Milestone 4 AST5220

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The final milestone builds on the previous background and evolution calculations in order to calculate the power spectrum of the cosmic microwave background that is observed today. As the end of this report will show, comparisons between the observed power spectrum and the theoretical power spectrum serve as a useful tool in constraining cosmic parameters, which are, of course, centrally important to the advancement of theoretical and observational cosmology.

The first necessary calculation for the power spectrum is the source function, which is an equation that solves for the photon moments at the present time by solving for terms of the monopole and dipole at the time of their release–near recombination when the universe became transparent. The equation is given by

$$\begin{aligned}\tilde{S}(k,x) &= \tilde{g}\left[\Theta_0 + \Psi + \frac{1}{4}\Theta_2\right] + e^{-\tau}\left[\Psi' - \Phi'\right] \\ &\quad - \frac{1}{ck}\frac{d}{dx}\left(\mathcal{H}\tilde{g}v_b\right) + \frac{3}{4c^2k^2}\frac{d}{dx}\left[\mathcal{H}\frac{d}{dx}\left(\mathcal{H}\tilde{g}\Pi\right)\right]. \end{aligned} \tag{1}$$

As seen in the equation, the evolution module parameters are needed to calculated the source function. The source function is then calculated based on the existing x and k grids with dimensions $[500, 100]$. The source function is then splined and recalculated for a high resolution $[5000, 5000]$ grid. The first term of the source function is the contribution of the Sachs-Wolfe effect, where the photons are gravitationally redshifted near recombination. The second term is the contribution of the integrated Sachs-Wolfe effect and has the largest effect on the large scales (small l values). The effect is caused by the gravitational potential changing during the recent dark energy dominated cosmic epoch within regions over the long time it takes the photon to transit through the region. For example, a photon transiting through a void whose gravitational potential is altered by universal expansion will be redshifted upon exiting the void. The third term is the contribution of the relativistic Doppler shift of the photons and raises the troughs of the power spectrum and generally has more effect at smaller scales (larger l values). The fourth term influences the larger scales more than the smaller but contributes less to the shape of the power spectrum than the previous three components.

The integrand of the transfer function $\Theta_l(k, x = 0)$ consists of the source function multiplied by the Bessel function $j_l[k[\eta_0 - \eta(x)]]$, which translate the 3D CMB temperature field projected onto the spherical surface as determined by l.

$$\Theta_l(k, x = 0) = \int_{-\infty}^{0} \tilde{S}(k,x) j_l[k(\eta_0 - \eta(x))]dx \,, \tag{2}$$

The transfer function is then calculated and is plotted here for six different values of l.

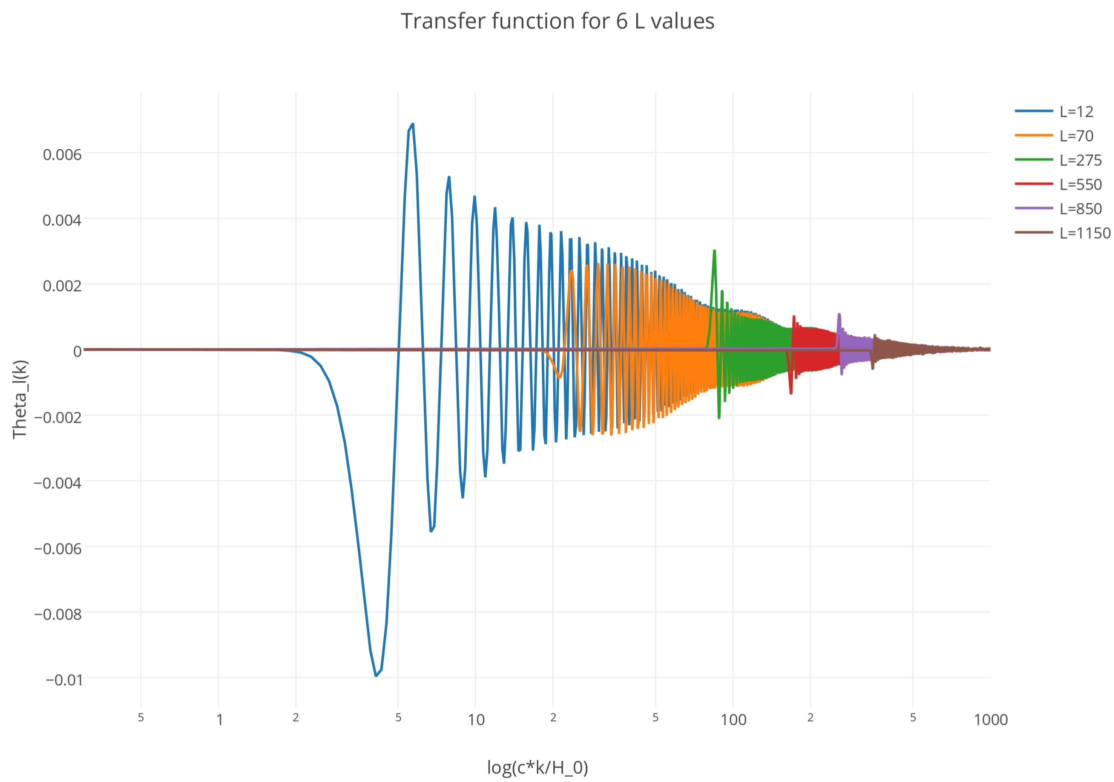

Figure 1: This plot shows 6 values of l from across the $[1, 1200]$ spectrum to illustrate the differences for different angular scales. The transfer function is a function of wave number and time (k, x). We see the shape of the Bessel function in each of the plots and the different l's peak for different regions of k. The l's and the k's are proportional: low l's correspond to peak in low k's and high l's peak at high k's.

The next integration will calculate the power spectrum itself for different values of l. The spectral integrand is given in the equation:

$$C_l = \int_0^\infty \left(\frac{k}{H_0}\right)^{n-1} \Theta_l^2(k) \frac{dk}{k} \,. \tag{3}$$

n is set to 0.96 which corresponds to the best fit to the WMAP data. The spectral integrand is shown in the plot below.

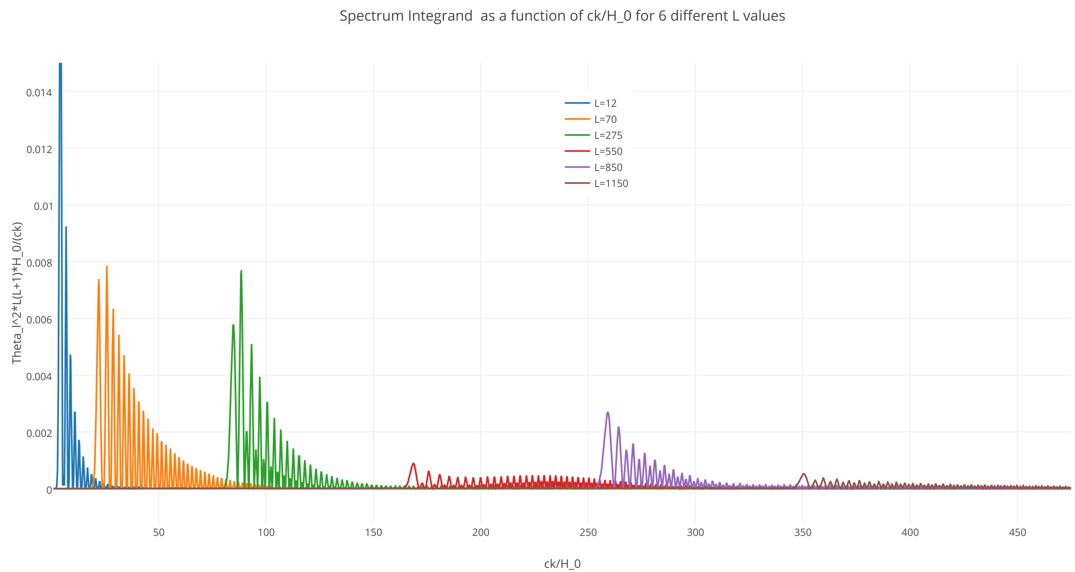

Figure 2: The l's and k's are proportional for the spectral integrand as well.

Finally, the power spectrum values C_l are splined on the form $C_l \cdot \frac{l(l+1)}{2\pi}$ and stored. The final power spectrum plot is then normalized to $5775\mu K^2$ and interpolated for $l = [1, 1200]$.

All the components that contributed to this plot are dependent upon the cosmic parameters in the params.mod module in the code. I've changed each of the relative energy densities to gauge their influence on the power spectrum and to fit the model with the actual Planck spectrum shown in the plot. The default parameters have a good fit to the Planck data for the first peak but the resulting peaks and valleys are significantly higher and shifted to the left from the Planck data. Also, the second peak is higher than the third in the model, which is not the case in the Planck data. When the baryonic density increases, the odd peaks grow relative to the even peaks as the low frequency of massive baryonic oscillations increase. I found that doubling the baryonic energy density had this effect and was a better fit to the actual data. When the cold dark matter energy density is also increased, we see small scale anisotropies decrease, and there is also less ISW contribution, which should lower the power spectrum in the smaller scales. This is due to the Θ_0 being lower as matter radiation equality occurred further away from recombination so the gravitational potentials are more equal during the timescale of the integrations. The dark energy density affects more recent time periods and so most affects the free streaming photons. Setting the dark energy to zero shifted the power spectrum to the right and lined up the model with the power spectrum rather well. I also found that increasing the radiation energy density by an order of magnitude caused the peaks of the power spectrum to spread out very significantly and raise the power spectrum values drastically for smaller scales. Lowering the radiation density by an order of magnitude smoothed out most of oscillations besides the first peak. After some trial and error I found the closest fit to the actual data included the following energy density values:

parameter	default	best fit
Ω_b	0.046	0.1
Ω_m	0.224	0.4
Ω_r	8.3e-5	8.3e-5
Ω_Λ	0.73	0

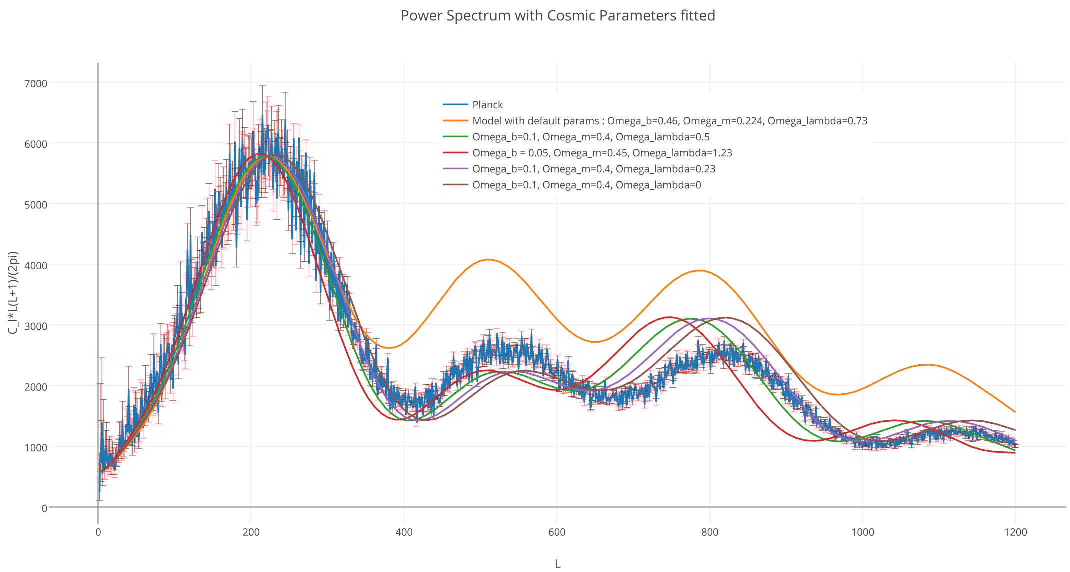

Figure 3: The actual Planck power spectrum is in blue with red error bars. The power spectra are plotted for $L = [2, 1200]$. Increasing the baryonic and CDM energy densities have brought the modeled spectrum to with the error range for most of the graph. However the the third peak never really matches up. The best fit power spectrum overlaps with the solid brown line in the plot above. The peak value occurs at $l = 225$. The code does not include polarization or neutrinos so a perfect match here would not dial in the parameters conclusively in any case.

The datafile powspec.dat is included as an attachment to the digital version of this report. The values for the power spectrum data here reflect the best fit parameters as detailed above in this report. The code is attached below: the subroutine for computing the source function in the evolution module, and the cl module.