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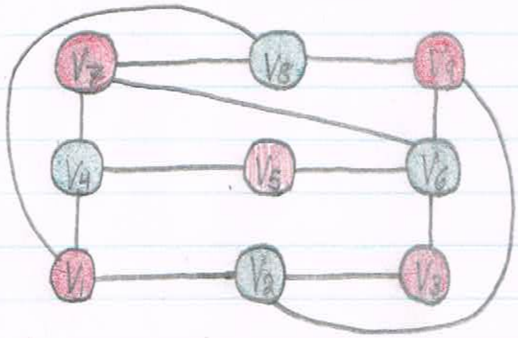
Acknowledgment: No help receaved

MATH 3802 Assignment Nine:

① (1 Point) Show that G is bipartite by partitioning the nodes into two sets X & Y so that every edge has one end in X & the other end in Y & that $V_1 \in X$:

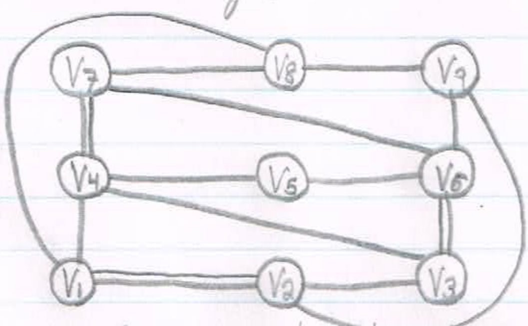
We can use edge colourings to show this:
let Red be X & Blue be Y



thus:
$X = \{V_1, V_3, V_5, V_7, V_9\}$
$Y = \{V_2, V_4, V_6, V_8\}$
& $V_1 \in X$; thus the requirments are satisfied as every edge has one end in X & the other end in Y

② (3 Point) Let $M = \{V_4V_7, V_3V_6, V_1V_2\}$, obtain an M-alternating tree by applying Algorithm 9.1 with $r = V_5$. Is the tree frustrated? Explain:

We see the following:



Note:
M is given by thick double edges.

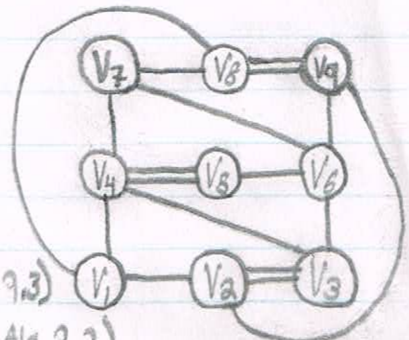
thus; there are 3 M-exposed nodes; $V_5, V_8, \& V_9$.
Choose V_5 as the node r.
Use the following table to track Progress:

Action	T	$\mathcal{E}(T)$	$\mathcal{O}(T)$
Initialization	$(\{V_5\}, \emptyset)$	$\{V_5\}$	$\emptyset$
Add V_4V_5, V_4V_7	$(\{V_4, V_5, V_7\}, \{V_4V_5, V_4V_7\})$	$\{V_5, V_7\}$	$\{V_4\}$
Add V_5V_6, V_6V_3	$(\{V_3, V_4, V_5, V_6, V_7\}, \{V_4V_5, V_4V_7, V_5V_6, V_6V_3\})$	$\{V_5, V_7, V_3\}$	$\{V_4, V_6\}$
Add V_2V_3, V_1V_2	$(\{V_1, V_2, V_3, V_4, V_5, V_6, V_7\}, \{V_4V_5, V_4V_7, V_5V_6, V_6V_3, V_2V_3, V_1V_2\})$	$\{V_1, V_5, V_7, V_3\}$	$\{V_2, V_4, V_6\}$

The matching $M = \{V_4V_7, V_3V_6, V_1V_2\}$ has edges $V_4, V_2, V_6 \in \mathcal{O}(T)$ & $V_7, V_3, V_1 \in \mathcal{E}(T)$:
Thus, each edge in matching has one end in $E(T)$ & one end in $O(T)$ so we are frustrated

$\therefore T = \{\{V_1, V_2, V_3, V_4, V_5, V_6, V_7\}, \{V_4V_5, V_4V_7, V_5V_6, V_6V_3, V_2V_3, V_1V_2\}\}$ & T is frustrated

③ (6 Points) Obtain a maximum-Cardinality matching & a minimum-Cardinality node Cover by applying Algorithm 9.3 Starting with $M=\{V_2V_3, V_4V_5, V_8V_9\}$. Choose the M-exposed node with the Smallest index whenever an M-exposed node needs to be chosen:



let M be given by thick double edges

Iter. 1: (Alg. 9.3)

Iter. 1: (Alg. 9.2)

thus, there are three M-exposed nodes; V_7, V_6, V_1.

Choose V_1 as the node r. use the table below to track progress:

(Alg. 9.1)

Action	T	$\mathcal{E}(T)$	$\mathcal{O}(T)$
Initialization	$(\{V_1\}, \emptyset)$	$\{V_1\}$	$\emptyset$
Add V_1V_8, V_8V_9	$(\{V_1, V_8, V_9\}, \{V_1V_8, V_8V_9\})$	$\{V_1, V_9\}$	$\{V_8\}$
Add V_9V_2, V_2V_3	$(\{V_1, V_8, V_9, V_2, V_3\}, \{V_1V_8, V_8V_9, V_9V_2, V_2V_3\})$	$\{V_1, V_9, V_3\}$	$\{V_8, V_2\}$
Add V_3V_4, V_4V_5	$(\{V_1, V_8, V_9, V_2, V_3, V_4, V_5\}, \{V_1V_8, V_8V_9, V_9V_2, V_2V_3, V_3V_4, V_4V_5\})$	$\{V_1, V_9, V_3, V_5\}$	$\{V_8, V_2, V_4\}$

So, $V(T) = \mathcal{E}(T) + \mathcal{O}(T) = \{V_1, V_2, V_3, V_4, V_5, V_8, V_9\}$

$G/V(T) = \{V_6, V_7\}$ yet V_7 is NOT connected to a node in $V(T)$

↳ Thus, we see that $V_6V_3 \in E$ Such that $V_3 \in \mathcal{E}(T)$ & $V_6 \notin V(T)$

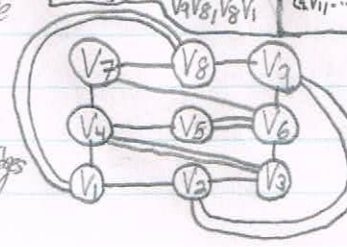
Iter. 2: (Alg. 9.2) So, we notice that we have a matching of $\{V_1V_8, V_9V_2, V_3V_4, V_5V_6\} = M'$

Choose $V_7 = r$ & let $M = M'$ be as above; Note only V_7 is M-exposed:

(Alg. 9.1)

Action	T	$\mathcal{E}(T)$	$\mathcal{O}(T)$
Initialization	$(\{V_7\}, \emptyset)$	V_7	$\emptyset$
Add V_7V_6, V_6V_5	$(\{V_7, V_6, V_5\}, \{V_7V_6, V_6V_5\})$	V_7, V_5	V_6
Add V_5V_4, V_4V_3	$(\{V_7, V_6, V_5, V_4, V_3\}, \{V_7V_6, V_6V_5, V_5V_4, V_4V_3\})$	V_7, V_5, V_3	V_6, V_4
Add V_3V_2, V_2V_9	$(\{V_7, V_6, V_5, V_4, V_3, V_2, V_9\}, \{V_7V_6, V_6V_5, V_5V_4, V_4V_3, V_3V_2, V_2V_9\})$	V_7, V_5, V_3, V_9	V_6, V_4, V_2
Add V_9V_8, V_8V_1	$(\{V_1, \ldots, V_9\}, \{V_7V_6, V_6V_5, V_5V_4, V_4V_3, V_3V_2, V_2V_9, V_9V_8, V_8V_1\})$	V_7, V_5, V_3, V_9, V_1	V_6, V_4, V_2, V_8

thus, we see G w/ M as double Edges



Thus, $G/V(T) = \emptyset$:

So: a matching of $M' = \{V_7V_6, V_4V_5, V_2V_3, V_9V_8\}$

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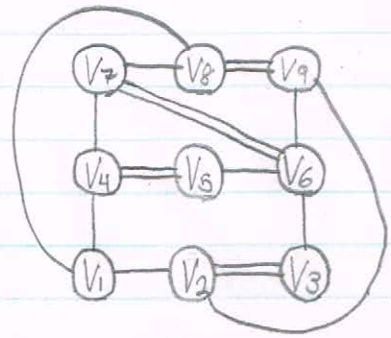
Iter. 2 Cont.: (Alg. 9.2)

As $M'=\{V_7V_6, V_4V_5, V_2V_3, V_9V_2\}$ (using M' from above) we see:



thus, we return (M,T) since there exists no more edges $uw \in E, u \in E(T), w \notin V(T)$ since $V(T)=V(G)$.

$C=O(T) \implies |C|=4=|M|$

$\implies$ V_1 is M-exposed so $C'=\emptyset$

With (M',T):

$M \leftarrow M \cup M' \cap E(T) = \{V_2V_3, V_4V_5, V_8V_9\} \cup \{V_7V_6, V_4V_5, V_2V_3, V_9V_8\} \cap \{V_7V_6, \cancel{V_6V_5}, V_5V_4, \cancel{V_4V_3}, V_3V_2, \cancel{V_2V_9}, V_8V_9, \cancel{V_8V_1}\}$

$= \{V_2V_3, V_4V_5, V_7V_6, V_8V_9\}$

$C \leftarrow C' \cup O(T) = \emptyset \cup \{V_6, V_4, V_2, V_8\} = \{V_6, V_4, V_2, V_8\}$

$|M|=|C|=4$

$G' \leftarrow G' \setminus V(T) = (\emptyset, E)$

Iter. 2: (Alg. 9.3)

$G'=(\emptyset,E), M=\{V_2V_3, V_4V_5, V_7V_6, V_9V_2, V_8V_9\}, C=\{V_6, V_4, V_2, V_8\}$

Iter. 1:

there are no more M-exposed nodes in $G'=(\emptyset,E)$ as $V(G')=\emptyset$

Return the Perfect matching M

With matching M' (Perfect):

$M \leftarrow M \cup M' = M \cup \emptyset = M$

$C \leftarrow C \cup X \cap V(G') = \{V_2, V_4, V_6, V_8\} \cup \cancel{\{V_1, V_3, V_5, V_7, V_9\} \cap \emptyset}$

$= C = \{V_2, V_4, V_6, V_8\}$

Return M, C

$\therefore$ We get a max.-card. matching of $\{V_2V_3, V_4V_5, V_6V_7, V_8V_9\}$ & a min.-card. node cover of $\{V_2, V_4, V_6, V_8\}$ after applying Algorithm 9.3