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MATH 3802 Tutorial one:

Tutorial Information:

1. Show a Path joining $V_1 \to V_5$:

Dipath:

A Dipath means the graph contains only forward arcs.

$V_1 e_1 V_2 e_4 V_3 e_7 V_5$ is a dipath

Non-Dipath: (we must go against the flow)

$V_1 e_2 V_4 e_8 V_5$ is a non-dipath

2. Find a $\delta^+(S)$ & $\delta^-(S)$ where $S = \{V_1, V_2\}$:

$\delta^+(S) = \{e_2, e_4, e_6\}$ leave the set of S

$\delta^-(S) = \{e_3\}$ enters the set of S

3. Show the cost of the V_1-V_4 dipath $V_1 e_1 V_2 e_4 V_3 e_7 V_5 e_8 V_4$:

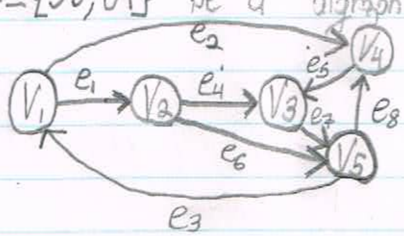
given $c \in \mathbb{R}^4$ & $c = [1\ 3\ 2\ 1\ 0\ 1\ 0\ -1]^T$

thus, we must add the cost of the paths along the dipath:

$1 + 1 + 0 + (-1) = 1$ $\therefore$ the cost of the path is $\boxed{1}$

Tutorial Problems:

Let $G = \{N, A\}$ be a digraph depicted below:



1. Find a path joining V_1 to V_5 that is not a dipath, & a V_1-V_5 dipath

Non-Dipath: Against flow

$V_1 e_3 V_5$ is a non-Dipath

Dipath: With flow

$V_1 e_2 V_4 e_5 V_3 e_7 V_5$ is a dipath

2. Find $\delta^+(S)$ & $\delta^-(S)$ for $S = \{V_2, V_4\}$

$\delta^+(S) = \{e_4, e_5, e_6\}$ leave the set of S

$\delta^-(S) = \{e_1, e_2, e_8\}$ enters the set of S

3. Show the cost of V_1-V_4 dipath with minimum cost: ($c \in \mathbb{R}^4$ & $c = [1\ 3\ 2\ 1\ 0\ 1\ 0\ -1]^T$)

there are 3 dipaths from V_1 to V_4, with their respective costs: (add costs of e_i)

$V_1 e_2 V_4 \longrightarrow$ Cost: $3 = 3$

$V_1 e_1 V_2 e_4 V_3 e_7 V_5 e_8 V_4 \longrightarrow$ Cost: $1 + 1 + 0 + (-1) = 1$

$V_1 e_1 V_2 e_6 V_5 e_8 V_4 \longrightarrow$ Cost: $1 + 1 + (-1) = 1$

thus, the minimum cost is 1 with dipaths of $V_1 e_1 V_2 e_4 V_3 e_7 V_5 e_8 V_4$ & $V_1 e_1 V_2 e_6 V_5 e_8 V_4$