

```

> restart;
> with(plots):

> first := Y1 = (U*b1 - a1*Y3) * (c1 / z-1);
first := Y1 = (U b1 - a1 Y3)  $\left(\frac{c1}{z} - 1\right)$  (1)

> second := Y2 = (Y1 + U*b2 - a2*Y3) * (c2*z / (z-1)); # (c2 / (z-1))
second := Y2 =  $\frac{(Y1 + U b2 - a2 Y3) c2 z}{z - 1}$  (2)

> third := Y3 = (Y2 + U*b3 - a3*Y3) * (c3 / (z-1));
third := Y3 =  $\frac{(Y2 + U b3 - a3 Y3) c3}{z - 1}$  (3)

> first := solve(first, Y1);
first :=  $\frac{(U b1 - a1 Y3) (c1 - z)}{z}$  (4)

> second := subs(Y1=first, second);
second := Y2 =  $\frac{\left(\frac{(U b1 - a1 Y3) (c1 - z)}{z} + U b2 - a2 Y3\right) c2 z}{z - 1}$  (5)

> second := solve(second, Y2);
second :=  $\frac{(U b1 c1 - U b1 z - a1 Y3 c1 + a1 Y3 z + U b2 z - a2 Y3 z) c2}{z - 1}$  (6)

> ue_f := subs(Y2=second, third);
ue_f := Y3
=  $\frac{\left(\frac{(U b1 c1 - U b1 z - a1 Y3 c1 + a1 Y3 z + U b2 z - a2 Y3 z) c2}{z - 1} + U b3 - a3 Y3\right) c3}{z - 1}$  (7)

> ue_f := solve(ue_f, Y3);
ue_f :=  $\frac{U (c2 b1 c1 - c2 b1 z + c2 b2 z + b3 z - b3) c3}{c3 c2 a1 c1 - c3 c2 a1 z + c3 c2 a2 z + c3 a3 z - c3 a3 + z^2 - 2 z + 1}$  (8)

> ue_f := ue_f / U;
ue_f :=  $\frac{(c2 b1 c1 - c2 b1 z + c2 b2 z + b3 z - b3) c3}{c3 c2 a1 c1 - c3 c2 a1 z + c3 c2 a2 z + c3 a3 z - c3 a3 + z^2 - 2 z + 1}$  (9)

> nenner := denom(ue_f);
nenner :=  $c3 c2 a1 c1 - c3 c2 a1 z + c3 c2 a2 z + c3 a3 z - c3 a3 + z^2 - 2 z + 1$  (10)

> pole := solve(nenner, z);
pole :=  $\frac{1}{2} c3 c2 a1 - \frac{1}{2} c3 c2 a2 + 1 - \frac{1}{2} c3 a3$  (11)
+  $\frac{1}{2} (c3^2 c2^2 a1^2 - 2 c3^2 c2^2 a1 a2 + 4 c3 c2 a1 - 2 c3^2 c2 a1 a3 + c3^2 c2^2 a2^2$ 
-  $4 c3 c2 a2 + 2 c3^2 c2 a2 a3 + c3^2 a3^2 - 4 c3 c2 a1 c1)^{1/2}, \frac{1}{2} c3 c2 a1 - \frac{1}{2} c3 c2 a2$ 
+  $1 - \frac{1}{2} c3 a3$ 

```

$$-\frac{1}{2} (c_3^2 c_2^2 a_1^2 - 2 c_3^2 c_2^2 a_1 a_2 + 4 c_3 c_2 a_1 - 2 c_3^2 c_2 a_1 a_3 + c_3^2 c_2^2 a_2^2 - 4 c_3 c_2 a_2 + 2 c_3^2 c_2 a_2 a_3 + c_3^2 a_3^2 - 4 c_3 c_2 a_1 c_1)^{1/2}$$

```
> numerator := numer(ue_f);
      numerator := (c2 b1 c1 - c2 b1 z + c2 b2 z + b3 z - b3) c3 (12)
```

```
> nullst := solve(numerator, z);
      nullst := \frac{c2 b1 c1 - b3}{c2 b1 - c2 b2 - b3} (13)
```

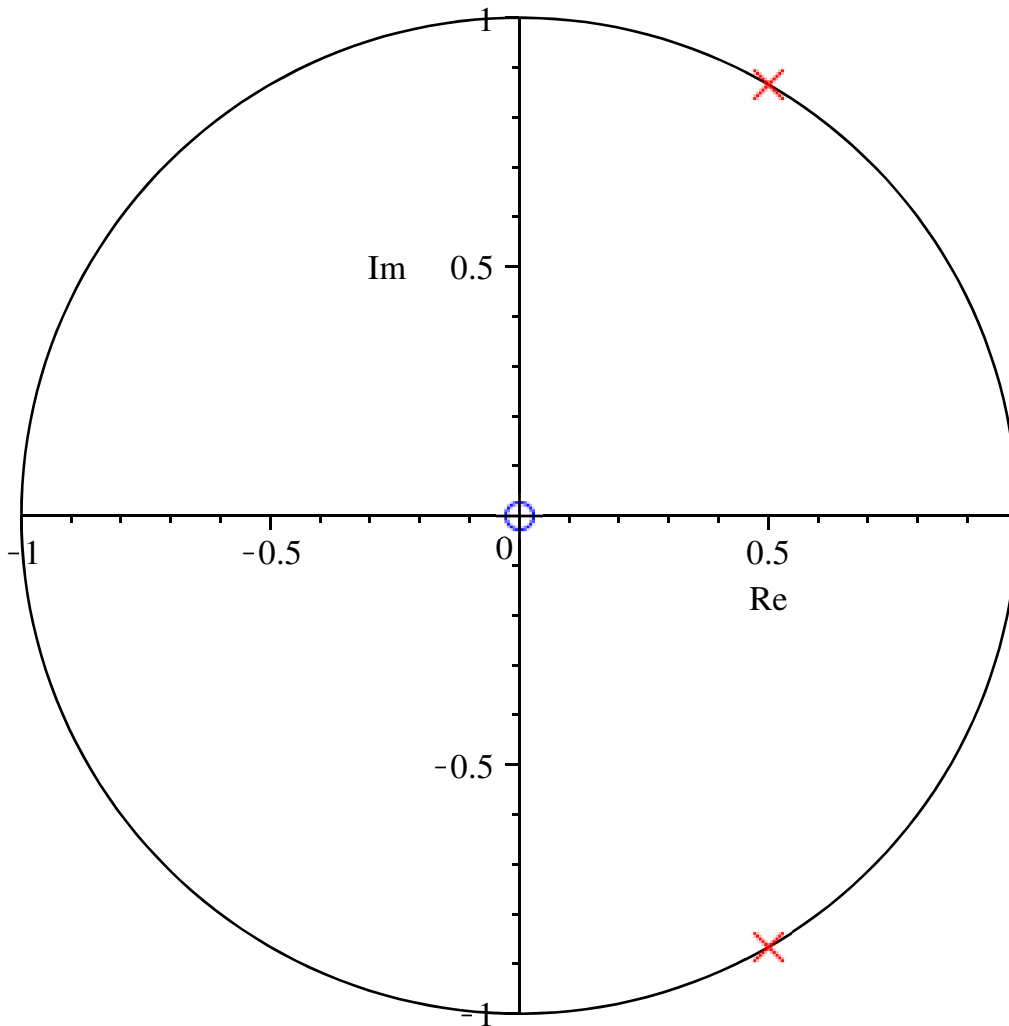
```
> params := [a1=1, a2=1, a3=1, b1=1, b2=1, b3=1, c1=1, c2=1, c3=1];
      params := [a1 = 1, a2 = 1, a3 = 1, b1 = 1, b2 = 1, b3 = 1, c1 = 1, c2 = 1, c3 = 1] (14)
```

```
> nullst1:=eval(nullst, params);
      nullst1 := 0 (15)
```

```
> pole1:=eval(pole, params);
      pole1 := \frac{1}{2} + \frac{1}{2} \sqrt{-3}, \frac{1}{2} - \frac{1}{2} \sqrt{-3} (16)
```

```
> ue_f_num := subs(params, ue_f);
      ue_f_num := \frac{z}{1 - z + z^2} (17)
```

```
> #ue_f_num := unapply(ue_f_num, z);
> complexplot([pole1], style=point,
      color=red, labels = ["Re", "Im"],
      symbol="diagonalcross", symbolsize=20, thickness=10,
      scaling=constrained): #, view=[-2..1, -2..2]):
> complexplot([nullst1], style=point,
      color="blue", labels = ["Re", "Im"],
      symbol="circle", symbolsize=20, thickness=10,
      scaling=constrained): #, view=[-2..1, -2..2]):
> complexplot(cos+I*sin, -Pi .. Pi, labels = ["Re", "Im"], color=
      black, scaling=constrained):
> display(% ,%%, %%%);
```



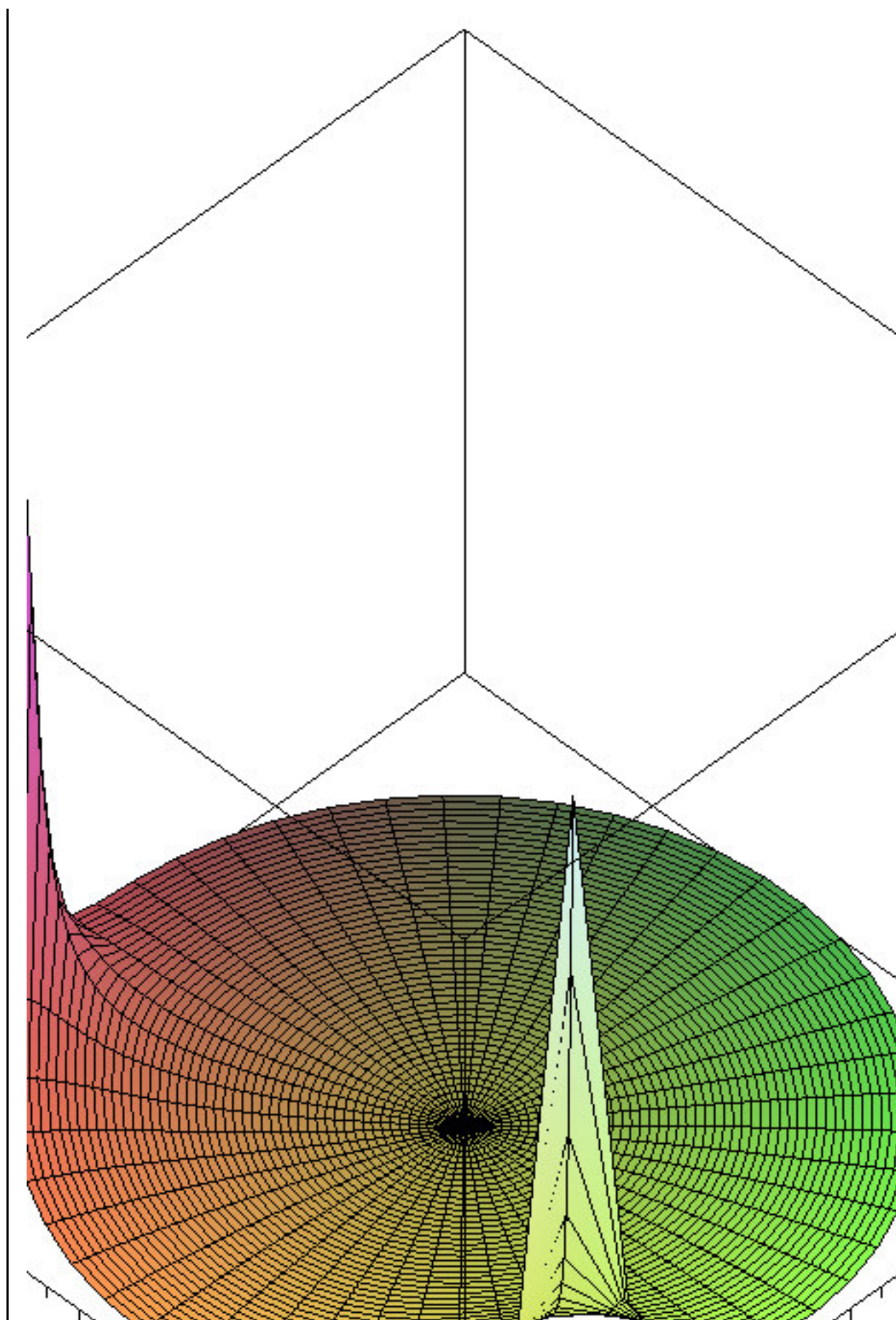
```
> #ue_f_abs := subs(params, ue_f_abs);
> #ue_f := subs(params, ue_f_abs);
> ue_f_num := subs(z=r*exp(I*phi), ue_f_num);
```

$$ue_f_num := \frac{r e^{I\phi}}{1 - r e^{I\phi} + r^2 (e^{I\phi})^2} \quad (18)$$

```
> ue_f_abs := abs(ue_f_num);
```

$$ue_f_abs := e^{-\Im(\phi)} \left| \frac{r}{1 - r e^{I\phi} + r^2 (e^{I\phi})^2} \right| \quad (19)$$

```
> #ue_f_abs := unapply(ue_f_num, r, phi);
> changecoords(plot3d(ue_f_abs, r=0..1, phi=0..2*Pi, grid=[50,50],
  axes=boxed), polar);
```

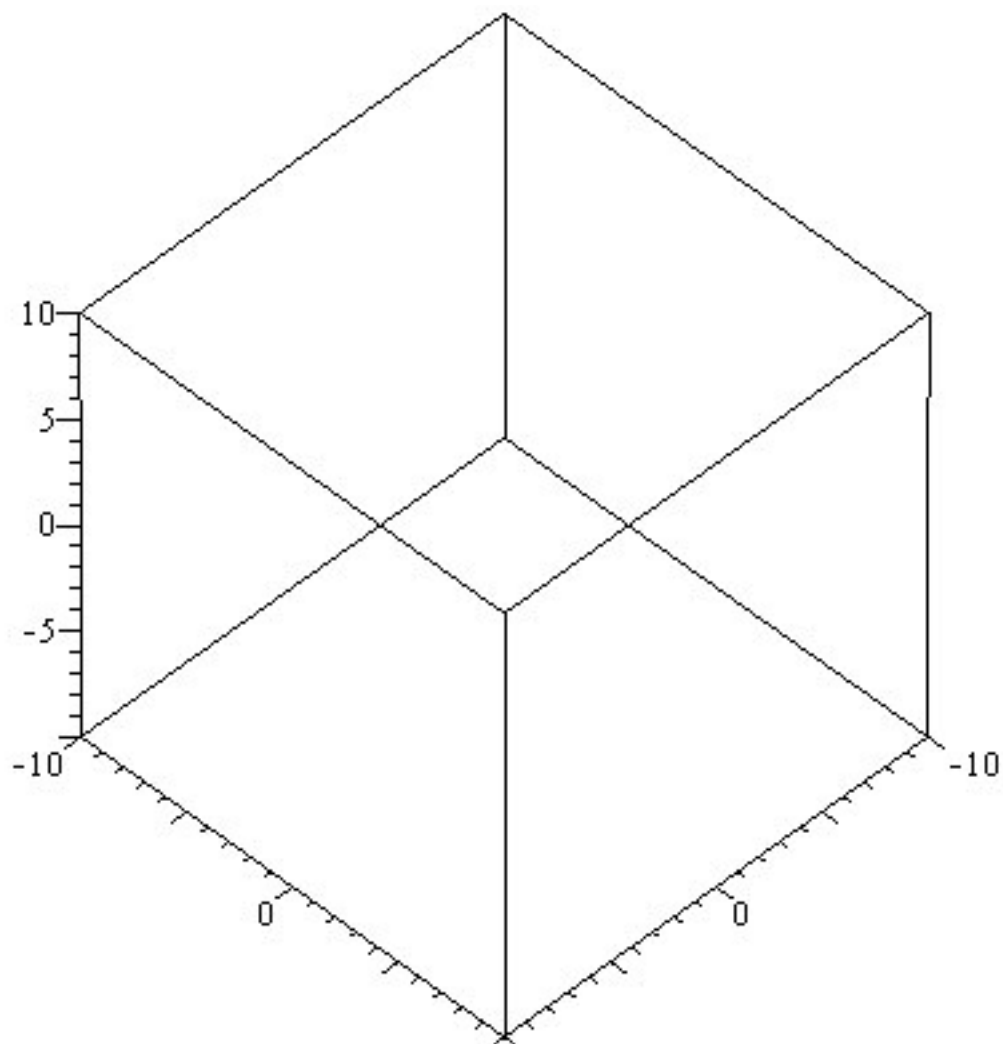


```

> #ue_f_abs := unapply(ue_f_abs, z);
> #ue_f_c := subs(z=r+i*I, ue_f);
> #ue_f_c := unapply(ue_f_c, r, i);
> #ue_f_pol:=convert(ue_f_c, polar);
> #ue_f_pol:=unapply(ue_f_pol, r, i);
> #plot3d(ue_f_pol, r=0..1, i=0..2*Pi, coords=cylindrical);
> #ue_f_pol(1, 2);
> #ue_f := unapply(ue_f, z);
> #complexplot3d(ue_f, z=0*I..2*Pi*I, axes = boxed, coords=
cylindrical);

> #####
#####
> #Workaround!
> #wolfgang:= piecewise(abs(z)<=1, 1, abs(z)>1, NaN);
> #ue_f:=ue_f*wolfgang;
> #ue_f := unapply(ue_f, z);
> #complexplot3d(ue_f, -1-I..1+I, axes=boxed, grid=[50,50]);
> #Workaround end
> #####
#####
> #ue_f := unapply(ue_f, z);
> #changecoords(complexplot3d(ue_f_num, -1-I..1+I, axes=boxed,
grid=[50,50]), polar);
> #changecoords(complexplot3d(ue_f, -1-I..1+I, axes=boxed, grid=
[50,50]), polar);
> #g := proc(z) local w; w := Re(z)*exp(Im(z)*I); w end proc;
> g := proc(z) local w; w := Re(z)*exp(Im(z)*I); subs(z=w, ue_f) end
proc;
    g := proc(z) local w; w :=  $\Re(z)$  * exp( $I * \Im(z)$ ); subs(z=w, ue_f) end proc (20)
> #g(2+I);
> changecoords(complexplot3d(g, 0..1+I, axes=boxed), cylindrical);
Warning, unable to evaluate the function to numeric values in
the region; see the plotting command's help page to ensure the
calling sequence is correct

```



```
[> #complexplot3d(ue_f_c, -Pi-I..Pi+I, axes = boxed);
```

```
[> test:=2+3*I;
```

```
test:= 2 + 3 I
```

```
[> Re(test);
```

```
2
```

```
[> test:=convert(test,polar);
```

```
test:= polar( $\sqrt{13}$ , arctan( $\frac{3}{2}$ ))
```