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MATH 3801 Problem Set Three:

① let (P) denote the following linear Programming Problem:

$$\begin{aligned} \min\ & 2x_1 + 3x_3 \\ \text{s.t. } & x_1 + x_2 + 3x_3 \geq 6 \\ & -x_1 + 2x_2 + x_3 \leq 1 \\ & 3x_2 - 4x_3 = 0 \\ & x_1, \quad x_3 \geq 0 \\ & x_2 \text{ is free} \end{aligned}$$

a) Write down the dual Problem for (P):

we know:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 2 & 1 \\ 0 & 3 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}, \quad \& \quad c = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

So, for the duel Problem we use b as the objective, c for constraints, & flip min to max

$$\boxed{\begin{aligned} \max\ & 6y_1 + y_2 \\ \text{s.t. } & y_1 - y_2 \leq 2 \\ & y_1 + 2y_2 + 3y_3 = 0 \\ & 3y_1 + y_2 - 4y_3 \leq 3 \\ & y_1 \geq 0 \\ & y_2 \leq 0 \\ & y_3 \text{ is free} \end{aligned}}$$

We also get constraint variables by selecting columns from A.

P(min) to D(max)

≤Var → ≥ Const.

≥Var → ≤ Const.

=Const → free

(P) Since x_1 & $x_3 \geq 0$:

(D) Const. 1 & 3 is ≤ Const.

(P) x_2 is free so:

(D) Constraint 2's = Const.

(P) Since Consts. are ≥, ≤, =:

y_1 is ≥

y_2 is ≤

y_3 is free

b) With the help of software, find an optimal solution to (P) & its duel. Express all numbers as exact rational values, not decimals:

(P) After running in microsoft excel, we see:

$x_1 = \frac{53}{24}, x_2 = \frac{7}{6}, x_3 = \frac{7}{8}$ is the optimal variable values for a minimum of $2\left(\frac{53}{24}\right) + 0\left(\frac{7}{6}\right) + 3\left(\frac{7}{8}\right) = \boxed{\frac{169}{24}}$

(D) After running in Microsoft Excel, we see:

$x_1 = \frac{31}{24}, x_2 = -\frac{17}{24}, x_3 = \frac{1}{24}$

optimal variables values for a maximum of $6(^{31}/_{24}) + 1\left(-\frac{17}{24}\right) + \cancel{0\left(\frac{1}{24}\right)} = \boxed{\frac{169}{24}}$

Primal Problem Linear Program

Variables	x1	x2	x3	value		
Solution	2.20833333	1.16666667	0.875			
Objective	2	0	3	7.04166667		
Constraint 1	1	1	3	6	>=	6
Constraint 2	-1	2	1	1	<=	1
Constraint 3	0	3	-4	-8.882E-16	=	0
Constraint 4	1	0	0	2.20833333	>=	0
Constraint 5	0	0	1	0.875	>=	0

Primal Problem Answer Report

Microsoft Excel 16.0 Answer Report
Worksheet: [lp (1).xlsx]Sheet1
Report Created: 10/20/2020 5:59:39 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.031 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$E$3	Objective value	0	7.041666667

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Solution x1	0	2.208333333	Contin
$C$2	Solution x2	0	1.166666667	Contin
$D$2	Solution x3	0	0.875	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$5	Constraint 1 value	6	$E$5>=$G$5	Binding	0
$E$6	Constraint 2 value	1	$E$6<=$G$6	Binding	0
$E$7	Constraint 3 value	-8.88178E-16	$E$7=$G$7	Binding	0
$E$8	Constraint 4 value	2.208333333	$E$8>=$G$8	Not Binding	2.208333333
$E$9	Constraint 5 value	0.875	$E$9>=$G$9	Not Binding	0.875

Duel Problem Linear Program

Variables	x1	x2	x3	value		
Solution	1.291667	-0.70833	0.041667			
Objective	6	1	0	7.041667		
Constraint	1	-1	0	2	<=	2
Constraint	1	2	3	-1.9E-16	=	0
Constraint	3	1	-4	3	<=	3
Constraint	1	0	0	1.291667	>=	0
Constraint	0	1	0	-0.70833	<=	0

Microsoft Excel 16.0 Answer Report
Worksheet: [lp (1).xlsx]Sheet2
Report Created: 10/20/2020 6:10:06 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.046 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$3	Objective value	0	7.041666667

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Solution x1	0	1.291666667	Contin
$C$2	Solution x2	0	-0.708333333	Contin
$D$2	Solution x3	0	0.041666667	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$5	Constraint 1 value	2	$E$5<=$G$5	Binding	0
$E$6	Constraint 2 value	-1.94289E-16	$E$6=$G$6	Binding	0
$E$7	Constraint 3 value	3	$E$7<=$G$7	Binding	0
$E$8	Constraint 4 value	1.291666667	$E$8>=$G$8	Not Binding	1.291666667
$E$9	Constraint 5 value	-0.708333333	$E$9<=$G$9	Not Binding	0.708333333

② let (P) denote the following linear programming Problem:

$$\min \quad 2x_1 + 3x_2$$
$$\text{s.t.} \quad 2x_1 + 3x_2 + x_3 = 10$$
$$x_1 + 2x_2 = 5$$
$$x_1, x_2, x_3 \geq 0$$

a) Obtain the tableau associated with the basis $B = \{1,2\}$:

Convert to system:

$$Z - 2x_1 - 3x_2 = 0$$
$$2x_1 + 3x_2 + x_3 = 10$$
$$x_1 + 2x_2 = 5$$

Set Columns 1 & 2 as Pivots:

$$\left[\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 0 \\ 0 & 2 & 3 & 1 & 10 \\ 0 & 1 & 2 & 0 & 5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 10 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 5 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

thus,

$$Z \quad + x_3 = 10$$
$$x_1 \quad + 2x_3 = 5$$
$$x_2 - x_3 = 0$$

is the tabeau associated with the basis

b) Preform one complete iteration of the tableau starting with the tableau in the Previous Part:

We know $B = \{1,2\}$:

$$x_1 \begin{vmatrix} 1 \\ 0 \end{vmatrix} + x_2 \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 5 \\ 0 \end{vmatrix} \quad \text{thus, } 5 \begin{vmatrix} 1 \\ 0 \end{vmatrix} + 0 \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 5 \\ 0 \end{vmatrix}$$

$$x^* = \begin{vmatrix} 5 \\ 0 \end{vmatrix}$$

Choose $K=3$ as the coefficient of x_3 in the Z-Row's Positive:

$\overline{a_{1,3}} = 2$ & $\cancel{\overline{a_{2,3}} = -1}$

↳ Choose $\overline{a_{1,3}}$ as it's the only Positive coefficient

Since $B=\{1\}$, $\bar{b}_i=\{5\}$ for $i=1$, therefore:

$$\min\left\{\left[\frac{\bar{b}_1}{\bar{a}_{1,3}}\right]\right\}=\min\left\{\left[\frac{5}{2}\right]\right\}=\frac{5}{2} \quad \therefore \underline{r=1}$$

So, the new basis has r replaced with K

new basis is: $B=\{2,3\}$

(3) Find all basic feasible Solutions to the System:

$$4x_1+x_2+x_3-x_4=2$$
$$3x_1+x_2+6x_4=2$$
$$x_1,x_2,x_3,x_4 \geq 0$$

For each basic feasible Solution, list all the basis that determine it:

We Know:

$$A=\begin{bmatrix}4&1&1&-1\\3&1&0&6\end{bmatrix},\ b=\begin{bmatrix}2\\2\end{bmatrix}$$

Since A has two Rows, for all B of length 2:

Basis	Basic Solution
$\{1,2\}$	$\begin{bmatrix}0\\2\\0\\0\end{bmatrix}$
$\{1,3\}$	$\begin{bmatrix}2/3\\0\\-2/3\\0\end{bmatrix}$ $\rightarrow x_3<0$ So false
$\{1,4\}$	$\begin{bmatrix}14/27\\0\\0\\2/27\end{bmatrix}$ $\rightarrow$ Since: $\left\|\begin{matrix}4&-1\\3&6\end{matrix}\right\|\begin{matrix}2\\2\end{matrix}\right\| \rightsquigarrow \left\|\begin{matrix}1&0\\0&1\end{matrix}\right\|\begin{matrix}14/27\\2/27\end{matrix}\right\|$
$\{2,3\}$	$\begin{bmatrix}0\\2\\0\\0\end{bmatrix}$
$\{2,4\}$	$\begin{bmatrix}0\\2\\0\\0\end{bmatrix}$
$\{3,4\}$	$\begin{bmatrix}0\\0\\7/3\\1/3\end{bmatrix}$

Since $x_i \geq 0$ for i from 1 to 4, the basic feasible Solutions are:

$$\begin{bmatrix}0\\2\\0\\0\end{bmatrix},\begin{bmatrix}14/27\\0\\0\\2/27\end{bmatrix},\begin{bmatrix}0\\0\\7/3\\1/3\end{bmatrix}$$