Exercise 1

I Counting and basic laws of probability

I1.

a) There are $\binom{52}{5} = 2598960$ atomic events

b) $\frac{\text{Possible occurrences}}{\text{All occurrences}} = \frac{1}{\binom{52}{5}} = \underline{\underline{\frac{1}{2598960}}}$

c) Royal Straight Flush: $\frac{4}{\binom{52}{5}} = \underline{\underline{\frac{1}{649740}}}$

- There are 4 possibilities, one for each suit.

Four of a kind: $\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}} = \underline{\underline{\frac{1}{4165}}}$

- First we pick 4 cards from one of 13 kinds, the final card can be any kind except for the one chosen, thus 12 choices from one of 4 suits.

I2.

a) Pair: $\frac{\binom{13}{1}\binom{4}{2}}{\binom{52}{2}} = \underline{\underline{\frac{1}{17}}}$

- We pick one of 13 kinds and two of 4 suits.

b) $A = $ Is pair $= \frac{1}{17}$, $B = $ Different suit

$B = 1 - \frac{\binom{13}{2}\binom{4}{1}}{\binom{52}{2}} = \frac{13}{17}$

$P(B|A) = 1$ (If they are a pair then they have different suits)

$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{1}{17}}{\frac{13}{17}} = \underline{\underline{\frac{1}{13}}}$

Correction: P(A) = 1/17 and P(B) = 13/17

I3.

1) Yes, as shown by:

$$P(A|B) > P(A)$$

$$\frac{P(A \cup B)}{P(B)} > P(A) \quad , \quad P(A|B) = \frac{P(A \cup B)}{P(B)}$$

$$\frac{P(A \cup B)}{P(A)} > P(B) \quad , \quad \text{Multiplied by } \frac{P(B)}{P(A)}$$

$$\underline{\underline{P(B|A) > P(B)}} \quad , \quad \frac{P(A \cup B)}{P(A)} = P(B|A)$$

Thus we have shown that if $P(A|B) > P(A)$ then $P(B|A) > P(B)$.

2) $P(S=0) = \frac{6}{10}$, $P(S=1) = \frac{4}{10}$, $P(R=1 \mid S=0) = P(R=0 \mid S=1) = \frac{1}{3}$

We want to know:

i) $$P(S=0 \mid R=0) = \frac{P(R=0 \mid S=0)P(S=0)}{P(R=0)}$$

Finding $P(R=0 \mid S=0)$:

$$P(R=0 \mid S=0) = 1 - P(R=1 \mid S=0) = 1 - \frac{1}{3} = \frac{2}{3}$$

Finding $P(R=0)$:

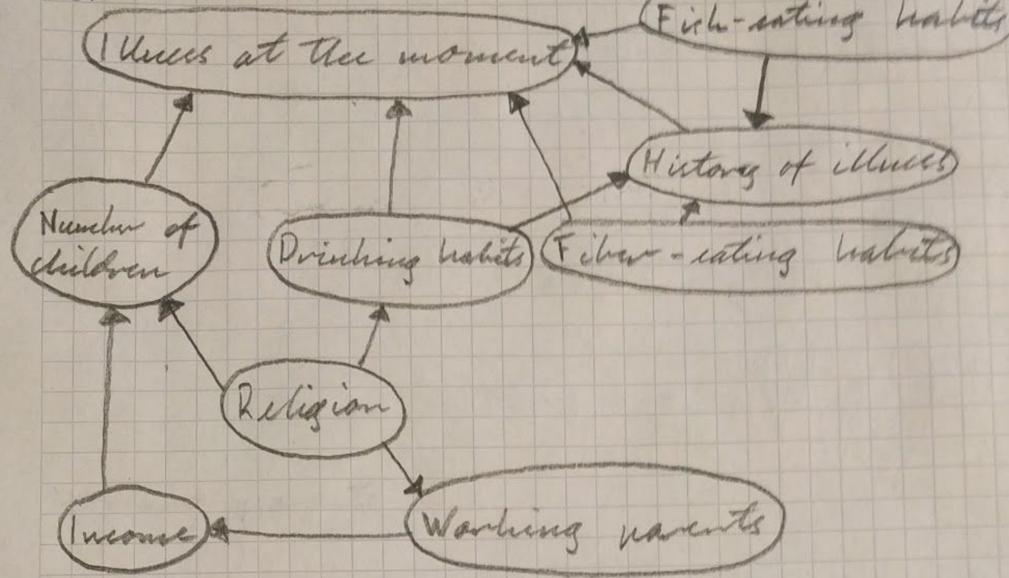
$$P(R=0) = P(R=0 \mid S=0)P(S=0) + P(R=0 \mid S=1)P(S=1) = \frac{2}{3} \cdot \frac{6}{10} + \frac{1}{3} \cdot \frac{4}{10} = \frac{8}{15}$$

Solving $P(S=0 \mid R=0)$:

i) $$\frac{P(R=0 \mid S=0)P(S=0)}{P(R=0)} = \frac{\frac{2}{3} \cdot \frac{6}{10}}{\frac{8}{15}} = \underline{\underline{\frac{3}{4}}}$$

II Bayesian Network Construction

II.



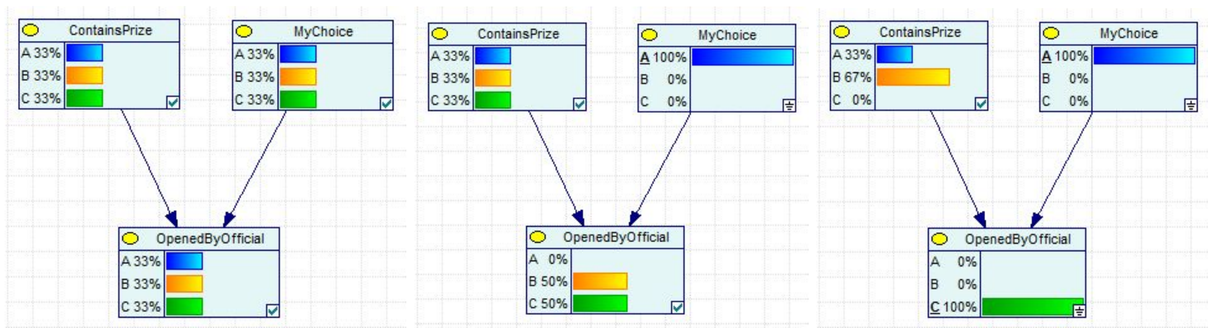
conditional independence:

- Religion
- Fish-eating habits
- Fiber-eating habit

Given any other variable will have no effect on their condition. This seems reasonable as they are usually affected by social and geo-graphical context, which we do not have in the network.

It is hard to say how sound the model is however this is the best model I could make with my perception of the world.

III Bayesian Network Application



Before I choose **After my choice (A)** **After door is eliminated (C)**

OpenedByOfficial table:

ContainsPrize	A			B			C		
MyChoice	A	B	C	A	B	C	A	B	C
A	0	0	0	0	0.5	1	0	1	0.5
B	0.5	0	1	0	0	0	1	0	0.5
C	0.5	1	0	1	0.5	0	0	0	0

The table for MyChoice and ContainsPrize is simply ⅓ chance on all options as seen on first figure (Before I choose).

Conclusion:

As seen in the demonstration above we first pick door A and then door C is opened. The resulting probability says there is a ⅔ chance of B (the door I did not initially select) containing the prize. Testing with other choices yields the same result as in the remaining door that was not initially picked had the highest probability of containing the prize. Thus we conclude that we should switch door once a door has been opened.