Chapter 20

stevenjin8

June 26, 2021

Comments and Proofs

I did not retain much from this chapter...

Exercises

Exercise 3

a. We first use Bayes law to "flip" the conditionals and then compute the marginals:

$$
\begin{aligned}
p(G_1|X_2 = 50) &\propto p(X_2 = 50|G_1)p(G_1) \\
&\propto p(X_2 = 50|G_1) \\
&\propto p(X_2 = 50|G_2 = 1)p(G_2 = 1|G_1) + p(X_2 = 50|G_2 = 2)p(G_2 = 2|G_1)
\end{aligned}
$$

Plugging in the numbers, we get $p(G_1|X_2 = 50) = [0.895, 1.05]$.
b. The key realization here is that $p(X_2|G1) = p(X_3|G_1)$ since the transition matrices are equal. Thus,

$$
\begin{aligned}
p(G_1 = 1|X_2 = 50, X_3 = 50) &= p(X_2 = 50, X_3 = 50|G_1 = 1) \\
&= p(X_2 = 50|G_1 = 1)p(X_3 = 50|G_1 = 1) \\
&= p(X_2 = 50|G_1 = 1)^2, \\
p(G_1|X_2 = 50, X_3 = 50) &= [0.986, 0.014].
\end{aligned}
$$

It makes sense that $p(G_1 = 1|X_2 = 50, X_3 = 50) > p(G_1 = 1|X_2 = 50)$, since in the former case, there is more evidence for $G_1 = 1$.
c. Now, there is equal evidence that $G = 0$ as there was for $G = 1$ in part b, and the prior distribution for G_1 is uniform. Thus, we can flip the distribution and $p(G_1|X_2 = 60, X_3 = 60) = [0.014, 0.986]$.
d. I think the author meant to ask for $p(G_1|X_2 = 50, X_3 = 60)$. If that is the case, there is equal evidence for $G = 1$ and $G = 0$. Thus, the posterior of G_1 is just its prior: $p(G_1|X_2 = 50, X_3 = 60) = [0.5, 0.5]$.