Milestone 1 AST5220

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This milestone parametrizes the expansion and evolution of a flat universe. For this project, we begin with $a(t)$, which represents the scale factor of the universe's expansion as a function of cosmic time t. a scales with time as $a \propto t^{1/2}$ during the radiation domination period and as $a \propto t^{2/3}$ during the matter dominated epoch.

a relates closely with Hubble rate, describing how quickly $a(t)$ changes: $H(t) \equiv \frac{da/dt}{a}$. H and a along with the relative energy densities of baryonic matter Ω_b, dark matter Ω_m, radiation Ω_r, and dark energy (cosmological constant) $Omega_\Lambda$, give us the Friedmann equation:

$$H = H_0 \sqrt{\Omega_b + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda} \tag{1}$$

H_0 represents the value of the Hubble parameter at the present day and whose value is $\sim 70 kms^{-1} Mpc^{-1}$. This project utilizes the logarithm of the scale factor $x \equiv \ln a$ as the time integration units for solving for the cosmological parameters. The normalized relative energy densities sum to 1 and have shifted over the cosmological era. The relative energy densities are fixed today at

$$(\Omega_b, \Omega_m, \Omega_r, \Omega_\Lambda) = (0.046, 0.224, 0.000083, 0.730)$$

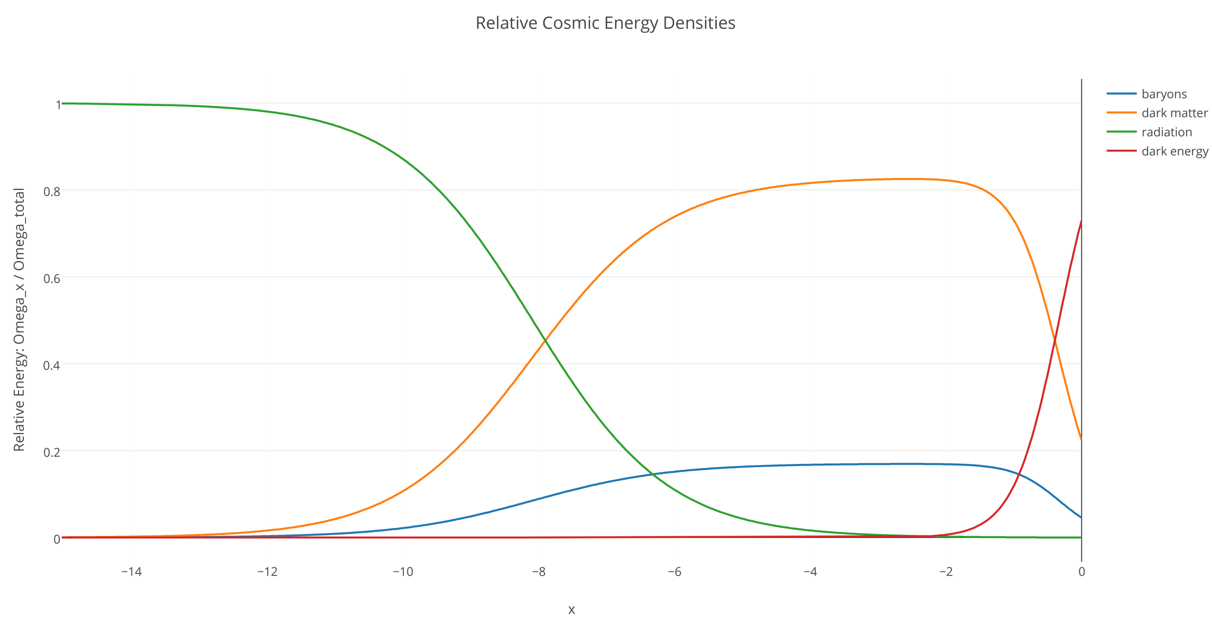

Figure 1: The above plot shows the 4 cosmological energy densities shifting over time. The logarithmic scale factor x varies between $[-23, 0]$, where $x = -23$ corresponds to an initial time of $a = 10^{-10}$ (approximately t $\sim$560 seconds after the big bang and ends with the values today corresponding to $x = 0$ or $a = 1$. Notice the early universe was radiation dominated and fell off as its dependence on the scale factor is -4 while matter density has a scale factor dependence of -3. This lead to a matter-radiation equality around $x = -8$. The dark energy density does not depend on scale factor and has come to dominate the relative cosmological composition in the present epoch.

Solving the Friedmann equation yields different values for the Hubble parameter at different stages of cosmological evolution. A related time variable is the redshift, defined by $(1+z) = a_0/a(t)$. Following are two representations of H: the first shows H as a function of the redshift z and the second shows H as a function of x.

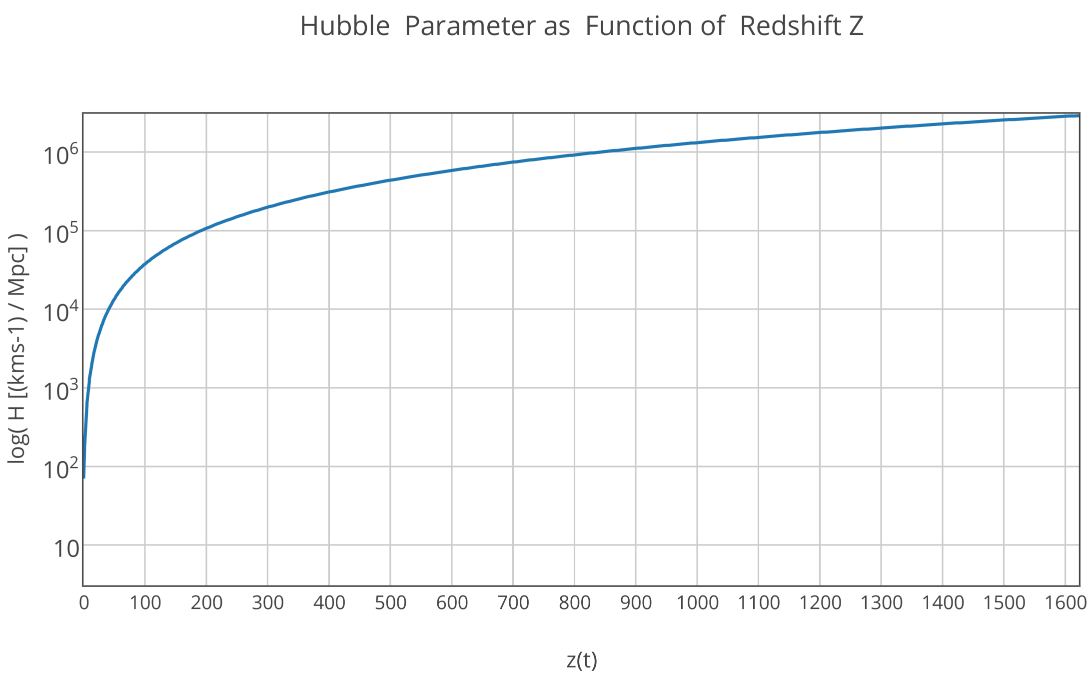

Figure 2: This plot shows the Hubble parameter as a function of the redshift z. The Hubble parameter is shown between redshift $z = [1630, 0]$. H_0 as noted is 70 $kms^{-1}Mpc^{-1}$ and the graph converges to this as $z \to 0$.

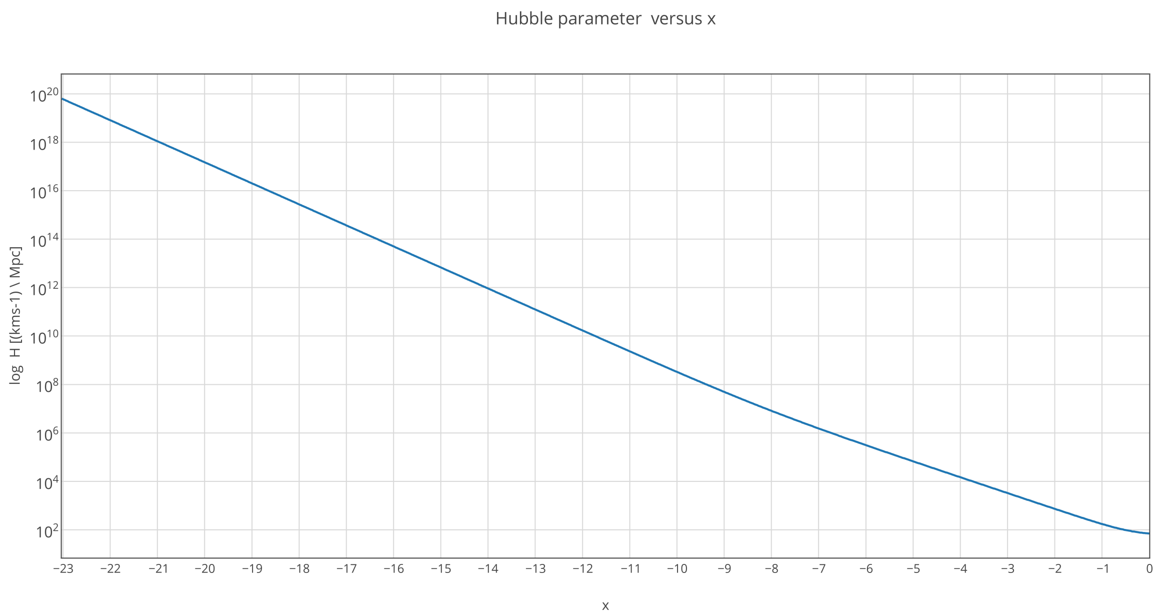

Figure 3: The Hubble parameter is shown further back in time in this plot as a function of x as the range corresponds to $a = [a^{-10}, 0]$.

The attached code utilizes the Hubble parameter calculations based on the logarithmic scale factor to compute the conformal time η which is defined as the distance that light has traveled since $t = 0$. In order to solve for η in terms of x, we define a first order ordinary differential equation:

$$\frac{d\eta}{dx} = \frac{d\eta}{dt}\frac{dt}{da}\frac{da}{dt} = \frac{da/dt}{a} = \frac{c}{aH} = \frac{c}{\mathbf{H}} \tag{2}$$

Where $\mathbf{H} = \mathbf{a}H$.

The code solves this first order equation numerically using the Bulirsch-Stoer algorithm. This method is quite successful for integrating smooth functions. As mentioned previously, the units of integration are the values of x ranging from $[-23, 0]$. In the code, x is an array of 1000 grid points and the BS-step method finds the values of η at each step. $\frac{d^2\eta}{dx^2}$ is determined by calculating a spline for the values of η. The code then computes a spline interpolation between the values so that the value of η can be approximated for any intermediate values of η between the endpoints of the array. The paper "How to Calculate the CMB Background" by Petter Callin outlines the initial condition for η as this point lies in the early universe where a is very small $(a = 10^{-10})$. As Callin states, as $a \to 0$, $a\mathbf{H} \to \frac{\mathbf{c}}{\mathbf{H_0}\sqrt{\mathbf{\Omega_r}}}$. The program iterates over the integration algorithm from this initial point solving for η. The results are shown in figure 4. The code for $time_mod.90$ is attached at the end of the document.

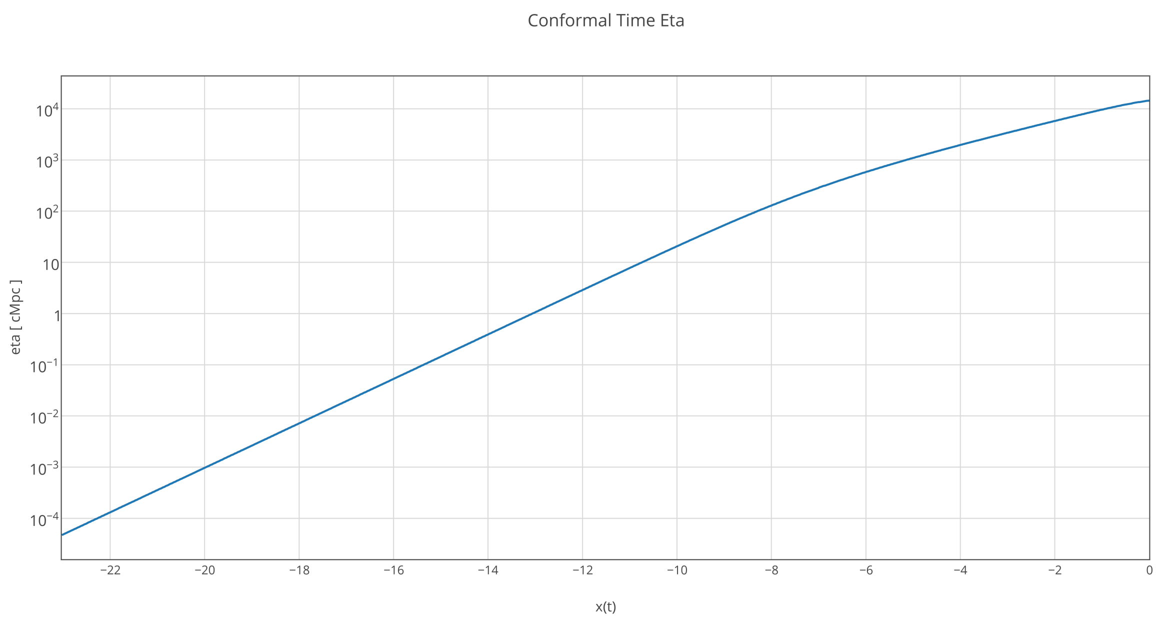

Figure 4: Here the integrated values of the conformal time η are plotted against the logarithmic scale factor x.