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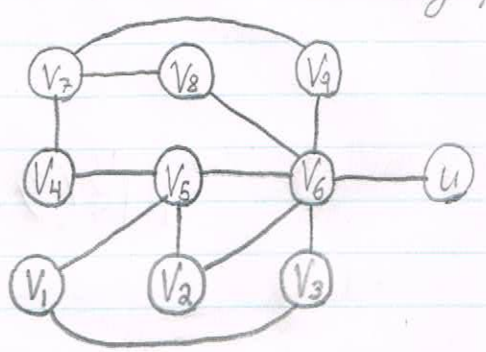
Acknowledgment: No Help Receaved

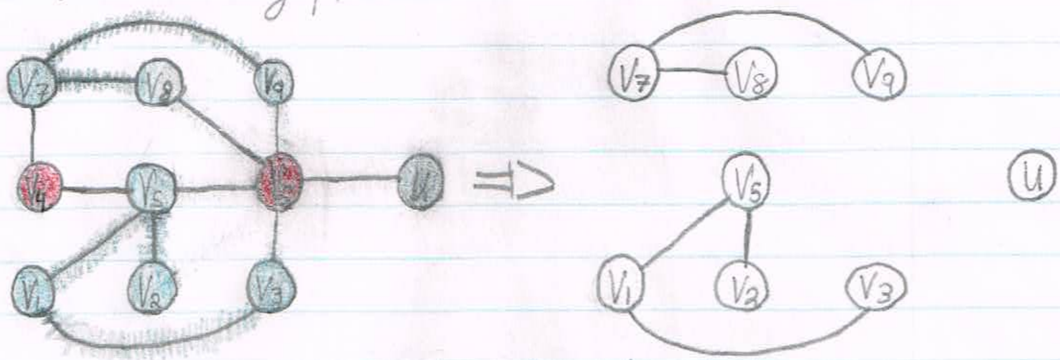
MATH 3802 Assignment # 10:

Let $G=(V,E)$ be the undirected graph depicted below:



① Let $S=\{V_4,V_6\}$:

a) (2 Points) Sketch the graph $G\setminus S$:



b) (2 Points) Compute the Value $|V|-o(G\setminus S)+|S|$. Can you Conclude from this Value that G has no perfect matching? Explain:

- $|V|$ is the Cardinality of the set of nodes in G
  ↳ $|V| = 9+1 = 10$
- $|S|$ is the Cardinality of the set of nodes S in G
  ↳ $|S| = 2$
- $o(G\setminus S)$ is the the number of Components in $G\setminus S$ having an odd number of nodes
  ↳ $o(G\setminus S) = 1+1 = 2$ Since $\{V_7, V_8, V_9\}$ & $\{u\}$ are odd Subgraphs
- $|V|-o(G\setminus S)+|S| = 10-2+2 = 10$

$\nu(G)$ is the max. Cardinality of a matching in G

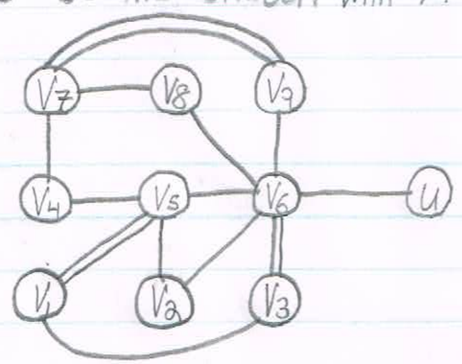
$$\hookrightarrow \nu(G) = \min\left\{\left[\frac{|V|-o(G\setminus S)+|S|}{2}\right] : \{V_4, V_6\}\in V\right\}$$

$$= \min\left\{\left[\frac{10-2+2}{2}\right]\right\} = \min\{5\} = 5$$

∴ We cannot use the Value to Conclude that G has no perfect matching as it informs us that the max Cardinality of a matching in G is 5, which is sufficient for a Perfect matching of 10 nodes.

(2) (6 Points) Suppose that Algorithm 10.1 is applied to G with $M=\{V_1V_5, V_3V_6, V_7V_9\}$ &, in the first iteration, V_4 is chosen in Step C.I & T in Step C.II is $(\{V_4,V_1,V_3,V_5,V_6,V_7,V_9\},\{V_4V_5, V_1V_5, V_4V_7, V_7V_9, V_9V_6, V_3V_6\})$. Continue execution of the algorithm until termination to obtain S so that $o(G\setminus S)>|S|$:

let G be the sketch with M represented as double lined edges:



Begin Algo. 10.1:

Iter. 1:

A. $M' \leftarrow M$, $G' \leftarrow G$

B. There are exposed nodes

C.

I) Choose V_4 as the M-exposed node

II) Using Algorithm 9.1; we track progress with the following table:

Action	T	$\mathcal{E}(T)$	$\mathcal{O}(T)$
Initialization	$(\{V_4\},\emptyset)$	$\{V_4\}$	$\emptyset$
Add V_4V_5, V_1V_5	$(\{V_1,V_4,V_5\},\{V_4V_5,V_1V_5\})$	$\{V_4,V_1\}$	$\{V_5\}$
Add V_4V_7, V_7V_9	$(\{V_1,V_4,V_5,V_7,V_9\},\{V_4V_5,V_1V_5,V_4V_7,V_7V_9\})$	$\{V_4,V_1,V_9\}$	$\{V_5,V_7\}$
Add V_9V_6, V_3V_6	$(\{V_1,V_3,V_4,V_5,V_6,V_7,V_9\},\{V_4V_5,V_1V_6,V_4V_7,V_7V_9, V_9V_6,V_3V_6\})$	$\{V_1,V_3,V_4,V_9\}$	$\{V_5,V_6,V_7\}$

III) While $uw \in E$ with $u \in \mathcal{E}(T)$ & $w \notin \mathcal{O}(T)$ exists:

As $\mathcal{E}(T)=\{V_1,V_3,V_4,V_9\}$ & $\mathcal{O}(T)=\{V_5,V_6,V_7\}$

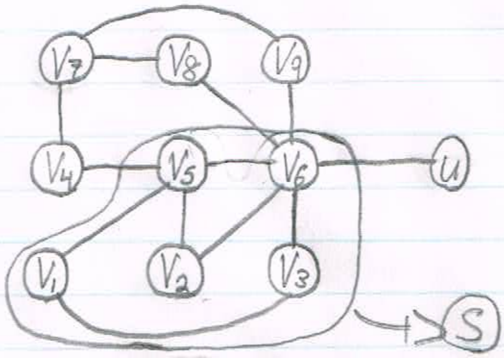
↳ $\{V_1V_3\} \in E$ & $V_1 \in \mathcal{E}(T)$ & $V_3 \notin \mathcal{O}(T)$

$V(T)=\{V_1,V_3,V_4,V_5,V_6,V_7,V_9\}$ & $V_1V_3 \in V(T)$

a) IF $w \notin V(T) \rightarrow$ False

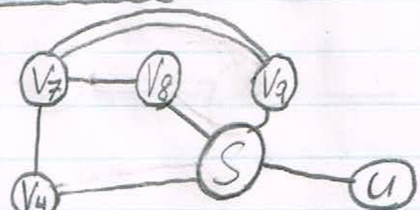
b) ELSE:

i) let S be the M' blossom associated with uw:



ii) $M' \leftarrow M'/S$, $G' \leftarrow G'/S$:

$G' \leftarrow G'/S$:



$M' \leftarrow M'/S$:

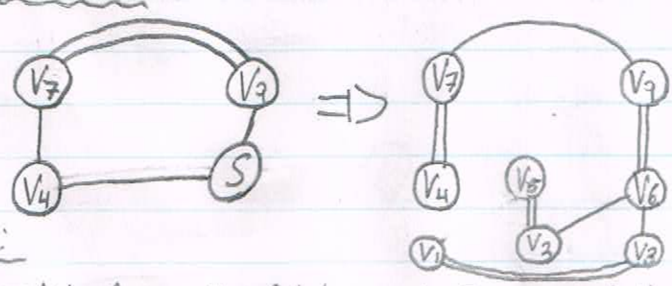
$$M' = M/S = \{V_7V_9\}$$

Augmenting Path:

$S, SV_9, V_9, V_9V_7, V_7, V_7V_4$

iii) Extend T/S to an M'-alternating tree & replace T with this M'-alternating tree:

$T' \leftarrow T/S$:



So:

M'-Augmenting Path has node sequence of $V_2, V_5, V_1, V_3, V_6, V_9, V_7, V_4$

We obtain a matching of $\{V_2V_5, V_1V_3, V_6V_9, V_4V_7\}$

iv) Return $O(T) = \{V_7, V_6, V_1, V_5\}$

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This gives a G/S of:

thus, $o(G/S)=6$ & $|S|=5$

$\therefore o(G/S) > |S|$ with $O(T)=\{V_1, V_5, V_6, V_7\}$