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COMP1805A
Discrete Structures I

Assignment 2

1.

A) s = “it is snowing”, h = “I bring my hat to work”

$s \rightarrow h$

h

$\therefore s$

I could bring my hat to work even when it I not snowing, even when it isn’t raining, because I may bring my hat to work every day. If I always bring my hat, then I can’t use the hat to conclude if it is raining.

b) p(x) = “x has a PC”, q(x) = “x plays games”, r(x) = “x is taking COMP 1501”, UoD = “all students”

$\forall x p(x) \rightarrow q(x)$

b) p(x) = “x has a PC”, q(x) = “x plays games”, c(x) = “x is taking COMP 1501”

$\forall x\ P(x) \rightarrow q(x)$ universal instantiation allows this line to represent any student (say students)

$\forall x\ C(x) \rightarrow q(x)$

$\therefore \forall x\ C(x) \rightarrow P(x)$

The premises in these arguments are true. In the first scenario ($P(x) \rightarrow q(x)$), if I do not have a PC but play games, the statement is true because the hypothesis is false. in the second scenario ($C(x) \rightarrow q(x)$), if I am a student and I play games, then this also true as the hypothesis and conclusion are both true, making the statement true. However, the conclusion of the original proposition would be false, $C(x) \rightarrow P(x)$, as the hypothesis is true and the conclusion is (quite possibly) false. The hypothesis is true but the conclusion is false, making the argument invalid as a result. Universal instantiation allows this line to represent any student (say students) or me (I am a student).

2. a) $\sqrt{9} + \sqrt{7}$, do both separately

1. Suppose: $\sqrt{9}$ using contradiction
2. Assume both variables are odd and that both a and b are odd and b is a factor of a
3. b cannot be 0,
4. $\sqrt{9} = a/b$ (math)
5. $\sqrt{9}^2 = a^2/b^2$ (math)
6. $9b^2 = a^2$ (math)
7. $9b^2 = (9x)^2$ (math)
8. $9b^2/9 = 81x^2/9$ (math)
9. $\sqrt{b^2} = \sqrt{9x^2}$ (math)
10. $b = 3x$ therefore, $b = 3x$, $a = 9x$ (by definition)
11. $\sqrt{9} = 9x/3x$ (math)
12. $\sqrt{9} = 3$ (math)
13. $3 + \sqrt{7}$ (replaced $\sqrt{9}$ with 3, line 12 above)

1 Assume $\sqrt{7}$ is a rational number

2 $\sqrt{7}$ = rational (defined)

3 $(\sqrt{7} = a/b) \wedge$ (a/b is in lowest form) (by definition)

4 $\sqrt{7} = a/b$ (by simplification)

5 $7 = a^2/b^2$ (math)

6 $7b^2 = a^2$ (math)

a. Lemma:

i. a^2 being divisible by 7 → a is also divisible by 2 (by definition)

ii. ¬a^2 divide 7 V a divide 7 (implication equivalence)

iii. ¬(¬a^2 divide 7 V a divide 7) (de Morgan's law)

1. $a^2 = 7k \wedge (a = 7k+1 \vee a=7k+2)$ (math)

iv. Case 1:

v. $a = 7k+1$ (math)

vi. $a^2= (7k+1)^2$ (math)

vii. $a^2= (49k^2 + 14k) + 1$ (math)

viii. $a^2= 7(7k^2 + 2k) + 1$ (math)

ix. $a^2= 7J+1$ (math)

x. Case 2:

xi. $a = 7k + 2$ (math)

xii. $a^2 = (7k+2)^2$ (math)

xiii. $a^2= 49k^2 + 28k + 4$ (math)

xiv. $a^2 = 7(7k^2 + 4k) + 4$ (math)

xv. $a^2 = 7J + 4$ (math)

xvi. Case 3:

xvii. $a = 7k + 3$ (math)

xviii. $a^2 = (7k+3)^2$ (math)

xix. $a^2 = 49k^2 + 42k + 9$ (math)

xx. $a^2 = 7(7k^2 + 6k) + 9$ (math)

xxi. $a^2 = 7J + 9$ (math)

xxii. Case 4:

xxiii. $a = 7k + 4$ (math)

xxiv. $a^2 = (7k+4)^2$ (math)

xxv. $a^2 = 49k^2 + 56k + 16$ (math)

xxvi. $a^2 = 7(7k^2 + 8k) + 16$ (math)

xxvii. $a^2 = 7J + 16$ (math)

xxviii. Case 5:

xxix. $a = 7k + 5$ (math)

xxx. $a^2 = (7k+5)^2$ (math)

xxxi. $a^2 = 49k^2 + 70k + 25$ (math)

xxxii. $a^2 = 7(7k^2 + 10k) + 25$ (math)

xxxiii. $a^2 = 7J + 25$ (math)

xxxiv. Case 6:

xxxv. $a = 7k + 6$ (math)

xxxvi. $a^2 = (7k+6)^2$ (math)

xxxvii. $a^2 = 49k^2 + 84k + 36$ (math)

xxxviii. $a^2 = 7(7k^2 + 12k) + 36$ (math)

xxxix. $a^2 = 7J + 36$ (math)

xl. $\neg a^2$ divisible by 7 V a divide 7 (from case 1-6)

xli. False (by negation)

xlii. there is a contradiction, The lemma has not been proven (by definition)

b. Proof by exhaustion:

i. a^2 being divisible by 7 → a is also divisible by 7 (by definition)

1. Proving the division of 7:
2. 1/7 = 0 R1 (math)
3. 2/7 = 0 R2 (math)
4. 3/7 = 0 R3 (math)
5. 4/7 = 0 R4 (math)

6. 5/7 = 0 R5 (math)
7. 6/7 = 0 R6 (math)
8. 7/7 = 1 R0 (math)
9. 8/7 = 1 R1 (math)
10. 9/7 = 1 R2 (math)
11. 10/7 = 1 R3 (math)
12. 11/7 = 1 R4 (math)
13. 12/7 = 1 R5 (math)
14. 13/7 = 1 R6 (math)
15. 14/7 = 2 R0 (math)

ii. Therefore, a is only divisible by factors of 7.

c. a^2 is divisible by 7 (from lemma[a] and exhaustion[b])

d. a is divisible by 7 (exhaustion[b])

e. $a = 7k$ (math)

f. $7b^2 = (7k)^2$ (math)

g. $7b^2 = 49k^2$ (math)

h. $b^2 = 7k^2$ (math)

i. b^2 is divisible by 7 (by definition)

j. b is divisible by 7 (lemma[a] and exhaustion[b])

k. a is divisible by 7 ∧ b is divisible by 7 (by addition)

l. a/b is not lowest form (by definition)

m. (a/b is in lowest form) ∧ (a/b is not in lowest form) (by conjunction)

n. False (negation)

o. ∴ √7 is an irrational number

7 3 + √7 (from √9 reduction)

8 the sum of an irrational number and a rational one is irrational, and the above sum is reduced to its lowest form

a. proof that the number is irrational:

b. √7 = x – 3 (math)

c. x is rational (by definition)

d. rational numbers are closed for subtraction (can't be reduced any more)

e. x = rational (by definition)

f. √7 = irrational (by definition)

g. rational = irrational (logic)

h. contradiction, if x is rational then √7 must be irrational

i. (√7 is irrational) ∧¬(√7 is irrational) (by conjunction)

j. T ∧ F (negation)

k. False (domination)

9 3 + √7 is irrational (from 8)

10 √9 + √7 is irrational (from 9, sub 3 for √7 from original proof at start line 12)

3. Consider √25 + √5

Lemma:

1 Consider: √25 using prime factorization

2 25/5 = 5 - divisor and resultant are equal, numbers are factors (math)

3 5 is drivable by only 1 and 5 and are prime (by definition)

4 if 5^2 = 25 then $\sqrt{5^2}$ = √25 = 5 (math)

5 5^2 = 25 → $\sqrt{5^2}$ = √25 = 5 (tuned into implication)

6 True (result of implication)

7 the √25 equals 5 (from 6)

8 5 + √5 (from 7, swapped + √25 with 5)

9 consider : assume √5 is a rational number

10 √5 = rational

11 (√5 = a/b) ∧ (a/b is in lowest form) (by definition)

12 √5 = a/b (by simplification)

13 $5 = a^2/b^2$ (math)

14 $5b^2 = a^2$ (math)

15 Lemma:

a. a^2 being divisible by 5 → a is divisible by 5 (by definition)

b. $a = P_1 * P_2 * P_3 * P_4$ (definition)

c. $a = 5 * P_2 * P_3 * P_4$ (sub prime 1 for factor coefficient of 5)

d. $a = 5k$ (by definition)

e. $k = P_1 * P_2 * P_3$ (by definition)

f. $5b^2 = a^2$ (by definition)

g. $a = P_1 * P_2 * P_3 * \ldots..$ (math, substitution)

h. $a^2 = (P_1 * P_2 * P_3 * \ldots)(P_1 * P_2 * P_3 * \ldots)$ (math, square)

i. $= P_1 * P_1 * P_2 * P_3$ (result for a^2, math)

j. a^2 and a are only divisible by factors of 5, the lemma is true (logic)

k. True

l. Proof by exhaustion:

i. a^2 being divisible by 5 → a is also divisible by 5 (by definition)

1. Proving the division of 7:

2. $1/5 = 0$ R1 (math)

3. $2/5 = 0$ R2 (math)
4. $3/5 = 0$ R3 (math)
5. $4/5 = 0$ R4 (math)
6. $5/5 = 1$ R0 (math)
7. $6/5 = 1$ R1 (math)
8. $7/5 = 1$ R2 (math)
9. $8/5 = 1$ R3 (math)
10. $9/5 = 1$ R4 (math)
11. $10/5 = 2$ R0 (math)

ii. Therefore, a is only divisible by factors of 5.

16 a^2 is divisible by 5 (from lemma[line 15] and exhaustion[b])

17 a is divisible by 5 (from lemma[line 15], and exhaustion[b])

18 $a = 5k$ (math)

19 $5b^2 = (5k)^2$ (math)

20 $5b^2 = 25k^2$ (math)

21 $b^2 = 5k^2$ (math)

22 b^2 is divisible by 5 (by definition)

23 b is divisible by 5 (lemma[a] and exhaustion[b])

24 a is divisible by 5 $\wedge$ b is divisible by 5 (by addition)

25 a/b is not lowest form (by definition)

26 (a/b is in lowest form) $\wedge$ (a/b is not in lowest form) (by conjunction)

27 False (negation)

28 $\therefore$ $\sqrt{5}$ is an irrational number

29 $5 + \sqrt{5}$ (from 14)

30 the sum of an irrational number and a rational one is irrational, and the above sum is reduced to its lowest form

a. proof that the number is irrational:

b. $\sqrt{5} = x - 3$ (math)

c. x is rational (by definition)

d. rational numbers are closed for subtraction (can't be reduced any more)

e. x = rational (by definition)

f. $\sqrt{5}$ = irrational (by definition)

g. rational = irrational (by conjunction)

h. contradiction, if x is rational then $\sqrt{5}$ must be irrational

i. ($\sqrt{5}$ is irrational) $\wedge \neg$($\sqrt{5}$ is irrational)

j. T $\wedge$ F (negation)

k. False (domination)

31 $5 + \sqrt{5}$ is irrational (from 16)

32 $\sqrt{25} + \sqrt{7}$ is irrational (from 15 and sub 5 for $\sqrt{7}$ from line 7)

4.

Assume integer n such that $n^3 + 5$ is odd, then n is even thus

indirect proof (Contraposition proof):

n is odd thus

$n^3 + 5$ is even, for example:

Let k be the integer such that $n = 2k + 1$

$n^3 + 5 = (2k + 1)^3 + 5$ (math)

$= (2k+1)(2k+1)(2k+1) + 5$ (math)

$= 8k^3 + 6k^2 + 6k + 1 + 5$ (math)

$= 8k^3 + 6k^2 + 6k + 6$ (math)

$= 2(4k^3 + 3k^2 + 3k +3)$ (math)

= an even number for every integer value used

(An even(odd) number is always even, and an even(even) is also even, therefore 2(n) = even)

By Contraposition, if $n^3 + 5$ is odd, then n is even

5. There is no base case

6. $-2 \leq x \leq 2$, prove that $y < 0$, where $y = x^4-36x^2+9x-5$

Proof by exhaustion; test all integer values:

(-2): $y = (-2)^4 – 36(-2)^2 + 9(-2) – 5$ (math)

$= 16 – 144 – 18 - 5$ (math)

$= -151$ (math)

(-1): $y = (-1)^4 – 36(-1)^2 + 9(-1) – 5$ (math)

$= 1 – 36 – 9 – 5$ (math)

$= -49$ (math)

(0): $y = (0)^4 – 36(0)^2 + 9(0) – 5$ (math)

$= -5$ (math)

(1): $y = (1)^4 – 36(1)^2 + 9(1) – 5$ (math)

$= 1 – 36 + 9 – 5$ (math)

$= -31$ (math)

(2): $y = (2)^4 – 36(2)^2 + 9(2) – 5$ (math)

$= 16 – 144 + 18 – 5$ (math)

$= -115$ (math)

All the possible integer values in the domain: $-2 \leq x \leq 2$ result in a range less than zero, therefore it is true that $y<0$.

7. This problem has a multistep approach:

(1): define P(n), Let P(n) be a property that is defined by integers, n

(2): show n = 1 is true,

$2n - 1 = n^2$ (math)

Let n = 1:

$2(1) – 1 = 1^2$ (math)

2-1 = 1 (math)

1 = 1 (math)

The statement is true for n=1

(3): inductive hypothesis, assume n = k is True for some K e N:

Assume that: $1+3+5+7 +…. + (2k-1) = k^2$ for some k e N

(4): Inductive procedure step: (show n = k is true – n = k+1 is also true)

1+3+5+7 +…. + (2k+1) + (2(k+1)-1) (by definition)

$k^2 + (2k(k+1)-1)$ (substitute k^2 for sequence, as defined in step 3)

$k^2 + (2k+2-1)$ (math)

$k^2 + 2k + 1$ (math)

$k^2 + 2k + 1 = (k+1)(k+1) = (k+1)^2$ (factorization, math)

$1+3+5+7+…+ (2k-1) + [2(k+1)-1] = (k+1)^2$ (original statement equals conclusion, math)

Truth for n=k implies that n = k+1 (step 4)

The statement is true for n=1 (step 2)

the steps were completed, by strong induction the identity is true for any value of n, thus n e N.

$1 + 1+3+5+7+\ldots+ (2n-1) = n^2$

8. The power set of A = {1, 6, 8} is:

P(A) = {1, 6, 8} = {∅, {1}, {6}. {8}, {1, 6}, {1, 8}, {6, 8}, {1, 6, 8}}

9.

a) The set of {“one”, “two”, “three”, “eight”, “ten”, “eleven”, “twelve”, “thirteen”, “eighteen”, “twenty”}

b) The set of {“Enzo”}

10.

a) Both cardinalities are the same size, 5

b) S ∩ T = {4}

c) S ∪ T = {4, 7, {1,7}, {monkey, banana}, {5,6,7,8,9,10,11,12,13,14,15}} ∪ {{1,7,20}, {5}, 4, {fruit}, monkey}

S ∪ T = {4, 7, {1,7}, {monkey, banana}, {5,6,7,8,9,10,11,12,13,14,15}, {1,7,20}, {5}, {fruit}, monkey}

therefore, 9

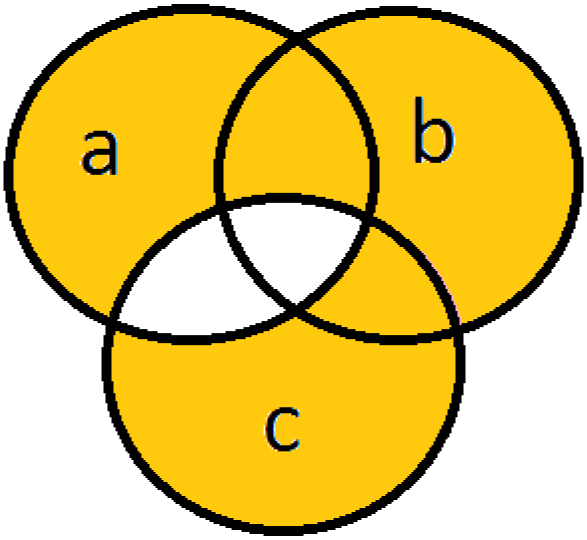
11. The following is not valid:

A	B	C	(A-C)	(B-A)	(A-C) ∪ (B-A)

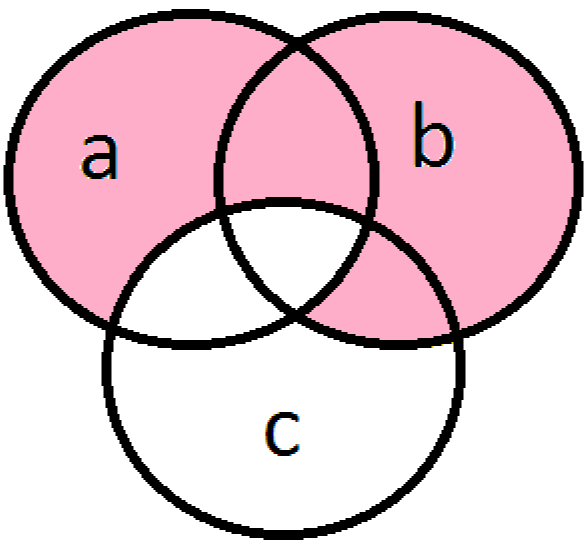
1	1	1	0	0	0
1	1	0	1	0	1
1	0	1	0	0	0
1	0	0	1	0	1
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	0	0

A	B	C	(A∪B∪C)	(C∩A)	(A∪B∪C) - (C∩A)
1	1	1	1	1	0
1	1	0	1	0	1
1	0	1	1	1	0
1	0	0	1	0	1
0	1	1	1	0	1
0	1	0	1	0	1
0	0	1	1	0	1
0	0	0	0	0	0

12.



Orange: (a∪b∪c) – (c∩a)



Pink: (a – c) ∪ (b – a)

13. The intersection is:

A = intersection of B and C = B∩C

B = all digits in student number = {1, 0, 1, 0, 4, 1, 1, 2, 5}

C = even numbers = {0, 0, 4, 2}

A = B∩C

A = {1,0,1,0,4,1,1,2,5} ∩ {0,0,4,2}

A = {0, 4, 2}

(Set "A" doesn't contain two zeros because it simply contains elements of a that are also elements of b, it doesn't double list them)