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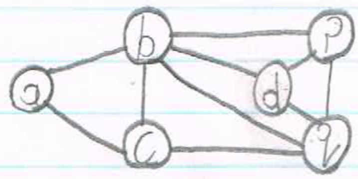
Acknowledgement: No Help Received

MATH 3802 Assignment Seven:

Let $G=(V,E)$ be the undirected graph depicted below:



Important: In your answers, include depictions for parts that ask for cycles, trees, forests, or matchings.

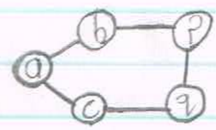
① (1 Point) Give the degrees of the nodes b & q:

b is incident to five nodes so ⑤, $deg(b)=5$

q is incident to four nodes so ④, $deg(q)=4$

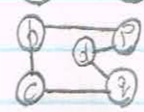
② (1 Point) Give two cycles of length 5. (The length of a cycle is the number of edges in the cycle.)

First cycle:
$a \to b \to p \to q \to c \to a$



dipiction one

Second Cycle:
$b \to p \to d \to q \to c \to b$



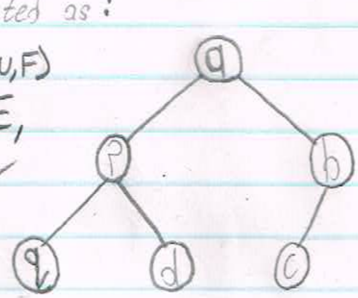
dipiction two

③ (2 Points) Give a spanning tree that contains the edges ab & pq:

We can use the tree: ab, bc, ap, pq, pd

this is depicted as:

Since the tree $T=(U,F)$ has $U=V$ & $F \subseteq E$, T is a spanning tree of G & contains edges ab & pq.



trees have n-1 edges, there are 6 nodes in the tree & 5 edges $\therefore$ $6-1=5$ so it's a tree

Each node is connected by an edge to the tree, & nodes all have a 1-1 connection at most. All nodes are in tree:

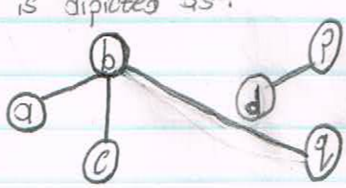
$\therefore$ Valid tree

④ (2 Point) Give a spanning forest having exactly two components containing the edges bc & bq:

An undirected graph with no cycles is a forest (A tree is a connected forest)

We use the following edges: ab, bc; pd, bq

this is dipicted as:



No cycles exist as no walk of length three or greater results in us ending up at a starting node $\therefore$ forest

Since the forest $F=(U,T)$ has $U=V$ & $T \subseteq E$, T is a spanning forest of G & contains edges bc & bq.

(5) Let $M=\{ab,dp\}$. Let $N=\{ac,bq,dp\}$. List all M-exposed nodes.

a) (1 Point) List all M-exposed nodes:

M-exposed nodes are not endnodes of some edge in a matching M

Since M has nodes $a,b,d,\&\ p$ it does not have c or q as endpoints

So, we know the M-exposed nodes are $\boxed{c\ \&\ q}$

b) (1 Point) Give a depiction of the graph $(V, N\Delta M)$:

$N\Delta M$: Symmetric difference

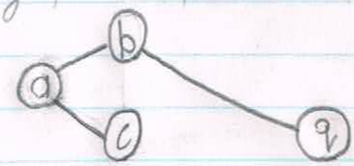
takes edges from either N or M, but not **both**

V: All verticies in G

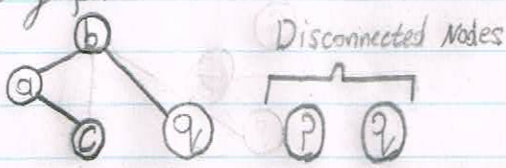
We note that:

$$N\Delta M = \{ab,\cancel{dp}\}\setminus\{ac,bq,\cancel{dp}\} \cup \{ac,bq,\cancel{dp}\}\setminus\{ab,\cancel{dp}\}$$
$$=\{ab,ac,bq\}$$

thus, the graph is depicted as:



Note; since V contains all nodes in G we can also display singleton nodes as the following graph:



where these nodes become excluded from the resulting graph

c) (1 Point) Use the result in Part (b) to obtain an M-augmenting Path:

we know $N\Delta M=\{ab,ac,bq\}$, $M=\{ab,dp\}$, & $N=\{ac,bq,dp\}$

M-augmenting Path: If the endnodes of P are both M-exposed

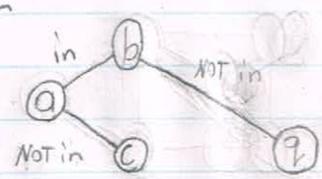
M-Alternating Path: A matching M in G such that its edges are alternatly in & not in M.

M contains nodes $a,b,d,\&\ p$ so these cannot be endpoints, this leaves $c\ \&\ q$

The Path from c to q in $N\Delta M$ is alternating as $ab\in M$ & $ac,bq\in N$

dp is in M so it can be used as an alternating Path:

<u>Depiction:</u>



↳ Alternates between edges NOT in & edges in M

$c\ \&\ q$ are both M-exposed nodes & ab is in M, whereas the other two edges are not

$\therefore$ Path $P=\{ab,ac,bq\} = N\Delta M$ is M-augmenting

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d) (1 Point) Give a perfect matching in G not Containing the edge ac:

Matching: No two edges incident to the Same node

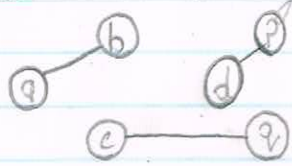
Perfect matching: Every node in G is M-Covered

M-Covered: A node that is the endnode of Some node in a matching M

Thus, we use the following:

ab, Pd, cq

this gives us the following dipiction:



So, our Perfect matching is: $\boxed{M=\{ab, Pd, cq\}}$