```
> restart;
> with(plots):
```

```
> first := Y1 = (U*b1 - a1*Y3) * (c1 / z-1);
```

$$first := Y1 = (U\,b1 - a1\,Y3)\left(\frac{c1}{z} - 1\right) \tag{1}$$

```
> second := Y2 = (Y1 + U*b2 - a2*Y3) * (c2*z / (z-1)); # (c2 /(z-1)
  ); #
```

$$second := Y2 = \frac{(Y1 + U\,b2 - a2\,Y3)\,c2\,z}{z-1} \tag{2}$$

```
> third := Y3 = (Y2 + U*b3 - a3*Y3) * (c3 / (z-1));
```

$$third := Y3 = \frac{(Y2 + U\,b3 - a3\,Y3)\,c3}{z-1} \tag{3}$$

```
> first := solve(first, Y1);
```

$$first := \frac{(U\,b1 - a1\,Y3)\,(c1 - z)}{z} \tag{4}$$

```
> second := subs(Y1=first, second);
```

$$second := Y2 = \frac{\left(\frac{(U\,b1 - a1\,Y3)\,(c1 - z)}{z} + U\,b2 - a2\,Y3\right)c2\,z}{z-1} \tag{5}$$

```
> second := solve(second, Y2);
```

$$second := \frac{(U\,b1\,c1 - U\,b1\,z - a1\,Y3\,c1 + a1\,Y3\,z + U\,b2\,z - a2\,Y3\,z)\,c2}{z-1} \tag{6}$$

```
> ue_f := subs(Y2=second, third);
```

$$ue\_f := Y3 = \frac{\left(\frac{(U\,b1\,c1 - U\,b1\,z - a1\,Y3\,c1 + a1\,Y3\,z + U\,b2\,z - a2\,Y3\,z)\,c2}{z-1} + U\,b3 - a3\,Y3\right)c3}{z-1} \tag{7}$$

```
> ue_f := solve(ue_f, Y3);
```

$$ue\_f := \frac{U\,(c2\,b1\,c1 - c2\,b1\,z + c2\,b2\,z + b3\,z - b3)\,c3}{c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1} \tag{8}$$

```
> ue_f := ue_f / U;
```

$$ue\_f := \frac{(c2\,b1\,c1 - c2\,b1\,z + c2\,b2\,z + b3\,z - b3)\,c3}{c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1} \tag{9}$$

```
> nenner := denom(ue_f);
```

$$nenner := c3\,c2\,a1\,c1 - c3\,c2\,a1\,z + c3\,c2\,a2\,z + c3\,a3\,z - c3\,a3 + z^2 - 2\,z + 1 \tag{10}$$

```
> pole := solve(nenner, z);
```

$$pole := \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 + \frac{1}{2}\left(c3^2\,c2^2\,a1^2 - 2\,c3^2\,c2^2\,a1\,a2 + 4\,c3\,c2\,a1 - 2\,c3^2\,c2\,a1\,a3 + c3^2\,c2^2\,a2^2 - 4\,c3\,c2\,a2 + 2\,c3^2\,c2\,a2\,a3 + c3^2\,a3^2 - 4\,c3\,c2\,a1\,c1\right)^{1/2}, \frac{1}{2}\,c3\,c2\,a1 - \frac{1}{2}\,c3\,c2\,a2 + 1 - \frac{1}{2}\,c3\,a3 \tag{11}$$

$$-\frac{1}{2}\left(c3^2\,c2^2\,a1^2-2\,c3^2\,c2^2\,a1\,a2+4\,c3\,c2\,a1-2\,c3^2\,c2\,a1\,a3+c3^2\,c2^2\,a2^2\right.$$

$$\left.-4\,c3\,c2\,a2+2\,c3^2\,c2\,a2\,a3+c3^2\,a3^2-4\,c3\,c2\,a1\,c1\right)^{1/2}$$

```
> numerator := numer(ue_f);
```

$$numerator := (c2\,b1\,c1-c2\,b1\,z+c2\,b2\,z+b3\,z-b3)\,c3 \tag{12}$$

```
> nullst := solve(numerator, z);
```

$$nullst := \frac{c2\,b1\,c1-b3}{c2\,b1-c2\,b2-b3} \tag{13}$$

```
> params := [a1=1, a2=1, a3=1, b1=1, b2=1, b3=1, c1=1, c2=1, c3=1];
```

$$params := [a1=1, a2=1, a3=1, b1=1, b2=1, b3=1, c1=1, c2=1, c3=1] \tag{14}$$

```
> nullst1:=eval(nullst, params);
```

$$nullst1 := 0 \tag{15}$$

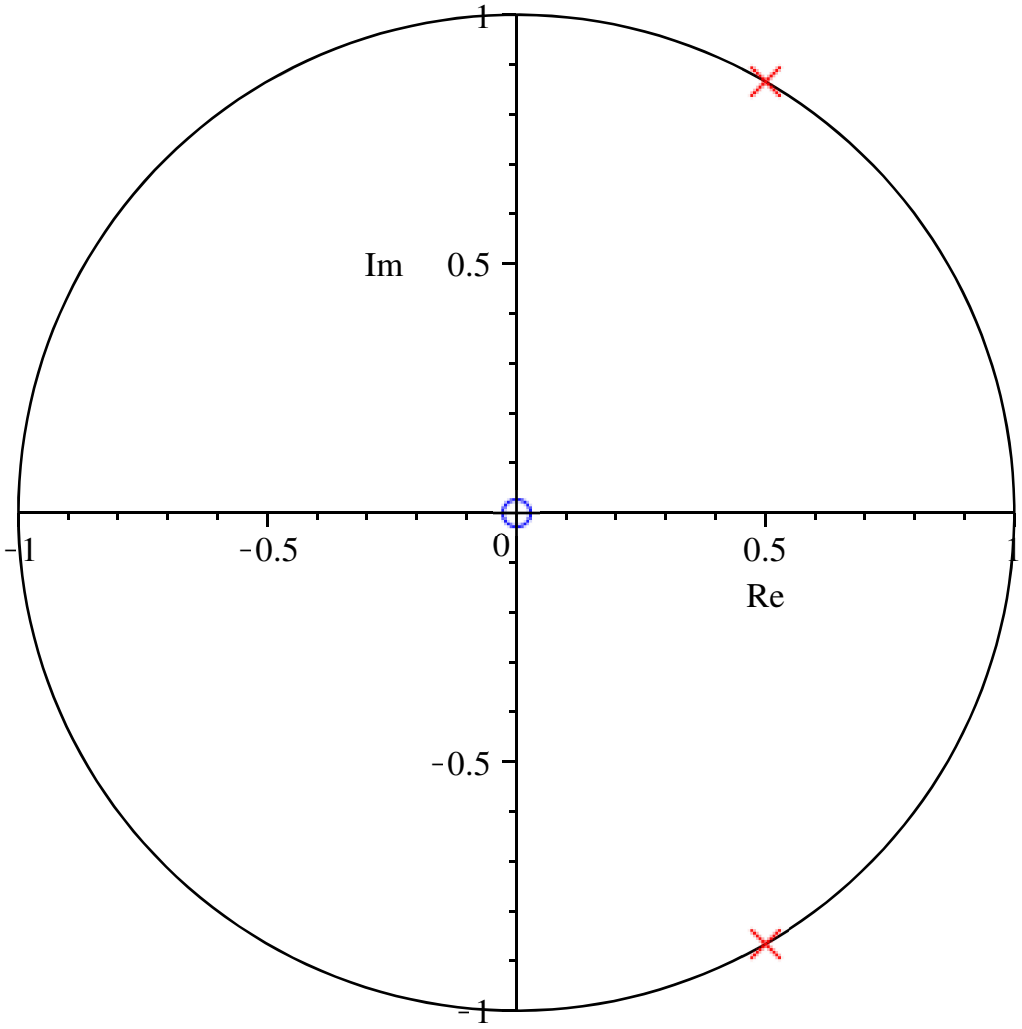
```
> pole1:=eval(pole, params);
```

$$pole1 := \frac{1}{2}+\frac{1}{2}\sqrt{-3},\ \frac{1}{2}-\frac{1}{2}\sqrt{-3} \tag{16}$$

```
> ue_f_num := subs(params, ue_f);
```

$$ue\_f\_num := \frac{z}{1-z+z^2} \tag{17}$$

```
> #ue_f_num := unapply(ue_f_num, z);
> complexplot([pole1],style=point,
    color=red, labels = ["Re", "Im"],
    symbol="diagonalcross", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
> complexplot([nullst1], style=point,
    color="blue", labels = ["Re", "Im"],
    symbol="circle", symbolsize=20,thickness=10,
    scaling=constrained): #,view=[-2..1,-2..2]):
> complexplot(cos+I*sin, -Pi .. Pi, labels = ["Re", "Im"], color=
  black, scaling=constrained):
> display(%,%%, %%%);
```



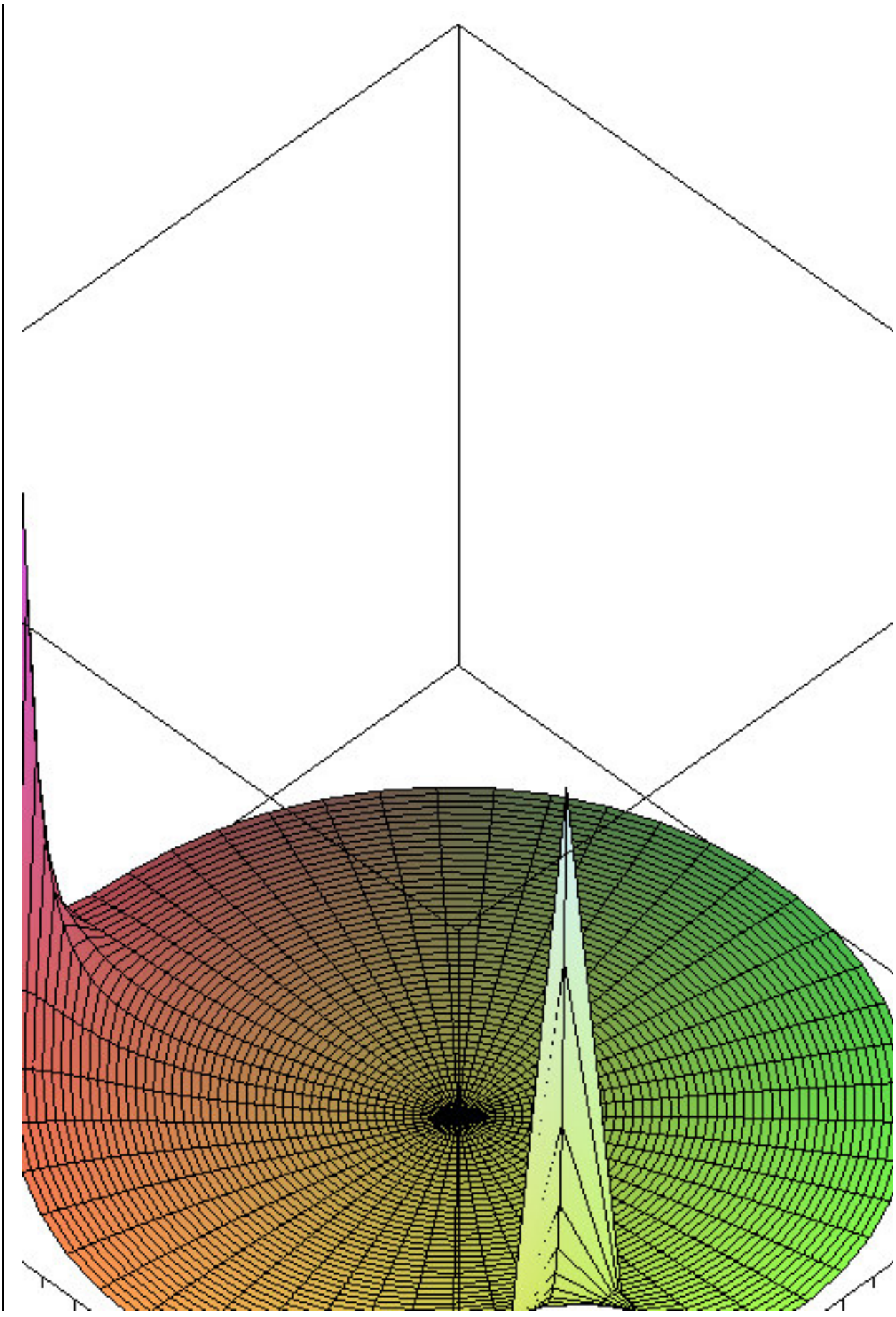

```
> #ue_f_abs := subs(params, ue_f_abs);
> #ue_f := subs(params, ue_f_abs);
> ue_f_num := subs(z=r*exp(I*phi),ue_f_num);
```

$$ue\_f\_num := \frac{r\,e^{I\phi}}{1 - r\,e^{I\phi} + r^2\left(e^{I\phi}\right)^2} \tag{18}$$

```
> ue_f_abs := abs(ue_f_num);
```

$$ue\_f\_abs := e^{-\Im(\phi)}\left|\frac{r}{1 - r\,e^{I\phi} + r^2\left(e^{I\phi}\right)^2}\right| \tag{19}$$

```
> #ue_f_abs := unapply(ue_f_num, r, phi);
> changecoords(plot3d(ue_f_abs, r=0..1, phi=0..2*Pi, grid=[50,50],
  axes=boxed), polar);
```



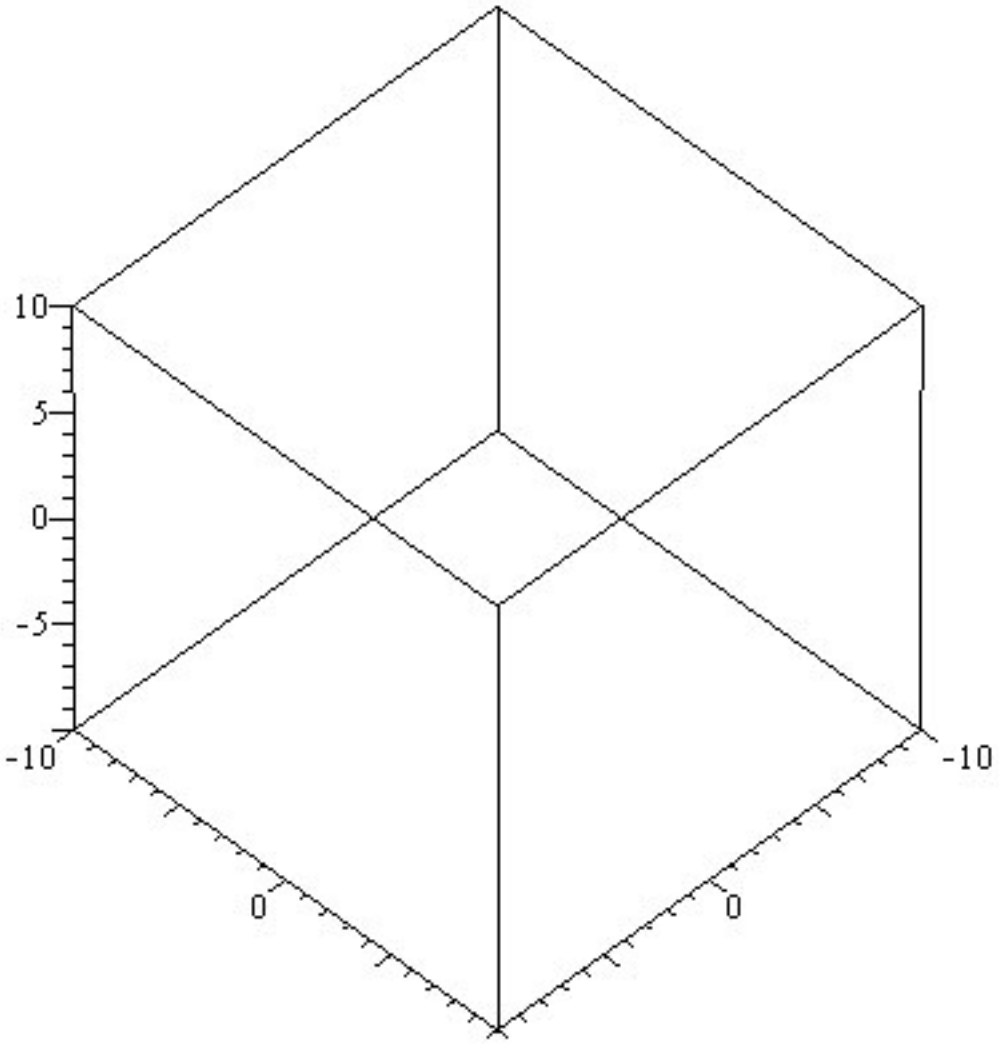
```
> #ue_f_abs := unapply(ue_f_abs,z);
> #ue_f_c := subs(z=r+i*I,ue_f);
> #ue_f_c := unapply(ue_f_c, r, i);
> #ue_f_pol:=convert(ue_f_c,polar);
> #ue_f_pol:=unapply(ue_f_pol,r,i);
> #plot3d(ue_f_pol,r=0..1,i=0..2*Pi,coords=cylindrical);
> #ue_f_pol(1,2);
> #ue_f := unapply(ue_f,z);
> #complexplot3d(ue_f, z=0*I..2*Pi*I, axes = boxed,coords=
  cylindrical);

> ##################################################################
  ##############################################
> #Workaround!
> #wolfgang:= piecewise(abs(z)<=1,1,abs(z)>1,NaN);
> #ue_f:=ue_f*wolfgang;
> #ue_f := unapply(ue_f,z);
> #complexplot3d(ue_f, -1-I..1+I, axes=boxed, grid=[50,50]);
> #Workaround end
> ##################################################################
  ##############################################
> #ue_f := unapply(ue_f,z);
> #changecoords(complexplot3d(ue_f_num, -1-I..1+I, axes=boxed,
  grid=[50,50]), polar);
> #changecoords(complexplot3d(ue_f, -1-I..1+I, axes=boxed, grid=
  [50,50]), polar);
> #g := proc(z) local w; w := Re(z)*exp(Im(z)*I); w end proc;
> g := proc(z) local w; w := Re(z)*exp(Im(z)*I); subs(z=w,ue_f) end
  proc;
```

$$g := \mathbf{proc}(z)\ \mathbf{local}\ w;\ w := \Re(z) * \exp(\mathrm{I} * \Im(z));\ subs(z = w, ue\_f)\ \mathbf{end\ proc} \qquad (20)$$

```
> #g(2+I);
> changecoords(complexplot3d(g, 0..1+I,axes=boxed),cylindrical);
```

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct




```
> #complexplot3d(ue_f_c, -Pi-I..Pi+I, axes = boxed);
```

```
> test:=2+3*I;
```

$$test := 2 + 3\,\mathrm{I} \tag{1}$$

```
> Re(test);
```

$$2 \tag{2}$$

```
> test:=convert(test,polar);
```

$$test := \mathrm{polar}\left(\sqrt{13}, \arctan\left(\frac{3}{2}\right)\right) \tag{3}$$