

a)

We are given the following lambda expression:

$$\lambda a.(\lambda b.(\lambda c.(b((a\ b)c)))(\lambda d.(\lambda e.(d(d\ d\ e)))))$$

We can start by replacing all instances of the letter 'a' in  $(\lambda b.(\lambda c.(b((a\ b)c))))$  with the expression  $(\lambda d.(\lambda e.(d(d\ d\ e))))$ . This removes the leading ' $\lambda a.$ ' from the overall expression.

$$\lambda b.(\lambda c.(b(((\lambda d.(\lambda e.(d(d\ d\ e))))\ b)c)))$$

Next, ' $\lambda b.$ ' and ' $\lambda c.$ ' get skipped since there is no expression outside of their encapsulating brackets. Thus, ' $\lambda d.$ ' is the outermost bracket which can be resolved, and in this case, all instances of 'd' within the expression will be replaced with 'b'. Therefore, all instances of the letter 'd' in  $(\lambda d.(\lambda e.(d(d\ d\ e))))$  will be replaced with b.

$$\lambda b.(\lambda c.(b((\lambda e.(b(b\ b\ e))))c)))$$

Finally, since ' $\lambda b.$ ' and ' $\lambda c.$ ' still can't be reduced, we must move and reduce ' $\lambda e.$ '. all instances of the letter 'e' in  $(b(b\ b\ e))$  will be replaced with 'c' as it is the expression directly outside ' $\lambda e.$ ', as can be seen in this simplified portion taken from the expression above,  $(\lambda e.(b(b\ b\ e))))c)$ .

$$\lambda b.(\lambda c.(b(b\ b\ c))))$$

The final result cannot be reduced further, as there are no expressions outside of the bracket scope of the variables or expressions. Therefore, the final solution is:

$$\lambda b.(\lambda c.(b(b\ b\ c))))$$

b)

We are given the lambda expression:

$$\lambda a.(\lambda b.(\lambda f.(\lambda x.((a\ f)((b\ f)x)))(\lambda f.(\lambda x.(f\ x)))(\lambda f.(\lambda x.x)))$$

Replace all instances of 'a' in  $(\lambda b.(\lambda f.(\lambda x.((a\ f)((b\ f)x)))(\lambda f.(\lambda x.f\ x))))$  with  $(\lambda f.(\lambda x.x))$

$$\lambda b.(\lambda f.(\lambda x.(((\lambda f.(\lambda x.x)\ f)((b\ f)x)))(\lambda f.(\lambda x.(f\ x))))$$

Replace all instances of 'b' in  $\lambda b.(\lambda f.(\lambda x.(((\lambda f.(\lambda x.x)\ f)((b\ f)x))))$  with  $(\lambda f.(\lambda x.(f\ x)))$

$$\lambda f.(\lambda x.((\lambda f.(\lambda x.x)\ f)((\lambda f.(\lambda x.(f\ x))\ f)x))$$

Replace all instances of 'f' in  $(\lambda x.x)$  with f, this results in no change as there's no f's

$$\lambda f.(\lambda x.((\lambda x.x)((\lambda f.(\lambda x.(f\ x))\ f)x))$$

Replace all instances of 'x' in x with  $((\lambda f.(\lambda x.(f\ x)))\ f)x$

$$\lambda f.(\lambda x.(((\lambda f.(\lambda x.(f\ x))\ f)x))$$

Replace all instances of 'f' in  $(\lambda x.(f\ x))$  with f

$\lambda f.(\lambda x.((\lambda x.(f\ x))x))$

Replace all instances of 'x' in (f x) with x

$\lambda f.(\lambda x.(f\ x))$

This is the representation for one

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$\lambda f.(\lambda x.(f\ x))$  which is the representation for one