

# CSC 212: Data Structures and Abstractions

Spring 2019

University of Rhode Island

## Weekly Problem Set #2

Due Thursday 2/14 at the beginning of class. Please turn in neat, and organized, answers hand-written on standard-sized paper **without any fringe**. At the top of each sheet you hand in, please write your name, and ID.

1. Simplify the following:

- $\log_2 xy^2 - \log_2 x^2 - 2 \log_2 y$

$$\log_2 \frac{y^2}{x} - \log_2 y^2$$

$$\log_2 \frac{1}{x}$$

- $\log_2 (16x^2)^{\frac{1}{3}}$

$$\frac{1}{3} \log_2 2^4 + \frac{1}{3} \log_2 x^2$$

$$\frac{4}{3} + \frac{2}{3} \log_2 x$$

- $\log_3 (9x^4) - \log_3 (3x)^2$

$$\log_3 \frac{9x^4}{(3x)^2}$$

$$2 \log_3 (x)$$

2. Solve for x:  $\log_2 \frac{x^2}{2} = 5$

$$\frac{x^2}{2} = 2^5$$

$$x^2 = 2^6$$

$$x = 2^3$$

$$x = 8$$

3. Evaluate:  $\sum_{x=0}^3 (5 + \sqrt{4^x})$

$$6 + 7 + 9 + 13 = 35$$

4. Solve the following:  $\sum_{n=0}^{10} (-n)$

$$-55$$

5. Prove that:  $\sum_{i=1}^x i = \frac{(x+1)x}{2}$

To prove by induction:

Check the base case:  $n = 1$ , verify that  $((1 + 1) * 1)/2 = 1$

State the induction hypothesis:  $\sum_{i=1}^{x-1} i = (((x - 1) + 1)(x - 1))/2 == ((x - 1)x)/2$

Thus  $\sum_{i=1}^x i = \left(\sum_{i=1}^{x-1} i\right) + x = ((x-1)x)/2 + x = (x^2 - x + 2x)/2 = (x(x+1))/2$

6. Rewrite the following expression into its closed form (i.e. without the sigma):  $\sum_{i=1}^n (2+i)$ .

$$\frac{n(n+1)}{2} + 2n$$

7. For each of the following, give an exact formula  $T(n)$  for the number of times `//op` is run: (assume `i+=1` each iteration unless otherwise specified)

Ex.        `T(n) = n`  
           `for i = 1 to n do`  
               `// constant time operation`

(a)        `for i = 1 to 4n do`  
               `// constant time operation`

$$T(n) = 4n$$

(b)        `for i = 1 to n*n*n do`  
               `// constant time operation`

$$T(n) = n^3$$

(c)        `for i = 1 to 4n do`  
               `for j = 1 to i do`  
                   `// constant time operation`

$$T(n) = 4n * \frac{4n+1}{2}$$

(d)        `for i = 1 to n*n do`  
               `for j = 1 to i do`  
                   `// constant time operation`

$$T(n) = n^2 * \frac{n^2+1}{2}$$

(e)        `for i = 1 to n do`  
               `for j = 1 to n do`  
                   `for k = 1 to n do`  
                       `// constant time operation`

$$T(n) = n^3$$

(f)        `for(int i=0; i<n; i+=2) {`  
               `// constant time operation`  
               `}`

$$T(n) = \frac{n}{2}$$

(g)        `for(int i=0; i<n; i+=4) {`  
               `// constant time operation`  
               `}`

$$T(n) = \frac{n}{4}$$

(h)        `for(int i = 1; i < 2^n; i*=2) {`  
               `// constant time operation`  
               `}`

$$T(n) = n$$

(i)        `for(int i = n; i > 1 ; i/=2) {`  
               `// constant time operation`  
               `}`

$$T(n) = \log_2 n$$

8. Rank the following functions by their asymptotic growth rate in ascending order. In your solution, group those functions that are big-Theta of one another (all log functions are base 2):

$$\begin{array}{cccccc} 6 \cdot n \log n & 2^{100} & \log \log n & \log^2 n & 2^{\log n} & \\ 2^{2^n} & \lceil \sqrt{n} \rceil & n^{0.01} & 1/n & 4n^{3/2} & \\ 4^n & n^3 & n^2 \log n & 4^{\log n} & \sqrt{\log n} & \end{array}$$

$$\begin{array}{cccccc} 1/n & & & & & \textit{Sub - Constant} \\ 2^{100} & & & & & \textit{Constant} \\ \log \log n & \sqrt{\log n} & \log^2 n & & & \textit{Logarithmic} \\ n^{0.01} & \lceil \sqrt{n} \rceil & & & & \textit{SquareRoot} \\ 2^{\log n} & & & & & \textit{Linear} \\ 6 \cdot n \log n & & & & & \textit{Linearithmic} \\ 4n^{3/2} & 4^{\log n} & n^2 \log n & n^3 & & \textit{Polynomial}(c > 1) \\ 4^n & 2^{2^n} & & & & \textit{Exponential} \end{array}$$

9. Based on the given data, please classify each of the following as linear, quadratic, logarithmic, or none of the above. If it is none of the above, try to reason what type of curve it may be.

- $f(0) = 4, f(1) = 6, f(2) = 9$

quadratic

- $f(0) = 6, f(10) = 8, f(20) = 10$

linear

- $f(0) = 80, f(0.1) = 60, f(0.2) = 45$

quadratic

- $f(1) = 10, f(10) = 20, f(100) = 30$

logarithmic

- $f(0) = 2, f(3) = 12, f(5) = 240$

none (factorial)