Algorithms and Data Structures (DAT3/SW3)

Exam Assignments

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This exam consists of three problems and there are three hours to solve them. When answering the questions in problem 1, mark or fill in the boxes on this paper. Remember also to put your name and your CPR number on any additional sheets of paper you will use for problems 2 and 3.

- *Read* carefully *the text of each problem before solving it!*
- *For problems 2 and 3, it is important that your solutions are presented in a readable form. Note that problems 2.B and 3.B have different requirements for the level of detail of the solution. If you don't have enough time to give full solutions for problems 2 and 3, then give a solution outline in a few lines of text.*
- *Make an effort to use a readable handwriting and to present your solutions neatly.*

[ItoA] refers to T.H. Cormen, Ch. E. Leiserson, R. L. Rivest, C. Stein, *Introduction to Algorithms* (both 2nd and 3rd editions).

During the exam you are allowed to consult books, printed lecture slides, and notes. The use of any kind of electronic devices, including calculators, is not permitted.

Problem 1 [50 points in total]

1. (*8 points*)

1.1. $\frac{1}{3}n^3 + n^2 \lg n + \sqrt{n^5}$ is:

☐ **a)** $\Theta(n^3)$ ☐ **b)** $\Theta(n \lg n)$ ☐ **c)** $\Theta(n^5)$ ☐ **d)** $\Theta(n^3 \lg n)$

1.2. $n\sqrt{\lg n}$ is:

☐ **a)** $\Omega(n \lg n)$ ☐ **b)** $\Theta(n \lg n)$ ☐ **c)** $O(n \lg n)$ ☐ **d)** $O(n)$

2. (*6 points*)
Consider the following recurrence relation:

$$T(n) \quad = \quad 2T(n/4) + n^{1/3} \qquad (n > 1).$$

Mark the correct solution. $T(n) =$

☐ **a)** $\Theta(\sqrt{n})$ ☐ **b)** $\Theta(n^{1/3})$ ☐ **c)** $\Theta(n^3)$ ☐ **d)** $\Theta(\sqrt{n} \lg n)$

3. (*8 points*)
Consider the following algorithm:

```
input  : An Integer n
output: An Integer Z
COMPUTE(n)
S = newStack()
for i=1 to n do
    S.push(i)
Z = 0
while S.isEmpty() == false do
    n = S.pop()
    for i=1 to n do
        Z = Z + i
return Z
```

3.1 Enter the return value of COMPUTE(4) in the box below:

COMPUTE(4) = $\boxed{}$

3.2 Complete the following statement by entering the correct expression within the parentheses: the complexity of COMPUTE as a function of n is

$$\Theta\Big(\quad \Big)$$

4 (*7 points*)

Let A be a Max-Heap given by the array

3	8	6	4	2	1	5
1	2	3	4	5	6	7

and $A.heap\text{-}size = 7$. Write into the boxes below the contents of the array A after the operation MAX-HEAPIFY$(A, 1)$ has been performed:

1	2	3	4	5	6	7

5 (*7 points*)

Consider the result of inserting the keys

$$9,\ 5,\ 13,\ 12,\ 16$$

(in this order) into a hash table of size $m = 7$ using open addressing with auxiliary hash function

$$h'(k) = k \bmod 7$$

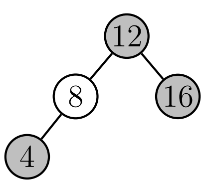
and linear probing. Write into the boxes below the contents of the hash table after the keys have been inserted:

0	1	2	3	4	5	6

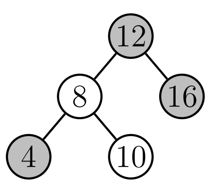
6 (*6 points*)

Which of the following is a valid red-black tree (are black nodes, are red nodes)?

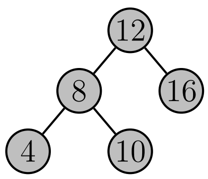
☐ **a)**



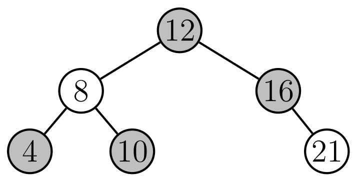
☐ **b)**



☐ **c)**

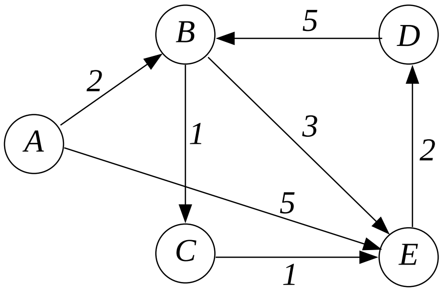


☐ **d)**



7 (*8 points*)

Consider the operation of the Bellman-Ford algorithm as given in [ItoA] on the following weighted graph with A as the source node:



Assume that in line 3 the algorithm enumerates the edges of the graph in the following order:

$$B \to C,\ A \to B,\ B \to E,\ A \to E,\ E \to D,\ C \to E,\ D \to B$$

Fill in the values that the $.d$ attributes of the nodes have after the completion of the second iteration of the **for** loop of lines 2-4 (i.e. the iteration $i = 2$ is completed):

$A.d =$ $\boxed{}$

$B.d =$ $\boxed{}$

$C.d =$ $\boxed{}$

$D.d =$ $\boxed{}$

$E.d =$ $\boxed{}$

Problem 2 [25 points]

Let us define an abstract data type *Integer Bag (IB)* as follows: the data that an IB stores consists of *multisets of integers*, i.e. collections of integer values where the same value can appear more than once. For example:

$$myMoI = [0, 3, 1, 0, 1, 2, 1]$$

is a multiset of integers. The following operations must be supported by a data structure implementing the IB abstract data type:

Name	Specification
void *add*(*Integer* i)	add i to the multiset
Integer *number_of*(*Integer* i)	returns the number of occurrences of i in the multiset
void *delete*(*Integer* i)	deletes one occurrence of i from the multiset (does nothing if the multiset does not contain any i)
Boolean *same_elements*(*IB* C)	returns *true* if this multiset and the multiset C contain the same elements (but maybe with different number of occurrences)

For example, if B is a new IB object that is initialized as the empty multiset, then after the sequence of operations

$$B.add(0), B.add(1), B.add(2), B.add(1), B.add(3), B.add(0), B.add(1)$$

B will represent the set *myMoI* from above. $B.number\_of(1)$ should now return 3. When next the operation $B.delete(1)$ is performed, then afterwards $B.number\_of(1)$ must return 2. $B.same\_elements(C)$ must return *true*, e.g. for $C = [0, 1, 2, 2, 2, 3, 3]$, and *false* for $C = [0, 2, 2, 3]$.

Following steps **A**-**C** below, describe a data structure that provides an efficient implementation of the IB abstract data type. You may make use of standard data structures described in [ItoA], and the operations they support. In that case, precisely show how the existing data structures are used, and what modifications or extensions of them you need to make.

A Give a description of how a multiset of integers is stored in your data structure, and make a sketch of how the example multiset myMoI would look like in your data structure.

B Give pseudo-code descriptions for the implementations of the *add* and *same_elements* operations.

C What is the complexity of the *add* and *same_elements* operations in your implementation? Depending on what is more appropriate for your implementation, you may give the complexity in terms of the total number of elements of a multiset, or the number of distinct elements in a multiset.

Problem 3 [25 points]

Suppose *Instwitr* is a social network with registered users who can *follow* one another.

Consumer electronics company *Beans* wants to market a new product to the users of *Instwitr*. To this end, *Beans* buys advertising space on *Instwitr*, so that a *Beans* ad will be displayed to a selected set of *Instwitr* users. *Beans* wants to maximize the effect of the advertising campaign at minimal cost. The cost of the campaign is proportional to the number of *Instwitr* users to whom the ad is displayed on their *Instwitr* home pages. To assess the effect of the campaign, *Beans* defines the *reach* of the campaign as follows: every user to whom the ad is displayed directly is in the *reach* of the campaign. If user U is in the *reach* of the campaign, and user V *follows* user U, then V is in the *reach* of the campaign. Users to whom none of the above two rules apply are not in the *reach* of the campaign.

A subset *MinSeed* of users is a *minimal seed set*, if the the following two conditions are satisfied: if the ad is displayed to all users in *MinSeed*, then the *reach* of the campaign is the set of all *Instwitr* users. And: there exists no smaller subset of users than *MinSeed* for which the first condition also holds.

A

Propose a formalization of this scenario in terms of a mathematical model or abstract data type. Give an illustration (drawing) of your formalization for the following example: *Instwitr* has 6 users A, B, C, D, E, F, which *follow* each other according to the table:

User	*follows* users
A	B
B	A
C	D
D	C
E	B, F
F	D, E

What is a minimal seed set in this network?

B

Give a description of an algorithm that computes a minimal seed set. A clear textual description (no detailed pseudo-code) is sufficient. What is the complexity of your algorithm?