```
> restart;
> with(plots):
```

```
> first := Y1 = (U*b1 - a1*Y3) * (c1 / z-1);
```

$$first := Y1 = (U\,b1 - a1\,Y3)\left(\frac{c1}{z} - 1\right) \tag{1}$$

```
> second := Y2 = (Y1 + U*b2 - a2*Y3) * (c2*z / (z-1)); # (c2 /(z-1)
  ); #
```

$$second := Y2 = \frac{(Y1 + U\,b2 - a2\,Y3)\,c2\,z}{z-1} \tag{2}$$

```
> third := Y3 = (Y2 + U*b3 - a3*Y3) * (c3 / (z-1));
```

$$third := Y3 = \frac{(Y2 + U\,b3 - a3\,Y3)\,c3}{z-1} \tag{3}$$

```
> first := solve(first, Y1);
```

$$first := \frac{(U\,b1 - a1\,Y3)\,(c1 - z)}{z} \tag{4}$$

```
> second := subs(Y1=first, second);
```

$$second := Y2 = \frac{\left(\frac{(U\,b1 - a1\,Y3)\,(c1 - z)}{z} + U\,b2 - a2\,Y3\right)c2\,z}{z-1} \tag{5}$$

```
> second := solve(second, Y2);
```

$$second := \frac{(U\,b1\,c1 - U\,b1\,z - a1\,Y3\,c1 + a1\,Y3\,z + U\,b2\,z - a2\,Y3\,z)\,c2}{z-1} \tag{6}$$

```
> ue_f := subs(Y2=second, third);
```

$$ue\_f := Y3 = \frac{\left(\frac{(U\,b1\,c1 - U\,b1\,z - a1\,Y3\,c1 + a1\,Y3\,z + U\,b2\,z - a2\,Y3\,z)\,c2}{z-1} + U\,b3 - a3\,Y3\right)c3}{z-1} \tag{7}$$

```
> ue_f := solve(ue_f, Y3);
```

$$ue\_f := \frac{U\,(c2\,b1\,c1 - c2\,b1\,z + c2\,b2\,z + b3\,z - b3)\,c3}{c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1} \tag{8}$$

```
> ue_f := ue_f / U;
```

$$ue\_f := \frac{(c2\,b1\,c1 - c2\,b1\,z + c2\,b2\,z + b3\,z - b3)\,c3}{c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1} \tag{9}$$

```
> nenner := denom(ue_f);
```

$$nenner := c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1 \tag{10}$$

```
> pole := solve(nenner, z);
```

$$pole := \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 + \frac{1}{2}\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,c2\,a2\,a3 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2}, \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 \tag{11}$$

$$-\frac{1}{2}\left(c3^2\,c2^2\,a1^2-2\,c3^2\,c2^2\,a1\,a2+4\,c3\,c2\,a1-2\,c3^2\,c2\,a1\,a3+c3^2\,c2^2\,a2^2\right.$$

$$\left.-4\,c3\,c2\,a2+2\,c3^2\,c2\,a2\,a3+c3^2\,a3^2-4\,c3\,c2\,a1\,c1\right)^{1/2}$$

```
> numerator := numer(ue_f);
```

$$numerator := (c2\,b1\,c1-c2\,b1\,z+c2\,b2\,z+b3\,z-b3)\,c3 \tag{12}$$

```
> nullst := solve(numerator, z);
```

$$nullst := \frac{c2\,b1\,c1-b3}{c2\,b1-c2\,b2-b3} \tag{13}$$

Find pole and null

```
> fs := 1500000;
```

$$fs := 1500000 \tag{14}$$

```
> f_nutz := 20000;
```

$$f\_nutz := 20000 \tag{15}$$

```
> redim := Pi / fs;
```

$$redim := \frac{1}{1500000}\,\pi \tag{16}$$

```
> f_nutz_norm := f_nutz * redim;
```

$$f\_nutz\_norm := \frac{1}{75}\,\pi \tag{17}$$

```
> pol_r := 0.7:
> nul_r := 0:
> nul_phi := 0*Pi:
> pol_def := [pol_r*exp(f_nutz_norm*I), pol_r*exp(-f_nutz_norm*I)];
```

$$pol\_def := \left[0.7\,e^{\frac{1}{75}\,I\pi},\ 0.7\,e^{-\frac{1}{75}\,I\pi}\right] \tag{18}$$

```
> null_def := [nul_r*exp(nul_phi*I), nul_r*exp(-nul_phi*I)];
```

$$null\_def := [0, 0] \tag{19}$$

```
> eq1 := null_def[1] = nullst;
```

$$eq1 := 0 = \frac{c2\,b1\,c1-b3}{c2\,b1-c2\,b2-b3} \tag{20}$$

```
> eq1_simpl := null_def[1] = numer(nullst);
```

$$eq1\_simpl := 0 = c2\,b1\,c1-b3 \tag{21}$$

```
> eq2 := pol_def[1] = pole[1];
```

$$eq2 := 0.7\,e^{\frac{1}{75}\,I\pi} = \frac{1}{2}\,c3\,c2\,a1-\frac{1}{2}\,c3\,c2\,a2+1-\frac{1}{2}\,c3\,a3 \tag{22}$$

$$+\frac{1}{2}\left(c3^2\,c2^2\,a1^2-2\,c3^2\,c2^2\,a1\,a2+4\,c3\,c2\,a1-2\,c3^2\,c2\,a1\,a3+c3^2\,c2^2\,a2^2\right.$$

$$\left.-4\,c3\,c2\,a2+2\,c3^2\,c2\,a2\,a3+c3^2\,a3^2-4\,c3\,c2\,a1\,c1\right)^{1/2}$$

```
> pol_1_re := Re(pol_def[1]) = (1/2)*c3*c2*a1 - (1/2)*c3*c2*a2 + 1
  - (1/2)*c3*a3;
```

$$pol\_1\_re := 0.7\cos\left(\frac{1}{75}\,\pi\right) = \frac{1}{2}\,c3\,c2\,a1-\frac{1}{2}\,c3\,c2\,a2+1-\frac{1}{2}\,c3\,a3 \tag{23}$$

```
> pol_1_im := Im(pol_def[1]) = 0.5*sqrt(c3^2*c2^2*a1^2 - 2*c3^2*
  c2^2*a1*a2 + 4*c3*c2*a1 - 2*c3^2*c2*a1*a3 + c3^2*c2^2*a2^2 - 4*
```

```
  c3*c2*a2 + 2*c3^2*c2*a2*a3 + c3^2*a3^2 - 4*c3*c2*a1*c1);
```

$$pol\_1\_im := 0.7 \sin\left(\frac{1}{75}\,\pi\right) = 0.5\,\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,c2\,a2\,a3 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2} \tag{24}$$

```
> eq3 := pol_def[2] = pole[2];
```

$$eq3 := 0.7\,e^{-\frac{1}{75}\,I\pi} = \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 - \frac{1}{2}\,\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,c2\,a2\,a3 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2} \tag{25}$$

```
> pol_2_re := Re(pol_def[2]) = (1/2)*c3*c2*a1 - (1/2)*c3*c2*a2 + 1
  - (1/2)*c3*a3;
```

$$pol\_2\_re := 0.7 \cos\left(\frac{1}{75}\,\pi\right) = \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 \tag{26}$$

```
> pol_2_im := Im(pol_def[2]) = -(1/2)*sqrt(c3^2*c2^2*a1^2 - 2*c3^2*
  c2^2*a1*a2 + 4*c3*c2*a1 - 2*c3^2*c2*a1*a3 + c3^2*c2^2*a2^2 - 4*
  c3*c2*a2 + 2*c3^2*c2*a2*a3 + c3^2*a3^2 - 4*c3*c2*a1*c1);
```

$$pol\_2\_im := -0.7 \sin\left(\frac{1}{75}\,\pi\right) = -\frac{1}{2}\,\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,c2\,a2\,a3 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2} \tag{27}$$

Define Coefficients

```
> #b1 := 1;
> #a1 := 1;
> #eq1_simpl;
> #pol_1_re;
> #pol_1_im;
> #pol_2_re;
> #pol_2_im;
> #c2 := solve(eq1_simpl,c2);
> #pol_1_re;
> #c1 := solve(pol_1_re, c1);
> #pol_1_im;
> #c3 := solve(pol_1_im, c3);
> #pol_2_re;

> #a3 := solve(pol_2_re, a3);

> #pol_2_im;
> params := [a1=0.53711, a2=0.60174, a3=1.06183, b1=1.93711, b2=
  0.72150, b3=9.12712, c1=1.27265, c2=1.05154, c3=1];
```

$$params := [a1 = 0.53711,\ a2 = 0.60174,\ a3 = 1.06183,\ b1 = 1.93711,\ b2 = 0.72150,\ b3 \tag{28}$$

$= 9.12712,\ c1 = 1.27265,\ c2 = 1.05154,\ c3 = 1\ ]$

Plotting of found coefficients

```
> nullst1:=eval(nullst, params);
```

$$nullst1 := 0.8325794339 \tag{29}$$

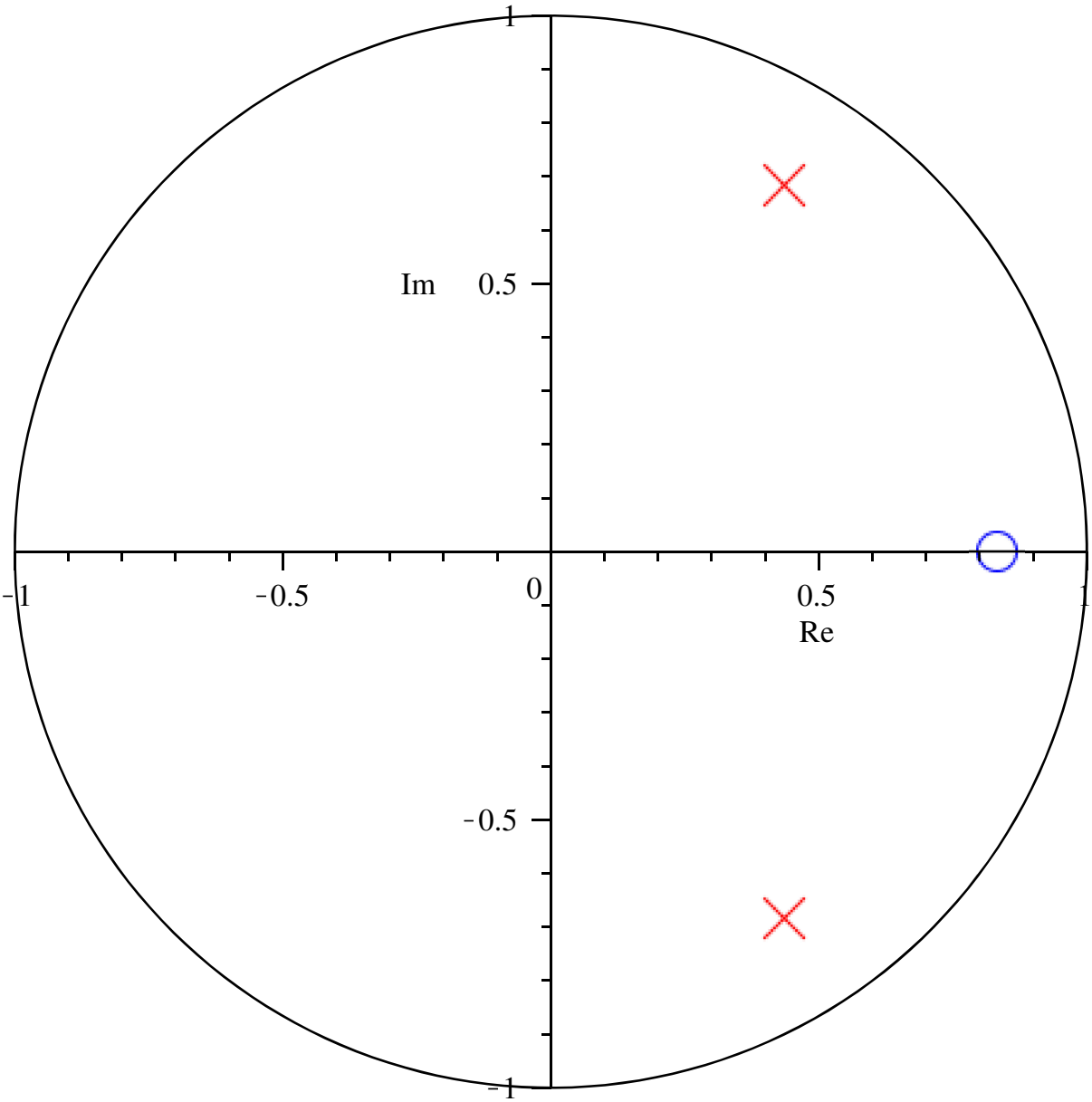
```
> pole1:=eval(pole, params);
```

$$pole1 := 0.4351044849 + 0.6838402245\ \mathrm{I},\ 0.4351044849 - 0.6838402245\ \mathrm{I} \tag{30}$$

```
> ue_f_num := subs(params, ue_f);
```

$$ue\_f\_num := \frac{-6.534797302 + 7.848857461\ z}{0.6569533653 - 0.8702089694\ z + z^2} \tag{31}$$

```
> #ue_f_num := unapply(ue_f_num, z);
> complexplot([pole1],style=point,
    color=red, labels = ["Re", "Im"],
    symbol="diagonalcross", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
> complexplot([nullst1], style=point,
    color="blue", labels = ["Re", "Im"],
    symbol="circle", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
> complexplot(cos+I*sin, -Pi .. Pi, labels = ["Re", "Im"], color=
  black, scaling=constrained):
> display(%,%%, %%%);
```


1
Im
0.5
-1
-0.5
0
0.5
1
Re
-0.5
-1

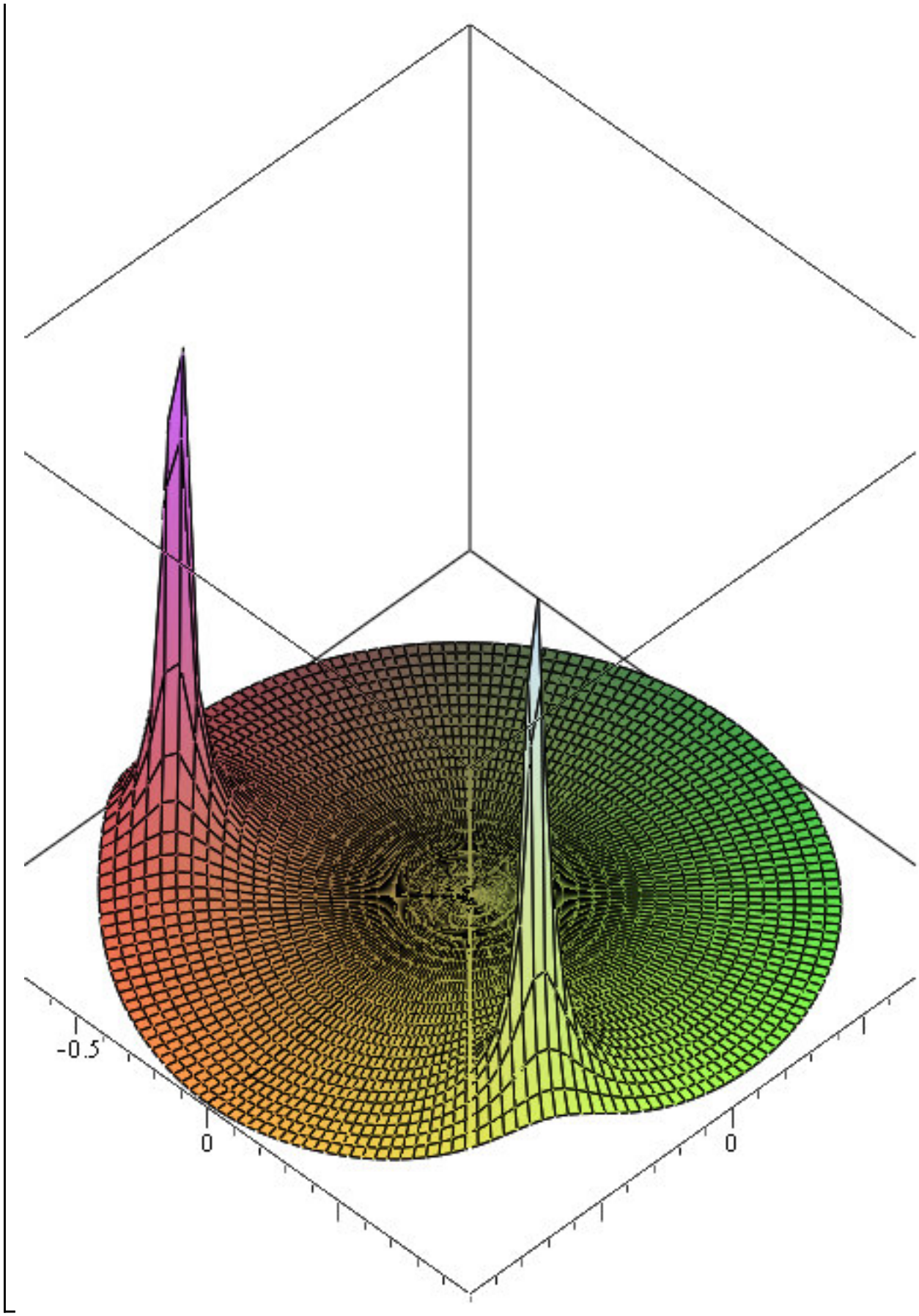
```
> #ue_f_abs := subs(params, ue_f_abs);
> #ue_f := subs(params, ue_f_abs);
> ue_f_num := subs(z=r*exp(I*phi),ue_f_num);
```

$$ue\_f\_num := \frac{-6.534797302 + 7.848857461\, r\, e^{I\phi}}{0.6569533653 - 0.8702089694\, r\, e^{I\phi} + r^2 \left(e^{I\phi}\right)^2} \tag{32}$$

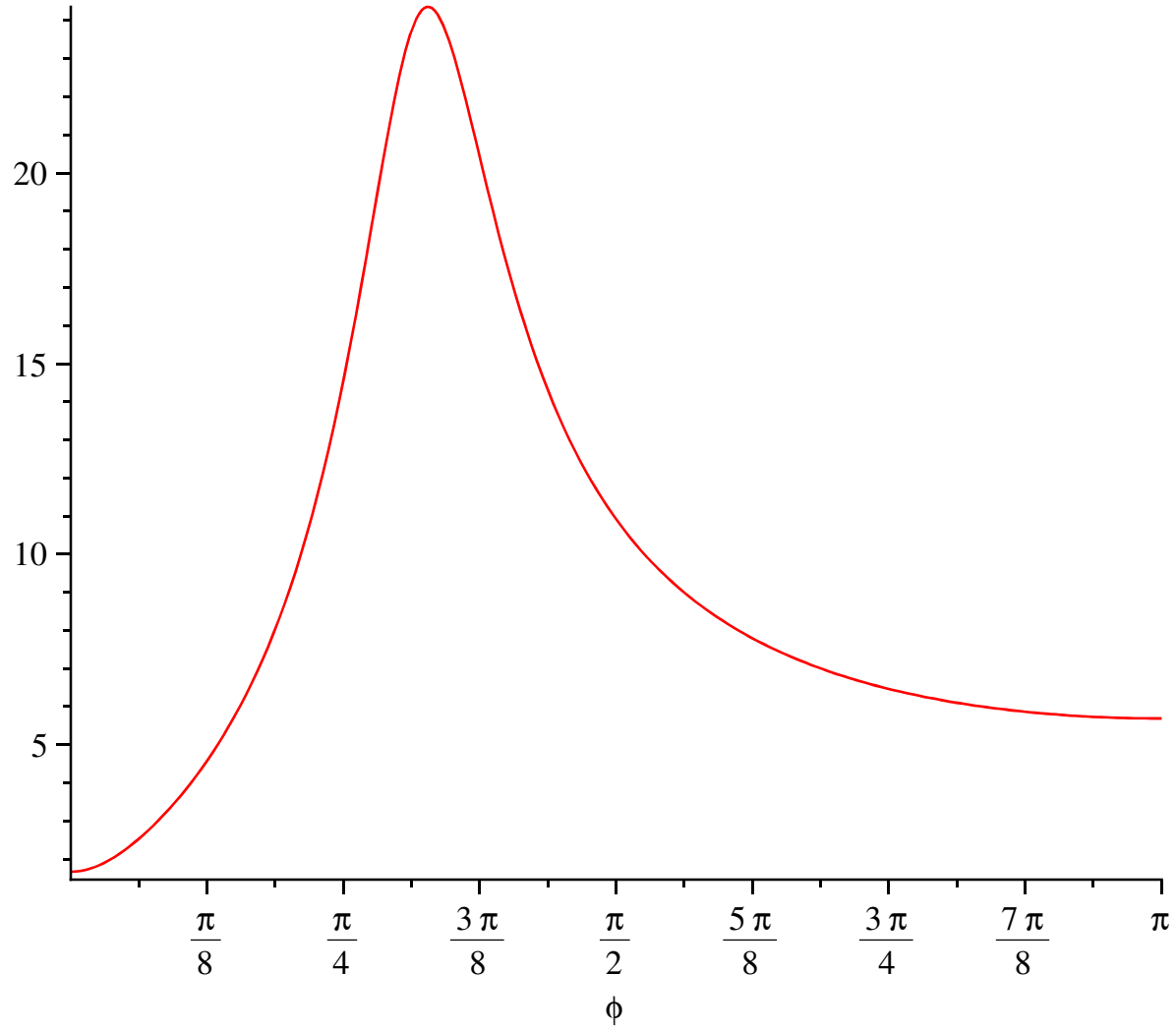
```
> ue_f_abs := abs(ue_f_num);
```

$$ue\_f\_abs := \left| \frac{-6.534797302 + 7.848857461\, r\, e^{I\phi}}{0.6569533653 - 0.8702089694\, r\, e^{I\phi} + r^2 \left(e^{I\phi}\right)^2} \right| \tag{33}$$

```
> #ue_f_abs := unapply(ue_f_num, r, phi);
> changecoords(plot3d(ue_f_abs, r=0..1, phi=0..2*Pi, grid=[25,180],
  axes=boxed), polar);
```


-0.5
0
0

```
> plot(subs(r=1, ue_f_abs), phi=0..Pi);
```




```
> 20222000020020325503204302840
Warning, inserted missing semicolon at end of statement
```

$$20222000020020325503204302840 \quad \textbf{(34)}$$

```
> 111111111111
Warning, inserted missing semicolon at end of statement
```

$$111111111111 \quad \textbf{(35)}$$