TTK4190 Guidance and Control of Vehicles

Assignment 3 - Part 1: Marine Craft Modeling

October 14, 2020 Group 02

1 Problem 1: Rigid-Body Kinetics of a Rectangular Prism

1.a Moments of Intertia about CG

$$\rho_m = \frac{m}{LBH} = \frac{17.0677 \times 10^6}{161 \times 21.8 \times 15.8} = 307.78 \tag{1}$$

Now we can calculate the moments of inertia:

$$\begin{aligned}
\boldsymbol{I}_z^{CG} &= \int_V (x^2 + y^2)\rho_m \, \mathrm{d}V &(2)\\
&= \rho_m H \int_{-B/2}^{B/2} \int_{-L/2}^{L/2} (x^2 + y^2) \, \mathrm{d}x \, \mathrm{d}y &(3)\\
&= 3.7544 \times 10^{10} &(4)
\end{aligned}$$

With similar formulas it's easy to see why all the cross term inertias are 0:

$$\begin{aligned}
\boldsymbol{I}_{yx}^{CG} = \boldsymbol{I}_{xy}^{CG} &= \int_V xy\rho_m \, \mathrm{d}V &(5)\\
&= \rho_m H \int_{-B/2}^{B/2} \int_{-L/2}^{L/2} xy \, \mathrm{d}x \, \mathrm{d}y &(6)\\
&= 0 &(7)
\end{aligned}$$

Intuitively this also makes sense as the "ship" is symmetric about CG.

1.b Moment of Inertia about CO

With the distance vector given by

$$\boldsymbol{r}_{bg}^b = [-3.7 \quad 0 \quad H/2]^\mathsf{T}$$

the moment of inertia about the z in CO is given by:

$$\begin{aligned}
\boldsymbol{I}_z^{CO} &= \boldsymbol{I}_z^{CG} + m(x^2 + y^2) &(8)\\
&= 3.7544 \times 10^{10} + 17.0677 \times 10^6 \times (-3.7)^2 &(9)\\
&= 3.7778 \times 10^{10} &(10)
\end{aligned}$$

1.c Mass and Coriolis matrices

The analytical expressions for the rigid body mass, M_{RB}, and the rigid body Coriolis, C_{RB} for the three DOGs surge, sway and yaw, using $\boldsymbol{r_{bg}^b}$ from task b, were found to be

$$\boldsymbol{M_{RB}} = \begin{bmatrix} m & 0 & -my_g \\ 0 & m & mx_g \\ -my_g & mx_g & I_z \end{bmatrix} \tag{11}$$

$$
\boldsymbol{C_{RB}} = \begin{bmatrix} 0 & 0 & -m(x_g r + v) \\ 0 & 0 & -m(y_g r - u) \\ m(x_g r + v) & m(y_g r - u) & 0 \end{bmatrix}, \tag{12}
$$

where u and v describes the linear velocity of the CO relative to o_n expressed in the body frame.

1.d Skew-Symmetric Coriolis

We see that $\boldsymbol{C_{RB}} = \boldsymbol{C_{RB}^T}$, which implies that the rigid body Coriolis matrix is skew symmetric. This is desired because the quadratic form $\boldsymbol{\nu^T C_{RB}(\nu)\nu} = 0$, which is exploited in energy based designs where the Lyaounov function is important.

1.e Other property of Coriolis matrix

The expression for $\boldsymbol{C_{RB}(\nu)}$ can be made independent of linear velocity $\boldsymbol{v_1}$. This is done by using the cross-product property $\boldsymbol{S(v_1)v_2} = \boldsymbol{-S(v_2)v_1}$, and move $\boldsymbol{S(v_1)v_2}$ from $\boldsymbol{C_{RB}^{12}}$ to $\boldsymbol{C_{RB}^{11}}$. This property becomes useful when irrational ocean currents enter the EoM, as $\boldsymbol{C_{RB}(\nu)}$ is independent on linear velocity $\boldsymbol{v_1}$ and only uses the angular velcoity $\boldsymbol{v_2}$ and lever arm $\boldsymbol{r_{bg}^b}$.

2 Problem 2: Hydrostatics

2.a Volume displacement

With the constant draft, we assume the ship is neutrally buoyant so the volume displacement can be calculated with Archimedes' principle:

$$
\begin{aligned}
mg &= \rho g \nabla \qquad (13) \\
\implies \nabla &= \rho / m \qquad (14)
\end{aligned}
$$

Seeing as our ship model is a prism an easier way of calculating the volume displacement would simply be:

$$
\nabla = LBT = 1.66 \times 10^4 \tag{15}
$$

2.b Waterplane and hydrostatic forces

From (4.14) in Fossen we have:

$$
\begin{aligned}
Z_{HS} &\approx -\rho g A_{wp} z^n \qquad (16) \\
&= -\rho g LBT = -3.14 \times 10^8 \qquad (17)
\end{aligned}
$$

2.c Heave period

$$
T_3 \approx 2\pi \sqrt{\frac{2T}{g}} = 8.46 \tag{18}
$$

2.d Metacentric heights

The metacentric heights are given by:

$$GM_T = \frac{I_T}{\nabla} + z_g - z_b, \quad GM_L = \frac{I_L}{\nabla} + z_g - z_b \tag{19}$$

Where the intertias are given by:

$$I_T = \frac{1}{A_{wp}} \int\int_{A_{wp}} y^2 \, \mathrm{d}A = L \int_{-B/2}^{B/2} y^2 \, \mathrm{d}y = \frac{1}{12} BL^3 \tag{20}$$

$$I_L = \frac{1}{12} B^3 L \tag{21}$$

With the center of bouyancy in $T/2$ we have that $z_g - z_b = T/2 - H/2$. Inserting this in equation 19 together with equation 15 we get:

$$GM_T = \frac{L^2}{12T} + \frac{T}{2} - \frac{H}{2} = 0.9998 \tag{22}$$

$$GM_L = \frac{B^2}{12T} + \frac{T}{2} - \frac{H}{2} = 239.26 \tag{23}$$

2.e Metacentric stability

By looking at the numerical values for GM_T and GM_L in d) we see that the vessel is more metacentrically stable in the longitudinal plane than the transverse plane. The value for GM_L implies stiffness in this plane, which makes logical sense considering the shape of the vessel. GM_T on the other hand is much lower, implying less metacentric stability, however as it is not negative or excessively low there is not a risk of the vessel capsizing in rough weather.

3 Problem 3: Added Mass and Coriolis

3.a Reduced added mass

With the assumption of no coupling between subsystems the added mass matrix for surge, sway and yaw will simply be:

$$\boldsymbol{M}_A = \boldsymbol{M}_A^{\mathsf{T}} = -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{r}} \end{bmatrix} = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{r}} \\ 0 & N_{\dot{v}} & N_{\dot{r}} \end{bmatrix} \tag{24}$$

3.b Coriolis forces

From Property 6.2 in Fossen we have the full Coriolis matrix given by:

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} \boldsymbol{0}_{3\times3} & -\boldsymbol{S}(\boldsymbol{A}_{11}\boldsymbol{\nu}_1 + \boldsymbol{A}_{12}\boldsymbol{\nu}_2) \\ -\boldsymbol{S}(\boldsymbol{A}_{11}\boldsymbol{\nu}_1 + \boldsymbol{A}_{12}\boldsymbol{\nu}_2) & -\boldsymbol{S}(\boldsymbol{A}_{21}\boldsymbol{\nu}_1 + \boldsymbol{A}_{22}\boldsymbol{\nu}_2) \end{bmatrix} \tag{25}$$

Picking out the components corresponding to surge, sway and yaw we get:

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & a_2 \\ 0 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{26}$$

Where a_1 and a_2 are given by:

$$a_1 = X_{\dot{u}}u + X_{\dot{v}}v + X_{\dot{r}}r = X_{\dot{u}}u \tag{27}$$
$$a_2 = Y_{\dot{u}}u + Y_{\dot{v}}v + Y_{\dot{r}}r = Y_{\dot{v}}v + Y_{\dot{r}}r \tag{28}$$

4 Problem 4: Implementing the Maneuvering Model

4.a Mass and Coriolis matrices

4.b Diagonal linear damping

4.c Simulation Plots