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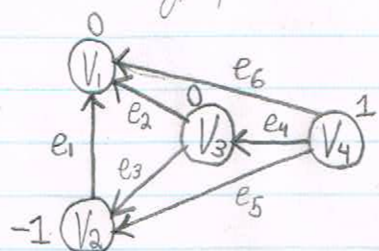
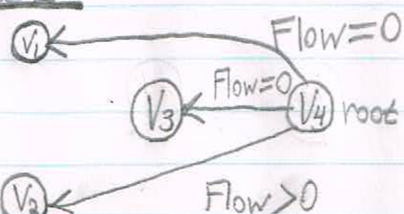
MATH 3802 Assignment #4:

① Consider the digraph $G=(\mathcal{N},\mathcal{A})$ & b depicted below:

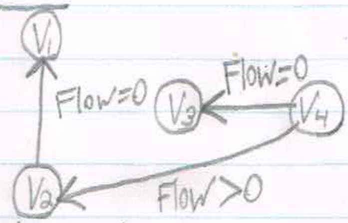


a) (2 Points) Give two feasible tree solutions determined by strongly feasible trees (with V_4 as the root):

Tree I:

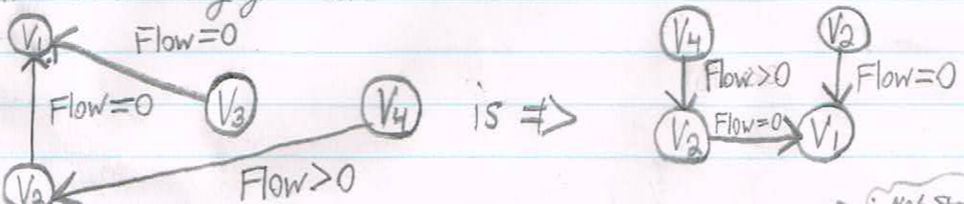


Tree II:



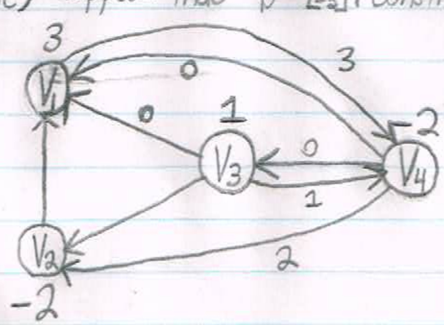
In both trees I & II, we can see that (Flow=0) arcs are orientated away from the root, these are strongly feasible trees.

b) (2 Points) Give one feasible tree solution that is not determined by a strongly feasible tree. Justify your answer:

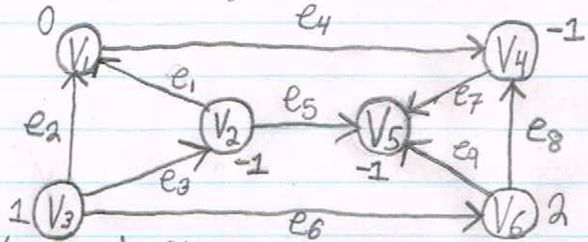


As can be seen, V_2 to V_1 is an arc of flow=0 orientated towards the root. ∴ Not strongly feasible trees!

c) (1 Point) Suppose that $b=\begin{bmatrix}3\\-2\\1\\-2\end{bmatrix}$. Construct the auxiliary network for G, b:

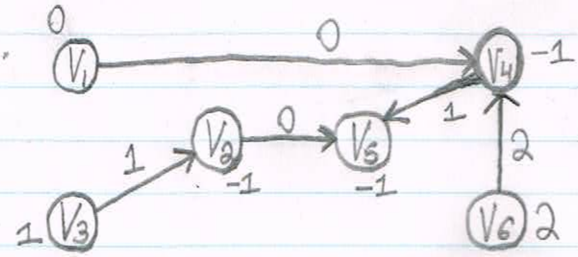


② Consider the digraph $G=(\mathcal{N}, A)$ and b depicted below:



let $T=\{e_3, e_4, e_5, e_7, e_8\}$

a) (1 Point) Give the tree Solution determined by the tree with arc-set T:



$\Rightarrow$ thus, $x^* = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

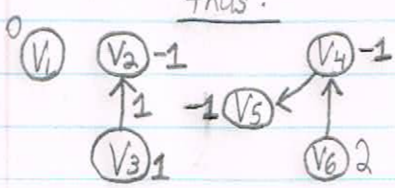
b) (2 Point) Construct the Set $S(T)$ of nodes joined by a path (not necessarily directed) to V_6 using only arcs in T that are not bad and Conclude that the network is decomposable:

e_4 is bad since it is a Zero Path Pointing towards the root

e_5 is also bad for the Same reason

$\Rightarrow$ Since e_5 is bad, we Can't use e_3 as it uses e_5 as an intermediate to the root

thus:



Works, So $S(T)=\{e_7, e_8\}$

$\Sigma b_i = 0$ & $S(T) \neq \emptyset$. Now $S(T)=\mathcal{N}$ So we are done.

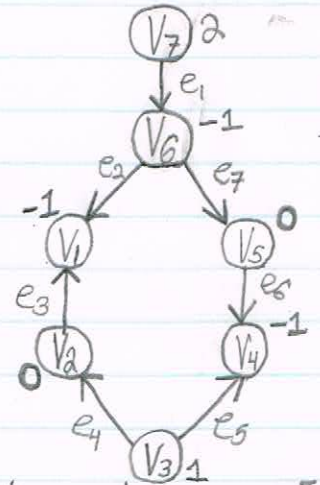
thus, the problem $S(T)$ Can be decomposed as $\delta(T) \cap T \neq \emptyset$

So, $\delta^+(T)=\emptyset$ meaning T was decomposable.

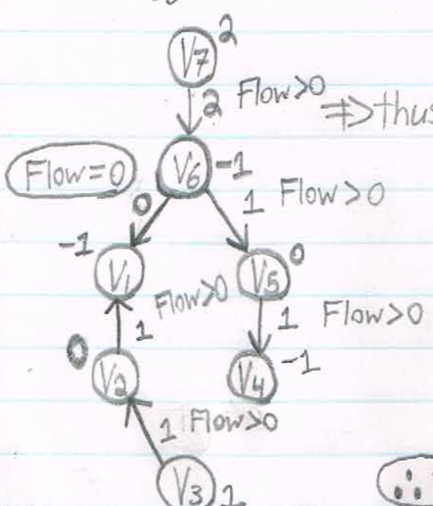
This leaves us with the decomposed graph above, with V_3 & V_2 & V_1 removed from the roots Segment & $S(T)=\{e_7, e_8\}$

$\boxed{\mathcal{N}=S(T)=\{e_7, e_8\} \neq T \therefore \text{the Network's decomposable}}$

③ Consider the digraph $G=(\mathcal{N},\mathcal{A})$ and b depicted below:



a) (1 Point) Let $T=\{e_1,e_2,e_3,e_4,e_6,e_7\}$. Show that the tree with arc-set T is a Strongly feasible tree (with V_7 as the root) determining a feasible tree Solution:



$\Rightarrow$ thus; $x^* = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

— Since the arc with Flow=0 (arc e_2) is orientated away from the root (V_7), this is a Strongly feasible tree.

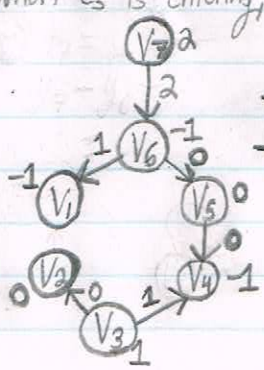
— Since all $x_i \geq 0$ for all i from 1 to 7 in x^*, we see that there is a feasible flow. Since T has $n-1=6$ arcs, it is a tree. Thus, T determines a feasible tree Solution.

∴ T is both a Strongly feasible tree & has a feasible tree Solution

b) (1 Point) Suppose that the tree Solution in the previous part is encountered in an iteration of the Network Simplex Method. If e_5 is Chosen as the entering arc, what would be Chosen as the leaving arc if Cunningham's anti-cycling rule is used:

$T=\{e_1,e_2,e_3,e_4,e_6,e_7\}$

When e_5 is entering, we see that $C=\{e_2,e_3,e_4,e_5,e_6,e_7\}$, So:



— e_3 & e_6 have the Same flow value of 1

— We encounter e_3 when walking along the tree, thus drop it

↳ The tree is Still Strongly feasible as all leaving arcs are orientated away from the root when their Flows equal Zero.

∴ New $T=\{e_1,e_2,e_4,e_5,e_6,e_7\}$