Connor Stewart

COMP 2804 Assignment 3:

Question 1:

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Question 2:

To see what option has the largest probability, all three options should be first analized.

① All four kids are of the same gender:
For this to occur, all kids must be all boys OR all girls.
Let B be boys & G be girls, & S be the set of all children.
$S = \{B,B,B,B\} = \{G,G,G,G\}$ are ∴ the only 2 valid sets.
- The chance of having a boy OR girl is $1/2$ for both
- Each child birth is an independent event ∴ use Product Rule
- The chance of having all Boys: $(1/2)^4$
The chance of all Girls: $(1/2)^2$
The sum of both is the total chance: $(2)(1/2)^4$
∴ The chance of having all boys or girls is:
$(2)(1/2)^4 = (2)(1/16) = \boxed{1/8} = 0.125$ OR 12.5% Chance

② Three kids are of the same gender & the fourth kid is of the opposite gender.
- Using the same principles as above, the set is:
$S = \{G,G,G,B\} = \{G,G,B,G\} = \{G,B,G,G\} = \{B,G,G,G\}$
$= \{B,B,B,G\} = \{B,B,G,B\} = \{B,G,B,B\} = \{G,B,B,B\}$
- Each set of 4 children has a $(1/2)^4$ chance of occuring, with 8 sets
∴ $(8)(1/2)^4$ is the probability of ②.
$(8)(1/2)^4 = (8)(1/16) = \boxed{1/2} = 0.5$ OR 50% Chance

③ Two kids are boys & two kids are girls.
- Using the principles set in ①
$S = \{B,G,B,G\} = \{B,G,G,B\} = \{G,B,B,G\} = \{G,B,G,B\} = \{B,B,G,G\} = \{G,G,B,B\}$
- Each set has a $(1/2)^4$ chance of occuring, theres 6 sets
∴ $(6)(1/2)^4 = (6)(1/16) = \boxed{3/8} = 0.375$ OR 37.5% Chance

In Conclusion: option ② has the greatest chance of occuring, as it has the most possibilities. Option ② has a 1/2 OR 50% chance of occuring.

Question 3:

— to awnser this question, we must find the probability that a boy will be born Given atleast one boy is born on a Sunday.
— To model this, the Conditional rule must be used as we are given some information (atleast one boy was born on a Sunday).
- Conditional Rule:

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

□ A = Another boy being born
□ B = Given (boy on a Sunday)

— To begin, we should find $Pr(B)$ first. The probability of a boy being born is 1/2 & the probability of a birth on Sunday is 1/7. Therefore, the probability of these two, unrelated events occuring is the product of both (Product Rule).
— A boy may be born on a Sunday whether he is the first born child, the second, or both.
- This can be modeled with inclusion–exclusion as; if the child is born on a Sunday & is a boy in both cases, this is contained within the set of the child being born first or second, & ∴ must be removed.
  - Theres only 1 way a boy can be born on a Sunday, let this be the first child.
  - Now we must conclude how many ways the second kid can be born.
    - □ A boy (2 choices) can be born any day of the week (7 choices)
    - □ The product rule will give the number of possibilities: $(2)(7) = 14$
      - There are ∴ 14 possibilities for the second child
    - □ If we reverse the scenario, the the given child is the second born, then theres 14 more possibilities.
    - □ If both children, the given & variable, are born on Sunday, then theres 1 possibility.
    - □ According to inclusion exclusion, we must add the possibilities & remove the intersection. Formula: $|A \cup B| = |A| + |B| - |A \cap B|$

$$= 14 + 14 - 1 = 28 - 1 = 27 \text{ Possibilities}$$

□ The Sample Space is the total number of possibilities for both children 14^2. This is the Num. of days (7) by the Num. of genders (2) for both cases (14^2)

□ The number of possibilities divided by the sample space is the total probability: $27/14^2 = 27/196 \doteq 0.137755$.

□ This is $Pr(B)$, $Pr(B) = 27/196$.

- Now we Should find $Pr(A \cap B)$, the probability of the intersect between a boy being born & the Given (a boy born on Sunday).
  - Now there both boys (1 choice) & they can be born any day of the week (7 choices). The Product of this is: $7 \cdot 1 = 7$
  - If we reverse the Scenario again, we find 7 more possibilities.
  - Like before, one boy on either day is given to be born on Sunday.
  - There is 1 Case where both Children are boys whome are born on Sunday.
  - the inclusion-exclution for this: $7+7-1 =$ 13 Possibilities
  - The Sample Space (All cases) is Still 14^2.
  - The Probability is $\therefore 13/14^2 = 13/196.$ This is $Pr(A \cap B)$, $Pr(A \cap B) = \frac{13}{196}$
  - We Substitute the values into the Conditional Rule to find $Pr(A|B)$.

$$Pr(A|B) = \frac{(13/\cancel{196})}{(27/\cancel{196})} = 13/27$$

$\therefore$ The Probability of Anil having two boys using the given Conditions is $\boxed{13/27}$.

Question 4:

- To Define the events we must determine the probabilities of each event:

For A:
- There are 2 b's per each face, & 2 dice. The Pr of this not happening is:
- $Pr(A) = 1 - \left[\underbrace{\left(\tfrac{1}{6}\right)^2}_{\text{roll } A,A} + \underbrace{\left(\tfrac{3}{6}\right)^2}_{\text{roll } B,B} + \underbrace{\left(\tfrac{1}{6}\right)\left(\tfrac{3}{6}\right)(2)}_{\text{roll } A,C \text{ or } C,A} \right] = 5/9$

For B:
- There is 1 a, 2 b's, & 3C's per face. There is 2 Die.
- $Pr(B) = \underbrace{\left(\tfrac{1}{6}\right)^2}_{\text{roll } A,A} + \underbrace{\left(2/6\right)^2}_{\text{roll } B,B} + \underbrace{\left(3/6\right)^2}_{\text{roll } C,C} = 7/18$

For A & B:
- Same rules as above, but now its A Given B:
- $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$
- $Pr(A \cap B)$ is the probability that both A & B occur.
- $Pr(A \cap B) = (2/6)^2 = 1/9$ (Only b must roll & both must be Same letter, $\therefore$ 2 b's must roll)
- $Pr(A|B) = \frac{1/9}{7/18} = 2/7$

Question 5:

- First we must count the number of ways to make a Stright.
  - Given any Suit, the number of ways to count Suits consecutivly are:

    Let S = the number of ways to order 5 cards into a Stright

    $S = \{A,2,3,4,5\}, \{2,3,4,5,6\}, \{3,4,5,6,7\}, \{4,5,6,7,8\}, \{5,6,7,8,9\}, \{6,7,8,9,10\}, \{7,8,9,10,J\}, \{8,9,10,J,Q\}, \{9,10,J,Q,K\}, \{10,J,Q,K,A\}$

    $\therefore$ There are $\boxed{10}$ ways to order 5 cards consecutivly into a Stright.
  - We must now factor in the number of Suits per hand of Cards.

    There are 4 Suits & 5 Cards per hand

    If there was only 1 Suit, there would be only 1 Combo of hands.

    Therefore, the number of Combination of Cards equals the number of Suits to the Power of the Size of the hand.

    $\therefore \boxed{4^5}$ = # of Card Combos per each Stright
  - The Cards Cannot all be of the Same Suit; & each Suit has only 1 Combo. of Cards per Stright that are all of the Same Suit.

    Ex. for Combo. $\{A,2,3,4,5\} \longrightarrow$ Diamonds has only 1 combo of this that can't be used: $\{A\diamond, 2\diamond, 3\diamond, 4\diamond, 5\diamond\}$

    Theres 4 Suits, $\therefore$ 1 Card per Stright multiplied by the number of Suits equals the intersect between Strights & Stright-flushes. This value should be removed per each Stright: $-(4)(1) = -4$
  - The Probibility that the hand is a Stright is based on the above info., the number of Straights is multiplied by the number of Card Combos per Stright Subtracted by the number of flushes (all the Same Suit) per Stright.

    $\therefore \boxed{10(4^5-4)}$ is the number of hands that are Strights
- We must now find the number of hands possible: we must Choose 5 Cards from the deck of 52. $\therefore \binom{52}{5}$ = # of 5 Card hands.

$$\boxed{\binom{52}{5}} = \frac{A!}{B!(A-B)!} = \frac{52!}{5!(52-5)!} = 2\,598\,960$$

- the $Pr(\text{hand is Stright}) = \dfrac{\text{Strights}}{\text{Hands}} = \dfrac{10(4^5-4)}{2598960} = \dfrac{10200}{2598960} = \dfrac{5}{1274}$

$$\doteq 0.00392465$$

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Question 6:

- To determine if these events independent, we must use the formula:
$Pr(A \wedge B) = Pr(A) \cdot Pr(B)$, this determines if there independent (if they equal).

$Pr(A)$:

There are 52 cards & 4 Aces (1 per Suit) in the deck:
$\therefore Pr(A) = Pr(4/52) = Pr(1/13)$ OR ($\doteq 0.0769$)

$Pr(B)$:

1/4 of the deck of 52 is diamonds $\therefore$ $52/4 = 13$ cards are diamonds.
$\therefore Pr(B) = Pr(13/52) = \boxed{Pr(1/4) = Pr(0.25)}$

$Pr(A \wedge B)$:

This is the prob. that the card is an ace & is of diamond suit.
There is only 1 card like this per 52.
$Pr(A \wedge B) = Pr(1/52)$ OR $\doteq 0.0192$

- We must find if $Pr(A \wedge B) = Pr(A) \cdot Pr(B)$ holds True:

$Pr(A \wedge B) = Pr(A) \cdot Pr(B)$

$Pr(1/52) = Pr(1/13) \cdot Pr(13/52)$

$= 1/52 \longrightarrow Pr(1/52) = 1/52$ $\therefore$ $\boxed{\text{They are independent}}$

- Events A & B are $\boxed{\text{independent}}$.

- To determine this, we use the same formula as above:

$Pr(A)$:

like above, theres 51 cards & 4 aces, 1 per suit. $\therefore Pr(A) = Pr(4/51)$

$Pr(B)$:

There are 51 cards & 13 are diamonds; No Diamonds have been removed prior.
$Pr(B) = Pr(13/51)$

$Pr(A \wedge B)$: The prob. that a cards an ace & is of diamonds is still 1, but now out of 51 cards. $Pr(A \wedge B) = Pr(1/51)$

— We must find is $Pr(A \wedge B) = Pr(A) \cdot Pr(B)$ holds True:

$Pr(A \wedge B) = Pr(A) \cdot Pr(B)$

$= Pr(4/51) \cdot Pr(13/51)$

$= 52/2601$

$1/51 \neq 52/2601$ $\boxed{\therefore \text{They are NOT independend}}$

— Events C & D are NOT independent

Question 7:

— There are 10 boxes & n balls, any ball may equally land in any box leaving a 1/10 chance a ball lands in a specific box.

— We may model this using the formula:

$q_n = \frac{d!}{(d-n)!\,d^n}$ Where: $d = \text{Size} = 10$ boxes, $n =$ balls, $q_n =$ Preprobability

$p_n = 1 - q_n \rightarrow 1 - p_n = q_n$

— The Smallest value of n for which $p_n \geq 1/2$:

$$q_n = \frac{d!}{(d-n)!\,d^n} \rightarrow 1 - 0.5 = \frac{10!}{(10-n)!(10^n)} \Rightarrow \frac{1}{2} \leq \frac{10!}{(10-n)!(10^n)}$$

Substitute for n: Find n such that the resutent is 1/2

Let $n=1$: $\frac{10!}{(10-1)!(10^1)} = \frac{10!}{9!\,10^1} = 1$

Let $n=2$: $9/10$

Let $n=3$: $18/25$ OR $= 0.72$

Let $\boxed{n=4}$: $63/125$ OR $= 0.504$

Let $n=5$: $189/625$ OR $= 0.3024$

$1 - 0.3024 = 0.6972$

$0.6927 \geq 1/2 \therefore n=5$

$\therefore$ 5 is the Smallest Value of n Such that $p_n \geq 1/2$.

— The Smallest Value of n for which $p_n \geq 2/3$:

The Probabilities for $n = 1, 2, 3, 4$ are the Same as above.

Continuing from $n=4$, $1 - 2/3 = q_n \therefore q_n = 1/3$

Let $n=5$: $\frac{10!}{(10-5)!(10^5)} \geq \frac{1}{3} \rightarrow \frac{3628800}{120 \cdot 100000} = \boxed{\frac{189}{625}} = 0.3024$

if $n=4$ then $0.504 \rightarrow 1 - 0.504 =$ too Small

if $n=5$ then $1 - 0.3024 = 0.6976$

$0.6927 \geq 2/3 \therefore n=5$

$\therefore$ 5 is the Smallest Value of n Such that $p_n \geq 2/3$.

Question 8:

— If Nick Knows the awnser, he will Say True
- If he dosn't, he makes a Even Guess between True or false
- Initially, if he Knows the awnser he will Say its true otherwise he guesses. To model his probability of getting true for the question we must use the following:

$Pr(B) = 0.8$ (B = nick Knows the answer)

$Pr(A|B) = 1$ (Prob. of getting right Knowing the awnser)

$Pr(A|\bar{B}) = 0.5$ (Prob. of getting right guessing)

$Pr(A) = Pr(A \wedge B) + Pr(A \wedge \bar{B})$ (Overall probability based on the 2 events above)

- These 2 independent events Can be added via. the Sum rule to find $Pr(A)$.

To proceed $Pr(A \wedge B)$ & $Pr(A \wedge \bar{B})$ must be determened.

$Pr(A|B) = \frac{Pr(A \wedge B)}{Pr(B)}$ Can be used to model this:

$Pr(A|B) \cdot Pr(B) = Pr(A \wedge B) \longrightarrow Pr(A \wedge B) = (1)(0.8) = 0.8$

$Pr(A \wedge \bar{B}) = Pr(A|\bar{B}) \cdot Pr(\bar{B}) \longrightarrow Pr(A \wedge \bar{B}) = (0.5)(0.2) = 0.1$

$\hookrightarrow Pr(\bar{B}) = 0.2$ (Chance of not Knowing anser)

$Pr(A) = Pr(A \wedge B) + Pr(A \wedge \bar{B})$ [Substitute]

$Pr(A) = 0.8 + 0.1$ [Add]

$\boxed{Pr(A) = 0.9}$ [final Solⁿ]

$\therefore Pr(A)$ equals 0.9

Question 9:

— We are given A_n & B_n, & that $P_n = Pr(A_n)$ & $Q_n = P_n - 1/2$

A_n = GenerateBit(1) returns 0:

flip coin 1 time $\longrightarrow Pr(head) = P$; $Pr(tail) = 1 - P$

Let K = # of heads in Sequence of 1 coin flip

if K = odd: $\longrightarrow$ If head Rolled $\hookrightarrow$ only heads OR tails

return 0

else: the total Pr is $\therefore \boxed{P}$

return 1

Endif

$P_1 = Pr(A_1)$

$P_1 = Pr(P)$

$Q_1 = P_1 - 1/2$

$Q_1 = P - 1/2$

— For any integer $n \geq 2$, Prove: $P_n = p + (1-2p) \cdot P_{n-1}$

- A_n is True if: A_{n-1} is False (Even) & B_n is True (head).
- if A_{n-1} was false then there was an even number of heads, as this would return a 1, not a 0. If B_n is True, then the last flip is heads.
  - This means A_n would equal A_{n-1} Plus B_n
  - It also means that if B_n is True & A_{n-1} is False, then A_n is True as A_{n-1} is Even & B_n (which is 1) is odd. A_n Even + odd = odd Number, ∴ A_n would be odd & thus True.
- P_n is the $Pr(A_n)$, if $A_n = (\overline{A_{n-1}} \wedge B_n) \vee (A_{n-1} \wedge \overline{B_n})$

  □ A_n is true if $(A_{n-1} \wedge \overline{B_n})$ becaus A_{n-1} is odd & $\overline{B_n}$ dosn't change this being tails

$P_n = Pr(A_n)$

- $A_n = (\overline{A_{n-1}} \wedge B_n) \vee (A_{n-1} \wedge \overline{B_n})$,

① $Pr(\overline{A_{n-1}} \wedge B_n) = ((1-P_{n-1}) \wedge (P)) = ([1-P_{n-1}][P])$

$= P - P_{n-1}P$

② $Pr(A_{n-1} \wedge \overline{B_n}) = ((P_{n-1}) \wedge (1-P)) = ([P_{n-1}][1-P])$

$= P_{n-1} - P_{n-1}P$

$P_n = (P - P_{n-1}P) + (P_{n-1} - P_{n-1}P)$ [Substitute ① & ②]

$P_n = P + P_{n-1} - 2P_{n-1}P$ [Arithmatic]

$P_n = P + (1-2p) \cdot P_{n-1}$ [factor out P_{n-1}]

— For any integer $n \geq 2$, Prove that:

- $Q_n = (1-2p) \cdot Q_{n-1}$ (Proove this)
- $P_n = P + (1-2p) \cdot P_{n-1}$
- $Q_n = P_n - 1/2$

[Substitution]

$Q_n = (P + (1-2p) \cdot P_{n-1}) - 1/2$ [Sub in P_n]

$Q_n = P + P_{n-1} - 2PP_{n-1} - 1/2$ [Expand]

$Q_n = P - 2PP_{n-1} + \underbrace{P_{n-1} - 1/2}$ [Algabra]

$Q_n = P_n - 1/2$ [By Def.]

$Q_{n-1} = P_{n-1} - 1/2$ [logic]

$Q_n = P - 2PP_{n-1} + Q_{n-1}$ [Substitution]

$Q_n = P(1 - 2P_{n-1}) + Q_{n-1}$ [factor]

$= -2P(P_{n-1} - 1/2) + Q_{n-1}$ [factor]

$= -2P(Q_{n-1}) + Q_{n-1}$ [Substitution]

$= Q_{n-1}(-2p(1) + 1)$ [factor out Q_{n-1}]

$Q_n = (1-2p) \cdot Q_{n-1}$ [final Sol$^{\underline{n}}$]

— For any integer $n \geq 1$, Prove that: $Q_n = (1-2p)^{n-1}(P - 1/2)$

- $Q_n = (1-2p) \cdot Q_{n-1}$
- $Q_1 = P - 1/2$
- $P_n = P + (1-2p) \cdot P_{n-1}$
- $Q_n = P_n - 1/2$

Let $n=2$: $Q_2 = (1-2p) \cdot Q_{2-1}$ [Sub. for n]

$Q_2 = (1-2p) \cdot Q_1$ [Simplify]

$Q_2 = (1-2p) \cdot (P - 1/2)$ [Substitute]

Let $n=3$: $Q_3 = (1-2p) \cdot Q_{3-1}$ [Sub. for n]

$Q_3 = (1-2p) \cdot Q_2$ [Sub. for Q_2]

$Q_3 = (1-2p) \cdot ([1-2p] \cdot [P - 1/2])$ [Simplify]

$Q_3 = (1-2P)^2 \cdot (P - 1/2)$ [Associative]

Let $n=n$: $Qn = (1-2p)\cdot Q_{n-1}$ [by Def.]

$Qn = (1-2p)\cdot([1-2p]^{n-2}[p-1/2])$ [from Q_2 & Q_3 Pattern]

$Qn = (1-2p)(1-2p)^{n-2}(p-1/2)$ [Associative]

$Qn = (1-2p)^{n-2+1}(p-1/2)$ [Simplify]

$Qn = (1-2p)^{n-1}(p-1/2)$ [final Sol^{n}]

□ As can be seen in Q_2 & Q_3, every time n increases by 1, Qn's proportion to $(1-2p)$ increased by a power of 1. This pattern can be expressed as $(1-2p)^{n-1}$ for Qn.

$\therefore\ Qn = (1-2p)^{n-1}(p-1/2)$

- If we were to use induction for this:

base case: when $n=1$

$Q_1 = (1-2p)^{1-1}(p-1/2) = (1)(p-1/2) = p-1/2$

Inductive Hypothesis:

$Q_{n-1} = (1-2p)^{n-2}(p-1/2)$

Given:

$Qn = (1-2p)(Q_{n-1})$

Inductive Step:

$Qn = (1-2p)(1-2p)^{n-2}(p-1/2)$ [Sub. Q_{n-1} for Given]

$Qn = (1-2p)^{n-2+1}(p-1/2)$ [Simplify]

$Qn = (1-2p)^{n-1}(p-1/2)$ [Simplify]

— Prove that:

- $\lim_{n\to\infty} Qn = 0$ △
- $0<p<1$
- $Qn \doteq (1-2p)^{n-1}$ when $\lim_{n\to\infty}$ because $n\to\infty$ isn't changing $p-1/2$ but does change $(1-2p)^{n-1}$.

- $-1 < 1-2p < 1$ as $0 < p < 1$:

for $0 < p < 1$ find highest and lowest possibility for $\lim_{n \to \infty} = (1-2p)^{n-1}$

high: $1-2(0)$ low: $1-2(1)$ – we are plugging 0 & 1

$= 1$ $= 1-2 = -1$ from $0 < p < 1$ into $1-2p$.

$\therefore -1 < 1-2p < 1$

- $\lim_{n \to \infty} Q_n = (1-2p)^{n-1}$ where $-1 < 1-2p < 1$:

We have a decimel approching infinity. As $n \to \infty$

$(1-2p)^{n-1} \to 0$.

$\therefore \lim_{n \to \infty} Q_n = (1-2p)^{n-1}$

- Prove: $\lim_{n \to \infty} P_n = 1/2$:

$P_n = Q_n + 1/2$ [from $Q_n = P_n - 1/2$]

$\lim_{n \to \infty} P_n = \lim_{n \to \infty} (Q_n + 1/2)$

$\lim_{n \to \infty} (P_n) = 1/2$

Question 10:

- Determine $Pr(A)$:
- We should first write out all the possibilities:

$C_1 = \{H, T\}$ – both H & T have a 1/2 probability

$C_2 = \{H, T\}$ – H has p probability, & T has $1-p$ prob.

$C = \{C_1, C_2\}$ – Picking either coin has a 1/2 chance each

$A =$ first flip is heads

Let $D =$ pick C_1

□ Write A in terms of:

These two terms are disjoint.

$$A \Leftrightarrow (A \wedge D) \vee (A \wedge \bar{D})$$

$$Pr(A) = Pr(A \wedge D) + Pr(A \wedge \bar{D})$$

$$Pr(A|D) = \frac{Pr(A \wedge D)}{Pr(D)} \text{ (by Def.)}$$

$$Pr(A \wedge D) = Pr(A|D) \cdot Pr(D)$$

$$Pr(A) = Pr(A|D) \cdot Pr(D) + Pr(A|\bar{D}) \cdot Pr(\bar{D})$$

Substitute values:

- The prob of picking the first coin is $D = 1/2$, & thus $\bar{D} = 1/2$. The prob. of picking the right coin given D is $A = 1/2$.
- $Pr(A) = Pr(0.5|0.5) \cdot Pr(1/2) + Pr(p) \cdot Pr(1/2)$

$Pr(A) = 1/2\,p + 1/4$

- Determine if A & B are independent assuming $p = 1/4$:

$Pr(A \wedge B) = Pr(A) \cdot Pr(B)$

$Pr(B) = Pr(A) = 1/2\,p + 1/4$, B follows same logic.

$A \wedge B \Leftrightarrow (A \wedge B \wedge D) \vee (A \wedge B \wedge \bar{D})$

disjoint, same as above.

$Pr(A \wedge B) = Pr(A \wedge B \wedge D) + Pr(A \wedge B \wedge \bar{D})$

- $Pr(A \wedge B \wedge D) = \dfrac{Pr(A \wedge B \wedge D)}{Pr(D)}$
- $Pr(A \wedge B \wedge D) = Pr(A \wedge B \mid D) \cdot Pr(D)$

Sub.

$Pr(A \wedge B) = Pr(A \wedge B \mid D) \cdot Pr(D) + \underbrace{Pr(A \wedge B \mid \bar{D}) \cdot Pr(\bar{D})}_{\text{Same as above}}$

- ~~Pr of D~~ $= 1/2$ as its 50:50 of getting right coin. Given that D is 50:50 there is a 1/2 chance of picking the right coin given D, for a fair coin (C_1). $\bar{D}$ is 50:50 given above. the prob. of flipping heads on the unfair coin is p because its unfare.
- $Pr(A \wedge B) = (1/2 \cdot 1/2)(1/2) + (p)(p)(1/2)$

  $Pr(A \wedge B) = \frac{1}{8} + p^2(1/2)$
- $Pr(A) \cdot Pr(B) = Pr(A \wedge B)$ is its independent

  $(1/4 + 1/2(p))(1/4 + 1/2(p)) = 1/8 + 1/2 p^2$

  Let $p = 1/4$:

  $(1/4 + (1/2)(1/4))(1/4 + 1/2(1/4)) = 1/8 + 1/2(1/4)^2$

  $(3/8)(3/8) = 1/8 + 1/32$

  $9/64 = 5/32$

  Clearly, $9/64 \neq 5/32$ $\therefore$ A & B are not independent

- determine all events for p for which events A & B independent:

  $Pr(A) \cdot Pr(B) = Pr(A \wedge B)$

  $(1/4 + 1/2(p))(1/4 + 1/2(p)) = 1/8 + 1/2 p^2$

  $1/16 + 2/8(p) + 1/4(p^2) - 1/8 - 1/2 p^2 = 0$

  $0 = \frac{1}{2}p^2 + \frac{1}{8} - 1/16 - 2/8 p - 1/4 p^2$

  $(0 = 1/4 p^2 - 2/8 p + 1/16)$

Quadradic Formula:

$$p = \frac{-(-2/8) \pm \sqrt{(-2/8)^2 - 4(1/4)(1/16)}}{2(1/4)}$$

$$= \frac{2/8 \pm \sqrt{0}}{(1/2)}$$

$$= \frac{2/8}{1/2}$$

$$= 1/2$$

$\therefore$ The only value for p such that A & B are independent is $1/2$.