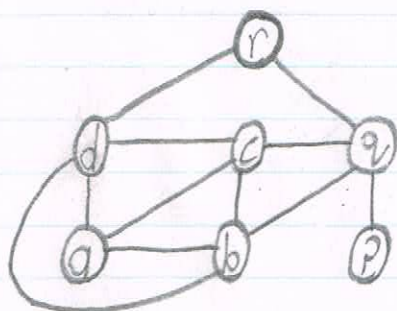


MATH 3802 Assignment #11:

Let  $G=(V,E)$  be the undirected graph depicted below:



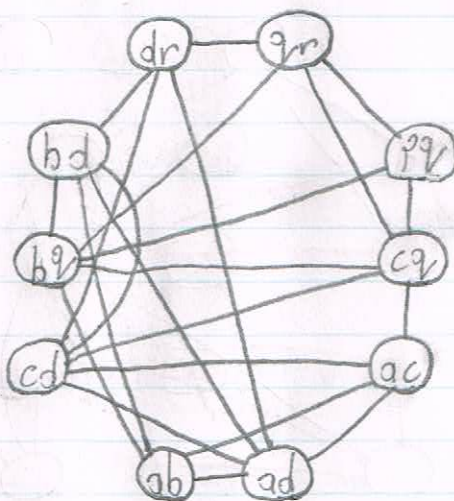
① Let  $H$  denote the line graph of  $G$ :

a) (2 points) Give a sketch of  $H$ . Label all the nodes properly:

- $H$  has a node set of  $E$  & an edge-set of  $\{ef : e, f \in E, e \neq f, e \text{ and } f \text{ share an end}\}$
- The node set is:

$dr, qr, bd, pq, br, cr, cd, ac, ab, ad$

- We see the following graph:



b) (4 points) Give a stable set in  $H$  of cardinality 3:

A stable set exists if  $S \subseteq V$  if no two nodes in  $S$  are adjacent in  $G$  where  $G=(V,E)$ .

Since  $pq \rightarrow qr, br, cr$   
 $ac \rightarrow cr, cd, ab, ad$   
 $dr \rightarrow qr, ad, cd, bd$

Since  $pq, ac, \& dr$  are not adjacent in  $H$ , we see that:

$S = \{pq, ac, dr\}$  &  $|S| = 3$

## Acknowledgment: No Help Received

② Let (LP) denote the Problem:

$$\begin{aligned} \max \quad & \sum_{u \in V} X_u \\ \text{s.t.} \quad & X_u + X_w \leq 1 \quad \forall u, w \in E \\ & 0 \leq X_u \leq 1 \quad \forall u \in V \\ & X \in \mathbb{Z}^V \end{aligned}$$

a) (3 Points) Use Microsoft excel to Solve (LP) for the graph  $G$ :  
See Attachment for Excel Solution  
We formulate the following Linear Program:

$$\begin{aligned} \max \quad & X_r + X_a + X_b + X_c + X_d + X_p + X_q \\ \text{s.t.} \quad & X_r + X_d \leq 1 \\ & X_r + X_p \leq 1 \\ & X_b + X_d \leq 1 \\ & X_b + X_q \leq 1 \\ & X_c + X_d \leq 1 \\ & X_p + X_q \leq 1 \\ & X_c + X_q \leq 1 \\ & X_a + X_c \leq 1 \\ & X_a + X_b \leq 1 \\ & X_a + X_d \leq 1 \\ & 0 \leq X_r, X_a, X_b, X_c, X_d, X_p, X_q \leq 1 \end{aligned}$$

b) (2 Points) Give two maximal cliques in  $G$  having cardinality at least 3. (A Clique is maximal if it is not strictly contained in another clique. For example,  $\{b, c\}$  is a Clique that is not maximal because it is strictly contained in the Clique  $\{a, b, c\}$ .)

Clique I:

$$\{a, b, c, d\}$$

Clique II:

$$\{b, c, q\}$$

c) (2 Points) Add the Clique inequalities for the Cliques you gave in part (b) to (LP) & resolve. Provide a screenshot of the formulation & the answer report:

See Attachments

## Question 2 Part a Linear Program Formulation

Variables	x_r	x_a	x_b	x_c	x_d	x_p	x_q		
Values		1	0	1	1	0	1	0	
								sumprods	
Objective		1	1	1	1	1	1	1	4
dr		1				1			1 <= 1
qr		1						1	1 <= 1
bd				1		1			1 <= 1
bq				1				1	1 <= 1
cd					1	1			1 <= 1
pq							1	1	1 <= 1
cq					1			1	1 <= 1
ac			1		1				1 <= 1
ab			1	1					1 <= 1
ad			1			1			0 <= 1

**Microsoft Excel 16.0 Answer Report****Worksheet: [New Microsoft Excel Worksheet.xlsx]Sheet1****Report Created: 4/5/2021 12:47:58 AM****Result: Solver found a solution. All Constraints and optimality conditions are satisfied.****Solver Engine**

Engine: Simplex LP

Solution Time: 0.031 Seconds.

Iterations: 7 Subproblems: 0

**Solver Options**

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

## Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$I\$4	Objective sumprods	0	4

## Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$2	Values x_r	0	1	Binary
\$C\$2	Values x_a	0	0	Binary
\$D\$2	Values x_b	0	1	Binary
\$E\$2	Values x_c	0	1	Binary
\$F\$2	Values x_d	0	0	Binary
\$G\$2	Values x_p	0	1	Binary
\$H\$2	Values x_q	0	0	Binary

## Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$I\$6	dr sumprods	1	\$I\$6<=\$K\$6	Binding	0
\$I\$7	qr sumprods	1	\$I\$7<=\$K\$7	Binding	0
\$I\$8	bd sumprods	1	\$I\$8<=\$K\$8	Binding	0
\$I\$9	bq sumprods	1	\$I\$9<=\$K\$9	Binding	0
\$I\$10	cd sumprods	1	\$I\$10<=\$K\$10	Binding	0
\$I\$11	pq sumprods	1	\$I\$11<=\$K\$11	Binding	0
\$I\$12	cq sumprods	1	\$I\$12<=\$K\$12	Binding	0
\$I\$13	ac sumprods	1	\$I\$13<=\$K\$13	Binding	0
\$I\$14	ab sumprods	1	\$I\$14<=\$K\$14	Binding	0
\$I\$15	ad sumprods	0	\$I\$15<=\$K\$15	Not Binding	1
\$B\$2:\$H\$2=Binary					

## Question 2 Part c Linear Program Formulation

Variables	x_r	x_a	x_b	x_c	x_d	x_p	x_q		
Values	1	1	0	0	0	1	0		
								sumprods	
Objective	1	1	1	1	1	1	1	3	
dr	1					1		1 <=	1
qr	1						1	1 <=	1
bd			1			1		0 <=	1
bq			1				1	0 <=	1
cd				1	1			0 <=	1
pq							1	1 <=	1
cq				1			1	0 <=	1
ac		1		1				1 <=	1
ab		1	1					1 <=	1
ad		1			1			1 <=	1
abcd		1	1	1	1			1 <=	1
bcq			1	1			1	0 <=	1

## Microsoft Excel 16.0 Answer Report

Worksheet: [New Microsoft Excel Worksheet.xlsx]Question 2c

Report Created: 4/5/2021 12:52:44 AM

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

### Solver Engine

Engine: Simplex LP

Solution Time: 0.031 Seconds.

Iterations: 6 Subproblems: 0

### Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

### Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$I\$4	Objective sumprods	4	3

### Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$2	Values x_r	1	1	Binary
\$C\$2	Values x_a	0	1	Binary
\$D\$2	Values x_b	1	0	Binary
\$E\$2	Values x_c	1	0	Binary
\$F\$2	Values x_d	0	0	Binary
\$G\$2	Values x_p	1	1	Binary
\$H\$2	Values x_q	0	0	Binary

### Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$I\$6	dr sumprods	1	\$I\$6<=\$K\$6	Binding	0
\$I\$7	qr sumprods	1	\$I\$7<=\$K\$7	Binding	0
\$I\$8	bd sumprods	0	\$I\$8<=\$K\$8	Not Binding	1
\$I\$9	bq sumprods	0	\$I\$9<=\$K\$9	Not Binding	1
\$I\$10	cd sumprods	0	\$I\$10<=\$K\$10	Not Binding	1
\$I\$11	pq sumprods	1	\$I\$11<=\$K\$11	Binding	0
\$I\$12	cq sumprods	0	\$I\$12<=\$K\$12	Not Binding	1
\$I\$13	ac sumprods	1	\$I\$13<=\$K\$13	Binding	0
\$I\$14	ab sumprods	1	\$I\$14<=\$K\$14	Binding	0
\$I\$15	ad sumprods	1	\$I\$15<=\$K\$15	Binding	0
\$I\$16	abcd sumprods	1	\$I\$16<=\$K\$16	Binding	0
\$I\$17	bcq sumprods	0	\$I\$17<=\$K\$17	Not Binding	1
\$B\$2:\$H\$2=Binary					