CHAPTER

9 Nondeterministic Effects of Events

An event has *nondeterministic effects* if the event can have two or more alternative effects. For example, flipping a coin has two alternative effects: the coin may land with heads showing, or it may land with tails showing. We represent nondeterministic effects of events in the event calculus by allowing event occurrences to give rise to several classes of models. In this chapter, we describe two ways of representing nondeterministic effects in the event calculus: determining fluents and disjunctive event axioms. These techniques enable the representation of uncertainty about the effects of events.

9.1 DETERMINING FLUENTS

One way of representing nondeterministic effects is to use determining fluents. A *determining fluent* is a fluent that (1) is released from the commonsense law of inertia so that its truth value is permitted to vary arbitrarily from timepoint to timepoint and (2) is used in the condition of one or more effect axioms in order to determine the effects of events on other fluents.

Suppose that *DF* is a fluent that is released from the commonsense law of inertia:

$$ReleasedAt(DF, t)$$

Then, this fluent may be used to determine whether the effect of *E* is *F1*:

$$HoldsAt(DF, t) \Rightarrow Initiates(E, F1, t)$$

or whether the effect of *E* is *F2*:

$$\neg HoldsAt(DF, \tau) \Rightarrow Initiates(E, F2, t)$$

9.1.1 EXAMPLE: ROULETTE WHEEL

We consider the example of spinning a roulette wheel that can land on the numbers 1, 2, or 3. We use a determining fluent *WheelNumberDeterminer*. We make the wheel an argument of this fluent so that at any given timepoint different wheels may land on different numbers. This fluent is released from the commonsense law of inertia:

$$ReleasedAt(WheelNumberDeterminer(w, n), t) \tag{9.1}$$

For any given wheel, this fluent specifies one of the numbers 1, 2, or 3 at any given timepoint:

$$\begin{aligned} &HoldsAt(WheelNumberDeterminer(w,n),t) \Rightarrow \\ &n = 1 \vee n = 2 \vee n = 3 \end{aligned} \tag{9.2}$$

And for any given wheel, this fluent specifies a unique number at any given timepoint:

$$\begin{aligned} &HoldsAt(WheelNumberDeterminer(w,n_1),t) \wedge \\ &HoldsAt(WheelNumberDeterminer(w,n_2),t) \Rightarrow \\ &n_1 = n_2 \end{aligned} \tag{9.3}$$

We have several effect axioms. If the determining fluent specifies a particular number for a wheel and a dealer spins the wheel, then the wheel will be on that number:

$$\begin{aligned} &HoldsAt(WheelNumberDeterminer(w,n),t) \Rightarrow \\ &Initiates(Spin(d,w), WheelNumber(w,n),t) \end{aligned} \tag{9.4}$$

If a wheel is on one number, the determining fluent specifies another number for the wheel, the first number is different from the second number, and a dealer spins the wheel, then the wheel will no longer be on the first number:

$$\begin{aligned} &HoldsAt(WheelNumber(w,n_1),t) \wedge \\ &HoldsAt(WheelNumberDeterminer(w,n_2),t) \wedge \\ &n_1 \neq n_2 \Rightarrow \\ &Terminates(Spin(d,w), WheelNumber(w,n_1),t) \end{aligned} \tag{9.5}$$

If a dealer resets a wheel, then the wheel is no longer on any number:

$$Terminates(Reset(d,w), WheelNumber(w,n),t) \tag{9.6}$$

We have a state constraint that says that the wheel is on at most one number at a time:

$$\begin{aligned} &HoldsAt(WheelNumber(w,n_1),t) \wedge \\ &HoldsAt(WheelNumber(w,n_2),t) \Rightarrow \\ &n_1 = n_2 \end{aligned} \tag{9.7}$$

Suppose that a wheel that is not on any number is spun:

$$\neg HoldsAt(WheelNumber(Wheel,n),0) \tag{9.8}$$

$$\neg ReleasedAt(WheelNumber(Wheel,n),t) \tag{9.9}$$

$$Happens(Spin(Dealer,Wheel),0) \tag{9.10}$$

We can show that the wheel will be on one of the numbers 1, 2, or 3 and that, if it lands on a given number, it will still be on that number one timepoint later.

Proposition 9.1. *Let* $\Sigma = (9.4) \wedge (9.5) \wedge (9.6)$, $\Delta = (9.10)$, $\Omega = U[Spin, Reset] \wedge U[WheelNumberDeterminer, WheelNumber]$, $\Psi = (9.2) \wedge (9.3) \wedge (9.7)$, *and* $\Gamma = (9.1) \wedge (9.8) \wedge (9.9)$. *Then we have*

$$\begin{aligned} &CIRC[\Sigma; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \\ &\Omega \wedge \Psi \wedge \Gamma \wedge \mathrm{EC} \vdash (HoldsAt(WheelNumber(Wheel,1),1) \vee \\ &\qquad HoldsAt(WheelNumber(Wheel,2),1) \vee \end{aligned}$$

$$HoldsAt(WheelNumber(Wheel, 3), 1)) \land$$
$$(HoldsAt(WheelNumber(Wheel, k), 1) \Rightarrow$$
$$HoldsAt(WheelNumber(Wheel, k), 2))$$

Proof. From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$\begin{aligned}
&Initiates(e, f, t) \Leftrightarrow \\
&\exists d, w, n\,(e = Spin(d, w) \land \\
&f = WheelNumber(w, n) \land \\
&HoldsAt(WheelNumberDeterminer(w, n), t))
\end{aligned} \tag{9.11}$$

$$\begin{aligned}
&Terminates(e, f, t) \Leftrightarrow \\
&\exists d, w, n_1, n_2\ (e = Spin(d, w) \land \\
&f = WheelNumber(w, n_1) \land \\
&HoldsAt(WheelNumber(w, n_1), t) \land \\
&HoldsAt(WheelNumberDeterminer(w, n_2), t) \land \\
&n_1 \neq n_2) \lor \\
&\exists d, w, n\,(e = Reset(d, w) \land f = WheelNumber(w, n))
\end{aligned} \tag{9.12}$$

$$\neg Releases(e, f, t) \tag{9.13}$$

From $CIRC[\Delta; Happens]$ and Theorem 2.1, we have

$$Happens(e, t) \Leftrightarrow (e = Spin(Dealer, Wheel) \land t = 0) \tag{9.14}$$

First, we show

$$HoldsAt(WheelNumber(Wheel, 1), 1) \lor$$
$$HoldsAt(WheelNumber(Wheel, 2), 1) \lor$$
$$HoldsAt(WheelNumber(Wheel, 3), 1)$$

We consider three cases, which, from (9.2) and (9.3), are exhaustive.

Case 1: $HoldsAt(WheelNumberDeterminer(Wheel, 1), 0)$.

From this, (9.10) (which follows from (9.14)), (9.4) (which follows from (9.11)), $0 < 1$, $\neg StoppedIn(0, WheelNumber(Wheel, 1), 1)$ (which follows from (9.14) and EC3), $\neg ReleasedIn(0, WheelNumber(Wheel, 1), 1)$ (which follows from (9.14) and EC13), and EC14, we have $HoldsAt(WheelNumber(Wheel, 1), 1)$.

Case 2: $HoldsAt(WheelNumberDeterminer(Wheel, 2), 0)$.

By a similar argument, we have $HoldsAt(WheelNumber(Wheel, 2), 1)$.

Case 3: $HoldsAt(WheelNumberDeterminer(Wheel, 3), 0)$.

By a similar argument, we have $HoldsAt(WheelNumber(Wheel, 3), 1)$.

Second, we show

$$HoldsAt(WheelNumber(Wheel, k), 1) \Rightarrow HoldsAt(WheelNumber(Wheel, k), 2)$$

Let κ be an arbitrary integer. We must show

$$HoldsAt(WheelNumber(Wheel, \kappa), 1) \Rightarrow HoldsAt(WheelNumber(Wheel, \kappa), 2)$$

Suppose $HoldsAt(WheelNumber(Wheel, \kappa), 1)$. From this, $1 < 2$, $PersistsBetween(1, WheelNumber(Wheel, \kappa), 2)$ (which follows from (9.9) and EC7), $\neg Clipped(1, WheelNumber(Wheel, \kappa), 2)$ (which follows from (9.14) and EC1), and EC9, we have $HoldsAt(WheelNumber(Wheel, \kappa), 2)$, as required. ■

9.2 DISJUNCTIVE EVENT AXIOMS

Another way of representing nondeterministic effects is to use disjunctive event axioms.

Definition 9.1. If α and $\alpha_1, \ldots, \alpha_n$ are event terms and τ is a timepoint term, then

$$\begin{gathered} Happens(\alpha, \tau) \Rightarrow \\ Happens(\alpha_1, \tau) \vee \cdots \vee Happens(\alpha_n, \tau) \end{gathered}$$

is a ***disjunctive event axiom***.

9.2.1 EXAMPLE: RUNNING AND DRIVING

We start with a disjunctive event axiom that says that, if an agent goes to a location, then the agent runs or drives to that location:

$$\begin{gathered} Happens(Go(a, l), t) \Rightarrow \\ Happens(Run(a, l), t) \vee \\ Happens(Drive(a, l), t) \end{gathered} \tag{9.15}$$

We have an effect axiom that says that, if an agent runs to a location, then the agent will be tired:

$$Initiates(Run(a, l), Tired(a), t) \tag{9.16}$$

Suppose we are told that Nathan was initially not tired and went to the bookstore:

$$\neg ReleasedAt(f, t) \tag{9.17}$$

$$\neg HoldsAt(Tired(Nathan), 0) \tag{9.18}$$

$$Happens(Go(Nathan, Bookstore), 0) \tag{9.19}$$

The event $Go(Nathan, Bookstore)$ has a nondeterministic effect: Nathan may or may not be tired afterward. Now suppose we are told that Nathan was tired afterward:

$$HoldsAt(Tired(Nathan), 1) \tag{9.20}$$

We can then show that Nathan must have run to the bookstore.

We cannot use predicate completion and Theorem 2.1 to compute

$$CIRC[(9.15) \wedge (9.19); Happens] \tag{9.21}$$

(This can be handled by answer set programming—see Section 15.3.3.) Instead we replace (9.21) with

$$\begin{aligned} &(Happens(e,t) \Leftrightarrow \\ &(e = Go(Nathan, Bookstore) \wedge t = 0) \wedge \\ &(e = Run(Nathan, Bookstore) \wedge t = 0)) \vee \\ &(Happens(e,t) \Leftrightarrow \\ &(e = Go(Nathan, Bookstore) \wedge t = 0) \wedge \\ &(e = Drive(Nathan, Bookstore) \wedge t = 0)) \end{aligned} \tag{9.22}$$

Proposition 9.2. *Let* $\Sigma = (9.16)$, $\Delta = (9.22)$, $\Omega = U[Go, Run, Drive]$, *and* $\Gamma = (9.17) \wedge (9.18) \wedge (9.20)$. *Then we have*

$$\begin{gathered} CIRC[\Sigma; Initiates, Terminates, Releases] \wedge \Delta \wedge \Omega \wedge \Gamma \wedge \text{EC} \\ \vdash Happens(Run(Nathan, Bookstore), 0) \end{gathered}$$

Proof. Suppose, to the contrary, that

$$\neg Happens(Run(Nathan, Bookstore), 0) \tag{9.23}$$

From $CIRC[\Sigma; Initiates, Terminates, Releases]$ and Theorems 2.1 and 2.2, we have

$$Initiates(e,f,t) \Leftrightarrow \exists a, l\, (e = Run(a,l) \wedge Tired(a)) \tag{9.24}$$

$$\neg Terminates(e,f,t) \tag{9.25}$$

$$\neg Releases(e,f,t) \tag{9.26}$$

From (9.17) and EC7, we have

$$PersistsBetween(0, Tired(Nathan), 1) \tag{9.27}$$

From (9.24), (9.22), (9.23), and EC2, we have $\neg Declipped(0, Tired\ (Nathan), 1))$. From this, (9.18), $0 < 1$, (9.27), and EC10, we have $\neg HoldsAt(Tired(Nathan), 1)$, which contradicts (9.20). Therefore, we have $Happens(Run(Nathan, Bookstore), 0)$, as required. ∎

BIBLIOGRAPHIC NOTES

The method of determining fluents was introduced by Shanahan (1997b, pp. 294-297, 301; 1999a, pp. 419-420). The roulette wheel example is similar to other examples of nondeterminism such as flipping a coin (Kartha, 1994, pp. 386-387; Shanahan, 1999a, p. 419) and Raymond Reiter's scenario of throwing an object onto a chessboard (Baral, 1995, p. 2018; Elkan, 1996, p. 63; Kartha & Lifschitz, 1994, p. 344; F. Lin, 1996, p. 671; Shanahan, 1995a, pp. 264-265; 1997b, pp. 290-298; 1999a, pp. 419-420). Disjunctive event axioms were introduced by Shanahan (1997b, pp. 297-298, 342-345, 359-361). Our formalization of the example of running and driving using disjunctive event axioms is based on Shanahan's (1997b, pp. 359-361) formalization of Kartha's (1994, pp. 382-383) bus ride scenario.

EXERCISES

9.1 Formalize that a lightbulb burns out at a random time.

9.2 Formalize the operation of buttons and lights used in game shows. There are two contestants. Each contestant has a button and a light. The light goes on for the first contestant to push the button. If both contestants press their button at the same instant, the tie is broken at random and only one light goes on.

9.3 Extend the formalization in Exercise 9.2 for *n* contestants.

9.4 Formalize that a balloon may or may not pop when it comes into contact with a pin.

9.5 List three everyday scenarios involving nondeterminism, and sketch out event calculus representations of them.