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② Base Case: $n \geq 0$

RS: $f(0) = (0^2 - 3(0)) \cdot 2^0$ LS: $f(0) = 0$

$= (0) \cdot 1$ $\therefore RS = LS$

$= 0$

$\therefore$ Base Case is true for $f(n) = (n^2 - 3n) \cdot 2^n$

Assume: $f(n-1) = (([n-1])^2 - 3([n-1])) \cdot 2^{(n-1)}$ for Some $n \geq 1$

Prove: $f(n) = ((n)^2 - 3(n)) \cdot 2^n$

$f(n) = f(n-1) + (n^2 - n - 4) \cdot 2^{n-1}$ [remove f by inductive hypathesis]

$= ((n-1)^2 - 3(n-1)) \cdot 2^{n-1} + (n^2 - n - 4) \cdot 2^{n-1}$

$= 2^{n-1}(((n-1)^2 - 3(n-1)) + (n^2 - n - 4))$ [factor 2^{n-1}]

$= 2^{n-1}((n-1)((n-1) - 3(1)) + (n^2 - n - 4))$ [factor $n-1$]

$= 2^{n-1}((n-1)(n-4) + n^2 - n - 4)$ [Simplify $n-1-3$]

$= 2^{n-1}(n^2 - 5n + \cancel{4} + n^2 - n - \cancel{4})$ [multiply $(n-1)(n-4)$]

$= 2^{n-1}(2n^2 - 6n)$ [Simplify]

$= 2^{n-1}(2)(n^2 - 3n)$ [factor out 2]

$= (2^{n-1})(2^1)(n^2 - 3n)$ [exponent Addition $(2^{n-1})(2^1)$]

$= (2^{n-1+1})(n^2 - 3n)$ [Simplify]

$= (2^n)(n^2 - 3n)$ OR $(n^2 - 3n) \cdot 2^n$ [final anser]

$\therefore f(n) = f(n-1) + (n^2 - n - 4) \cdot 2^n = (n^2 - 3n) \cdot 2^n$ for all $n \geq 0$

Determine $f(n)$ for $n = 0, 1, 2, 3, 4, 5$

$f(n) = f(n-1) + (n^2 - n - 4) \cdot 2^{n-1}$ if $n \geq 1$

$f(0) = 0$ [by definition]

$f(1) = f(1-1) + (1^2 - 1 - 4) \cdot 2^{1-1} = f(0) + (1-1-4) \cdot 2^0 = 0 + -4(1) = \boxed{-4}$

$f(2) = f(2-1) + (2^2 - 2 - 4) \cdot 2^{2-1} = f(1) + (4-2-4) \cdot 2^1 = -4 - 4 = \boxed{-8}$

$f(3) = f(3-1) + (3^2 - 3 - 4) \cdot 2^{3-1} = f(2) + (9-3-4) \cdot 2^2 = -8 + 8 = \boxed{0}$

$f(4) = f(4-1) + (4^2 - 4 - 4) \cdot 2^{4-1} = f(3) + (16-4-4) \cdot 2^3 = 0 + 8 \cdot 2^3 = 64$

$f(5) = f(5-1) + (5^2 - 5 - 4) \cdot 2^{5-1} = f(4) + (25-5-4)(2^4) = 64 + (16)(2^4)$

$\boxed{= 320}$

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③ $f(0) = x$

$f(n) = f(n-1) + y$

Find the value of x & y given $f(n) = 7n^2 - 2n + 9$

$x = f(0) = \cancel{7(0)^2} - \cancel{2(0)} + 9$ [Simplify multiplication]

$x = 9$ [final anser]

$f(n) = f(n-1) + y$, if we rearrange this then, $f(n) - f(n-1) = y$

$\therefore y = f(n) - f(n-1)$ when $f(n) = 7n^2 - 2n + 9$, thus sub n for $n-1$ because $n \geq 1$, so $n-1 \geq 0$

$\therefore f(n-1) = 7(n-1)^2 - 2(n-1) + 9$ for $n-1 \geq 0$ [by substitution]

we now know what $f(n)$ & $f(n-1)$ are $\therefore$ we may substitute for y's equation:

$y = f(n) - f(n-1)$

$= 7n^2 - 2n + 9 - f(n-1)$ [Sub. $f(n)$ for $7n^2-2n+9$]

$= 7n^2 - 2n + 9 - [7(n-1)^2 - 2(n-1) + 9]$ [Sub in value]

$= 7n^2 - 2n + 9 - 7(n-1)^2 + 2(n-1) - 9$ [Simplify negative]

$= 7n^2 - 2n + 9 - 7(n-1)(n-1) + 2n - 2 - 9$ [multiply]

$= 7n^2 - 2n + 9 - 7(n^2 - 2n + 1) + 2n - 2 - 9$ [multiply]

$= \cancel{7n^2} - \cancel{2n} + \cancel{9} - \cancel{7n^2} + 14n - 7 + \cancel{2n} - 2 - \cancel{9}$ [multiply]

$\boxed{y = 14n - 9}$ [Simplify]

(4) By Definition a_0 & a_1 are defined as:

$a_0 = 5$ [by Def.]

$a_1 = 3$ [by Def.]

if $n \geq 2$ then $a_n = 6 \cdot a_{n-1} - 9 \cdot a_{n-2}$ by definition

$\therefore a_2 = (6)(a_{2-1}) - 9(a_{2-2})$ [Definition]

$= (6)(a_1) - 9(a_0)$ [Subtraction]

$= (6)(3) - 9(5)$ [Substitute a_1 & a_0]

$= 18 - 45$ [Subtraction]

$a_2 = -27$ [final Solution]

Repeat Same Steps

$a_3 = (6)(a_{3-1}) - 9(a_{3-2})$

$= (6)(a_2) - 9(a_1)$

$= (6)(-27) - 9(3)$

$= -162 - 27$

$= -189$

$a_4 = (6)(a_{4-1}) - 9(a_{4-2})$

$= (6)(a_3) - 9(a_2)$

$= (6)(-189) - 9(-27)$

$= -1134 + 243$

$= -891$

$a_5 = (6)(a_{5-1}) - 9(a_{5-2})$

$= (6)(a_4) - 9(a_3)$

$= (6)(-891) - 9(-189)$

$= -5346 + 1701$

$= -3645$

Base Case: $n \geq 0$

LS: $a_0 = 5$ $\therefore LS = RS$

RS: $a_n = (5-4n)(3^n)$

$a_0 = (5-4(0))(3^0)$

$= (5)(1)$

$= 5$

$\therefore$ Base case is true for $a_n = (5-4n)(3^n)$ as both equations equal five.

Assume 1: $a_{n-1} = (5-4(n-1))(3^{(n-1)})$ for Some $n \geq 2$

Assume 2: $a_{n-2} = (5-4(n-2))(3^{(n-2)})$ for Some $n \geq 2$

for both (1) & (2) Proove: $a_n = (5-4n)(3^n)$

$$a_n = (6)(a_{n-1}) - 9(a_{n-2})$$
$$= (6)([(5-4(n-1))(3^{(n-1)})]) - (9)([(5-4(n-2))(3^{(n-2)})])$$ [Substitution]
$$= (5-4(n-1))(6 \cdot 3^{n-1}) - (5-4(n-2))(9 \cdot 3^{n-2})$$ [multiplication]
$$= (5-4(n-1))(2 \cdot 3^1 \cdot 3^{n-1}) - (5-4(n-2))(3^2 \cdot 3^{n-2})$$ [factor]
$$= (5-4(n-1))(2 \cdot 3^{n-\cancel{1}+\cancel{1}}) - (5-4(n-2))(3^{n-\cancel{2}+\cancel{2}})$$ [Exponent Addition]
$$= (5-4n+4)(2 \cdot 3^n) - (5-4n+8)(3^n)$$ [find Sums]
$$= (9-4n)(2)(3^n) - (13-4n)(3^n)$$ [factor out 2]
$$= (3^n)[(9-4n)(2) - (13-4n)]$$ [factor out 3^n]
$$= (3^n)[18-8n-13+4n]$$ [multiply by 2]
$$= (3^n)[5-4n]$$ [find Sum of terms]
$$\boxed{= (5-4n)(3^n)}$$ [final anser]

$\therefore a_n = (5-4n)(3^n)$ by induction

⑤ I. — For E_1, we can only have an even # of C's if we have none, as 2 C's is over the limit & 1 C is odd.

$\therefore E_1 = |\{a\},\{b\}| = 2$

— For E_2, only 2 C's or No C's are even, thus only Strings with No C's and cc itself are counted:

$\therefore E_2 = |\{cc\},\{a,a\},\{b,b\},\{b,a\},\{a,b\}| = 5$

— For O_1, only C itself is odd, as no C is even:

$\therefore O_1 = |\{c\}| = 1$

— For O_2, Only a String with neither 0 nor 2 C's Count:

$\therefore O_2 = |\{c,a\},\{c,b\},\{a,c\},\{b,c\}| = 4$

II. $E_n + O_n = 3^n$ because we have 3 options for every space, the base $= 3$. Also, Since we have Strings of n length, we find the exponential value of the base (3) to the n (resulting in 3^n).

III. Prove that for every integer $n \geq 2$ for:

$$E_n = 2 \cdot E_{n-1} + O_{n-1}$$

aE_n Starts with a: $|E_{n-1}|$ Counts the Strings of length n Starting with a, with even C's.

bE_n Starts with b: $|E_{n-1}|$ Counts the Strings of length n Starting with b, with even C's

cE_n Starts with c: $|O_{n-1}|$ Counts the Strings of length n, with odd num. & odd num. of C's

$E_n = aE_n \cup bE_n \cup cO_n$ | ↳ Even − odd = odd number, therefore any String of length n Starting with c is odd.

$E_n = 2E_{n-1} + O_{n-1}$

$E_n = 2 \cdot E_{n-1} + O_{n-1}$

IV. Prove for every integer $n \geq 1$: $E_n = \frac{1+3^n}{2}$

Base Case: $n=1$

$E_1 = \frac{1+3^1}{2} = \frac{4}{2} = 2$ ∴ the Claim is true

Assume: $E_n + O_n = 3^n$ for O_{n-1}

Prove that for every int $n \geq 1$ $E_n = \frac{1+3^n}{2}$

We know that for E_n, where $n \geq 2$

Let $E_n = 2E_{n-1} + O_{n-1}$

$E_n = (2)\left(\frac{1-3^{n-1}}{2}\right) + (3^{n-1} - E_{n-1})$ [by Def.]

$= \frac{\cancel{2} - \cancel{(2)}(3^{n-1})}{\cancel{2}} + 3^{n-1} - E_{n-1}$ [multiplication]

$= 1 + 3^{n-1} + 3^{n-1} - E_{n-1}$ [Simplify]

$= 1 + (2)(3^{n-1}) - E_{n-1}$ [Simplify]

- $E_n = \frac{1+3^n}{2}$ [By Definition]
- $E_{n-1} = \frac{1+3^{n-1}}{2}$ [By Def.]

$= 1 + (2)(3^{n-1}) - \frac{1+3^{n-1}}{2}$ [Sub. E_{n-1} for defitiition]

$= \frac{(2)(1) + (2)(2)(3^{n-1})}{2} - \frac{1+3^{n-1}}{2}$ [Cos multiply]

$= \frac{2 + (4)(3^{n-1}) - (1+3^{n-1})}{2}$ [Subtraction]

$= \frac{1 + (3^1)(3^{n-1})}{2}$ [Simplify]

$= \frac{1 + (3^{n-1+1})}{2}$ [Subtract]

$\therefore E_n = \frac{1+3^n}{2}$ by induction [Simplify, final Sol$\underline{n}$]

the claim is true

For every int $n \geq 1$, $E_n = \frac{1+3^n}{2}$

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⑥ **I.** B_1 is the number of blocks in all the bitstrings of length one.

-- B_2 is the total number of blocks in all the bitstrings of length two.

__B1:__

0	0
1	1

— This is a bitstring of length 1
— There are only 2 blocks, one contains a 1 & 1 contains a 0, ∴ $\boxed{B_1 = 1}$, as theres only one "1".

__B2:__

1	0	1
0	1	1
1	1	1
0	0	0

— This is a bitstring of length 2
— There are 4 blocks, one with 0 consecutive ones & three with "1" consecutive one.
— $\underline{\therefore B_2 = 1+1+1 = 3}$, $\boxed{B_2 = 3}$

II. — The number of blocks of in the bitstring of length n thats starts with zero can be modeled using B2's matrix.

0	1	0	1
0	0	1	1
0	1	1	1
0	0	0	0

Add 0's

— As can be seen, the number of blocks with consecutive ones remains the same. For every value of n in B_n, Adding 0's in the first column of the matrix dosn't change the total number of consecutive ones. ∴ the total $= 2^n$ bitstrings of length n.

— Determine the number of blocks in the bitstring:

$\underbrace{1 \ldots 1}_{n}$ — There is no spacing between consecutive blocks, meaning that the number of blocks can only ever $= \boxed{1}$, & n is irrelevent.

__Ex.:__ $n = 6$; | 1 | 1 | 1 | 1 | 1 | 1 |, as can be seen, n dosn't matter, there will never be any more consecutive 1's after the first 1 placed.

— Determine the number of blocks in the bitstring:

$\underbrace{1 \ldots 1}_{n-1} 0$ — Same as above, no spacing between every 1 meaning theres no more consecutive ones after the first.
— The zero at the end value n does not change this, meaning the number of consecutive 1's is $\boxed{1}$ in this list.

$\therefore$ Blocks $= \boxed{1}$

— # of Blocks in the bitstring:

$\underbrace{1\ldots\ldots 1}_{K-1}0$ — K Cannot = '0' or '1' as that would end up in a bitstring of size −1 or 0, which isn't what we want, by definition.

— (n−1) is the max range for K, otherwise the bitstring could extend to length (n+1) due to the 0 at the end.

∴ Blocks = $\boxed{1}$

— In terms of the number of blocks, the problem is the same as the last two. There is no spacing between 1's, meaning there is only ever $\boxed{1}$ group of consecutive 1's. The zero at the end doesn't change this, as there are no 1's after it.

— Proove the total number of blocks in these bitstrings is equal to:

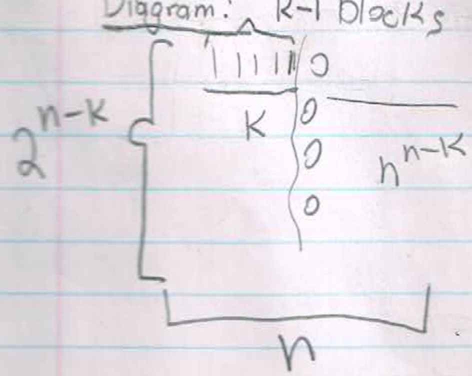
$2^{n-K} + B_{n-K}$

- The bitstring contains 2^n columns & has 2^K assigned terms. The bitstring cannot assign strings over already assigned strings, thus $2^n / 2^K = 2^{n-K}$ according to the logic that you cannot reassign. ∴ The # of block arrangments = 2^{n-K}. These blocks contain non consecutive 1's.
- The bitstring contains B_{n-K} assigned blocks, K is the assigned terms, & n is the bitstring. ∴ B_{n-K} = # of consecutive 1's.
- Finding the sum of the non-consecutive & consecutive blocks results in a sum which is the value of the total # of blocks.

— It can be said that simply, there are 2^{n-K} possible arrangments of B_{n-K} blocks.

∴ Thus, the number of bitstrings in these blocks = $2^{n-K} + B_{n-K}$

Diagram:



- Prove that:

$$B_n = 2 + B_{n-1} + \sum_{K=2}^{n-1} (2^{n-K} + B_{n-K})$$

- If $n-1$ is smaller then anything $K=2$ then stop.
- $n \geq 3$ always

Example:

$$B_6 = 2 + B_{6-1} + \sum_{K=2}^{5} (2^{6-K} + B_{6-K})$$

$$2^4 + B_4 + 2^3 + B_3 + \ldots\ldots + 2^1 + B_1$$

- If we change 6 to n:

$$B_n = 2 + B_{n-1} + \sum_{K=2}^{n} (2^{n-K} + B_{n-K})$$

$$f_1(n) = \sum_{K=2}^{n} (2^{n-K} + B_{n-K}) \quad \Delta \quad \text{Sub as } f_1(n)$$

$$2^{n-K} + B_{n-K} + 2^{(n-K)-1} + B_{(n-K)-1} \ldots\ldots 2^1 + B_1$$

Sub $f_1(n)$ for $\sum_{K=2}^{n} (2^{n-K} + B_{n-K})$:

$$2 + B_{n-1} + 2^{n-K} + B_{n-K} + 2^{(n-K)-1} + B_{(n-K)-1} \ldots\ldots 2^1 + B_1$$

- B_n is the total number of blocks bitstrings for n, we must find the sum of all blocks of size B_{n-x} where $x \geq 2$, explaning $\sum_{K=2}^{n-1} (2^{n-K} + B_{n-K})$ which does that.
- The blocks in a bitstring can be modeled exponentially:
  - for example a 3 file matrix contains $2^3 = 8$ bitstrings,
  - Adding exponents of value $(2^{n-1} + 2^{n-1}) = 2^n$ as $(2^2 + 2^2 = 4 + 4 = 8 = 2^3 = 2 \cdot 2 \cdot 2 = 8)$.
    - This explanes B_{n-1} as $\sum_{K=2}^{n-1} (2^{n-K} + B_{n-K}) = B_{n-1}$.
    - Therfore $B_{n-1} + \sum_{K=2}^{n-1} (2^{n-K} + B_{n-K}) = B_n -$ first 2 elements
- 2^{n-K} models the # of possible block combinations (n) minus the # of assigned terms. This $\geq (K)$ combined with B_{n-K} (the # of bitstrings) gives B_n (the # of all blocks).
- The coefficient 2 is added to the function because the 2 base cases: 1,1,1,1,1,1,1,1 & 1,1,1,1,1,1,1,0 result in a total # of bitstrings of 2, which must be added to the formula.

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$\therefore$ this info could be used to generate the formula Above.

— Prove that: $B_n = 2^{n-1} + B_1 + B_2 + \dots + B_{n-1}$ using $1+2+2^2+2^3+\dots+2^{n-2} = 2^{n-1}-1$

Given 1: $1+2+2^2+2^3+\dots+2^{n-2} = 2^{n-1}-1$

Given 2: $B_n = 2^{n-1} + B_1 + B_2 + \dots + B_{n-1}$ [Simplify]

Given 1:

$\cancel{1}+1+2+2^2+2^3+\dots+2^{n-2} = 2^{n-1}\cancel{-1}\cancel{+1}$

$2+2+2^2+2^3+\dots+2^{n-2} = 2^{n-1}$ [Simplify]

Given 2:

$B_n - (B_1 + B_2 + \dots + B_{n-1}) = 2^{n-1}$

— use given 2's 2^{n-1} & sub. into Given 1:

$2+2+2^2+2^3+\dots+2^{n-2} = B_n - (B_1 + B_2 + \dots + B_{n-1})$ [Sub.]

$2+2^1+2^2+2^3+\dots+2^{n-2} + (B_1+B_2+\dots+B_{n-1}) = B_n$ [Algebra]

$2^{n-1} + (B_1 + B_2 + \dots + B_{n-1}) = B_n$ [Sub]

$2^{n-1} + B_1 + B_2 + \dots + B_{n-1} = B_n$ [Remove brakets]

$\therefore$ It its proven to be true

— Prove (1) holds for $n=2$:

$B_2 = 2^{2-1} + B_1$ [plug in numbers]

$= 2^1 + B_1$ [Simplify]

$= 2^1 + 1$ [plug in value B_1]

$= 2+1$ [Simplify]

$\boxed{B_2 = 3}$ [Add]

B_2 does indeed $= 3$ $\therefore$ (1) is true for $n=2$.

— $n \geq 3$, Prove:

① $B_n = 2^{n-2} + 2 \cdot B_{n-1}$

② Given: $B_n = 2^{n-1} + B_1 + B_2 + \dots + B_{n-1}$

② $B_{n-1} = 2^{(n-1)-1} + B_1 + B_2 + \dots + B_{(n-1)-1}$

① $\dfrac{B_n - 2^{n-2}}{2} = B_{n-1}$

We may now sub Eq ② & Eq ① as they both $= B_{n-1}$

$\dfrac{B_n - 2^{n-2}}{2} = 2^{(n-1)-1} + B_1 + B_2 + \dots + B_{(n-1)-1}$ [Sub.]

$\dfrac{B_n - 2^{n-2}}{2} = 2^{n-2} + B_1 + B_2 + \dots + B_{n-2}$ [Simplify]

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$B_n = 2(2^{n-2}) + 2^{n-2} + (B_1 + B_2 + \dots + B_{n-2})(2)$ [Algabra]

$B_n = 3(2^{n-2}) + (B_1 + B_2 + \dots + B_{n-2})(2)$ [Algabra]

• Eq. 2 [Make equation, Eq. 3]

$B_{n-1} = 2^{n-2} + B_1 + B_2 + \dots + B_{n-2}$ [Alg.]

$B_{n-1} - (B_1 + B_2 + \dots + B_{n-2}) = 2^{n-2}$ [Alg.]

• Sub into above :

[Sub. eq. 3]

[Sub.] $B_n = 2(B_{n-1} - (B_1 + B_2 + \dots + B_{n-2})) + (1)(2^{n-2}) + (B_1 + B_2 + \dots + B_{n-2})(2)$

[Cancel] $B_n = 2(B_{n-1}) - \cancel{(2)(B_1 + B_2 + \dots + B_{n-2})} + \cancel{(2)(B_1 + B_2 + \dots + B_{n-2})} + 2^{n-2}$

[Simplify] $B_n = 2^{n-2} + 2 \cdot B_{n-1}$

$\therefore$ It is proven that $B_n = 2^{n-2} + 2 \cdot B_{n-1}$ for $n \geq 3$

— Prove that for every $n \geq 1$:

$$B_n = \left(\frac{n+1}{4}\right)(2^n)$$

Base Step: $B_2 = \left(\frac{2+1}{4}\right)(2^2) = 3$ $\therefore$ Held true for both B_1 & B_2

$B_1 = \left(\frac{1+1}{4}\right)(2^1) = 2$

Assume: B_{n-1} is true per some $n > 3$:

Prove: $B_n = \left(\frac{n+1}{4}\right)(2^n)$

$\therefore B_{n-1} = \left(\frac{(n-1)+1}{4}\right)(2^{n-1})$

Given: $B_n = 2^{n-2} + 2B_{n-1}$

• We may now sub B_{n-1} from above with the given:

$B_n = 2^{n-2} + 2\left(\frac{n}{4} \cdot 2^{n-1}\right)$

[Simplify] $= 2^{n-2} + \left(\frac{n}{4} \cdot 2^1 \cdot 2^{n-1}\right)$

[Simp.] $= 2^{n-2} + \left(\frac{n}{4} \cdot 2^{n-\cancel{1}+\cancel{1}}\right)$

[Simp.] $= 2^{n-2} + \left(\frac{n}{4} \cdot 2^n\right)$

[Expand] $= \frac{(4)(2^{n-2})}{4} + \left(\frac{n}{4} \cdot \frac{2^n}{1}\right)$

[Simp.] $= \frac{(4)(2^{n-2}) + (n2^n)}{4}$

[expand] $= \frac{2^2 \cdot 2^{n-2} + n2^n}{4}$

[Simp.] $= \frac{2^{n-2+2} + n2^n}{4}$

Continue

$= \frac{2^n(1+n)}{4}$ [factor out]

$B_n = 2^n \frac{(1+n)}{4}$ [Simp.]

$\therefore$ It is proven that

$B_n = \left(\frac{n+1}{4}\right)(2^n)$

⑦

— There is one bottle, which can only be grouped only one way: $\{B_1\}$ has only 1 grouping, there is only 1 bottle.

— If theres more then 1 bottle, there is more then one grouping: $\{B_2, B_6\}$ has many groupings.

- — W_1 has $\boxed{1}$ grouping, there is only bottle to be catigorized agains't nothing else.

— W_2 has : $W_2 = \{B_1\}, \{B_2\} = \{B_1, B_2\} = 2$ arrangments, as can be seen there is only 2 ways to arrange the B_1 & B_2 groups.

$|\{B_1\}, \{B_2\}| = 1$ grouping $|| \{B_1, B_2\} = 1$ grouping

1 group + 1 group = $\boxed{2}$ grouping.

— W_3 has: $W_3 = \{B_1\}, \{B_2\}, \{B_3\} = \{B_1, B_2\}, \{B_3\} = \{B_1\}, \{B_2, B_3\} = \{B_1, B_3\}, \{B_2\}$

$= 4$

this is saying the groupings $= 1+1+1+1 = \boxed{4}$

— W_4 has : $W_4 = \{B_1\}, \{B_2\}, \{B_3\}, \{B_4\} = \{B_1, B_2\}, \{B_3\}, \{B_4\} = \{B_1\}, \{B_2, B_3\}, \{B_4\} =$
$\{B_1\}, \{B_2\}, \{B_3, B_4\} = \{B_1, B_2\}, \{B_3, B_4\} = \{B_1, B_3\}, \{B_2, B_4\}$
$= \{B_1, B_4\}, \{B_2, B_3\} = \{B_1\}, \{B_3\}, \{B_2, B_4\}$

$= 8$

this is saying the groupings $= 1+1+1+1+1+1+1+1 = \boxed{8}$

- Prove that for every integer $n \geq 3$: $W_n = W_{n-1} + (n-1) W_{n-2}$

— If there are only 2 groupings maximum then if $\{B_n\}$ is in a single group W_{n-1} must consist the remainder of bottles, 1's removed.

$\therefore W_{n-1}$ is the ways to group after a single category

— If there is a grouping of 2 bottles, then the group can be $\{B_1, B_{n-1}\}$ bottles, as 1 bottle is in position index 1 & any other bottle is in position 2.

— Since 1 bottle must be removed for this to occur, the # of arrangments is multiplied by $(n-1)$. This also stops duplicates.

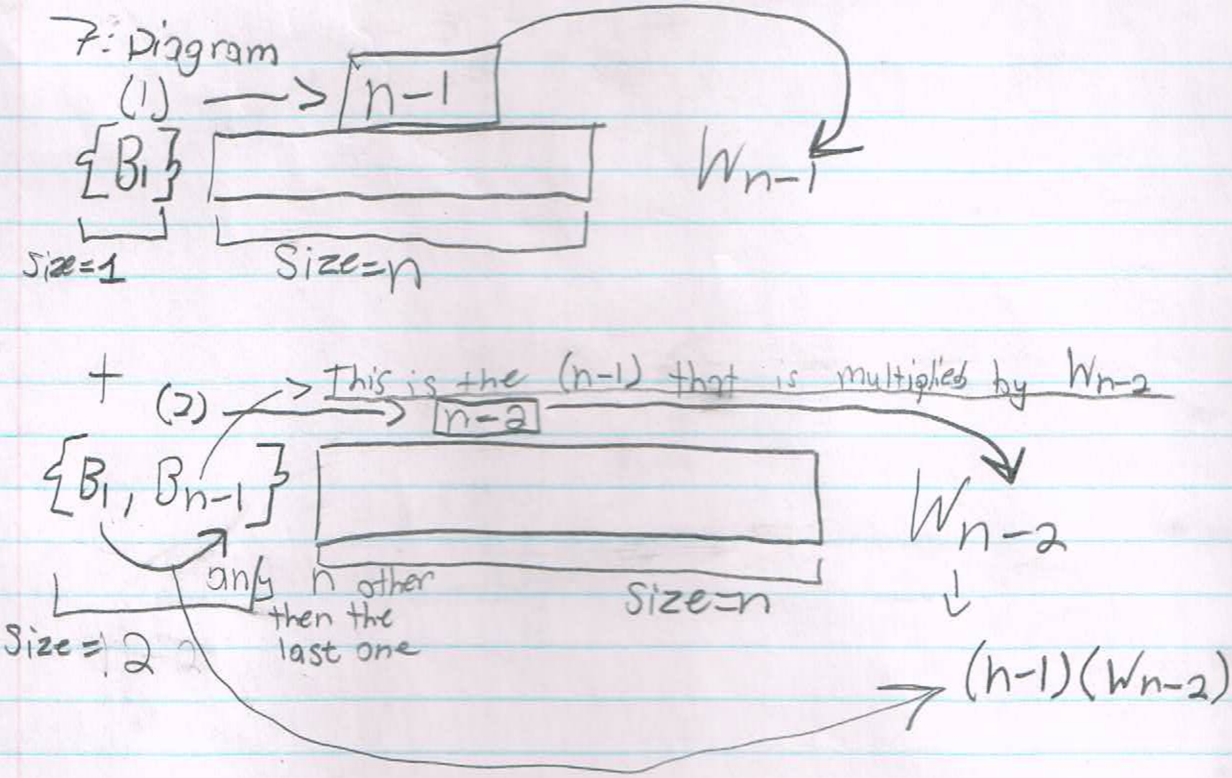
— 2 bottles must be removed to make a group of 2, thus W_{n-2} are the ways to group a double group. W_{n-2} must be multplied by $(n-1)$ as stated above (1 bottle is removed before the second in the group).

— there are 2 groupings, 1 of size 1 & 1 of size 2, the W_n = the sum of both.

$\therefore W_n = W_{n-1} + (n-1)(W_{n-2})$

Diagram →

7: Diagram



— this illistrates Q. 7

⑧ - Prove by induction ~~the~~ size of the list
- Assume it works for the list of length $n-1$
- Prove it will be reverse a list of length n

- Prove by induction ~~the~~ Size of the list:

base $n=1$:

- This algorithm will return a list of length 1, ∴ it would return a_1. [by observation]

- Assume it works for the list of length $n-1$;

Prove: the list is the reverse

Assume: list of length $n-1$

$$(a_{n-1}, \ldots, a_1) = Mystery(a_1, a_2, \ldots, a_{n-1})$$

return:

$$(a_n, a_{n-1}, \ldots, a_1)$$

- by observation we can see, the Algorithm is simply reversing the list. This can be seen via observation.

(7)

- Determine S_1:
  - There is only one way to climb up 1 floor, ∴ $S_1 = \boxed{1}$

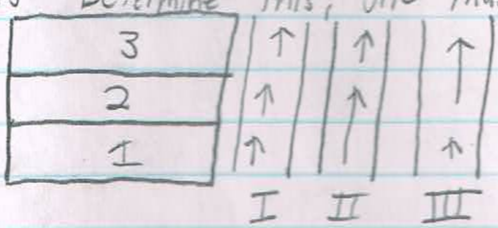
Determine S_2:

- There are only two ways to climb up 2 floors, you either climb both in one step, or climb one step 2 times.

  ∴ $S_2 = 1 + 1 = \boxed{2}$

Determine S_3:

- To determine this, one must visualize the floors:

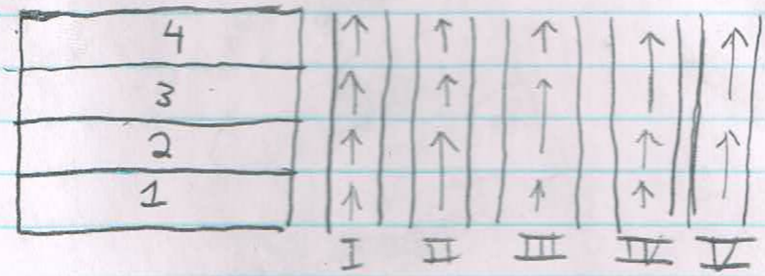


- There are ∴ 3 ways to climb the building to the 3rd floor.

∴ $S_3 = 1+1+1 = \boxed{3}$

Determine S_4:

- once again, visualize the floors:



- there are ∴ 5 ways to climb the 4th floor.

∴ $S_4 = 1+1+1+1+1 = \boxed{5}$

- This is clearly a fibonacci sequence, except there is only one 1. We have seen this in class.

Fib Sequence: $F_n = F_{n-1} + F_{n-2}$

- If we were to build up a recurence for this, we would be accounting for Nick's choice to jump one or two floors. His first move determines the sequence, if he jumps for example 1 floor, then for the remaing S_{n-1} floors he must climb picking from S_{n-1} choices. n is the # of floors.
- Nicks moves are thus split, between 1 or 2 jumps. If he jumps 2 floors he needs to chose for the remaing S_{n-2} floors.
- ∴ the sum of ways to climb n floors = the # of ways to jump after Nicks first jump. ∴ $S_n = S_{n-1} + S_{n-2}$ Models this Sol^n