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MATH 3801 Problem Set One:

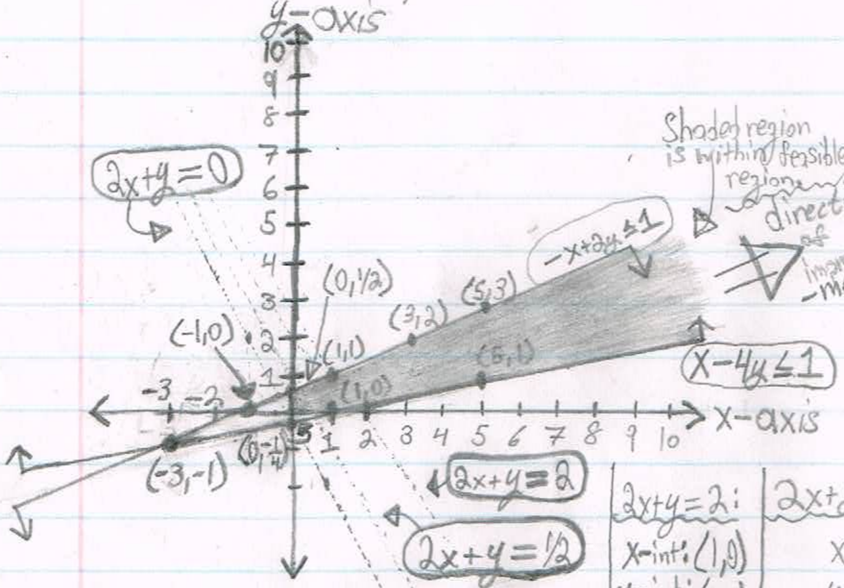
① Consider the linear programming problem:

$$\max \; 2x+y$$
$$\text{s.t. } -x+2y \le 1$$
$$x-4y \le 1$$
$$x,y \geq 0$$

$-2z+z=-z$

$2z-2z=0$

a) Give a hand drawn sketch of the feasible region. Label the constraints & the intercepts:



line: $-x+2y\le1$
x-int: $(-1,0)$
y-int: $(0,\frac{1}{2})$

line: $x-4y\le1$
x-int: $(1,0)$
y-int: $(0,-1/4)$

the lines intercept at: $(-3,-1)$

$2x+y=2$:	$2x+y=1/2$:	$2x+y=0$:
x-int: $(1,0)$	x-int: $(1/4,0)$	x-int: $(0,0)$
y-int: $(0,2)$	y-int: $(0,1/2)$	y-int: $(0,0)$

b) Show **algebraically** that the problem **is** unbounded:

the line defined by $2x+y=z$ has an x-intercept of $z/2$

For $z\geq0$, $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2z\\z/2\end{bmatrix}$ satisfies both inequalities & the value of the objective function at $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2z\\z/2\end{bmatrix}$ is z.

Note:

Since $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2z\\z/2\end{bmatrix}$ is subject to:

$-x+2y\le1$; $x-4y\le1$

then;

$-2z+2(z/2)=-z\le1$; $2z-4(z/2)=0\le1$

Since $0\le1$ is always true $x-4y\le1$ is satisfied &

$-z\le1$ is always true since $z\geq0$

Also, as $x,y\geq0$, z must be greater or equal to 0 or else the line will NOT be in the feasible region thus:

As $Z \to \infty$ the objective function tends to ∞ ✓

$\therefore$ there is no maximum value for the objective function

$\therefore$ The Problem is unbounded

(2) Convert the optimization Problem:

$$\min \; 8^x \cdot 2^{-y}$$
$$\text{s.t.} \quad 3^{4x-y} \geq 9$$
$$|9x+y| \leq 7$$

to an equivalent linear programming problem:

the Constraint $|9x+y| \leq 7$ is equivalent to:

$9x+y \leq 7$ & $9x+y \geq -7$

taken together, & the Constraint $3^{4x-y} \geq 9$ is equivalent to $4x-y \geq \log_3(9)$ Since e^u is an increasing function in u with $\log_3 9 = 2$ thus $4x-y \geq 2$

Also, minimizing $8^x \cdot 2^{-y}$ is the Same as minimizing $2^{3x} \cdot 2^{-y} = 2^{3x-y}$ which is the Same as minimizing $3x-y$ Since 2^u is an increasing function in u.

So, the equivalent is:

$$\text{minimize} \; 3x-y$$
$$\text{s.t.} \quad 4x-y \geq 2$$
$$9x+y \leq 7$$
$$9x+y \geq -7$$

(3) the following Sheet Shows the Solution to the linear Programming Problem:

$$\min \; 4x_1+9x_2+1x_3+x_4$$
$$\text{s.t.} \quad x_1+x_2+x_3+x_4 = 7$$
$$3x_1 - x_2 \geq 2$$
$$x_2+x_3-x_4 \geq 1$$
$$x_1, x_2, x_3, x_4 \geq 0$$

When using the excel Solver add-in to Solve the problem.

3. (1 point) Consider the linear programming problem:

$$\min\ 4x_1 + 9x_2 + 1x_3 + x_4$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 = 7$$

$$3x_1 - x_2 \geq 2$$

$$x_2 + x_3 - x_4 \geq 1$$

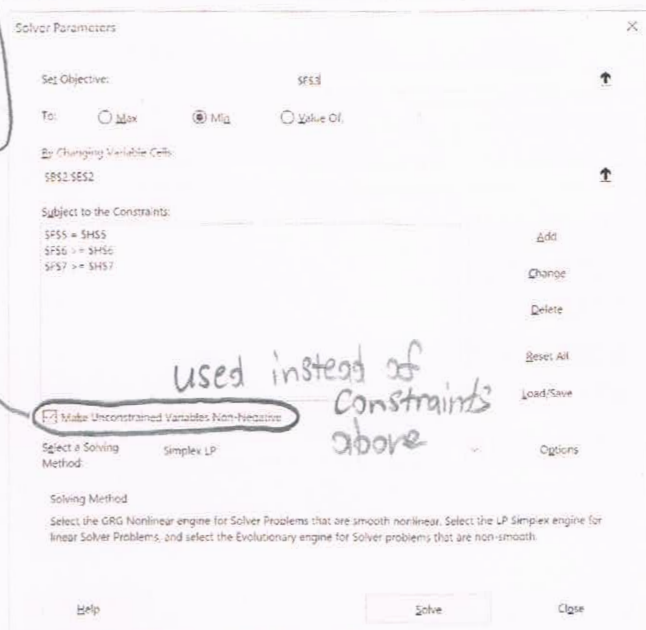
$$x_1, x_2, x_3, x_4 \geq 0$$

Use Microsoft Excel with the Solver add-in to solve this problem. Submit a screenshot of the spreadsheet that you create for the problem showing the final solution and a screenshot of the answer report.

Excel Spreadsheet used for the problem

F3 =SUMPRODUCT(B3:E3,B2:E2)

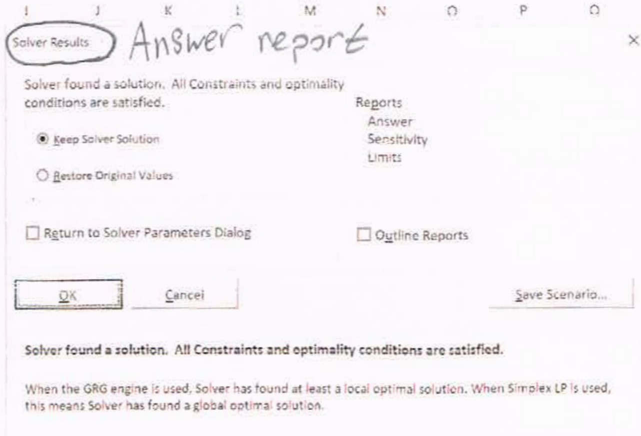
	A	B	C	D	E	F	G	H
1	Variables	x	y	z	w	value		
2	Solution							
3	Objective	4	9	1	1	0		
4								
5	Constraint	1	1	1	1	0 =		7
6	Constraint	3	-1	0	0	0 >=		2
7	Constraint	0	1	1	-1	0 >=		1



used instead of Constraints above

	A	B	C	D	E	F	G	H
1	Variables	x	y	z	w	value		
2	Solution	0.666667	0	6.333333	0			
3	Objective	4	9	1	1	9		
4								
5	Constraint	1	1	1	1	7 =		7
6	Constraint	3	-1	0	0	2 >=		2
7	Constraint	0	1	1	-1	6.333333 >=		1

final Solution



Answer report

Note: (for the excel Sheets)
x is X_1
y is X_2
z is X_3
w is X_4

Thus, the Solution is $\left(\frac{2}{3}, 0, \frac{19}{3}, 0\right) = (X_1, X_2, X_3, X_4)$ for the minimum point & this occurs when $4X_1 + 9X_2 + 1X_3 + X_4 = 9$.

Acknowledgements

As was required for question 3, I used Microsoft excels optimization solver to help solve question 3.

No other outside help was used for the problem set.