Chapter 26

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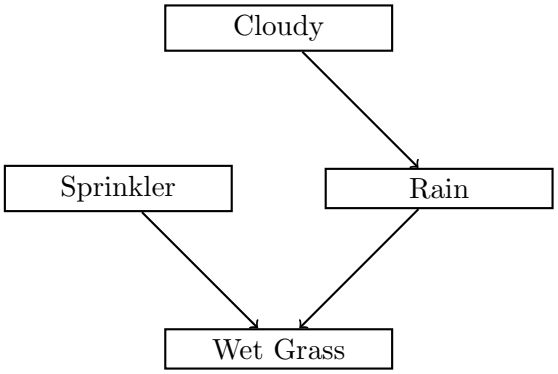
Comments and Proofs

Exercises

Exercise 1

a. By conditioning on $\text{do}(S = T)$, the graph becomes



The marginal for R uniform (but if we were in Vancouver, $p(R = T) = 1$).
Thus,

$$\begin{aligned}
p(W = T|\,\text{do}(S = T)) &= p(W = T, R = T|\,\text{do}(S = T)) + p(W = T, R = F|\,\text{do}(S = T)) \\
&= p(W = T|R = T, \text{do}(S = T))p(R = T|\,\text{do}(S = T)) \\
&\quad + p(W = T|R = F, \text{do}(S = T))p(R = F|\,\text{do}(S = T))) \\
&= p(W = T|R = T, \text{do}(S = T))p(R = T) \\
&\quad + p(W = T|R = F, \text{do}(S = T))p(R = F) \\
&= 0.9 \times 0.5 + 0.99 \times 0.5 \\
&= 0.945
\end{aligned}$$

b. Similarly, we have

$$\begin{aligned} p(W = T|\,\mathrm{do}(S = F)) &= p(W = T|R = T, \mathrm{do}(S = F))p(R = T) \\ &\quad + p(W = T|R = F, \mathrm{do}(S = F))p(R = F) \\ &= 0 \times 0.5 + 0.9 \times 0.5 \\ &= 0.45 \end{aligned}$$

c. Since C is the root of the DAG, performing do-calculus makes no difference. Thus

$$p(S = T|\,\mathrm{do}(C = T)) = p(S = T|C = T) = 0.1.$$

Unsurprisingly, we find out that sprinklers cause grass to become wet.