z}, v_{2x}, v_{2y}, v_{2z} \\
&(e = Collide(o_1, o_2) \wedge \\
&f = Flying(o_1, v_{2x}, v_{2y}, v_{2z}) \wedge \\
&HoldsAt(Velocity(o_1, v_{1x}, v_{1y}, v_{1z}), t) \wedge \\
&HoldsAt(Velocity(o_2, v_{2x}, v_{2y}, v_{2z}), t))
\end{aligned}
$$

$$
\begin{aligned}
&Terminates(e,f,t) \Leftrightarrow \\
&\exists a, o, v_x, v_y, v_z\ (e = Throw(a, o, v_x, v_y, v_z) \wedge \\
&f = IN(o,a) \wedge \\
&HoldsAt(IN(o,a),t)) \vee \\
&\exists o_1, o_2, v_x, v_y, v_z\ (e = Collide(o_1, o_2) \wedge \\
&f = Flying(o_1, v_x, v_y, v_z) \wedge \\
&HoldsAt(Velocity(o_1, v_x, v_y, v_z), t))
\end{aligned}
\tag{10.52}
$$

$$
\begin{aligned}
&Releases(e,f,t) \Leftrightarrow \\
&\exists a, o, v_x, v_y, v_z\ (e = Throw(a, o, v_x, v_y, v_z) \wedge \\
&f = AT(o,x,y,z) \wedge \\
&HoldsAt(IN(o,a),t)) \vee \\
&\exists a, o, v_{x1}, v_{y1}, v_{z1}, v_{x2}, v_{y2}, v_{z2} \\
&(e = Throw(a, o, v_{x1}, v_{y1}, v_{z1}) \wedge \\
&f = Velocity(o, v_{x2}, v_{y2}, v_{z2}) \wedge \\
&HoldsAt(IN(o,a),t))
\end{aligned}
\tag{10.53}
$$

From $CIRC[\Delta; Happens]$ and Theorem 2.1, we have

$$
\begin{aligned}
&Happens(e,t) \Leftrightarrow \\
&\exists o_1, o_2, x, y, z, v_{1x}, v_{1y}, v_{1z}, v_{2x}, v_{2y}, v_{2z} \\
&(e = Collide(o_1, o_2) \wedge \\
&o_1 \neq o_2 \wedge \\
&HoldsAt(Flying(o_1, v_{1x}, v_{1y}, v_{1z}), t) \wedge \\
&HoldsAt(Flying(o_2, v_{2x}, v_{2y}, v_{2z}), t) \wedge \\
&HoldsAt(AT(o_1, x, y, z), t) \wedge \\
&HoldsAt(AT(o_2, x, y, z), t)) \vee \\
&(e = Throw(Nathan, Baseball1, 1, 0, 0) \wedge t = 0) \vee \\
&(e = Throw(Ryan, Baseball2, -1, 0, 0) \wedge t = 0)
\end{aligned}
\tag{10.54}
$$

We can show

$$
\neg \exists o, t\ (0 < t < 0.5 \wedge Happens(Collide(Baseball1, o), t)) \tag{10.55}
$$

To see this, suppose to the contrary that

$$\exists o, t\,(0 < t < 0.5 \wedge \mathit{Happens}(\mathit{Collide}(\mathit{Baseball1}, o), t))$$

Let $\mathit{Happens}(\mathit{Collide}(\mathit{Baseball1}, \omega), \tau)$ be the first such event. That is, we have

$$0 < \tau < 0.5 \tag{10.56}$$

$$\mathit{Happens}(\mathit{Collide}(\mathit{Baseball1}, \omega), \tau) \tag{10.57}$$

$$\neg\exists \omega', \tau'\,(0 < \tau' < \tau \wedge \mathit{Happens}(\mathit{Collide}(\mathit{Baseball1}, \omega'), \tau')) \tag{10.58}$$

From (10.57) and (10.54), we have

$$\mathit{Baseball1} \neq \omega \tag{10.59}$$

$$\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball1}, X1, Y1, Z1), \tau) \tag{10.60}$$

$$\mathit{HoldsAt}(\mathit{AT}(\omega, X1, Y1, Z1), \tau) \tag{10.61}$$

for some $X1$, $Y1$, and $Z1$. From (10.59), (10.60), (10.61), and (10.42), we have $\omega = \mathit{Baseball2}$. From (10.52), (10.58), and EC3, we have $\neg\mathit{StoppedIn}(0, \mathit{Flying}(\mathit{Baseball1}, 1, 0, 0), \tau)$. From this, (10.39) (which follows from (10.54)), (10.37), (10.21) (which follows from (10.51)), $0 < \tau$ (which follows from (10.56)), (10.33), (10.25), and EC5, we have $\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball1}, \tau, 0, \frac{1}{2}G(1-\tau^2)), \tau)$. From this, (10.60), (10.61), and $\omega = \mathit{Baseball2}$, we have

$$\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball2}, \tau, 0, \frac{1}{2}G(1-\tau^2)), \tau) \tag{10.62}$$

From (10.58), we have

$$\neg\exists \tau'\,(0 < \tau' < \tau \wedge \mathit{Happens}(\mathit{Collide}(\mathit{Baseball1}, \mathit{Baseball2}), \tau')) \tag{10.63}$$

From (10.54), we have $\mathit{CIRC}[(10.29); \mathit{Happens}]$. From this, (10.63), and Lemma 10.1, we have

$$\neg\exists \tau'\,(0 < \tau' < \tau \wedge \mathit{Happens}(\mathit{Collide}(\mathit{Baseball2}, \mathit{Baseball1}), \tau'))$$

From this, (10.52), and EC3, we have $\neg\mathit{StoppedIn}(0, \mathit{Flying}(\mathit{Baseball2}, -1, 0, 0), \tau)$. From this, (10.40) (which follows from (10.54)), (10.38), (10.21) (which follows from (10.51)), $0 < \tau$ (which follows from (10.56)), (10.35), (10.25), and EC5, we have $\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball2}, 1-\tau, 0, \frac{1}{2}G(1-\tau^2)), \tau)$. From this, (10.56), and MS1, we have $\neg\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball2}, \tau, 0, \frac{1}{2}G(1-\tau^2)), \tau)$, which contradicts (10.62).

From (10.52), (10.55), and EC3, we have

$$\neg\mathit{StoppedIn}(0, \mathit{Flying}(\mathit{Baseball1}, 1, 0, 0), 0.5) \tag{10.64}$$

From this, (10.39) (which follows from (10.54)), (10.37), (10.21) (which follows from (10.51)), $0 < 0.5$, (10.33), (10.25), and EC5, we have

$$\mathit{HoldsAt}(\mathit{AT}(\mathit{Baseball1}, 0.5, 0, \frac{3}{8}G), 0.5) \tag{10.65}$$

From (10.53) and EC13, we have $\neg\mathit{ReleasedIn}(0, \mathit{Flying}(\mathit{Baseball1}, 1, 0, 0), 0.5)$. From this, (10.39) (which follows from (10.54)), (10.37), (10.21) (which follows from (10.51)), $0 < 0.5$, (10.64), and EC14, we have

$$\mathit{HoldsAt}(\mathit{Flying}(\mathit{Baseball1}, 1, 0, 0), 0.5) \tag{10.66}$$

Using similar arguments we can show

$$HoldsAt(AT(Baseball2, 0.5, 0, \frac{3}{8}G), 0.5) \tag{10.67}$$

$$HoldsAt(Flying(Baseball2, -1, 0, 0), 0.5) \tag{10.68}$$

From (10.41), (10.66), (10.68), (10.65), (10.67), and (10.29) (which follows from (10.54)), we have $Happens(Collide(Baseball1, Baseball2), 0.5)$. ■

10.3 OBJECT IDENTITY

We can express object identity in first-order logic using the atom $o_1 = o_2$, which represents that o_1 and o_2 are the same object. The commonsense world of space and time places certain restrictions on object identity, which we can reason about using the event calculus. In this section, we consider the problem of observing the locations of two objects over time and determining whether the two objects must be the same, may be the same, or must be different.

We use the following spatial theory. The predicate $Adjacent(l_1, l_2)$ represents that location l_1 is adjacent to location l_2. The event $Move(o, l_1, l_2)$ represents that object o moves from location l_1 to location l_2. The fluent $At(o, l)$ represents that object o is located at location l. An object is at exactly one location at a time:

$$HoldsAt(At(o, l_1), t) \wedge HoldsAt(At(o, l_2), t) \Rightarrow l_1 = l_2 \tag{10.69}$$

$$\exists l\ HoldsAt(At(o, l), t) \tag{10.70}$$

Two objects cannot occupy the same location at the same time:

$$HoldsAt(At(o_1, l), t) \wedge HoldsAt(At(o_2, l), t) \Rightarrow o_1 = o_2 \tag{10.71}$$

The *Adjacent* predicate is symmetric:

$$Adjacent(l_1, l_2) \Leftrightarrow Adjacent(l_2, l_1) \tag{10.72}$$

If an object is at location l_1, which is adjacent to location l_2, and the object moves from l_1 to l_2, then the object will be at l_2 and will no longer be at l_1:

$$\begin{gathered} HoldsAt(At(o, l_1), t) \wedge Adjacent(l_1, l_2) \Rightarrow \\ Initiates(Move(o, l_1, l_2), At(o, l_2), t) \end{gathered} \tag{10.73}$$

$$\begin{gathered} HoldsAt(At(o, l_1), t) \wedge Adjacent(l_1, l_2) \Rightarrow \\ Terminates(Move(o, l_1, l_2), At(o, l_1), t) \end{gathered} \tag{10.74}$$

10.3.1 EXAMPLE: ONE SCREEN

Consider the following scenario involving three locations: The location *L1* is to the left of *L2*, which is to the left of *L3*:

$$\begin{gathered} Adjacent(l_1, l_2) \Leftrightarrow \\ (l_1 = L1 \wedge l_2 = L2) \vee \end{gathered} \tag{10.75}$$

$$(l_1 = L2 \wedge l_2 = L1) \vee$$
$$(l_1 = L2 \wedge l_2 = L3) \vee$$
$$(l_1 = L3 \wedge l_2 = L2)$$

Our view of location *L2* is blocked by a screen.

Suppose we observe the following: At timepoint 0, we observe an object, let us call it *O1*, at *L1* and nothing at *L3*. At timepoint 1, we observe no objects at *L1* or *L3*. At timepoint 2, we observe an object, let us call it *O2*, at *L3* and nothing at *L1*. We observe nothing about *L2* because it is blocked by a screen.

Thus, we have the following observations:

$$HoldsAt(At(O1, L1), 0) \tag{10.76}$$

$$\neg HoldsAt(At(o, L3), 0) \tag{10.77}$$

$$\neg HoldsAt(At(o, L1), 1) \tag{10.78}$$

$$\neg HoldsAt(At(o, L3), 1) \tag{10.79}$$

$$HoldsAt(At(O2, L3), 2) \tag{10.80}$$

$$\neg HoldsAt(At(o, L1), 2) \tag{10.81}$$

$$\neg ReleasedAt(f, t) \tag{10.82}$$

We can then show that *O1* and *O2* must be the same object.

Proposition 10.3. *Let* $\Sigma =$ (10.73) $\wedge$ (10.74), $\Psi =$ (10.69) $\wedge$ (10.70) $\wedge$ (10.71) $\wedge$ (10.72), *and* $\Gamma =$ (10.75) $\wedge$ (10.76) $\wedge$ (10.77) $\wedge$ (10.78) $\wedge$ (10.79) $\wedge$ (10.80) $\wedge$ (10.81) $\wedge$ (10.82). *Suppose*

$$CIRC[\Sigma; Initiates, Terminates, Releases] \wedge \Psi \wedge \Gamma \wedge \text{DEC}$$

Then $O1 = O2$.

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned} &Initiates(e, f, t) \Leftrightarrow \\ &\exists o, l_1, l_2\, (e = Move(o, l_1, l_2) \wedge \\ &f = At(o, l_2) \wedge \\ &HoldsAt(At(o, l_1), t) \wedge \\ &Adjacent(l_1, l_2)) \end{aligned} \tag{10.83}$$

$$\begin{aligned} &Terminates(e, f, t) \Leftrightarrow \\ &\exists o, l_1, l_2\, (e = Move(o, l_1, l_2) \wedge \\ &f = At(o, l_1) \wedge \\ &HoldsAt(At(o, l_1), t) \wedge \\ &Adjacent(l_1, l_2)) \end{aligned} \tag{10.84}$$

$$\neg Releases(e, f, t) \tag{10.85}$$

From (10.80) and the contrapositive of DEC6, we have

$$\begin{gathered} HoldsAt(At(O2, L3), 1) \lor \\ ReleasedAt(At(O2, L3), 2) \lor \\ \exists e\, (Happens(e, 1) \land Initiates(e, At(O2, L3), 1)) \end{gathered}$$

From this, (10.79), and (10.82), we have

$$\exists e\, (Happens(e, 1) \land Initiates(e, At(O2, L3), 1))$$

From this, (10.83), and (10.75), we have

$$HoldsAt(At(O2, L2), 1) \tag{10.86}$$

From (10.78), we have $\neg HoldsAt(At(O1, L1), 1)$. From this and the contrapositive of DEC5, we have

$$\begin{gathered} \neg HoldsAt(At(O1, L1), 0) \lor \\ ReleasedAt(At(O1, L1), 1) \lor \\ \exists e\, (Happens(e, 0) \land Terminates(e, At(O1, L1), 0)) \end{gathered}$$

From this, (10.76), and (10.82), we have

$$\exists e\, (Happens(e, 0) \land Terminates(e, At(O1, L1), 0))$$

From this, (10.84), and (10.75), we have $Happens(Move(O1, L1, L2), 0)$. From this, (10.76), (10.75), (10.83), and DEC9, we have

$$HoldsAt(At(O1, L2), 1)$$

From this, (10.86), and (10.71), we have $O1 = O2$. ■

10.3.2 EXAMPLE: TWO SCREENS

Now consider the following scenario involving five locations: The location $L1$ is to the left of $L2$, which is to the left of $L3$, which is to the left of $L4$, which is to the left of $L5$:

$$\begin{gathered} Adjacent(l_1, l_2) \Leftrightarrow \\ (l_1 = L1 \land l_2 = L2) \lor \\ (l_1 = L2 \land l_2 = L1) \lor \\ (l_1 = L2 \land l_2 = L3) \lor \\ (l_1 = L3 \land l_2 = L2) \lor \\ (l_1 = L3 \land l_2 = L4) \lor \\ (l_1 = L4 \land l_2 = L3) \lor \\ (l_1 = L4 \land l_2 = L5) \lor \\ (l_1 = L5 \land l_2 = L4) \end{gathered} \tag{10.87}$$

Locations $L2$ and $L4$ are blocked by screens.

This time we observe the following. At timepoint 0, we observe *O1* at *L1* and nothing at *L5*. At timepoint 4, we observe *O2* at *L5* and nothing at *L1*. We never observe an object at *L3*, and we never observe anything about *L2* or *L4*.

Thus, we have the following observations:

$$HoldsAt(At(O1, L1), 0) \tag{10.88}$$

$$\neg HoldsAt(At(o, L5), 0) \tag{10.89}$$

$$HoldsAt(At(O2, L5), 4) \tag{10.90}$$

$$\neg HoldsAt(At(o, L1), 4) \tag{10.91}$$

$$\neg HoldsAt(At(o, L3), t) \tag{10.92}$$

$$\neg ReleasedAt(f, t) \tag{10.93}$$

In this case, we can show that *O1* and *O2* cannot be the same object. (See Exercise 10.5.)

BIBLIOGRAPHIC NOTES

Hayes (1985) describes the following ontology for liquids. Liquids can be lazy or energetic. Lazy liquids can be still or moving. Rain is a lazy moving liquid, and a splash is an energetic liquid. Liquids can be bulk or divided. Bulk and divided liquids can be supported by a two-dimensional object, supported by a three-dimensional object, or unsupported. A mist is a lazy, still, divided, unsupported liquid. A river is a lazy, moving, bulk liquid supported by a three-dimensional object.

E. Davis (1990, pp. 241-310) reviews a number of spatial representations including geometric entities, occupancy arrays, constructive solid geometry, boundary representations, topological route maps, and configuration spaces. E. Davis (1995) describes a formal language for describing objects in two-dimensional space. The language provides the following entities: real number, point, length, angle, direction, area, vector, frame of reference, interval, arc, region, directed curve, scale mapping, and collection of regions. Partial and inexact information can be expressed in the language. For example, the language can be used to represent a circle whose diameter is between 1 and 2 inches, with a triangular hole somewhere inside it.

Randell, Cui, and Cohn (1992) introduce what has become known as the *region connection calculus* (RCC), a logic for spatial reasoning. RCC is based on a single predicate $C(x, y)$, which represents that region x connects with region y. A number of predicates can be defined in terms of C to represent that one region is disconnected from another region, one region is a part of another region, one region overlaps another region, and so forth. Cohn, Bennett, Gooday, and Gotts (1997) provide an introduction to RCC.

Kuipers (2000) describes the *spatial semantic hierarchy*, a collection of interacting representations of large-scale space. Representations are classified along two dimensions: (1) qualitative and quantitative, and (2) sensory, control, causal, topological, and metrical. In this hierarchy, our representation of relational space is classified as qualitative and topological, whereas our representation of metric space is classified as quantitative and metrical. Kuipers discusses several implementations of the spatial semantic hierarchy within real and virtual robots. Remolina and Kuipers (2004) define a theory of topological maps using many-sorted logic and circumscription. Topological and metric spaces are discussed by Marsden (1974).

Our axiomatizations of space use techniques developed in previous event calculus axiomatizations of space (Morgenstern, 2001; Shanahan, 2004, 1996). Our representation of metric space in the event calculus is based on that of Shanahan (1996), which is adapted from his previous situation calculus representation (Shanahan, 1995b). Shanahan (1995b, 1996) uses the predicate $Occupies(o, r)$ to represent that object o occupies region r, where a *region* is a set of points in two-dimensional space. Shanahan (1996) represents a robot that can rotate, move in the direction it is facing, and stop moving when it bumps into an object.

Morgenstern (2001) develops spatial representations in the event calculus for reasoning about cracking an egg and pouring its contents into a bowl. She uses the fluent $shape(o) = r$ to represent that object o occupies region r, the fluent $At(o, l)$ to represent that object o is at location l, and the event $Move(o, l)$ to represent that object o moves to location l. Objects and locations are related by the *Above* fluent. Morgenstern represents object parts, object sets, eggs, eggshells, egg insides, packages, open and closed containers, solid and liquid phases, material, falling, pouring, leaking, breaking, and cracking. Shanahan (2004) provides representations in the event calculus for reasoning about egg cracking, including representations of parts and wholes of objects. Synge (1960, pp. 94-96) provides equations for the velocities of two objects after collision. The object identity examples involving one and two screens are taken from Cassimatis (2002).

EXERCISES

10.1 Add doors to the formalization in Section 10.1. Modify Axioms RS14 and RS15 so that, in order for an agent to go from one room to another, the two rooms must have a door in common.

10.2 Extend the formalization in Section 10.1 to represent that one physical object is on another physical object. Because *in* and *on* are different spatial relations, one spatial relation IN is no longer sufficient. Use IN_1 to represent *in* and IN_2 to represent *on*. Write state constraints across IN_1 and IN_2. For example, an object cannot be both in and on another object.

10.3 Extend the formalization in Section 10.1 to represent that one physical object is a part of another physical object. Use IN_3 to represent "is a part of."

10.4 Extend the formalization in Section 10.1 to represent that an agent is wearing one or more items of clothing.

10.5 In the object identity example in Section 10.3.2, prove that $O1$ and $O2$ cannot be the same object.

10.6 Create an event calculus formalization of the grid-based space used in ThoughtTreasure (Mueller, 1998). There are many grids, each consisting of a rectangular array of cells. For any given grid, a given object is located at a unique cell $\langle row, column \rangle$ in that grid at a time. (An object may be in several grids at a time.) If an agent walks from one cell in a grid to another cell in that grid, then the agent will be at that other cell. In order for an agent to walk from one cell to another, the two cells must be adjacent.

10.7 Formalize riding in a car. Include getting into the car, getting out of the car, and driving from one location to another along a street.