Copula-Based Trading of Cointegrated Cryptocurrency Pairs

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Abstract

This research introduces a novel pairs trading strategy based on copulas for cointegrated pairs of cryptocurrencies. To identify the most suitable pairs, the study employs linear and non-linear cointegration tests along with a correlation coefficient measure and fits different copula families to generate trading signals formulated from a reference asset for analyzing the mispricing index. The strategy's performance is then evaluated by conducting back-testing for various triggers of opening positions, assessing its returns and risks. The findings indicate that the proposed method outperforms buy-and-hold trading strategies in terms of both profitability and risk-adjusted returns.

Keywords: Statistical arbitrage, Pairs trading, Cointegration, Copulas, Cryptocurrency market.

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1 Introduction

Pairs trading is a well-known algorithmic trading strategy that capitalizes on temporary abnormal relationships among two or multiple assets whose historical prices tend to move together. When this relationship begins to exhibit abnormal behavior, it triggers the opening of trading positions. These positions are closed as soon as the pairs return to their normal behavior (Vidyamurthy 2004). In the decentralized cryptocurrency market, pairs trading can be profitable, offering two potential arbitrage opportunities: exchange-to-exchange arbitrage and statistical arbitrage. However, implementing a statistical arbitrage strategy based on exchange-to-exchange arbitrage can be risky and pose numerous challenges. In contrast, statistical arbitrage opportunities present similar profit potentials with lower risk (Pritchard 2018).

As stated by Krauss (2017), pairs trading is characterized by a formation and a trading period. During the formation period, the objective is to identify pairs of assets that exhibit similar price movements. This is commonly achieved through co-movement criteria, which can be measured using various methods such as distance metrics (distance approach), for instance, the minimum sum of squared distances of normalized asset prices, or statistical relationships like cointegration rules (cointegration approach). Furthermore, the parameters of the trading period are estimated during the formation period. During the trading timeframe, irregularities in pairs' price movement are aimed to benefit from statistical arbitrage opportunities and create signals to open long/short positions. Advanced strategies may utilize a range of mathematical tools, such as stochastic processes, stochastic control techniques, copulas, and machine learning methods, to enhance the effectiveness of their outcomes.

This paper aims to implement a copula-based pairs trading strategy on cointegrated cryptocurrency pairs. First, we start with a literature review on existing pairs trading strategies. We also provide an overview of the cryptocurrency exchange and data sources used in this study. The theoretical framework of the strategy includes linear and non-linear cointegration tests to identify cointegrated pairs and the copula concept to model their dependence structure. We discuss different families of copulas and the methods for copula estimation. We then outline the methodology for implementing the copula-based trading strategy, including ranking, selection of the assets, and the generation of the trading signals. Next, we present the empirical results of back-testing the strategy to a dataset of historical cryptocurrency prices. We analyze the performance of the strategy using various metrics, including profitability, risk-adjusted return, and maximum drawdown. In addition, we perform a comparative analysis of our results with the buy-and-hold strategy.

2 Literature Review

There is extensive literature on pair trading strategies that use various concepts, such as the distance approach, cointegration analysis, or the concept of copulas. The distance approach involves calculating the historical price spread or price difference between two related assets and monitoring this spread over time. The spread is usually calculated as the difference between the prices of the two assets, either as a raw spread or as a normalized spread, such as the z-score or the percent difference. As an example, Gatev, Goetzmann, and Rouwenhorst (2006) defined the normalized spread, S_t^{ij}, between assets i and j at time t by

$$S_t^{ij} = P_t^{i*} - P_t^{j*} \tag{1}$$

$$P_t^{i*} = \frac{P_t^i}{P_0^i} = \prod_{\tau=1}^{t} \frac{P_\tau^i}{P_{\tau-1}^i} = \prod_{\tau=1}^{t} \left(1 + r_\tau^i\right) = cr_t^i, \tag{2}$$

where P_t^i is the price of the asset i at time t, r_t^i is the t-period's return on asset i at time t, and cr_t^i is the cumulative total return of asset i up to time t ($cr_0^i = 1$). The calculation of the spread sum of squared distance is performed using the following equation:

$$SSD_{i,j} = \sum_{t=1}^{T} \left(S_t^{ij}\right)^2 = \sum_{t=1}^{T} (cr_{i,t} - cr_{j,t})^2. \tag{3}$$

Pairs are chosen for the trading period based on their ascending order of $SSD_{i,j}$ during the formation stage. The initial pairs at the top of the list are selected, and basic non-parametric threshold rules are employed to generate trading rules. An alternative method was used by Chen et al. (2019), who identified the most suitable pairs for the trading period using Pearson correlation of assets' returns instead of finding the minimum sum of the squared distance. Krauss (2017) found that to maximize the profit of the distance approach strategy, the spread of each selected pair needs to have high volatility, indicating the potential divergence of the two assets. The pair's spread should also have a mean-reverting property. This approach's key advantage is its simplicity and transparency, making it suitable for large-scale empirical applications.

Huck (2015) found that the cointegration approach outperformed the distance approach in selecting effective pairs. The cointegration approach aims to identify a long-term equilibrium between non-stationary time series (e.g., asset prices) that move together. This equilibrium can be linear or nonlinear. Engle and Granger (1987) introduced the first cointegration test, which is based on linear regression and the unit-root test of residuals in the equilibrium. Typically, the augmented Dickey-Fuller test is used for the unit-root test. Other improvements of the Engle-Granger cointegration test were introduced by Phillips and Ouliaris (1990) and Johansen (1991). Highly volatile markets, such as cryptocurrency, usually exhibit nonlinear features. Therefore, we can extend the Engle-Granger cointegration test by adjusting the error correction model and applying nonlinear unit root tests to increase the reliability of the study. These extensions have been studied by Enders and Siklos (2001), Hansen and Seo (2002), and Kapetanios, Shin, and Snell (2006). Several research papers have been published that specifically explore the application of pairs trading in the cryptocurrency market. These include studies by Broek and Sharif (2018), Pritchard (2018), Kakushadze and Yu (2019), Leung and Nguyen (2019), and Tadi and Kortchemski (2021).

In addition to the commonly used methods discussed above, more advanced concepts such as copulas can be applied to enhance the empirical results of the strategy. Compared to correlation or linear cointegration-based methods, the copula approach provides more valuable information regarding the shape and characteristics of pairs' dependency (Ferreira 2008). Xie and Wu (2013) demonstrated that the two commonly used pairs trading methods, namely the distance and cointegration methods, can be generalized as special cases of the copula method under certain conditions, and the dependency structure of assets in the copula approach is more robust and accurate.

Moreover, Liew and Wu (2013) conducted a comparative study of a copula-based pairs trading strategy with other conventional approaches such as the distance and cointegration approach. They found that the copula approach for pairs trading has better empirical results than the others, as it provides more trading possibilities with higher confidence in practice and does not entail any rigid assumptions, such as the linearity association between assets' returns, in traditional approaches. Hence, the copula approach can provide closer estimations and predictions of reality. According to Stander, Marais, and Botha (2013), the copula approach can significantly demonstrate the pairs' dependency, as it can capture the asymmetry and heavy-tail characteristics of asset returns to model the marginal distribution functions instead of modeling them by Gaussian distribution. Additionally, their empirical analyses reveal that there are more trading opportunities when the market is highly volatile, and the profitability of the strategy strongly depends on the liquidity of the market.

According to Krauss and Stübinger (2017), the copula approach can be divided into two sub-streams: return-based and level-based. In the return-based copula method, the log-returns of two assets are calculated, and their marginal distributions are estimated. Then, an appropriate copula is chosen to represent the dependency relationship between the two assets. To generate trading signals, a mispricing index is defined to indicate the degree of abnormal relationship between the assets. Unlike the distance and cointegration methods which use spread-based mispricing definition, the copula method defines mispricing based on the copula's conditional probability distribution of the corresponding assets' log returns. The conditional distribution of copulas can be obtained by taking the partial derivative of the copula function, as shown below[1]:

$$\begin{aligned} h^{1|2} &:= h(u_1|u_2) = P(U_1 \leq u_1 | U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2} \\ h^{2|1} &:= h(u_2|u_1) = P(U_2 \leq u_2 | U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1} \end{aligned} \tag{4}$$

where $C(u_1, u_2)$ is the copula distribution function, $h^{1|2}$ and $h^{2|1}$ are conditional copula distribution functions, and U_1 and U_2 are the transformed uniform variables of the log returns. The values of $h^{1|2}$ and $h^{2|1}$ are between 0 and 1, and as much as their values get away from 0.5, we can consider it as a deviance from the expected relationship between the two assets. Ferreira (2008), Liew and Wu (2013), Stander, Marais, and Botha (2013), and Keshavarz Haddad and Talebi (2023) deployed their strategy in that way.

The usage of the return-based method has a drawback in that entry and exit signals are linked with only the previous period's return. To address this limitation, a new approach (hereafter referred to as the value-based method) was proposed by Xie and Wu (2013). They defined a new mispricing index by aggregating the surplus value of conditional probability in equation 4 from 0.5 across multiple periods to determine the extent to which assets are out of balance.

$$\begin{aligned} \text{CMI}_t^{1|2} &= \text{CMI}_{t-1}^{1|2} + \left(h_t^{1|2} - 0.5\right) \\ \text{CMI}_t^{2|1} &= \text{CMI}_{t-1}^{2|1} + \left(h_t^{2|1} - 0.5\right) \end{aligned} \tag{5}$$

The studies conducted by Xie, Liew, et al. (2016), Rad, Low, and Faff (2016), Krauss and Stübinger (2017), and Silva, Ziegelmann, and Caldeira (2023). According to Xie, Liew, et al. (2016), the copula method outperforms the distance approach in terms of describing the dependency relationship between assets and identifying more statistical arbitrage opportunities that generate greater profits. They also recommended utilizing the copula approach for high-frequency pairs trading. Rad, Low, and Faff (2016) compared the performance of the copula approach with that of traditional methods using daily US stocks. They found that while the profitability of the copula method could be weaker, it is more reliable in capturing arbitrage opportunities in the US stock market. They determined that the Student-t copula is more appropriate for modeling the dependence structure of pairs in the US stock market than other copulas.

The performance of a pairs trading strategy based on a weighted combination of copulas was evaluated by Silva, Ziegelmann, and Caldeira (2023) against a distance methodology, using a vast dataset of $S\&P500$ stocks over 25 years. The study

[1]For more details, see Section 4.2.

examined the effect of financial factors on profitability. The mixed copula approach yielded better results than the distance method, with higher alphas for fully invested capital and superior performances overall. The approach was notably effective under committed capital during both crisis and non-crisis periods and fully invested during non-crisis periods.

However, in practice, the cumulative mispricing indices (CMI) in equation 5 may not necessarily exhibit mean-reverting behavior, which can negatively impact the profitability of the strategy. Hence, a new methodology is proposed in this study to replace log-returns with stationary spread processes in order to overcome this issue[2].

3 Cryptocurrency Exchange and Data Source

The cryptocurrency market facilitates decentralized trading of cryptocurrencies across various exchanges. Binance, established in 2017, is the largest cryptocurrency exchange worldwide in terms of daily trading volume in both spot and derivatives markets. Binance provides three types of derivative contracts: Futures contracts, Options, and Binance Leveraged Tokens (BLVT).

Futures contracts are classified into two main categories: COIN-Margined contracts and USDⓈ-Margined contracts. COIN-Margined contracts are inverse Futures quoted in US dollars but denominated in an underlying cryptocurrency (e.g., Bitcoin). They include traditional quarterly Futures and perpetual Futures, also known as perpetual Swaps, which never expire or settle. Because they lack a settlement price, their price can deviate significantly from their spot contract price. Binance uses funding fees to address this issue for long and short sides. On the other hand, USDⓈ-Margined contracts are similar to COIN-Margined Futures, and they may have perpetual or quarterly expiration. However, they are quoted, denominated, and settled in stablecoins such as Tether (USDT) and Binance USD (BUSD), which are pegged to the US dollar value. For this study, all nominated cryptocurrency coins are USDT-Margined Futures (*Binance Crypto Derivatives* accessed 2022-11-03).

The minimum trade amount, also known as the minimum price increment, is the smallest possible change in the price of a contract at an exchange. This value is specific to each asset and can be adjusted over time. Assets with smaller increments have narrower bid/ask spreads. When limit orders are not entirely disclosed to traders, they tend to order contracts with smaller quote sizes to avoid slippage (Harris 1997).

To calculate profit and loss, all coins are valued in Tether (USDT), a stablecoin. Using the Binance API, we collected historical hourly closed prices for twenty

[2] For more details, see Section 5.

cryptocurrency coins from 01/01/2021 to 10/11/2022. We used Engle-Granger (EG) and Kapetanios-Shin-Shell (KSS) cointegration tests to identify cointegrated pairs and calculated Kendall's Tau. Then, we calibrated the method's parameters using the copula approach and generated trading signals for the strategy.

A pairs trading cycle comprises formation and trading periods. In this study, the entire process of pairs trading is conducted within a month, with three weeks allocated for the formation step and the remaining week designated for the trading step. We carry out 94 cycles that move dynamically over time, with each cycle sharing three-quarters of its data with its previous or subsequent cycle.

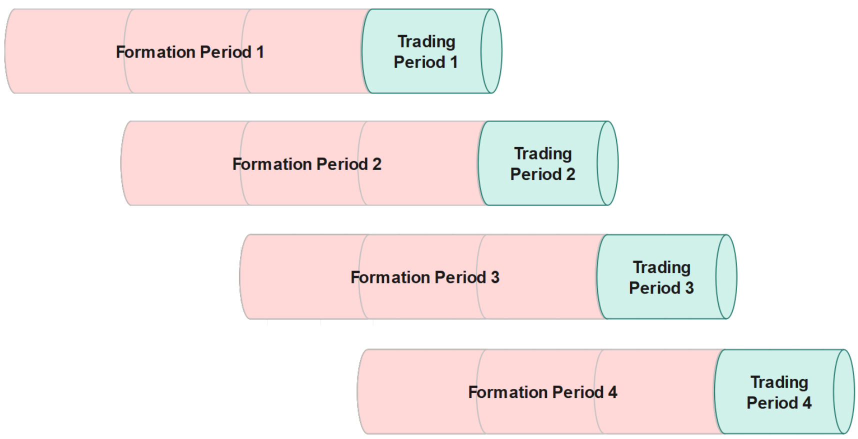

Figure 1: The scheme of Formation and Trading Periods

Furthermore, we implemented the entire methodology using Python and R, with Python being the preferred option for data handling and R for statistical tests and copula modeling.

Python		**R**	
Name	**Type**	**Name**	**Type**
NumPy	Numerical analysis	copula	Copula modeling
Pandas	Technical computing	VineCopula	Copula modeling
Matplotlib	Plotting	NonlinearTSA	Statistical package
Datetime	Date/time package	dpylr	Data handling
Statsmodels	Statistical package	lubridate	Date/time package
SciPy	Technical computing	scatterplot3d	Plotting

Table I: Python libraries and R packages used in this research

4 Theoretical Framework

4.1 Tests for Linear and Non-Linear Cointegration

The cointegration property can be employed to identify the most appropriate pair from multiple combinations of coins. Both linear and non-linear cointegration tests can be utilized for evaluation. Initially, the pair spread value is defined without an intercept in the following manner:

$$S_t = P_t^1 - \beta P_t^2 \tag{6}$$

Suppose that P_t^1 and P_t^2 are non-stationary time series. We use unit-root tests to study whether the spread, S_t, is also a non-stationary process or not. The augmented Dickey-Fuller (ADF) unit-root test, as described by Dickey and Fuller (1979), uses a test equation (without constant and trend) in the following form:

$$S_t = \beta S_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta S_{t-i} + \epsilon_t, \tag{7}$$

where β is the coefficient of the lagged level of the series, γ_i are the coefficients of the lagged differences, ϵ_t is the error term, and p is the number of lags used in the test. The null hypothesis of the ADF test is that the series has a unit root, i.e., $\beta = 0$. If the test statistic exceeds a critical value at a given significance level, the null hypothesis is rejected, indicating that the series is stationary and does not have a unit root. Conversely, if the test statistic is below the critical value, the null hypothesis cannot be rejected, implying that the series has a unit root and is non-stationary.

Traditional unit-root tests like the Augmented Dickey-Fuller (ADF) test assume that the data-generating process is linear. However, non-linear unit-root tests are designed to account for non-linearities in time series data and provide more accurate assessments of unit roots. There are several non-linear unit-root tests available in the literature, each with its own assumptions, methodologies, and advantages. Examples include the Teräsvirta (1994) test, the Zivot and Andrews (2002) test, the Kapetanios, Shin, and Snell (2003) test, and the Kapetanios (2005) test. These tests often involve estimating non-linear models like threshold auto-regressive (TAR) models, smooth transition auto-regressive (STAR) models, or other non-linear models, and computing test statistics to compare estimated model parameters with critical values.

The general self-exciting threshold auto-regressive (SETAR) model with n regimes is in the following form:

$$S_t = \sum_{i=1}^{p} \left(\phi_{i1} \mathbb{1}_{\{S_{t-d} \le c_1\}} + \sum_{j=1}^{n-1} \phi_{ij} \mathbb{1}_{\{c_j < S_{t-d} \le c_{j+1}\}} + \phi_{in} \mathbb{1}_{\{S_{t-d} > c_n\}} \right) S_{t-i} + \epsilon_t, \tag{8}$$

where d denotes the transition's delay, c_j represents the j-th threshold, and ϵ_t represents the error term. In special case Kapetanios, Shin, and Snell (2003) proposed a test equation where the indicator function is replaced by an exponential smooth transition function in the form:

$$S_t = S_{t-1} + \sum_{i=1}^{p} \left(\gamma_{1i} \left(1 - e^{-\theta(S_{t-1}-c)^2} \right) S_{t-i} \right) + \epsilon_t \tag{9}$$

When c is set to zero, and p is set to one, using Taylor approximation, equation 9 can be illustrated as

$$\Delta S_t = \delta \left(S_{t-1} \right)^3 + \epsilon_t^{'} \tag{10}$$

where $\delta = \gamma_1 \theta$ and $\epsilon_t^{'} = f(\epsilon_t)$. The null hypothesis assumes that δ is equal to zero, while the alternative hypothesis posits that δ is less than zero.

4.2 Copula Concept

Suppose that a continuous random variable X has a continuous distribution, and its probability distribution function is defined as $F_X(x) := \mathbb{P}(X \leq x)$. If F_X is strictly increasing, then F_X^{-1} is defined by $F_X^{-1}(u) = x \Leftrightarrow F_X(x) = u$. However, if F_X is constant on some interval, then the inverse function is not well defined by $F_X^{-1}(u) = x$. To avoid this problem, we can define $F_X^{-1}(u)$ for $0 < u < 1$ by the generalized inverse function such that

$$F_X^{-1}(u) = \inf\{x : F_X(x) \geq u\}, \tag{11}$$

Now, in the same way, we define another continuous random variable Y with distribution function F_Y and generalized inverse function F_Y^{-1} similar to equation 11. Given two continuous random variables X and Y, with distribution functions F_X and F_Y respectively, the joint distribution function $F_{X,Y}$ can be written as:

$$F_{X,Y}(x, y) = \mathbb{P}(X \leq x, Y \leq y) = \mathbb{P}(F_X(X) \leq F_X(x), F_Y(Y) \leq F_Y(y)) \tag{12}$$

where the last equality follows from the fact that F_X and F_Y are both increasing. Then we define random variables U and V such that $U := F_X(X)$ and $V := F_Y(Y)$. According to the probability integral transformation theorem, the probability distribution function of U, F_U, and the probability distribution function of V, F_V, are uniformly distributed on $[0, 1]$. (See Casella and Berger (2021, p. 54-55) for the proof).

Definition: A two-dimensional copula C is a function that maps the unit square $[0, 1]^2$ into the unit interval $[0, 1]$, satisfying the following requirements:

1. $C(0, v) = C(u, 0) = 0,$ for $0 \leq u, v \leq 1$.

2. $C(u,1) = u$, and $C(1,v) = v$, for $0 \le u, v \le 1$.
3. $C(u_1,v_1) - C(u_1,v_2) - C(u_2,v_1) + C(u_2,v_2) \ge 0$, for $1 \ge u_1 > u_2 \ge 0$, and $1 \ge v_1 > v_2 \ge 0$.

We can define several copula functions, but the three requirements above should be satisfied by $C(u,v)$ to have a well-defined joint distribution function (Cherubini et al. 2011). According to Sklar's theorem, there exists a Copula function C which could connect the uniform random variables U and V to the joint distribution function $F_{X,Y}$ as follows

$$F_{X,Y}(X,Y) = F_{X,Y}(F_X^{-1}(U), F_Y^{-1}(V)) := C(U,V), \tag{13}$$

Hence, we can rewrite the joint distribution function of X and Y in terms of standard uniform random variables U and V such that

$$F_{X,Y}(x,y) = F_{X,Y}(F_X^{-1}(u), F_Y^{-1}(v)) := C(u,v) = C(F_X(x), F_Y(y)), \tag{14}$$

where $u = F_X(x)$, and $v = F_Y(y)$. Knowing $F_{X,Y}(x,y) = C(u,v)$, we can determine the copula density function $c(u,v)$ by

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u\, \partial v} = \frac{\partial^2 F_{X,Y}(x,y)}{\partial F_X(x)\, \partial F_Y(y)} = \frac{\frac{\partial^2 F_{X,Y}(x,y)}{\partial x\, \partial y}}{\frac{\partial F_X(x)\, \partial F_Y(y)}{\partial x\, \partial y}} = \frac{f_{X,Y}(x,y)}{f_X(x) f_Y(y)} \tag{15}$$

Sklar's theorem enables us to separate the modeling of the marginal distributions $F_X(x)$ and $F_Y(y)$ from the dependence relation represented in C. We can define the conditional distribution functions using copula function. The conditional distribution of $Y|X = x$ is obtained by the first partial derivatives of the copula function as follows

$$\begin{aligned} F_{Y|X}(y) &= \mathbb{P}(Y \le y | X = x) = \mathbb{P}(F_Y(Y) \le F_Y(y) | F_X(X) = F_X(x)) \\ &= \mathbb{P}(V \le v | U = u) = \lim_{\delta \to 0^+} \frac{\mathbb{P}(V \le v, U \in (u-\delta, u+\delta)}{\mathbb{P}(U \in (u-\delta, u+\delta))} \\ &= \lim_{\delta \to 0^+} \frac{C(u+\delta, v) - C(u-\delta, v)}{2\delta} \\ &= \frac{\partial}{\partial u} C(u,v). \end{aligned} \tag{16}$$

Copulas are invariant concerning strictly increasing transformations of the marginal distributions. For more details about the characterization of invariant copulas, see Klement, Mesiar, and Pap (2002). Also, we can increase the range of dependence captured by copulas by rotating them. The following equations show how to rotate a copula by 90, 180, and 270 degrees:

$$\begin{aligned} C_{90}(u_1,u_2) &:= C(1-u_2, u_1) \\ C_{180}(u_1,u_2) &:= C(1-u_1, 1-u_2) \\ C_{270}(u_1,u_2) &:= C(u_2, 1-u_1) \end{aligned} \tag{17}$$

4.3 Families of copulas

There are several types of copulas. This research focuses on three popular families of copulas: elliptical, Archimedean, and extreme value copulas. Each type of copula is characterized by its properties, such as its dependence structure and tail behavior, and is often used in different areas of statistics and finance.

4.3.1 Elliptical Copulas

Elliptical copulas are widely used in finance and insurance applications due to their ability to model dependence with different levels of tail dependence. An elliptical copula is constructed from a multivariate elliptical distribution. The most commonly used elliptical distributions are the multivariate Gaussian and Student-t. The density function of any elliptical distribution $f_{\boldsymbol{X}}$ is in the form

$$f_{\boldsymbol{X}}(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{\Sigma}) = k_n|\boldsymbol{\Sigma}|^{-\frac{1}{2}} g\left((\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right), \tag{18}$$

where $k_n \in \mathbb{R}$ is the normalizing constant and dependents on the dimension n, $\boldsymbol{x}$ is an n-dimensional random vector with mean vector $\boldsymbol{\mu}$, and a positive definite matrix which is proportional to the covariance matrix $\boldsymbol{\Sigma}$, and some function $g(.)$ which is independent of the dimension n (Czado 2019). In the the case of bivariate Gaussian distribution $g(x) := e^{-x/2}$, $k_n := 1/(2\pi)$, and the probability density function of $\boldsymbol{X} = (X_1, X_2)$ is

$$\begin{aligned} f_{\boldsymbol{X}}(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{\Sigma}) &= \frac{1}{2\pi|\boldsymbol{\Sigma}|^{1/2}}\mathrm{e}^{-\frac{1}{2}\left[(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]} \\ \boldsymbol{\mu} &= \begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}, \end{aligned} \tag{19}$$

where ρ is the correlation between random variables X_1 and X_2, $\sigma_1 > 0$ and $\sigma_2 > 0$. If $\mu_1 = \mu_2 = 0$ and $\sigma_1 = \sigma_2 = 1$, then the density and distribution functions of the standard Bivariate Gaussian distribution are obtained by

$$\begin{aligned} \phi_{X_1X_2}(x_1, x_2; \rho) &= \frac{1}{2\pi\sqrt{1-\rho^2}}e^{-\frac{x_1^2-2\rho x_1x_2+x_2^2}{2(1-\rho^2)}} \\ \Phi_{X_1X_2}(u_1, u_2; \rho) &= \int_{-\infty}^{u_1}\int_{-\infty}^{u_2}\phi_{X_1X_2}(x_1, x_2; \rho)\,dx_1\,dx_2 \end{aligned} \tag{20}$$

Using Sklar's theorem in 13 and 20, the bivariate Gaussian copula is defined by

$$C(u_1, u_2; \rho) := \Phi_{X_1X_2}(\Phi_{X_1}^{-1}(u_1), \Phi_{X_2}^{-1}(u_2); \rho) \tag{21}$$

In the bivariate t distribution, $g(.)$ and k_n function in the formula 18 are defined by $g(x) := (1 + x/\nu)^{-(\nu+2)/2}$, $k_n := \Gamma(\frac{\nu+2}{2})/\left(\Gamma(\frac{\nu}{2})\nu\pi\right)$, and the probability density function of $\boldsymbol{T} = (T_1, T_2)$ is obtained by

$$f_{\boldsymbol{T}}(\boldsymbol{t}; \nu, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\nu\pi|\boldsymbol{\Sigma}|^{1/2}} \left[1 + \frac{1}{\nu}(\boldsymbol{t} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{t} - \boldsymbol{\mu})\right]^{-(\nu+2)/2} \tag{22}$$

where $\nu > 0$ is the degree of freedom parameter and $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are the same as 19. If $\mu_1 = \mu_2 = 0$ and $\sigma_1 = \sigma_2 = 1$, and knowing that $\Gamma\left(\frac{\nu+2}{2}\right)/\Gamma\left(\frac{\nu}{2}\right) = \frac{\nu}{2}$, the density and distribution function of the standard Bivariate Student-t distribution are obtained by

$$\begin{aligned} f_{T_1T_2}(t_1, t_2; \nu, \rho) &= \frac{(1-\rho^2)^{-1/2}}{2\pi} \left[1 + \frac{t_1^2 - 2\rho t_1 t_2 + t_2^2}{\nu(1-\rho^2)}\right]^{-(\nu+2)/2} \\ F_{T_1T_2}(u_1, u_2; \nu, \rho) &= \int_{-\infty}^{u_1}\int_{-\infty}^{u_2} f_{T_1T_2}(t_1, t_2; \nu, \rho)\, dt_1\, dt_2 \end{aligned} \tag{23}$$

Then the bivariate Student-t copula is defined by

$$C(u_1, u_2; \nu, \rho) := F_{T_1T_2}(F_{T_1}^{-1}(u_1), F_{T_2}^{-1}(u_2); \nu, \rho) \tag{24}$$

4.3.2 Archimedean Copulas

Archimedean copulas are based on a generator function and can model dependence with tail dependence that decreases logarithmically or exponentially. According to Nelsen (2007), Archimedean copulas are defined as follows:

$$C(u_1, \ldots, u_n) = \phi^{[-1]}(\phi(u_1) + \cdots + \phi(u_n)) \tag{25}$$

where $\phi : [0, 1] \to [0, \infty]$ is called a generator which is continuous, strictly decreasing, and convex function such that $\phi(1) = 0$. In addition, $\phi^{[-1]} : [0, \infty] \to [0, 1]$ is called the pseudo-inverse of ϕ function and is defined by

$$\phi^{[-1]}(t) = \begin{cases} \phi^{-1}(t) & , 0 \leq t \leq \phi(0) \\ 0 & , \phi(0) \leq t \leq \infty \end{cases} \tag{26}$$

If $\phi(0)$ equals infinity, then the pseudo-inverse function $\phi^{[-1]}$ is equivalent to the inverse function ϕ^{-1}. Archimedean copulas allow for a range of generator functions to be selected, which ultimately determines the type of tail dependence exhibited by the copula. For this research, both one-parameter copulas (such as the Gumbel, Clayton, Frank, and Joe copulas) and two-parameter copulas (such as BB1, BB6, BB7, and BB8) were utilized. Table X in the appendix provides a list of the bivariate Archimedean copulas utilized in this study.

4.3.3 Extreme-Value Copulas

Extreme value copulas are used to model dependence with strong tail dependence, making them suitable for modeling extreme events. Suppose that $X_i = (X_{i1}, X_{i2})^T$, $i \in \{1, 2, \cdots, n\}$, be independent and identically distributed random vectors with joint distribution function F, and marginal distributions F_1 and F_2. According to Gudendorf and Segers (2010), we can define the bivariate vector of component-wise maxima $M_n = (M_{n1}, M_{n2})^T$ such that

$$M_{nj} := \max_{i\in\{1,2,\cdots,n\}} (X_{ij}), \quad j = 1, 2. \tag{27}$$

Then, the bivariate copula C_n of M_n is obtained by

$$C_n(u_1, u_2) = C_F\left(u_1^{1/n}, u_2^{1/n}\right)^n, \quad (u_1, u_2) \in [0, 1]^2. \tag{28}$$

In equation 28, if C_F exists such that

$$\lim_{n\to\infty} C_F(u_1^{1/n}, u_2^{1/n})^n = C(u_1, u_2), \quad (u_1, u_2) \in [0, 1], \tag{29}$$

then the bivariate copula C in (29) is called an extreme-value copula. Bivariate extreme-value copulas can be demonstrated in terms of a function $A(t)$ in this form:

$$C(u_1, u_2) = (u_1 u_2)^{A(\ln(u_2)/\ln(u_1 u_2))}, \quad (u_1, u_2) \in (0, 1]^2 \setminus \{(1, 1)\}, \tag{30}$$

where the function $A : [0, 1] \to [1/2, 1]$, which is called the Pickands dependence function, is convex and satisfies $\max(1 - t, t) \leq A(t) \leq 1$ for all $t \in [0, 1]$ (Gudendorf and Segers 2010). Some Archimedean copulas, such as the Gumbel copula, can be expressed by the extreme-value family. These special copulas create a hybrid category, including both the Archimedean and the extreme-value copulas, and are called Archimax copulas.

Table II shows extreme-value copulas used in this research. Note that Tawn copula has three parameters and its Pickands's dependence function is in the form

$$A(t) = (1 - \beta) + (\beta - \alpha)t + \left[(\alpha(1 - t))^\theta + (\beta t)\theta\right]^{1/\theta}, \tag{31}$$

where $\theta \geq 1$ and $\alpha, \beta \in [0, 1]1$. The simplified Tawn copula cases with $\beta = 1$ and $\alpha = 1$ are respectively called Tawn Type 1 and Tawn Type 2 copula and have two parameters.

Name	Pickands function A(t)	Parameters
Gumbel	$\left[t^{\theta}+(1-t)^{\theta}\right]^{1/\theta}$	$\theta \geq 1$
Tawn Type 1	$(1-\alpha)t+\left[(\alpha(1-t))^{\theta}+t^{\theta}\right]^{1/\theta}$	$\theta \geq 1,\ 0 \leq \alpha \leq 1$
Tawn Type 2	$(1-\beta)(1-t)\left[(1-t)^{\theta}+(\beta t)\theta\right]^{1/\theta}$	$\theta \geq 1,\ 0 \leq \beta \leq 1$

Table II: Pickands dependence function of some extreme-value copulas

4.4 Copula Estimation

When the marginal probability density of X_1 and X_2 and their corresponding copula density $c(.)$ are given in their parametric with unknown parameters, we can estimate the vector of parameter vector $\theta = (\beta, \alpha)^T$ with the maximum likelihood estimation method, where $\beta = (\beta_1, \beta_2)^T$ represent the marginal parameters and α represents the copula parameters. The log-likelihood function for (X_1, X_2), where $X_i = (x_{i1}, \dots, x_{in})$, can be expressed as

$$l(\theta) = \sum_{j=1}^{n} \left[\log c\big(F_1(x_{1j};\beta_1), F_2(x_{2j};\beta_2);\alpha\big) + \log f_1(x_{1j};\beta_1) + \log f_2(x_{2j};\beta_2)\right], \quad (32)$$

and the maximum likelihood estimator of θ is

$$\hat{\theta}_{ML} = \underset{\theta}{\text{argmax}}\, l(\theta). \quad (33)$$

However, since this approach is computationally expensive, an alternative method called the Inference for the Margins (IFM) two-step method can be used instead. This method is computationally easier to obtain compared to the full maximum likelihood estimation approach. First, we estimate the margins' parameters by performing the estimation of the univariate marginal distributions using the log-likelihood function where the maximum likelihood estimator of β_i is

$$\hat{\beta}_i = \underset{\beta_i}{\text{argmax}} \sum_{j=1}^{n} \left[\log f_i(x_{ij};\beta_i)\right] \quad (34)$$

Then given estimated marginal parameters, we transform data to the copula scale, develop the copula model, and estimate the copula parameters α as follows

$$\hat{\alpha}_{ML} = \underset{\alpha}{\text{argmax}} \sum_{j=1}^{n} \left[\log c\left(F_1(x_{1j};\hat{\beta}_1), F_2(x_{2j};\hat{\beta}_2);\alpha\right)\right]. \quad (35)$$

We can also employ a semiparametric approach known as canonical maximum likelihood to estimate copula parameters without specifying the marginals. The empirical cumulative distribution function of $X_i = (x_{i1}, \ldots, x_{in})$ is

$$\widehat{F_n^i}(t) = \frac{\#\{X_i \leq x\}}{n} = \frac{1}{n} \sum_{j=1}^{n} \mathbb{1}_{\{x_{ij} \leq t\}}. \tag{36}$$

Then, the copula parameters are estimated using maximum likelihood estimation as follows:

$$\hat{\alpha}_{ML} = \underset{\alpha}{\operatorname{argmax}} \sum_{j=1}^{n} \left[\log c \left(\widehat{F_n^1}(x_{1j}), \widehat{F_n^2}(x_{2j}); \alpha \right) \right]. \tag{37}$$

5 Implementation Methodology

The study suggests a new approach to address the issue of cumulative mispricing indices (CMI) not exhibiting mean-reverting behavior in practice, which can negatively affect the profitability of trading strategies. The proposed methodology involves using stationary spread processes instead of log-returns. In the pairs trading strategy employed in this study, the spread process is defined as a linear combination of two assets, with a reference asset (BTCUSDT) selected and other cryptocurrency coins identified as cointegrated with it using a specific equation.

$$S_t^i = \text{BTCUSDT}_t - \hat{\beta}^i P_t^i \quad i = 1, 2, \ldots, 19 \tag{38}$$

where $\hat{\beta}$ is the estimated linear regression coefficient between BTCUSDT and the second coin, chosen from the other 19 coins. This allows us to identify 19 pairs and select the optimal pairs during the formation period for trading in the trading period. To determine the optimal pairs, we use the linear Engle-Granger (EG) two-step method and the non-linear Kapetanios-Shin-Snell (KSS) cointegration tests to identify cointegrated coins. However, since there may be multiple cointegrated pairs, we need to add another criterion to rank them. To do this, we calculate Kendall's Tau (τ), which is a measure of correlation for ranked data and defined

$$\tau(S^i, S^j) = \tau_{ij} = \frac{\text{Number of concordant pairs} - \text{Number of discordant pairs}}{\text{Total number of pairs}} \tag{39}$$

where

$$\text{Number of concordant pairs} = \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \text{sgn}(S_{[n]}^i - S_{[m]}^i)\text{sgn}(S_{[n]}^j - S_{[m]}^j),$$

$$\text{Number of discordant pairs} = \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \text{sgn}(S^i_{[n]} - S^i_{[m]})\text{sgn}(S^j_{[m]} - S^j_{[n]}),$$

and total number of pairs $= N(N-1)/2$. Here, N is the number of data points, $S^i_{[n]}$ and $S^j_{[n]}$ are the rankings of the n-th data point in two different variables, and sgn(x) is the sign function, which is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

After calculating τ for all spreads, we select the first couple of pairs with the highest τ. During the course of one week, the chosen pairs are traded and can be substituted with different pairs at the start of each trading period. Instead of trading a pair of coins, we employ a pair of spreads where each of them contains BTCUSDT. In this way, having a long position in one spread and a short position in the other one implies that BTCUSDT will not be traded at all and it only plays an intermediary role between two other coins.

Now, we should estimate the probability distribution function of spread processes. Suppose that $\widehat{F}^i_n$ is the empirical cumulative distribution function of the spread process S^i with n samples. We can compute $\widehat{F}^i_n$ for $\mathbf{S^i} = (S^i_1, S^i_2, \ldots, S^i_n)$ using equation 36. The Probability Integral Transform is employed to convert the spreads to random variables $\mathbf{U_1} := \widehat{F}^1_n(\mathbf{S^1})$ and $\mathbf{U_2} := \widehat{F}^2_n(\mathbf{S^2})$ with a standard uniform distribution. The next step is to determine a fitting copula model for $\mathbf{U_1}$ and $\mathbf{U_2}$. We select some potential copulas and estimate the corresponding parameters by the maximum likelihood method. Finally, using the Akaike information criterion (AIC), we distinguish the most fitted copula model.

During the trading period, using the hourly realizations of random variables U_1 and U_2, which are the transformed values of spread processes, we calculate the copula conditional probabilities $h^{1|2}$ and $h^{2|1}$ defined in equation 4, for the selected pairs of the week. when $h^{1|2}$ is higher (lower) than 0.5, the first coin can be considered to be overvalued (undervalued) relative to the second one. Similarly, when $h^{2|1}$ is higher (lower) than 0.5, the second coin can be considered to be overvalued (undervalued) relative to the first one. Therefore, we can interpret mispricing as conditional probabilities in equation 4 minus 0.5. We denote the trading trigger to be α_1 and α_2. We study the optimal triggers via back-testing. The opening and closing signals are generated by the following rules:

Trading Rule	**Signals**
If $h^{1\|2} < \alpha_1$ and $h^{2\|1} > 1 - \alpha_1$	open long S^1 and short S^2
If $h^{1\|2} > 1 - \alpha_1$ and $h^{2\|1} < \alpha_1$	open short S^1 and,long S^2
If $\|h^{1\|2} - 0.5\| < \alpha_2$ and $\|h^{2\|1} - 0.5\| < \alpha_2$	close both positions

Table III: Trading rules in terms S^1 and S^2

Trading Rule	Signals
If $h^{1\|2} < \alpha_1$ and $h^{2\|1} > 1 - \alpha_1$	open long $\beta^2 \times P^2$ and short $\beta^1 \times P^1$
If $h^{1\|2} > 1 - \alpha_1$ and $h^{2\|1} < \alpha_1$	open short $\beta^2 \times P^2$ and long $\beta^1 \times P^1$
If $\|h^{1\|2} - 0.5\| < \alpha_2$ and $\|h^{2\|1} - 0.5\| < \alpha_2$	close both positions

Table IV: Trading rules in terms P^1 and P^2

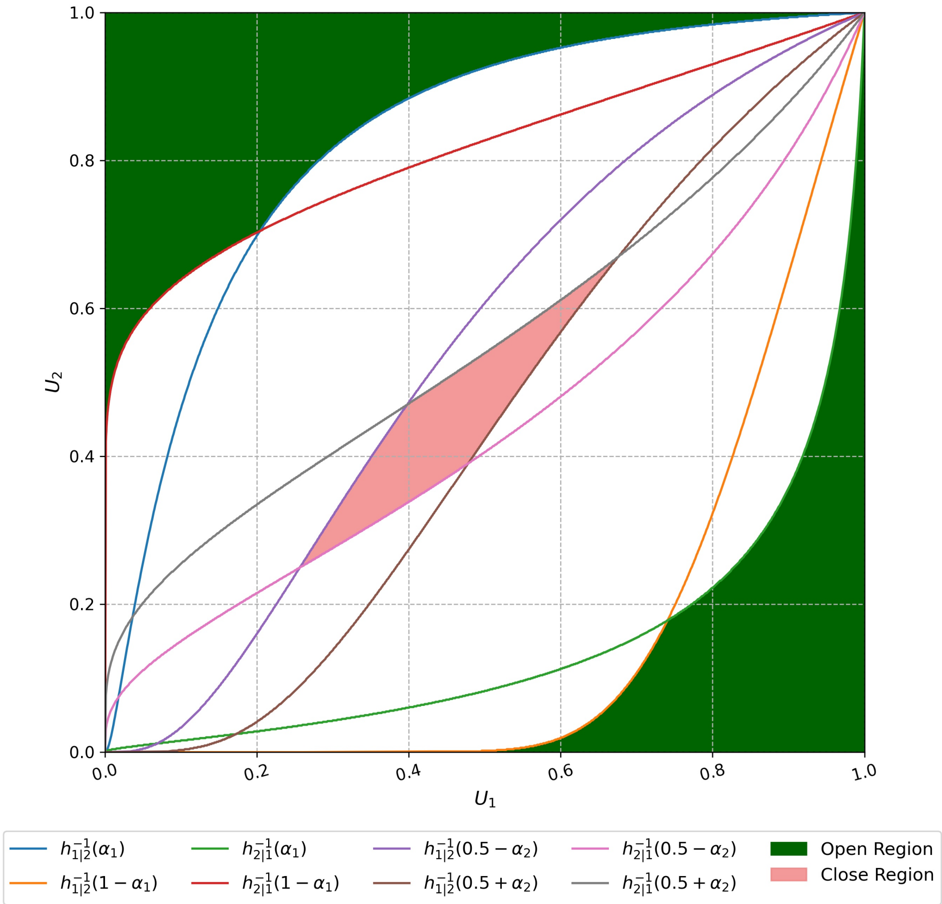

Figure 2: Confidence bands of Gumbel copula ($\theta = 2$) at $\alpha_1 = 5\%$ and $\alpha_2 = 10\%$

Figure 2 provides an illustrative depiction of the confidence bands in trading rules for the Gumbel copula with a parameter value of $\theta = 2$, under the conditions of $\alpha_1 = 5\%$ and $\alpha_2 = 10\%$. If the data point (u_{1t}, u_{2t}) falls within the top green (down green) area, it suggests that S_1 is undervalued (overvalued) and S_2 is overvalued (undervalued), which may indicate an opportunity to open a position. Conversely, if (u_{1t}, u_{2t}) falls within the red area, it may signal the need to close the positions.

6 Empirical Results

Table V presents the occurrence rate of selected copulas and their rotations over 94 trading weeks. The results indicate that copulas of extreme value, such as Tawn type 1 and 2, and certain two-parameter Archimedean copulas, particularly BB8, play a significant role in the process of selecting the appropriate model.

Copulas and their rotations	Strategy with EG Test	Strategy with KSS Test
Gaussian	2.1%	3.2%
Student-t	2.1%	2.1%
Clayton	3.2%	4.3%
Frank	3.2%	8.5%
Gumbel	7.4%	7.5%
Joe	4.3%	4.3%
BB1	3.2%	4.3%
BB6	2.1%	1.1%
BB7	11.7%	3.2%
BB8	28.7%	24.5%
Tawn type 1	20.2%	19.2%
Tawn type 2	11.7%	18.1%

Table V: Occurrence rate of copulas in the study

The selected coin pairs for each trading week, which may differ in each instance, are presented in Tables VIII and IX in the appendix. It is worth noting that sometimes the chosen coin pairs stay the same from week to week. For both unit-root tests, the significance level is set at 10%. The results of our trading strategy's profit and loss calculations for two strategies with varying entry thresholds (α_1) are illustrated in Figure 2. Realized profit and loss are determined by considering the commission fees and the difference between the opening and closing prices of a position. It should be noted that we initially invested 200,000 USDT, with the coins' weights set to ensure that each side has a maximum initial capital of around 200,000 USDT. It is also assumed that all trades are executed using market orders, which typically incur higher fees (known as taker fees) compared to the lower fees charged for limit orders (known as maker fees). For instance, on Binance, the maker fee is set at 0.02%, while the taker fee is 0.04%.

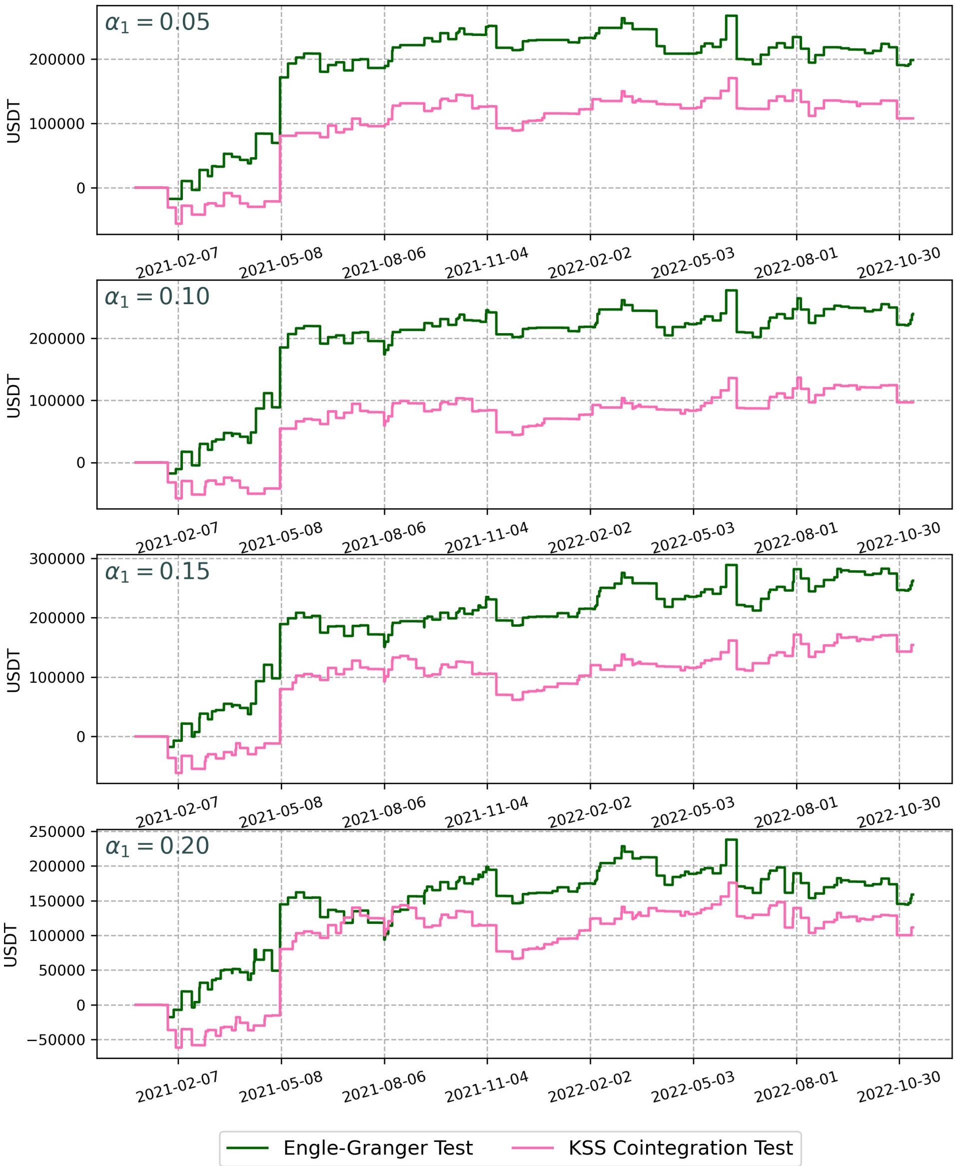

Figure 3: Copula-based pairs trading strategy P&L

We assess the risk-adjusted performance of a strategy using the Sharpe ratio. Table VI illustrates the yearly returns, volatility, and Sharpe ratios for different strategies, along with the maximum drawdown and the returns obtained over the maximum drawdown (RoMaD), which serves as an alternative to the Sharpe Ratio[3]. In pairs trading with the EG cointegration test, the annualized return increases with higher confidence levels,

[3]Transaction fees are taken into account in all calculations

ranging from 36.5% at $\alpha_1 = 0.05$ to 45.0% at $\alpha_1 = 0.20$. The annualized standard deviation also increases with higher confidence levels, ranging from 34.7% at $\alpha_1 = 0.05$ to 42.3% at $\alpha_1 = 0.20$. The annualized Sharpe Ratio is always around 1. In contrast, the maximum drawdown decreases with higher confidence levels, ranging from -24.3% at $\alpha_1 = 0.05$ to -22.2% at $\alpha_1 = 0.20$. The return over maximum drawdown increases with higher confidence levels, ranging from 1.50 at $\alpha_1 = 0.05$ to 2.03 at $\alpha_1 = 0.20$.

The annualized return in pairs trading using the KSS cointegration test initially drops as the confidence levels rise. Specifically, the return decreases from 32.8% when $\alpha_1 = 0.05$ to 21.7% when $\alpha_1 = 0.15$. However, it then rises to 30.7% when $\alpha_1 = 0.20$. As the confidence level increases, the annualized standard deviation also increases, starting from 39.4% at $\alpha_1 = 0.05$ and reaching 55.0% at $\alpha_1 = 0.20$. On the other hand, the annualized Sharpe Ratio exhibits a declining pattern as the threshold levels increase. It ranges from 0.83 at $\alpha_1 = 0.05$ to 0.39 at $\alpha_1 = 0.15$, but then it rises to 56.0% when $\alpha_1 = 0.20$. The maximum drawdown increases with higher threshold levels, ranging from -17.6% at $\alpha_1 = 0.05$ to -37.5% at $\alpha_1 = 0.15$. The return over maximum drawdown shows a similar pattern as the annualized Sharpe Ratio, decreasing with higher confidence levels.

Increasing the entering threshold α_1 augments the number of trading signals, leading to greater volatility in both EG and KSS strategies, but the return behaves differently in each of them. With $\alpha_1 = 0.05$, the strategy with the EG test exhibits risk-adjusted profitability higher than the KSS test, indicating that the Engle-Granger linear cointegration test can identify pairs with more profitable outcomes.

To compare our strategy's results with the buy-and-hold strategy, we use a passive investment approach that involves holding a relatively steady portfolio for an extended period, despite short-term market fluctuations. The Bitcoin buy-and-hold strategy shows a negative annualized return of -26.9% with a high annualized standard deviation of 77.8%. The annualized Sharpe Ratio is negative (-0.35), indicating poor risk-adjusted performance. The maximum drawdown is severe (-77.1%), indicating a high risk. On the other hand, the portfolio buy-and-hold strategy[4] shows a positive annualized return of 21.2% with a high annualized standard deviation of 104.5%. The annualized Sharpe Ratio is positive (0.20), indicating better risk-adjusted performance compared to the Bitcoin buy-and-hold strategy. The maximum drawdown is also severe (-82.6%), indicating a high risk. Both tests of the pairs trading strategy outperform the Bitcoin buy-and-hold strategy and the portfolio buy-and-hold strategy. The pairs trading strategy using the EG test outperforms the portfolio buy-and-hold by over five times with a significance level of 0.05. Similarly, the KSS strategy's

[4]The buy-and-hold strategy in the portfolio involves investing in all twenty cryptocurrency coins with equal weights at the start of the study, retaining them throughout the trading periods, and ultimately selling them at the end of the study period.

performance is more than four times better[5].

Pairs Trading with EG Test	$\alpha_1 = \mathbf{0.05}$	$\alpha_1 = \mathbf{0.10}$	$\alpha_1 = \mathbf{0.15}$	$\alpha_1 = \mathbf{0.20}$
Annualized Return	36.5%	37.0%	42.3%	45.0%
Annualized STD	34.7%	38.9%	41.0%	42.3%
Annualized Sharpe Ratio	1.05	0.95	1.03	1.06
Maximum Drawdown	-24.3%	-25.2%	-22.4%	-22.2%
Return Over Max. Drawdown.	1.50	1.47	1.89	2.03
Number of Transactions	156	186	218	246
Transaction Costs Over Gross P&L	11.3%	13.0%	12.6%	13.0%
Pairs Trading with KSS Test	$\alpha_1 = \mathbf{0.05}$	$\alpha_1 = \mathbf{0.10}$	$\alpha_1 = \mathbf{0.15}$	$\alpha_1 = \mathbf{0.20}$
Annualized Return	32.8%	23.6%	21.7%	30.7%
Annualized STD	39.4%	49.2%	56.2%	55.0%
Annualized Sharpe Ratio	0.83	0.48	0.39	0.56
Maximum Drawdown	-17.6%	-19.1%	-37.5%	-25.6%
Return Over Max. Drawdown.	1.86	1.24	0.58	1.20
Number of Transactions	150	170	198	228
Transaction Costs Over Gross P&L	12.6%	19.9%	24.3%	18.9%
Buy & Hold Strategy		Bitcoin Buy & Hold		Portfolio Buy & Hold
Annualized Return		-26.9%		21.2%
Annualized STD		77.8%		104.5%
Annualized Sharpe Ratio		-0.35		0.20
Maximum Drawdown		-77.1%		-82.6%
Return Over Max. Drawdown.		-0.35		0.26

Table VI: Results of pairs trading strategies Utilizing hourly closed prices of twenty cryptocurrencies from 01/01/2021 to 10/11/2022

7 Conclusion

In this paper, We develop a novel pairs trading framework for twenty Binance USDT-Margined Futures cryptocurrency coins, which combines copula-based and

[5]The Gross Profit and Loss (P&L) mentioned in table VI is the total profit and loss generated by transactions before deducting any transaction fees associated with them.

cointegrated-based approaches. The methodology involves setting a reference asset (in this case, BTCUSDT) and identifying other cryptocurrency coins that are cointegrated with it. To investigate the presence of cointegration, we utilize both the Engle-Granger two-step method and the KSS cointegration test. After ranking the cointegrated coins based on Kendall's Tau correlation coefficients, we select the two assets with the highest correlation. These selected coins are then traded during the one-week trading period and are updated weekly. Trading rules are generated based on the copula conditional probabilities of the spread processes corresponding to the selected assets. We establish various trading triggers and backtest the strategy.

The research discovered that different entry thresholds used in pairs trading produced varying results. Higher thresholds increased volatility, but returns were dependent on the cointegration test used. Our pairs trading strategy, which involves performing two cointegration tests, yielded a high risk-adjusted return and return over maximum drawdown compared to the buy-and-hold approach. This indicates that our method can be profitably utilized. The selection of coin pairs and copula models is critical, with extreme value copulas and certain two-parameter Archimedean copulas playing a significant role. The study highlights the effectiveness of pairs trading in the cryptocurrency market and underscores the importance of carefully selecting coin pairs and copula models.

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Appendix

Symbol	Underlying Crypto	Min. Trade Amount	Max. Leverage
BTCUSDT	Bitcoin	0.001 BTC	125x
ETHUSDT	Ethereum	0.001 ETH	100x
BCHUSDT	Bitcoin Cash	0.001 BCH	75x
XRPUSDT	Ripple	0.1 XRP	75x
EOSUSDT	EOS.IO	0.1 EOS	75x
LTCUSDT	Litecoin	0.001 LTC	75x
TRXUSDT	TRON	1 TRX	50x
ETCUSDT	Ethereum Classic	0.01 ETC	75x
LINKUSDT	Chainlink	0.01 LINK	75x
XLMUSDT	Stellar	1 XLM	50x
ADAUSDT	Cardano	1 ADA	75x
XMRUSDT	Monero	0.001 XMR	50x
DASHUSDT	Dash	0.001 DASH	50x
ZECUSDT	Zcash	0.001 ZEC	50x
XTZUSDT	Tezos	0.1 XTZ	50x
ATOMUSDT	Cosmos	0.01 ATOM	25x
BNBUSDT	Binance Coin	0.01 BNB	75x
ONTUSDT	Ontology	0.1 ONT	50x
IOTAUSDT	IOTA	0.1 IOTA	25x
BATUSDT	Basic Attention Token	0.1 BAT	50x

Table VII: Binance USDT-Margined Futures contracts used in the research

Week	ADF Unit-Root Test Result Pair	P-Value (S_1)	P-Value (S_2)	KSS Unit-Root Test Result Pair	P-Value (S_1)	P-Value (S_2)
1	ETH-LTC	0.061	0.071	ETH-BCH	0.087	0.007
2	LTC-BCH	0.024	0.010	ETH-LTC	0.057	0.003
3	ETH-LTC	0.058	0.037	ETH-LTC	0.039	0.004
4	ETH-LTC	0.095	0.021	ETH-LTC	0.019	0.075
5	LTC-EOS	0.092	0.078	ETH-LTC	0.099	0.011
6	LINK-TRX	0.098	0.069	LTC-BCH	0.017	0.014
7	LINK-TRX	0.099	0.057	LTC-BCH	0.013	0.012
8	ETH-LTC	0.010	0.010	ETH-LTC	0.005	0.061
9	ETH-LTC	0.043	0.022	ETH-LTC	0.067	0.006
10	ETH-BCH	0.097	0.072	ETH-LTC	0.080	0.024
11	LTC-BCH	0.085	0.010	LTC-BCH	0.020	0.008
12	ATOM-ADA	0.010	0.010	LINK-ATOM	0.024	0.004
13	BAT-ONT	0.084	0.017	ADA-XTZ	0.016	0.093
14	ADA-EOS	0.060	0.010	LTC-ADA	0.071	0.002
15	EOS-LTC	0.045	0.024	EOS-LTC	0.017	0.057
16	BAT-IOTA	0.023	0.010	TRX-BNB	0.030	0.042
17	BNB-TRX	0.057	0.029	BCH-BNB	0.079	0.003
18	ETH-BCH	0.095	0.073	TRX-IOTA	0.002	0.020
19	ETH-TRX	0.049	0.057	ETH-DASH	0.097	0.063
20	ETH-LINK	0.093	0.072	LTC-BCH	0.079	0.014
21	TRX-LTC	0.010	0.010	ETH-TRX	0.075	0.019
22	XMR-LTC	0.010	0.166	BNB-LTC	0.018	0.059
23	XMR-LTC	0.189	0.468	BNB-LINK	0.037	0.097
24	ETH-LINK	0.079	0.072	ETH-BNB	0.058	0.001
25	LTC-EOS	0.020	0.023	LTC-BNB	0.030	0.012
26	ETH-LTC	0.081	0.010	LTC-XRP	0.014	0.000
27	BNB-DASH	0.039	0.042	DASH-EOS	0.006	0.002
28	ETH-BCH	0.060	0.097	ETH-BCH	0.038	0.022
29	BNB-LTC	0.061	0.013	BNB-LTC	0.015	0.001
30	BNB-EOS	0.044	0.030	BNB-EOS	0.030	0.001
31	ETH-LINK	0.024	0.013	LTC-LINK	0.023	0.003
32	ETH-ONT	0.058	0.040	ETH-LTC	0.014	0.097
33	LTC-XRP	0.064	0.029	LTC-EOS	0.006	0.079
34	EOS-XRP	0.054	0.045	EOS-LTC	0.018	0.022
35	ETH-EOS	0.051	0.040	EOS-BNB	0.061	0.001
36	LINK-EOS	0.057	0.092	LTC-EOS	0.000	0.043
37	ETH-BNB	0.051	0.066	ETH-BNB	0.003	0.009
38	ETH-LINK	0.014	0.069	ETH-BNB	0.058	0.067
39	DASH-ONT	0.071	0.010	ETH-XRP	0.070	0.013
40	LTC-BNB	0.087	0.087	EOS-DASH	0.098	0.030
41	ETC-DASH	0.093	0.036	BCH-ETC	0.071	0.051
42	BCH-LTC	0.017	0.026	BCH-LTC	0.001	0.027
43	ETH-ETC	0.066	0.010	ETH-ETC	0.091	0.000
44	ETH-ETC	0.045	0.010	ETH-ETC	0.007	0.011
45	EOS-ETC	0.021	0.010	EOS-ETC	0.000	0.002
46	EOS-ETC	0.036	0.010	EOS-ETC	0.004	0.005
47	ETC-XRP	0.067	0.074	ETC-XRP	0.006	0.007

Table VIII: Selected pairs using unit-root tests and Kendall's τ coefficient (part I)

Week	ADF Unit-Root Test Result			KSS Unit-Root Test Result		
	Pair	P-Value (S_1)	P-Value (S_2)	Pair	P-Value (S_1)	P-Value (S_2)
48	ETH-DASH	0.089	0.042	LTC-DASH	0.001	0.002
49	ETH-ETC	0.025	0.010	ETH-ETC	0.016	0.023
50	ETH-ETC	0.072	0.010	ETH-ETC	0.002	0.006
51	ETC-LTC	0.013	0.020	ETH-ETC	0.015	0.001
52	LTC-EOS	0.010	0.052	LTC-EOS	0.001	0.027
53	EOS-XRP	0.032	0.010	EOS-XRP	0.015	0.006
54	ATOM-XRP	0.010	0.474	LTC-BNB	0.063	0.015
55	DASH-XLM	0.095	0.022	ETH-EOS	0.097	0.044
56	ETH-BNB	0.053	0.015	ETH-BNB	0.013	0.070
57	ETH-BNB	0.010	0.017	ETH-BNB	0.006	0.002
58	ETH-BNB	0.032	0.023	ETH-BNB	0.005	0.000
59	ETH-BAT	0.024	0.087	ETH-BAT	0.001	0.065
60	ETH-BNB	0.020	0.020	ETH-EOS	0.016	0.043
61	ETH-BNB	0.085	0.010	BNB-ADA	0.006	0.069
62	BNB-ONT	0.010	0.084	ETH-BNB	0.060	0.006
63	BNB-LTC	0.059	0.022	LTC-LINK	0.002	0.068
64	LINK-ONT	0.028	0.091	ETH-XRP	0.089	0.033
65	XRP-XLM	0.010	0.096	XRP-ADA	0.000	0.059
66	ETH-BNB	0.010	0.049	ETH-BNB	0.003	0.002
67	ETH-BNB	0.042	0.010	ETH-BNB	0.001	0.001
68	ETH-BNB	0.017	0.010	ETH-BNB	0.000	0.001
69	ETH-BNB	0.473	0.186	BNB-ZEC	0.012	0.011
70	BNB-ETC	0.033	0.041	BNB-ETC	0.026	0.003
71	ETC-ADA	0.066	0.098	LINK-ETC	0.065	0.017
72	LINK-EOS	0.046	0.027	BNB-LINK	0.000	0.020
73	DASH-ETC	0.038	0.043	DASH-ETC	0.002	0.028
74	ONT-ZEC	0.025	0.010	EOS-BNB	0.061	0.025
75	ZEC-BCH	0.048	0.050	ETH-EOS	0.011	0.098
76	ETH-XTZ	0.010	0.030	ETH-BNB	0.000	0.090
77	ETH-BNB	0.050	0.060	ETH-BNB	0.006	0.015
78	ETH-BNB	0.021	0.010	ETH-BNB	0.000	0.040
79	BNB-LTC	0.021	0.038	BNB-LTC	0.008	0.000
80	BNB-LTC	0.016	0.056	BNB-LTC	0.004	0.054
81	ETH-LTC	0.010	0.010	ETH-LTC	0.072	0.001
82	LTC-XRP	0.014	0.017	LTC-XRP	0.004	0.000
83	LTC-DASH	0.010	0.047	LTC-DASH	0.000	0.003
84	ETH-LTC	0.039	0.010	ETH-LTC	0.003	0.000
85	ETH-IOTA	0.091	0.010	ETH-IOTA	0.085	0.001
86	BAT-IOTA	0.010	0.010	BAT-DASH	0.000	0.049
87	BNB-ETC	0.023	0.066	BAT-BNB	0.050	0.069
88	BNB-TRX	0.020	0.084	DASH-BAT	0.017	0.012
89	DASH-TRX	0.010	0.040	ETH-DASH	0.008	0.000
90	ETH-DASH	0.010	0.014	ETH-DASH	0.003	0.011
91	ETH-BAT	0.095	0.033	ETH-DASH	0.041	0.036
92	ETH-BAT	0.010	0.089	ETH-BAT	0.007	0.011
93	BNB-LTC	0.014	0.023	LTC-BAT	0.001	0.080
94	BCH-DASH	0.010	0.046	ETH-BCH	0.076	0.001

Table IX: Selected pairs using unit-root tests and Kendall's τ coefficient (part II)

Name	Bivariate Copula Distribution $\mathbf{C(u_1, u_2)} \quad (\overline{\mathbf{u_i}} = \mathbf{1 - u_i})$	Generator $\phi(\mathbf{t})$	Parameters
Clayton	$\left[\max\left(u_1^{-\theta} + u_2^{-\theta} - 1, 0\right)\right]^{-1/\theta}$	$\frac{1}{\theta}\left(t^{-\theta} - 1\right)$	$\theta > 0$
Gumbel	$\exp\left[-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta\right]^{1/\theta}\right]$	$(-\ln(t))^\theta$	$\theta \geq 1$
Frank	$-\theta^{-1}\ln\left[1 + \left(e^{-\theta} - 1\right)^{-1}\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)\right]$	$-\ln\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\theta \in \mathbb{R} \setminus \{0\}$
Joe	$1 - \left[\left(1 - u_1\right)^\theta + \left(1 - u_2\right)^\theta - \left(1 - u_1\right)^\theta\left(1 - u_2\right)^\theta\right]^{1/\theta}$	$-\ln\left(1 - (1-t)^\theta\right)$	$\theta \geq 1$
BB1	$\left[1 + \left[\left(u_1^{-\theta} - 1\right)^\delta + \left(u_2^{-\theta} - 1\right)^\delta\right]^{1/\delta}\right]^{-1/\theta}$	$\left(t^{-\theta} - 1\right)^\delta$	$\theta > 0,\ \delta \geq 1$
BB6	$1 - \left[1 - \exp\left(-\left[\left[-\ln\left(1 - \overline{u_1}^\theta\right)\right]^\delta + \left[-\ln\left(1 - \overline{u_1}^\theta\right)\right]^\delta\right]^{1/\delta}\right)\right]^{1/\theta}$	$\left(\ln\left(1 - (1-t)^\theta\right)\right)^\delta$	$\theta \geq 1,\ \delta \geq 1$
BB7	$1 - \left[1 - \left(\left(1 - \overline{u_1}^\theta\right)^{-\delta} + \left(1 - \overline{u_2}^\theta\right)^{-\delta} - 1\right)^{-1/\delta}\right]^{1/\theta}$	$\left(1 - (1-t)^\theta\right)^{-\delta} - 1$	$\theta \geq 1,\ \delta > 0$
BB8	$\delta^{-1}\left[1 - \left(1 - \left[1 - (1-\delta)^\theta\right]^{-1}\left[1 - (1 - \delta u_1)^\theta\right]\left[1 - (1 - \delta u_2)^\theta\right]\right)^{1/\theta}\right]$	$-\ln\left(\frac{1-(1-\delta t)^\theta}{1-(1-\delta)^\theta}\right)$	$\theta \geq 1,\ 0 < \delta \leq 1$

Table X: Bivariate Archimedean copulas distributions