Digital Signal Processing

Edited by Stewart Nash

2026

ii

Preface

This material is extracted from "Digital Signal Processing: A System Design Approach", a text which was used heavily, if not exclusively, to create this library. The authors of the aforementioned text are David J. DeFatta, Joseph G. Lucas and William S. Hodgkiss. It will be useful to only extract the pertinent material from the text and replace the Fortran examples with examples in Python – a modern programming language for scientific computing – if the programming examples are used at all: the library speaks for itself as far as coding is concerned. It may be better to omit references altogether or perhaps just refer to specific parts of the library (if time allows).

Contents

Chapter 1

Introduction to Digital Signal Processing

Chapter 2

Discrete-Time Signal Analysis and Linear Systems

Chapter 3

The Z-Transform

Chapter 4

Infinite Impulse Response Digital Filter Design

An infinite impulse response (IIR) filter is characterized by an impulse response of infinite duration. We will discuss several methods of digital IIR filter design:

- Bilinear transformation

- Pole-zero placement

- Complex coefficients

- Computer aided design (CAD)

We will focus on bilinear transformation and treat the other methods only in passing. We will first discuss the bilinear transformation and then describe the filter design equations going from analog to digital filters.

4.1 Bilinear Transformation

One IIR digital filter (DF) design method is the bilinear transformation (BT) of classic analog filters. BT uniquely maps the entire left half of the s-plane into the interior of the unit circle in the z-plane. The design procedure is as follows:

1. Design formulas – generate analog poles and zeros of Butterworth, Chebyshev, and elliptic lowpass filters

2. Frequency band transformation formulas – converts analog lowpass filters into analog highpass, bandpass, and bandstop filters

3. Bilinear transformation – maps poles in the s-plane to poles in the z-plane

For the linear network to be causal and stable (1) the transfer function should be a rational function of s with real coefficients, (2) the poles of the analog filter must lie in the left half of the s-plane, and (3) the degree of the numerator (polynomial) must be less than or equal to the degree of the denominator (polynomial).

4.2   Design Equations

We refer to ω_p as the passband frequency and ω_s as the stopband frequency. In some texts, the passband frequency is called the cutoff frequency and denoted ω_0. The transition band is between ω_p and ω_s. Please note that the aforementioned frequencies can also be given as f_p and f_s, respectively, where $omega = 2\pi f$ is the angular frequency.

4.2.1   Butterworth Filters

(By definition) Butterworth filters have a magnitude response that is maximally flat in the passband. The magnitude squared function for an n-th order analog Butterworth filter is

$$|H_n(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2n}} \tag{4.1}$$

4.2.2   Chebyshev Filters

(By definition) A Chebyshev filter has a magnitude response that is equiripple in the passband and monotonic in the stopband for type 1 or monotonic in the passband and equiriple in the stopband for type 2. Regarding the analog filter, the analytic form for the squared magnitude function (for an n-th order filter) is

$$|H_n(j)|^2 = \frac{1}{1 + \varepsilon^2 T_n^2 \left(\frac{\omega}{\omega_p}\right)} \tag{4.2}$$

where T_n is the Chebyshev polynomial of the n-th order.

4.2.3 Elliptic Filters

(By definition) An elliptic filter has a magnitude response that is equiripple in both the passband and stopband. The magnitude response of the the filter is

$$|H_n(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 R_n^2(\omega)} \tag{4.3}$$

where R_n is the n-th order elliptic rational function (also known as the Chebyshev rational function). (Elliptic filters are optimum in the sense that for a given order and ripple specification it achieves the fastest transition between the passband and stopband - the narrowest transition bandwidth.)

4.3 Pole-Zero Placement

This design method involves direct placement of the poles and zeros in the z-plane to meet an arbitrary frequency response specification.

4.4 Complex Coefficients

4.5 CAD

This is a practical method for designing IIR digital filters with arbitrary, prescribed magnitude characteristics.

Chapter 5

Finite Impulse Response Digital Filter Design

A finite impulse response (FIR) filter is characterized by an impulse response of finite duration. An FIR filter may also be called a nonrecursive, moving average, transversal or tapped delay line filter. Let us consider the design techniques for FIR digital filters. These filters can be efficiently implemented in the frequency domain, but we will consider the time domain implementation of the FIR filter. These filters are usually solving by using Fourier series or numerical analysis techniques.

A disadvantage of FIR filters is that they must be higher order to achieve a specified magnitude response (as compared to IIR filters). Some characteristics (advantages) of FIR filters are

- FIR filters can be designed with exactly linear phase

- FIR filters realized nonrecursively are inherently stable

- Quantization noise can be negligible for nonrecursive realizations

- Coefficient accuracy problems inherent in sharp cutoff can be less severe

- FIR filters can be efficiently implemented in multirate DSP systems

The transfer function of an FIR causal filter is

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \qquad (5.1)$$

11

where $h(n)$ is the impulse response of the filter. The difference equation is obtained by taking the inverse Z-transform

$$y(iT) = \sum_{n=0}^{N-1} h(n)x(iT - nT) \tag{5.2}$$

which turns out to be a convolution summation. The Fourier transform of $h(n)$ is

$$y(iT) = \sum_{n=0}^{N-1} h(n)e^{-j\omega nT} = |H(e^{j\omega T})|e^{j\theta(\omega)} \tag{5.3}$$

which give the magnitude and phase response

$$M(\omega) = |H(e^{j\omega T})| \tag{5.4}$$

$$\theta(\omega) = \tan^{-1} \frac{-\operatorname{Im} H(e^{j\omega T})}{\operatorname{Re} H(e^{j\omega T})} \tag{5.5}$$

The phase delay and group (time) delay are

$$\tau_p = -\frac{\theta(\omega)}{\omega} \tag{5.6}$$

$$\tau_g = -\frac{d\theta(\omega)}{d\omega} \tag{5.7}$$

In linear phase (aka constant time delay) filters, τ_p and τ_g are constant. By definition, we have

$$\theta(\omega) = -\tau\omega \quad -\pi < \omega < \pi \tag{5.8}$$

We can show that FIR filters will have constant phase and group delays if

$$\tau = \frac{(N-1)T}{2} \tag{5.9}$$

$$h(n) = h[(N-1-n)] \quad 0 < n < N-1 \tag{5.10}$$

The symmetry conditions of the impulse response results in a transfer function that is a mirror-image polynomial.

Frequency response of constant-delay nonrecursive filters

$h(nT)$	N	$H(e^{j\omega T})$
Symmetrical	Odd	$e^{-j\omega(N-1)T/2} \sum_{k=0}^{(N-1)/2} a_k \cos \omega kT$
	Even	$e^{-j\omega(N-1)T/2} \sum_{k=1}^{N/2} b_k \cos [\omega(k-1/2)T]$
Antisymmetrical	Odd	$e^{-j[\omega(N-1)T/2-\pi/2]} \sum_{k=1}^{(N-1)/2} a_k \sin \omega kT$
	Even	$e^{-j[\omega(N-1)T/2-\pi/2]} \sum_{k=1}^{N/2} b_k \sin [\omega(k-1/2)T]$

$$a_0 = h\left[\frac{(N-1)T}{2}\right]$$
$$a_k = 2h\left[\left(\frac{N-1}{2} - k\right)T\right]$$
$$b_k = 2h\left[\left(\frac{N}{2} - k\right)T\right]$$

5.1 Fourier Series Design Method

The desired frequency response of an FIR digital filter can be represented by the Fourier series

$$H(e^{j2\pi fT}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j2\pi fnT} \tag{5.11}$$

The Fourier coefficients $h_d(n)$ are the desired impulse response sequence of the filter determined from

$$h_d(n) = \frac{1}{F}\int_{-F/2}^{F/2} H(e^{j2\pi fT})e^{j2\pi fnT}\,df \tag{5.12}$$

We substitute $e^{j\omega T} = z$ to obtain the transfer function

$$H(z) = \sum_{n=-\infty}^{\infty} h_d(n)z^{-n} \tag{5.13}$$

For N-odd we obtain

$$H(z) = z^{-(N-1)/2} \sum_{n=-(N-1)/2}^{(N-1)/2} h_d(n)z^{-n}$$
$$= z^{-(N-1)/2}\left[h_d(0) + \sum_{n=1}^{(N-1)/2} h_d(n)(z^n + z^{-n})\right] \tag{5.14}$$

A technique for the design of FIR digital filters is to multiply the desired impulse response $h_d(n)$ by a window function $a(n)$. (Window functions are a class of time-domain functions.)

$$h(n) = h_d(n)a(n) \tag{5.15}$$

We thus have

$$H_A(e^{j\omega T}) = \frac{1}{2\pi F}\int_0^{2\pi F} H(e^{j\omega T})A(e^{j(\omega-\Omega)T})\,d\Omega \tag{5.16}$$

5.1.1   Rectangular Window Function

The rectangular window function is

$$a_R(n) = \begin{cases} 1 & \text{for } |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5.17}$$

Its Fourier transform is

$$\begin{aligned} A_R(e^{j\omega T}) &= \sum_{n=-(N-1)/2}^{(N-1)/2} e^{-j\omega nT} \\ &= \frac{\sin(\omega NT/2)}{\sin(\omega T/2)} \end{aligned} \tag{5.18}$$

The causal rectangular window is

$$\begin{aligned} A_R(e^{j\omega NT}) &= \sum_{n=0}^{N-1} e^{-j\omega nT} \\ &= e^{-j\omega(N-1)T/2} \frac{\sin(\omega NT/2)}{\sin(\omega T/2)} \end{aligned} \tag{5.19}$$

5.1.2   Hamming Window Function

The Hamming window function is

$$a_H(n) = \begin{cases} 0.54 + 0.46\cos\frac{2\pi n}{N-1} & \text{for } |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5.20}$$

Note that

$$a_H(n) = a_R(n)\left[0.54 + 0.46\cos\frac{2\pi n}{N-1}\right] \tag{5.21}$$

$$\begin{aligned} A_H(e^{j\omega T}) &= 0.54\frac{\sin(\omega NT/2)}{\sin(\omega T/2)} \\ &\quad + 0.46\frac{\sin[\omega NT/2 - N\pi/(N-1)]}{\sin[\omega T/2 - \pi/(N-1)]} \\ &\quad + 0.46\frac{\sin[\omega NT/2 + N\pi/(N-1)]}{\sin[\omega T/2 + \pi/(N-1)]} \end{aligned} \tag{5.22}$$

5.1.3 Blackman Window Function

The noncausal Blackman window function is given by

$$a_B(n) = \begin{cases} 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}, & \text{for } |n| < \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5.23}$$

5.1.4 Kaiser Window Function

The Kaiser window function uses functions which approximate the prolate spheroidal functions (which are optimal in a certain sense). The functions given by Kaiser are in terms of the zero-order modified Bessel functions of the first kind, $I_0(x)$. The formula for the Kaiser function is

$$a_K(n) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)} & \text{for } |n| \leq \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5.24}$$

where α is an independent variable empirically determined by Kaiser and the parameter β is given by

$$\beta = \alpha \left[1 - \left(\frac{2n}{N-1} \right) \right]^{0.5} \tag{5.25}$$

The modified Bessel function of the first kind is

$$I_0(x) = 1 + \sum_{n=1}^{\infty} \left[\frac{1}{k!} \left(\frac{x}{2} \right)^k \right]^2 \tag{5.26}$$

The spectrum of the Kaiser window is given by

$$\sum_{(N-1)/2}^{-(N-1)/2} a_K(n) e^{-j\omega nT} = a_k(0) + 2 \sum_{n=1}^{(N-1)/2} a_K(n) \cos \omega nT \tag{5.27}$$

The actual passband peak-to-peak ripple A_p is

$$A_p = 20 \log_{10} \frac{1 + \delta_p}{1 - \delta_p} \tag{5.28}$$

The minimum stopband attenuation A_s is

$$A_s = -20 \log_{10} \delta_s \tag{5.29}$$

The transition bandwidth is

$$\Delta F = f_s - f_p \tag{5.30}$$

The specified passband ripple is A'_p. The minimum stopband attenuation is A'_s. We have

$$A_p \leq A'_p \tag{5.31}$$

$$A_s \geq A'_s \tag{5.32}$$

5.1.5   FIR Filter Design with the Kaiser Window Function

This section will describe how to design an FIR filter using the Kaiser window function. First, we must obtain the design specifications:

1. Filter type: LP, HP, BP, BS

2. Critical passband and stopband frequencies in hertz

   - LP/HP: f_p and f_s
   - BP/BS: f_{p1}, f_{p2}, f_{s1}, and f_{s2}

3. Passband ripple and minimum stopband attenuation in positive decibels: A'_p and A'_s

4. Sampling frequency in hertz: F

5. Filter order (N)-odd

 The design procedure is as follows:

1. Determine δ (the actual design parameter)

$$\delta = \min(\delta_p, \delta_s) \tag{5.33}$$

$$\delta_s = 10^{-0.05A'_s} \tag{5.34}$$

$$\delta_p = \frac{10^{0.05A'_p} - 1}{10^{0.05A'_p} + 1} \tag{5.35}$$

2. Calculate A_s

3. Determine the parameter α from the empirical design equation

$$\alpha = \begin{cases} 0 & \text{for } A_s \leq 21 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21) & \text{for } 21 < A_s \leq 50 \\ 0.1102(A_s - 8.7) & \text{for } A_s > 50 \end{cases} \tag{5.36}$$

4. Determine the parameter D from the empirical design equation

$$D = \begin{cases} 0.9222 & \text{for } A_s \leq 21 \\ \frac{A_s - 7.95}{14.36} & \text{for } A_s > 21 \end{cases} \tag{5.37}$$

5. Calculate the filter order for the lowest odd value of N

$$N \geq \frac{FD}{\Delta F} + 1 \tag{5.38}$$

6. COmpute the modified impulse response

$$h(n) = a_k(n)h_d(n) \quad \text{for } |n| \leq \frac{N-1}{2} \tag{5.39}$$

7. The transfer function is

$$H(z) = z^{-(N-1)/2} \left[h(0) + 2 \sum_{n=0}^{(N-1)/2} h(n)(z^n + z^{-n}) \right] \tag{5.40}$$

$$h(0) = a_K(0)h_d(0) \tag{5.41}$$

$$h(n) = a_K(n)h_d(n) \tag{5.42}$$

The magnitude response is

$$M(\omega) = h(0) + 2 \sum_{n=0}^{(N-1)/2} h(n) \cos 2\pi f n T \tag{5.43}$$

The FIR filter design equations are as follows:

- Lowpass FIR filter

$$h_d(n) = \begin{cases} \left(\frac{2f_c}{F}\right) \frac{\sin 2\pi n f_c / F}{2\pi n f_c / F} & \text{for } n > 0 \\ \frac{2f_c}{F} & \text{for } n = 0 \end{cases} \tag{5.44}$$

$$f_c = 0.5(f_p + f_s) \tag{5.45}$$

$$\Delta F = f_s - f_p \tag{5.46}$$

- Bandpass FIR filter

$$h_d(n) = \begin{cases} -\left(\frac{2f_c}{F}\right) \frac{\sin 2\pi n f_c / F}{2\pi n f_c / F} & \text{for } n > 0 \\ 1 - 2f_c / F & \text{for } n = 0 \end{cases} \tag{5.47}$$

$$f_c = 0.5(f_p + f_s) \tag{5.48}$$

$$\Delta F = f_p - f_s \tag{5.49}$$

- Bandpass FIR filter

$$h_d(n) = \begin{cases} \frac{1}{n\pi}[\sin(2\pi n f_{c2}/F) - \sin(2\pi n f_{c1}/F)] & \text{for } n > 0 \\ \frac{2}{F}(f_{c2} - f_{c1}) & \text{for } n = 0 \end{cases} \qquad (5.50)$$

$$f_{c1} = f_{p1} - \frac{\Delta F}{2} \qquad (5.51)$$

$$f_{c2} = f_{p2} + \frac{\Delta F}{2} \qquad (5.52)$$

$$\Delta F_1 = f_{p1} - f_{s1} \qquad (5.53)$$

$$\Delta F_h = f_{s2} - f_{p2} \qquad (5.54)$$

$$\Delta F = \min[\Delta F_1, \Delta F_h] \qquad (5.55)$$

- Bandstop FIR filter

$$h_d(n) = \begin{cases} \frac{1}{n\pi}[\sin(2\pi n f_{c1}/F) - \sin(2\pi n f_{c2}/F)] & \text{for } n > 0 \\ \frac{2}{F}(f_{c1} - f_{c2}) + 1 & \text{for } n = 0 \end{cases} \qquad (5.56)$$

$$f_{c1} = f_{p1} + \frac{\Delta F}{2} \qquad (5.57)$$

$$f_{c2} = f_{p2} - \frac{\Delta F}{2} \qquad (5.58)$$

$$\Delta F_1 = f_{s1} - f_{p1} \qquad (5.59)$$

$$\Delta F_h = f_{p2} - f_{s2} \qquad (5.60)$$

$$\Delta F = \min[\Delta F_1, \Delta F_h] \qquad (5.61)$$

5.2   Examples

Example 5.2.1.   Design an FIR lowpass digital filter with the following specifications

Example 5.2.2.   Design an FIR bandpass filter with the following specifications

Chapter 6

The Discrete Fourier Transform and Fast Fourier Transform Algorithms

Chapter 7

Multirate Digital Signal Processing

Chapter 8

Response of Linear Systems to Discrete-Time Random Processes, Power Spectrum Estimation, and Detection of Signals in Noise

8.1   Random Processes

In this section we look at the effect of a random process (RP) on a a system. A wide-sense stationary (WSS) process only requires that the first and second moments are not function of time, and that the autocorrelation function depends only on the time difference. An ergodic random process requires that any statistic calculated by averaging over all members of an ergodic ensemble at a a fixed time can be calculated by averaging over all time on a single representative member of the ensemble; that is, time averages equal ensemble averages.

Let the autocorrelation functions (ACF) of the input and output processes

be given by $\phi_{xx}(m)$ and $\phi_{yy}(m)$, or

$$\phi_{xx}(m) = E[x(n)x(n+m)] = \lim_{x\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)x(n+m) \qquad (8.1)$$

$$\phi_{yy}(m) = E[y(n)y(n+m)] = \lim_{x\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} y(n)y(n+m) \qquad (8.2)$$

If the process is assumed to be WSS, the ACF depends only on the time difference; in that case, the system output process ACF is given by

$$\phi_{yy}(m) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h(i)h(i+j)\phi_{xx}(m-j) \qquad (8.3)$$

The CCF of two discrete-time random processes $x(n)$ and $y(n)$ that are jointly WSS RPs is

$$\phi_{xy}(m) = E[x(n)y(n+m)] \qquad (8.4)$$

The CCF of the input $x(n)$ and response $y(n)$ of a discrete-time linear system can then be given as

$$\phi_{xy}(m) = \sum_{j=-\infty}^{\infty} h(j)\phi_{xx}(m-j) \qquad (8.5)$$

The autocovariance (ACVF) and cross-covariance (CCVF) functions of two stationary RP $x(n)$ and $y(n)$ are given by

$$\gamma_{xx}(m) = E\{[x(n) - m_x][(x(n+m) + m_x)]\} = \phi_{xx}(m) - m_x^2 \qquad (8.6)$$
$$\gamma_{xy}(m) = E\{[x(n) - m_x][(y(n+m) - m_y)]\} = \phi_{xy}(m) - m_x m_y \qquad (8.7)$$

respectively. The Z-transform of $\gamma_{xx}(m)$ and $\gamma_{xy}(m)$ are given by $\Gamma_{xx}(z)$ and $\Gamma_{xy}(z)$, respectively. The Z-transforms exist only when $m_x = -$.

8.2   Power Spectra

The Z-transform and the inverse Z-transform of the ACF of a zero-mean WSS discrete-time RP $x(n)$ form a transform pair $\mathbf{\Phi}_{xx}(z) \leftrightarrow \phi_{xx}(m)$ as shown by

$$\mathbf{\Phi}_{xx}(z) = \sum_{m=-\infty}^{\infty} \phi_{xx}(m)z^{-m} \qquad (8.8)$$

$$\phi_{xx}(m) = \frac{1}{2\pi j} \oint_c \mathbf{\Phi}_{xx}(z)z^{m-1}\,dz \qquad (8.9)$$

where the power spectral density (PSD) is defined as the Z-transform of the ACF with $z = e^{j2\pi fT}$ or

$$P_{xx}(f) = \mathbf{\Phi}_{xx}(e^{j2\pi fT}) = \sum_{m=-\infty}^{\infty} \phi_{xx}(m)e^{j2\pi fT} \tag{8.10}$$

The Z-transform of the CCF is given by

$$\mathbf{\Phi}_{xy}(z) = \sum_{m=-\infty}^{\infty} \phi_{xy}(m)z^{-m} \tag{8.11}$$

The cross-power spectral density (CPSD) of two functions is the Z-transform of their CCF with $z = e^{j2\pi fT}$ or

$$P_{xy}(f) = \mathbf{\Phi}_{xy}(e^{j2\pi fT}) = \sum_{m=-\infty}^{\infty} \phi_{xy}(m)e^{j2\pi fT} \tag{8.12}$$

For a linear system with response $y(n)$, input $x(n)$ and impulse response $h(n)$, we can show

$$\mathbf{\Phi}_{yy}(z) = H(z)H(z^{-1})\mathbf{\Phi}_{xx}(z) \tag{8.13}$$

$$P_{yy}(f) = \mathbf{\Phi}_{yy}(e^{j2\pi fT}) = |H(e^{j2\pi fT})|\mathbf{\Phi}_{xx}(e^{j2\pi fT}) \tag{8.14}$$

This says that the the PSD of the output process of a discrete-time linear system is equal to the PSD of the input process times the squared magnitude response of the system. We can then show that the CPSD is

$$\mathbf{\Phi}_{xy}(z) = H(z)\mathbf{\Phi}_{xx}(z) \tag{8.15}$$

$$P_{xy}(f) = H(e^{j2\pi fT})\mathbf{\Phi}_{xx}(e^{j2\pi fT}) = H(e^{j2\pi fT})P_{xx}(f) \tag{8.16}$$

Consider two linear time-invariant systems with outputs $v(nT)$ and $w(nT)$, respectively, inputs $x(nT)$ and $y(nT)$, respectively, and impulse responses $h_1(n)$ and $h_2(n)$, respectively. The CPSD is given by

$$\mathbf{\Phi}_{vw}(z) = H_1(z)H_2(z^{-1})\mathbf{\Phi}_{xy}(z) \tag{8.17}$$

8.3 Noise

The ACF and PSD of white noise are expressed by

$$\phi_{xx}(m) = \sigma_x^2\delta(m) \tag{8.18}$$

$$P_{xx}(f) = \mathbf{\Phi}_{xx}(e^{j2\pi fT}) = \sigma_x^2 \tag{8.19}$$

Therefore, the response of a digital filter to a white-noise input is

$$P_{yy}(f) = \sigma_x^2 |H(e^{j2\pi fT})|^2 \tag{8.20}$$

With an input with zero mean and the variance (second moment) σ_x^2, the response of a system with an impulse response $h(n)$ is

$$\sigma_y^2 = \sigma_x^2 \sum_{n=0}^{\infty} h^2(n) \tag{8.21}$$

Suppose that we would like to determine the average output power of a filter to a white-noise random process with zero mean and variance σ_x^2. For the ACF with $m = 0$, we find the average power in the input is

$$\phi_{xx}(0) = E[x^2(n)] = \frac{1}{2\pi j} \oint_c \Phi_{xx}(z) z^{m-1} \, dz = \sigma_x^2 \tag{8.22}$$

The average output power is then given by

$$\phi_{yy}(0) = \frac{1}{2\pi j} \oint_c \sigma_x^2 H(z^{-1}) H(z) z^{-1} \, dz \tag{8.23}$$

Chapter 9

Finite Register Length Effects in Digital Signal Processing

"DSP system design requires a thorough analysis of finite arithmetic effects to ensure that system performance is not degraded."

9.0   Introduction

Input signals encountered by sonar or radar signal processing systems are continuous and must first be sampled in time and digitized in amplitude prior to processing by digital systems. Since the hardware complexity of digital signal processing systems is directly related to the digital wordlength, it is important to limit the number of bits in the various processing elements of the system. This requires an error analysis at each point in the system to assure that the system implementation does not degrade system performance, that is, degrade dynamic range by increasing the system noise level.

It is assumed in the following sections that you are familiar with binary number representation. The following are some of the issues related to the implementation of algorithms in special-purpose hardware.

1. Analog-to-digital conversion.

2. IIR digital filter finite wordlength effects.

    (a) Product-quantization errors.

      (b) Coefficient-quantization errors.

      (c) Dynamic range considerations.

      (d) Zero input limit cycle behavior.

3. FIR digital filter wordlength effects.

4. FFT finite wordlength effects.

The error analysis presented herein will concentrate primarily on fixed-point arithmetic operations. The approach, however, can be extended to analyze the effects of floating-point operations.

9.1 Quantization Noise Introduced by Analog-to-Digital Conversion

Chapter 10

Signal Processing System Design

"Our signal processing system design methodology provides a systematic approach to specifying requirements, analyzing signals, and developing signal processing designs. The methodology results in efficient signal processing hardware and software implementations."

10.0   Introduction

The digital signal processing (DSP) techniques presented in the preceding chapter have application to many diverse fields such as acoustics, radar, sonar, seismology, speech, data communication, and biomedical engineering. This chapter presents a DSP system design methodology and illustrates the use of the methodology in solving DSP system application problems. The methodology uses design and analysis approaches discussed in the preceding chapters. Implementation of the methodologies using computer-aided design and analysis tools provide an efficient approach to handling complex application designs.

Chapter 11

Adaptive Filtering

"Filters that adapt to a changing environment are discussed. The stochastic Wiener filtering and deterministic least-squares problems are set up, and the similarity between them is pointed out. First, a block-processing approach is taken in the solution to these two problems. Second, a fading memory (recursive) approach is taken. Then the application of these algorithms to the adaptive beamforming (ABF) problem is considered. The chapter appendix provides background material on conventional beamforming (CBF)."

11.0   Introduction

Numerous applications exist that require a linear filtering operation. Often, the nature of that filtering task is time-varying in some nondeterministic fashion owing to nonstationarity of the underlying time series. In such situations, a filter that can adapt to a changing environment is needed. Examples include adaptive noise canceling, line enhancing, frequency tracking, and channel equalization. Beyond the discussion here, additional material on adaptive filtering can be found in references 1 through 16.

11.1   The Stochastic Wiener Filtering and Deterministic Least Squares Problem

The problem of interest is described in Figure 11.1. The goal is to filter the time series $x[n]$ with a causal, finite impulse response (FIR) digital filter

to yield an estimate of the time series $s[n]$. The error in that estimate is denoted $e[n]$

$$e[n] = s[n] - \hat{s}[n] = s[n] + \sum_{k=0}^{N} h_k x[n - k] \qquad (11.1)$$

Written in matrix form, Eq. 11.1 becomes

$$e[n] = s[n] + [h_0\ h_1\ \ldots\ h_N] \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-N] \end{bmatrix} \qquad (11.2)$$

or, in vector notation,

$$e[n] = s[n] + \mathbf{h}^T \mathbf{x} \qquad (11.3)$$