

# MATH 3801 Problem Set one:

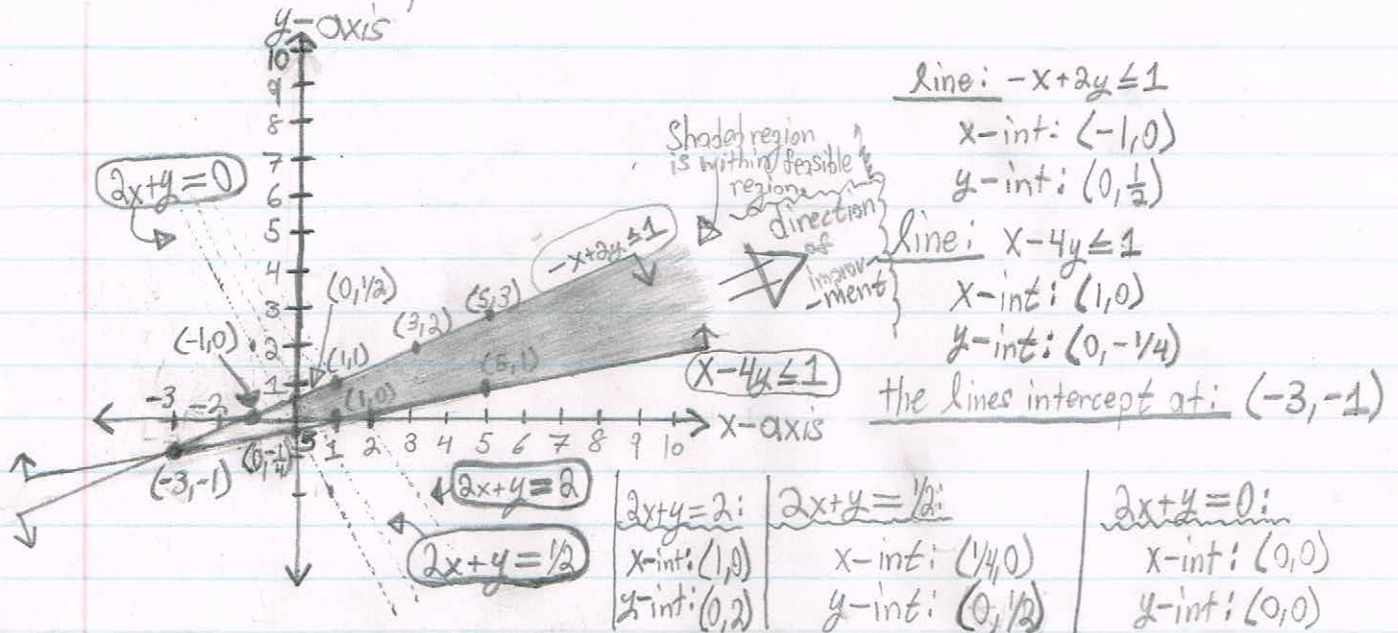
① Consider the linear programming problem:

$$\begin{aligned} \max \quad & 2x+y \\ \text{s.t.} \quad & -x+2y \leq 1 \\ & x-4y \leq 1 \\ & x, y \geq 0 \end{aligned}$$

$$-2Z + Z = -Z$$

$$2Z - 2Z = 0$$

a) Give a hand drawn sketch of the feasible region. Label the constraints & the intercepts:



b) Show algebraically that the problem is unbounded:

the line defined by  $2x+y=Z$  has an  $x$ -intercept of  $Z/2$ .  
 For  $Z \geq 0$ ,  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Z/2 \\ 0 \end{bmatrix}$  satisfies both inequalities & the value of the objective function at  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Z/2 \\ 0 \end{bmatrix}$  is  $Z$ .

Note:

Since  $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Z/2 \\ 0 \end{bmatrix}$  is subject to:

$$-x+2y \leq 1; x-4y \leq 1$$

then;

$$-2Z + 2(Z/2) = -Z \leq 1; 2Z - 4(Z/2) = 0 \leq 1$$

Since  $0 \leq 1$  is always true  $x-4y \leq 1$  is satisfied &

$-Z \leq 1$  is always true since  $Z \geq 0$

Also, as  $x, y \geq 0$ ,  $Z$  must be greater or equal to 0 or else the line will NOT be in the feasible region

thus:

As  $z \rightarrow \infty$  the objective function tends to  $\infty$

$\therefore$  there is no maximum value for the objective function

$\therefore$  The problem is unbounded

② Convert the Optimization Problem:

$$\min 8^x \cdot 2^{-y}$$

$$\text{s.t. } 3^{4x-y} \geq 9$$

$$|9x+y| \leq 7$$

to an equivalent linear programming problem:

the constraint  $|9x+y| \leq 7$  is equivalent to:

$$9x+y \leq 7 \quad \& \quad 9x+y \geq -7$$

taken together, & the constraint  $3^{4x-y} \geq 9$  is equivalent to

$$4x-y \geq \log_3(9) \text{ Since } e^u \text{ is an increasing function in } U \text{ with}$$

$$\log_3 9 = 2 \text{ thus } 4x-y \geq 2$$

Also, minimizing  $8^x \cdot 2^{-y}$  is the same as minimizing  $2^{3x-y}$   
 $= 2^{3x-y}$  which is the same as minimizing  $3x-y$  Since  $2^u$  is an increasing function in  $U$ .

So, the equivalent is:

$$\text{minimize } 3x-y$$

$$\text{s.t. } 4x-y \geq 2$$

$$9x+y \leq 7$$

$$9x+y \geq -7$$

③ the following sheet shows the solution to the linear programming problem:

$$\min 4x_1 + 9x_2 + 1x_3 + x_4$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 = 7$$

$$3x_1 - x_2 \geq 2$$

$$x_2 + x_3 - x_4 \geq 1$$

$$x_1, x_2, x_3, x_4 \geq 0$$

When using the excel Solver add-in to solve the problem.

3. (1 point) Consider the linear programming problem:

$$\min 4x_1 + 9x_2 + 1x_3 + x_4$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 = 7$$

$$3x_1 - x_2 \geq 2$$

$$x_2 + x_3 - x_4 \geq 1$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Use Microsoft Excel with the Solver add-in to solve this problem. Submit a screenshot of the spreadsheet that you create for the problem showing the final solution and a screenshot of the answer report.

Excel Spreadsheet used for the problem

F3	=SUMPRODUCT(B3:E3,B2:E2)					
1	Variables	x	y	z	w	value
2	Solution					
3	Objective	4	9	1	1	0
4						
5	Constraint	1	1	1	1	0 =
6	Constraint	3	-1	0	0	0 >=
7	Constraint	0	1	1	-1	0 >=

Solver Parameters

Set Objective: \$F\$3

To: ☐ Max ☒ Min ☐ Value Of.

By Changing Variable Cells: \$B\$2:\$E\$2

Subject to the Constraints:

\$F\$5 = \$H\$5  
 \$F\$6 >= \$H\$6  
 \$F\$7 >= \$H\$7

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help Solve Close

used instead of constraints above

A	B	C	D	E	F	G	H
1	Variables	x	y	z	w	value	
2	Solution	0.666667	0	6.333333	0	9	Final Solution
3	Objective	4	9	1	1	9	
4							
5	Constraint	1	1	1	1	7 =	7
6	Constraint	3	-1	0	0	2 >=	2
7	Constraint	0	1	1	-1	6.333333 >=	1

Note: (for the excel sheets)  
 x is  $x_1$   
 y is  $x_2$   
 z is  $x_3$   
 w is  $x_4$

Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

☒ Keep Solver Solution  
☐ Restore Original Values

☐ Return to Solver Parameters Dialog ☐ Outline Reports

OK Cancel Save Scenario...

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Thus, the Solution is  $(\frac{2}{3}, 0, \frac{13}{3}, 0) = (x_1, x_2, x_3, x_4)$  for the minimum point & this occurs when  $4x_1 + 9x_2 + 1x_3 + x_4 = 9$ .

**Acknowledgements**

As was required for question 3, I used Microsoft excels optimization solver to help solve question 3.

No other outside help was used for the problem set.