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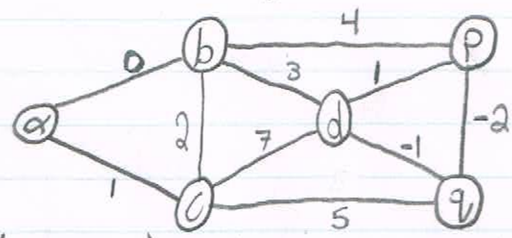
Acknowledgement: No Help Receaved

MATH 3802 Assignment Eight:

let $G=(V,E)$ be the undirected graph depicted below with edge weights Shown next to the edges:

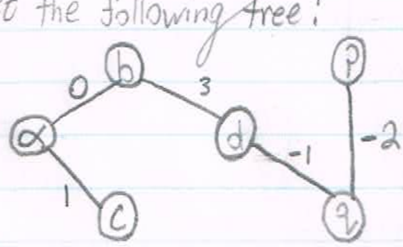


① (4 Points) Find a minimum-weight Spanning tree using Prim's algorithm with $r=\alpha$ in Step 1. Your answer should include the sequence of edges listed in the order they are Selected by the algorithm as well as an illustration of the Spanning tree:

Prims Algorithm:

Step	S	Node: α	b	c	d	p	q
	{}	0/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil
	α		0/α	1/α	∞/nil	∞/nil	∞/nil
	α,b			1/α	3/b	4/b	∞/nil
	α,b,c				3/b	4/b	5/c
	α,b,c,d					1/d	-1/d
	α,b,c,d,q					-2/q	
	α,b,c,d,p,q						

↳ Thus, nodes a,b,c,d,p,q are connected via. Paths αb, αc, bd, dq, & qp. We get the following tree:



thus, we select (In the following order):

① αb
② αc
③ bd
④ dq
⑤ qp

thus, the weight of the MST is: $1+0+3-1-2=1$

We get the following ordered sequence: {ab, ac, bd, dq, qp}

(2)

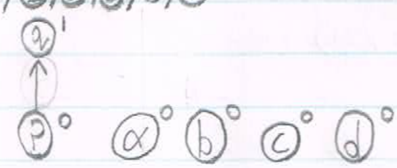
a) (1 Point) List the edges in non-decreasing order of edge weights:

edge (node-Pair)	Weight
pq	-2
dq	-1
ab	0
ac	1
dp	1
bc	2
bd	3
bp	4
cq	5
cd	7

b) (4 Points) Find a minimum-weight Spanning tree using Krukal's algorithm with respect to the ordering in part (a). Your answer should include the Sequence of edges listed in the order they are Selected by the algorithm as well as an illustration of the Spanning tree:

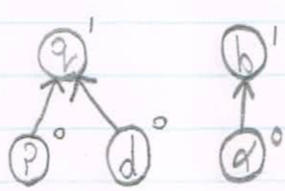
initialization: $a^0, b^0, c^0, d^0, p^0, q^0$

union (p, q):

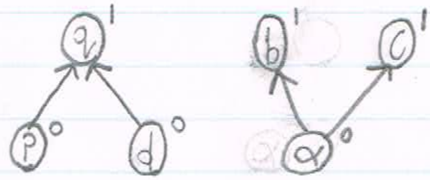


union(d, q):

union (a, b):

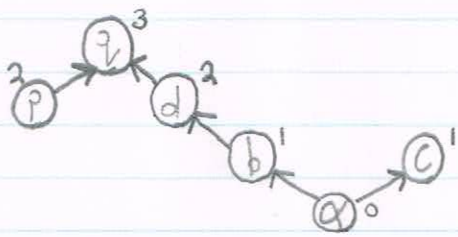


union(α, c):



union(d,p) forms a cycle so skip (its weight is higher then others in cycle)
union(b,c) forms a cycle so skip (its weight is higher)

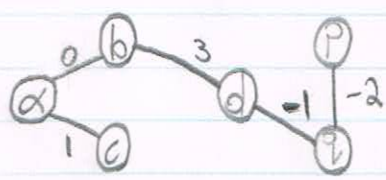
union(b,d):



Note that:
union(b,p), union(c,q), union(c,d) all form cycles & have higher weights then the other edges, & are thus skipped in this step.

thus, the edges chosen are:
① pq ② dq ③ αb ④ αc ⑤ bd

this gives a weight of: $-2-1+0+1+3=1$
we get the following tree:



We get the following ordered sequence:
{pq, dq, ab, ac, bd}

c) (1 Point) Construct an optimal solution to (DMST) in the Proof of Theorem 8.3 by making use of the answer in (b):

Note:

$$(\text{DMST}) \quad \begin{aligned} \max \ & \textstyle\sum_{A \in \mathcal{A}} \left((|V| - \kappa(A)) y_A \right) \\ \text{s.t.} \ & \textstyle\sum_{A \in \mathcal{A}: e \in A} (y_A) \leq c_e && \forall e \in E \\ & y_A \leq 0 && \forall A \in \mathcal{A}, A \neq E \\ & y_E \text{ is free} \end{aligned}$$

given:

$$y_A = \begin{cases} c_{e_i} - c_{e_{i+1}} & \text{if } A = \{e_1, \dots, e_i\} \\ c_{e_m} & \text{if } A = E \\ 0 & \text{otherwise} \end{cases}$$

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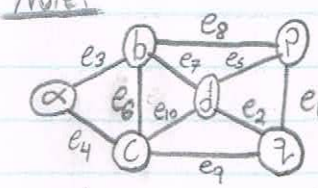
We see the following:

A	y_A	$K(A)$
$\{e_1\}$	$-2-(-1)=-1$	5
$\{e_1,e_2\}$	$-1-0=-1$	4
$\{e_1,e_2,e_3\}$	$0-1=-1$	3
$\{e_1,\dots,e_4\}$	$1-1=0$	2
$\{e_1,\dots,e_5\}$	$1-2=-1$	2
$\{e_1,\dots,e_6\}$	$2-3=-1$	2
$\{e_1,\dots e_7\}$	$3-4=-1$	1
$\{e_1,\dots,e_8\}$	$4-5=-1$	1
$\{e_1,\dots,e_9\}$	$5-7=-2$	1
$\{e_1,\dots,e_{10}\}$	7	1

Note:



edges are ordered such that $c_{e_1} \leq \dots \leq c_{e_m}$

So:

$$\max \sum_{A \in \mathcal{A}} ((|V|-K(A))y_A)$$

s.t.
$-2 \leq -2$
$-2 \leq -1$
$-2 \leq 0$
$-2 \leq 1$
$-2 \leq 1$
$-2 \leq 2$
$-2 \leq 3$
$-2 \leq 4$
$-2 \leq 5$
$-2 \leq 7$
$-1 \leq 0$
$-1 \leq 0$
$-1 \leq 0$
$0 \leq 0$
$-1 \leq 0$
$-1 \leq 0$
$-1 \leq 0$
$-1 \leq 0$
$-2 \leq 0$

Note:
$\sum_{A \in \mathcal{A}} (y_A) = -1-1-1+0-1-1-1-1-2+7 = \underline{-2}$
$|V|=6$ as there's six nodes

All constraints are satisfied, thus we see that:

$$\max \sum_{A \in \mathcal{A}} ((|V|-K(A))(y_A))$$
$$= (6-(5))(-1)+(6-(4))(-1)+(6-(3))(-1)$$
$$+(6-(2))(0)+(6-(2))(-1)+(6-(2))(-1)$$
$$+(6-(1))(-1)+(6-(1))(-1)+(6-(1))(-2)$$
$$+(6-(1))(7)$$
$$=(1)(-1)+(2)(-1)+(3)(-1)+\cancel{(4)(2)}+(4)(-1)+(4)(-1)$$
$$+(5)(-1)+(5)(-1)+(5)(-2)+(5)(7)$$
$$=-1-2-3-4-4-5-5-10+35$$
$$= 1$$

thus, an optimal solution to the DMST is 1