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Question 2: Best behavior

Base: when there is an empty list

(myLength []) – (myLength (filterPQ [])) = (countIf [])

LHS: (myLength []) – (myLength (filterPQ []))

= (myLength []) – (myLength (filterPQ []))

= 0 – myLength (filterPQ [])   [L1]

= 0 – (myLength [])   [F1]

= 0 – 0   [L1]

= 0   [Arithmetic]

RHS: (countIf [])

= (countIf [])

= 0   [C1]

Setting LHS to RHS:

0 = 0

Inductive Assumption:

(myLength t) – (myLength (filterPQ t)) = (countIf t)

Inductive Case:

(myLength (h:t)) – (myLength (filterPQ (h:t))) = (countIf (h:t))

Case 1: h = 'P'

RHS: countIf (h:t)

= countIf (h:t)

= 1 + (countIf t)   [C2A]

LHS: (myLength (h:t)) – (myLength (filterPQ (h:t)))

= (myLength (h:t)) – (myLength (filterPQ (h:t)))

= (1 + myLength t) – (myLength (filterPQ (h:t)))   [L2]

= (1 + myLength t) – (myLength (filterPQ t))   [F2C]

Setting LHS to equal RHS:

(1 + myLength t) – (myLength (filterPQ t)) = 1 + (countIf t)

(myLength t) – (myLength (filterPQ t)) = (countIf t)   [Arithmetic]

Case 2: h = 'Q'

RHS: countIf (h:t)

$= \text{countIf (h:t)}$

$= 1 + (\text{countIf t})$ [C2B]

LHS: (myLength (h:t)) – (myLength (filterPQ (h:t)))

$= (\text{myLength (h:t)}) - (\text{myLength (filterPQ (h:t))})$

$= (1 + (\text{myLength t})) - (\text{myLength (filterPQ (h:t))})$ [L2]

$= (1 + (\text{myLength t})) - (\text{myLength (filterPQ t)})$ [F2C]

Setting LHS to equal RHS:

$(1 + (\text{myLength t})) - (\text{myLength (filterPQ t)}) = 1 + (\text{countIf t})$

$(\text{myLength t}) - (\text{myLength (filterPQ t)}) = (\text{countIf t})$ [Arithmetic]

Case 3: h is a capital letter befire capital 'P'

RHS: countIf (h:t)

$= \text{countIf t}$ [C2C]

LHS: (myLength (h:t)) – (myLength (filterPQ (h:t)))

$= (1 + (\text{myLength t})) - (\text{myLength (filterPQ (h:t))})$ [L2]

$= (1 + (\text{myLength t})) - (\text{myLength (h:(filterPQ t))})$ [F2A, B1]

$= (1 + (\text{myLength t})) - (1 + \text{myLength (filterPQ t)})$ [L2]

$= (\text{myLength t}) - (\text{myLength (filterPQ t)})$ [Arithmetic]

Setting LHS to equal RHS:

$(\text{myLength t}) - (\text{myLength (filterPQ t)}) = (\text{countIf t})$

Case 4: h is a capital letter after capital 'Q'

RHS: countIf (h:t)

$= \text{countIf t}$ [C2C]

LHS: (myLength (h:t)) – (myLength (filterPQ (h:t)))

$= (1 + (\text{myLength t})) - (\text{myLength (filterPQ (h:t))})$ [L2]

$= (1 + (\text{myLength t})) - (\text{myLength (h:(filterPQ t))})$ [F2B, A1]

$= (1 + (\text{myLength t})) - (1 + \text{myLength (filterPQ t)})$ [L2]

$= (\text{myLength t}) - (\text{myLength (filterPQ t)})$ [Arithmetic]

Setting LHS to equal RHS:

$(\text{myLength t}) - (\text{myLength (filterPQ t)}) = (\text{countIf t})$

By inductive assumption, we prove that

$(\text{myLength (h:t)}) - (\text{myLength (filterPQ (h:t))}) = (\text{countIf (h:t)})$