```
> restart;
> with(plots):
```

Calculate Noise Transfer Function

```
> first := Y = e + (A*c2*z / (z-1) -a3*Y)*c3/(z-1);
```

$$first := Y = e + \frac{\left(\frac{A\,c2\,z}{z-1} - a3\,Y\right) c3}{z-1} \tag{1}$$

```
> second := A = -a1*Y*1/(z-1)-a2*Y;
```

$$second := A = -\frac{a1\,Y}{z-1} - a2\,Y \tag{2}$$

```
> second := solve(second, A);
```

$$second := -\frac{Y\,(a1 + a2\,z - a2)}{z-1} \tag{3}$$

```
> second := subs(A=second, first);
```

$$second := Y = e + \frac{\left(-\frac{Y\,(a1 + a2\,z - a2)\,c2\,z}{(z-1)^2} - a3\,Y\right) c3}{z-1} \tag{4}$$

```
> ue_f := solve(second, Y);
```

$$ue\_f := \frac{e\,(z-1)^3}{c3\,c2\,z\,a1 + c3\,c2\,z^2\,a2 - c3\,c2\,z\,a2 + c3\,a3\,z^2 - 2\,c3\,a3\,z + c3\,a3 + z^3 - 3\,z^2 + 3\,z - 1} \tag{5}$$

Final Noise Transfer Function

```
> ue_f := ue_f / e;
```

$$ue\_f := \frac{(z-1)^3}{c3\,c2\,z\,a1 + c3\,c2\,z^2\,a2 - c3\,c2\,z\,a2 + c3\,a3\,z^2 - 2\,c3\,a3\,z + c3\,a3 + z^3 - 3\,z^2 + 3\,z - 1} \tag{6}$$

Separate poles and zeros

```
> numerator := numer(ue_f);
```

$$numerator := (z-1)^3 \tag{7}$$

```
> nenner := denom(ue_f);
```

$$nenner := c3\,c2\,z\,a1 + c3\,c2\,z^2\,a2 - c3\,c2\,z\,a2 + c3\,a3\,z^2 - 2\,c3\,a3\,z + c3\,a3 + z^3 - 3\,z^2 + 3\,z - 1 \tag{8}$$

Calculate zeros

```
> solve(numerator=0);
```

$$1, 1, 1 \tag{9}$$

Define Poles

```
> fs := 1500000;
```

$$fs := 1500000 \tag{10}$$

```
> f_pol := 40000;
```

$$f\_pol := 40000 \tag{11}$$

```
> redim := Pi / fs;
```

$$redim := \frac{1}{1500000}\pi \tag{12}$$

```
> f_nutz_norm := f_pol * redim;
```

$$f\_nutz\_norm := \frac{2}{75}\pi \tag{13}$$

```
> p1 := 0.8;
```

$$p1 := 0.8 \tag{14}$$

```
> p2 := 0.8*exp(I*f_nutz_norm);
```

$$p2 := 0.8\,\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi} \tag{15}$$

```
> p3 := 0.8*exp(-I*f_nutz_norm);
```

$$p3 := 0.8\,\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi} \tag{16}$$

Calculate Poles

```
> nenner;
```

$$c3\,c2\,z\,a1 + c3\,c2\,z^2\,a2 - c3\,c2\,z\,a2 + c3\,a3\,z^2 - 2\,c3\,a3\,z + c3\,a3 + z^3 - 3\,z^2 + 3\,z - 1 \tag{17}$$

```
> subs(z=p1, nenner);
```

$$0.8\,c3\,c2\,a1 - 0.16\,c3\,c2\,a2 + 0.04\,c3\,a3 - 0.008 \tag{18}$$

```
> subs(z=p2, nenner);
```

$$0.8\,c3\,c2\,\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\,a1 + 0.64\,c3\,c2\left(\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\right)^2 a2 - 0.8\,c3\,c2\,\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\,a2 + 0.64\,c3\,a3\left(\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\right)^2 - 1.6\,c3\,a3\,\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi} + c3\,a3 + 0.512\left(\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\right)^3 - 1.92\left(\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi}\right)^2 + 2.4\,\mathrm{e}^{\frac{2}{75}\mathrm{I}\pi} - 1 \tag{19}$$

```
> subs(z=p3, nenner);
```

$$0.8\,c3\,c2\,\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\,a1 + 0.64\,c3\,c2\left(\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\right)^2 a2 - 0.8\,c3\,c2\,\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\,a2 + 0.64\,c3\,a3\left(\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\right)^2 - 1.6\,c3\,a3\,\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi} + c3\,a3 + 0.512\left(\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\right)^3 - 1.92\left(\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi}\right)^2 + 2.4\,\mathrm{e}^{-\frac{2}{75}\mathrm{I}\pi} - 1 \tag{20}$$

```
> solve(nenner,z):
> second := solve(second, Y2);
```

$$second := \tag{21}$$

```
> ue_f := subs(Y2=second, third);
```

$$ue\_f := third \tag{22}$$

```
> ue_f := solve(ue_f, Y3);
```

$$ue\_f := \tag{23}$$

```
> ue_f := ue_f / U;
```

$$ue\_f := \frac{}{U} \tag{24}$$

```
> nenner := denom(ue_f);
```

$$\tag{25}$$

$$nenner := U \quad (25)$$

```
> pole := solve(nenner, z);
```

$$pole := \quad (26)$$

```
> numerator := numer(ue_f);
```

$$numerator := \quad (27)$$

```
> nullst := solve(numerator, z);
```

$$nullst := 0 \quad (28)$$

Find pole and null

```
> fs := 1500000;
```

$$fs := 1500000 \quad (29)$$

```
> f_nutz := 20000;
```

$$f\_nutz := 20000 \quad (30)$$

```
> redim := Pi / fs;
```

$$redim := \frac{1}{1500000}\,\pi \quad (31)$$

```
> f_nutz_norm := f_nutz * redim;
```

$$f\_nutz\_norm := \frac{1}{75}\,\pi \quad (32)$$

```
> pol_r := 0.7:
> nul_r := 0:
> nul_phi := 0*Pi:
> pol_def := [pol_r*exp(f_nutz_norm*I), pol_r*exp(-f_nutz_norm*I)];
```

$$pol\_def := \left[0.7\,e^{\frac{1}{75}\,I\pi},\, 0.7\,e^{-\frac{1}{75}\,I\pi}\right] \quad (33)$$

```
> null_def := [nul_r*exp(nul_phi*I), nul_r*exp(-nul_phi*I)];
```

$$null\_def := [0, 0] \quad (34)$$

```
> eq1 := null_def[1] = nullst;
```

$$eq1 := 0 = 0 \quad (35)$$

```
> eq1_simpl := null_def[1] = numer(nullst);
```

$$eq1\_simpl := 0 = 0 \quad (36)$$

```
> eq2 := pol_def[1] = pole[1];
Error, invalid subscript selector
> pol_1_re := Re(pol_def[1]) = (1/2)*c3*c2*a1 - (1/2)*c3*c2*a2 + 1
  - (1/2)*c3*a3;
```

$$pol\_1\_re := 0.7\cos\left(\frac{1}{75}\,\pi\right) = \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 \quad (37)$$

```
> pol_1_im := Im(pol_def[1]) = 0.5*sqrt(c3^2*c2^2*a1^2 - 2*c3^2*
  c2^2*a1*a2 + 4*c3*c2*a1 - 2*c3^2*c2*a1*a3 + c3^2*c2^2*a2^2 - 4*
  c3*c2*a2 + 2*c3^2*c2*a2*a3 + c3^2*a3^2 - 4*c3*c2*a1*c1);
```

$$pol\_1\_im := 0.7\sin\left(\frac{1}{75}\,\pi\right) \quad (38)$$
$$= 0.5\,\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,a3\,c2\,a2 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2}$$

```
> eq3 := pol_def[2] = pole[2];
Error, invalid subscript selector
```

```
> pol_2_re := Re(pol_def[2]) = (1/2)*c3*c2*a1 - (1/2)*c3*c2*a2 + 1
  - (1/2)*c3*a3;
```

$$pol\_2\_re := 0.7\cos\left(\frac{1}{75}\pi\right) = \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 \qquad \textbf{(39)}$$

```
> pol_2_im := Im(pol_def[2]) = -(1/2)*sqrt(c3^2*c2^2*a1^2 - 2*c3^2*
  c2^2*a1*a2 + 4*c3*c2*a1 - 2*c3^2*c2*a1*a3 + c3^2*c2^2*a2^2 - 4*
  c3*c2*a2 + 2*c3^2*c2*a2*a3 + c3^2*a3^2 - 4*c3*c2*a1*c1);
```

$$pol\_2\_im := -0.7\sin\left(\frac{1}{75}\pi\right) = -\frac{1}{2}\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,a3\,c2\,a2 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2} \qquad \textbf{(40)}$$

Define Coefficients

```
> #b1 := 1;
> #a1 := 1;
> #eq1_simpl;
> #pol_1_re;
> #pol_1_im;
> #pol_2_re;
> #pol_2_im;
> #c2 := solve(eq1_simpl,c2);
> #pol_1_re;
> #c1 := solve(pol_1_re, c1);
> #pol_1_im;
> #c3 := solve(pol_1_im, c3);
> #pol_2_re;

> #a3 := solve(pol_2_re, a3);

> #pol_2_im;
> params := [a1=0.53711, a2=0.60174, a3=1.06183, b1=1.93711, b2=
  0.72150, b3=9.12712, c1=1.27265, c2=1.05154, c3=1];
```

$$params := [a1 = 0.53711, a2 = 0.60174, a3 = 1.06183, b1 = 1.93711, b2 = 0.72150, b3 = 9.12712, c1 = 1.27265, c2 = 1.05154, c3 = 1] \qquad \textbf{(41)}$$

Plotting of found coefficients

```
> nullst1:=eval(nullst, params);
```

$$nullst1 := 0 \qquad \textbf{(42)}$$

```
> pole1:=eval(pole, params);
```

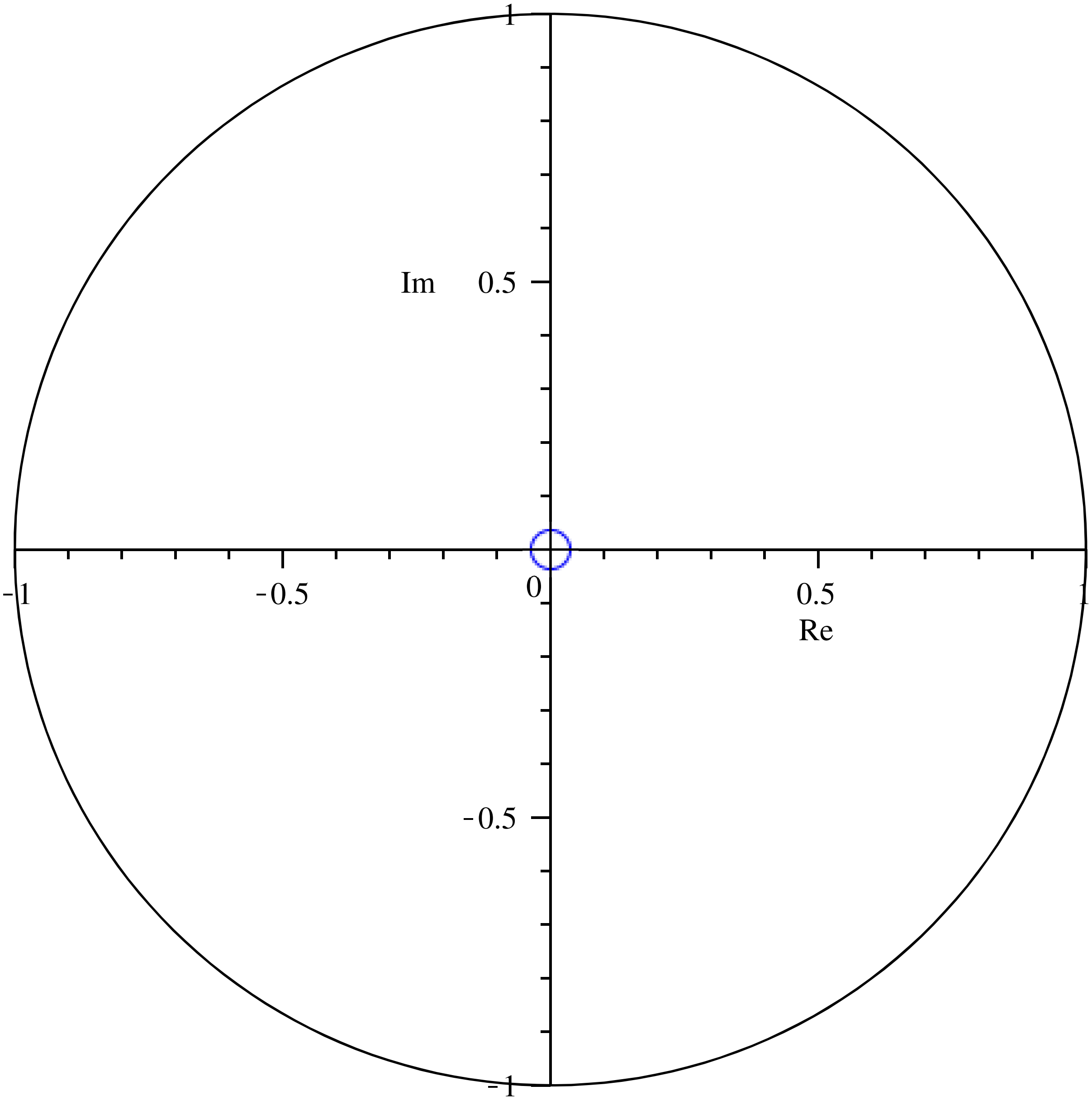
$$pole1 := \qquad \textbf{(43)}$$

```
> ue_f_num := subs(params, ue_f);
```

$$ue\_f\_num := \frac{}{U} \qquad \textbf{(44)}$$

```
> #ue_f_num := unapply(ue_f_num, z);
> complexplot([pole1],style=point,
    color=red, labels = ["Re", "Im"],
    symbol="diagonalcross", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
```

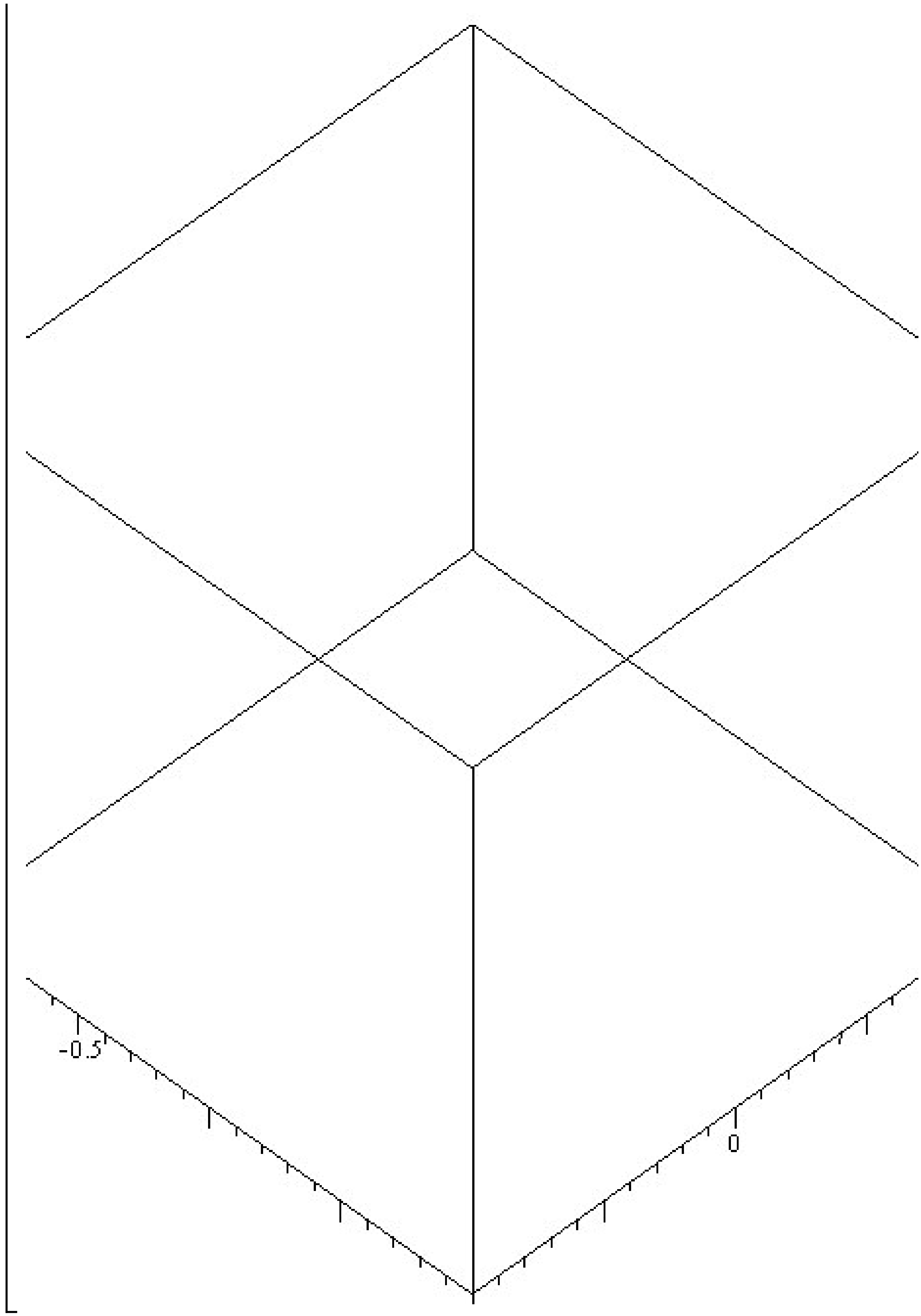
```
> complexplot([nullst1], style=point,
    color="blue", labels = ["Re", "Im"],
    symbol="circle", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
> complexplot(cos+I*sin, -Pi .. Pi, labels = ["Re", "Im"], color=
  black, scaling=constrained):
> display(%,%%, %%%);
```


1
Im
0.5
-1
-0.5
0
0.5
1
Re
-0.5
-1

```
> #ue_f_abs := subs(params, ue_f_abs);
> #ue_f := subs(params, ue_f_abs);
> ue_f_num := subs(z=r*exp(I*phi),ue_f_num);
```

$$ue\_f\_num := \frac{}{U} \tag{45}$$

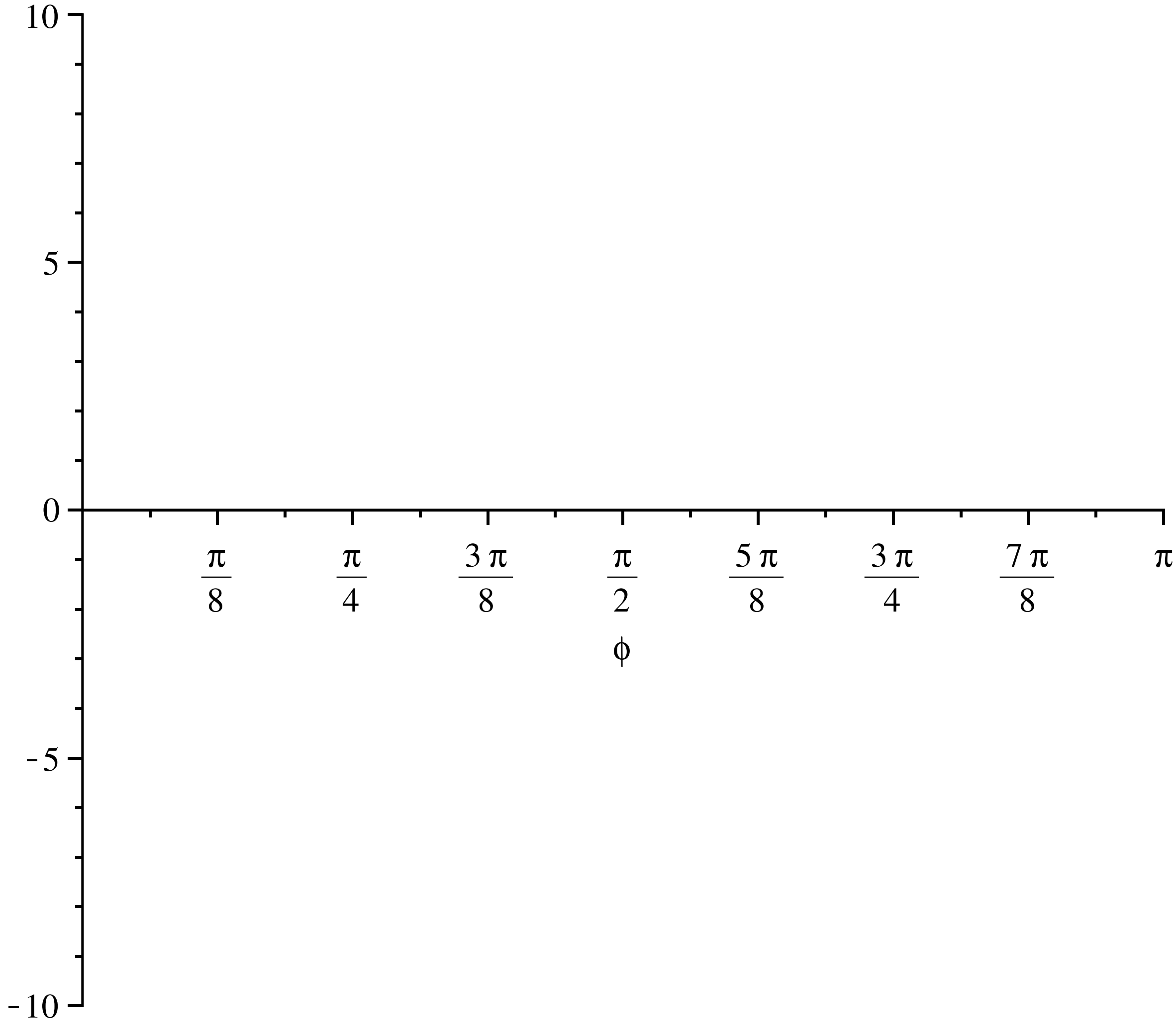
```
> ue_f_abs := abs(ue_f_num);
Error, (in simpl/abs) expecting 1 or 2 arguments, got 0
> #ue_f_abs := unapply(ue_f_num, r, phi);
> changecoords(plot3d(ue_f_abs, r=0..1, phi=0..2*Pi, grid=[25,180],
  axes=boxed), polar);
Warning, unable to evaluate the function to numeric values in
the region; see the plotting command's help page to ensure the
calling sequence is correct
```



-0.5
0

```
> plot(subs(r=1, ue_f_abs), phi=0..Pi);
```

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct




```
> 20222000020020325503204302840
```

Warning, inserted missing semicolon at end of statement

$$20222000020020325503204302840 \tag{46}$$

```
> 111111111111
```

Warning, inserted missing semicolon at end of statement

$$111111111111 \tag{47}$$