In this document, we verify the essential inequality necessary to prove 2M_T>mu(T;v) by considering the critical values of mu^bullet, when there are (essentially) 3 sub-k-trees

We first write the formulas in general

> $LHS := (N1, N2, N3, k, mu1, mu2, mu3) \rightarrow (N1 \cdot N2 \cdot N3 + 1) \cdot (1 + mu1 + mu2 + mu3 + k) + k \cdot (k - 2) + mu1 \cdot (1 + k + mu1) \cdot N1 + mu2 \cdot (1 + k + mu2) \cdot N2 + mu3 \cdot (1 + k + mu3) \cdot N3$

$$LHS := (N1, N2, N3, k, \mu1, \mu2, \mu3) \rightarrow (N1\, N2\, N3 + 1)\, (1 + \mu1 + \mu2 + \mu3 + k) + k\, (k - 2) + \mu1\, (1 + k + \mu1)\, N1 + \mu2\, (1 + k + \mu2)\, N2 + \mu3\, (1 + k + \mu3)\, N3 \quad \textbf{(1)}$$

> $RHS := (N1, N2, N3, k, mu1, mu2, mu3) \rightarrow (1 + mu1 + mu2 + mu3) \cdot ((1 + k + mu1) \cdot N1 + (1 + k + mu2) \cdot N2 + (1 + k + mu3) \cdot N3 + k - 3)$

$$RHS := (N1, N2, N3, k, \mu1, \mu2, \mu3) \rightarrow (1 + \mu1 + \mu2 + \mu3)\, ((1 + k + \mu1)\, N1 + (1 + k + \mu2)\, N2 + (1 + k + \mu3)\, N3 + k - 3) \quad \textbf{(2)}$$

> $Dif := (N1, N2, N3, k, mu1, mu2, mu3) \rightarrow LHS(N1, N2, N3, k, mu1, mu2, mu3) - RHS(N1, N2, N3, k, mu1, mu2, mu3)$

$$Dif := (N1, N2, N3, k, \mu1, \mu2, \mu3) \rightarrow LHS(N1, N2, N3, k, \mu1, \mu2, \mu3) - RHS(N1, N2, N3, k, \mu1, \mu2, \mu3) \quad \textbf{(3)}$$

>

> $f1(N1, N2, N3, k) := Dif(N1, N2, N3, k, 0, 0, 0)$

$$f1 := (N1, N2, N3, k) \rightarrow Dif(N1, N2, N3, k, 0, 0, 0) \quad \textbf{(4)}$$

> $f2(N1, N2, N3, k) := Dif\left(N1, N2, N3, k, \frac{N1}{2}, 0, 0\right)$

$$f2 := (N1, N2, N3, k) \rightarrow Dif\left(N1, N2, N3, k, \frac{1}{2}\, N1, 0, 0\right) \quad \textbf{(5)}$$

> $f3 := (N1, N2, N3, k) \rightarrow Dif\left(N1, N2, N3, k, \frac{N1 - 1}{2}, \frac{N2 - 1}{2}, 0\right)$

$$f3 := (N1, N2, N3, k) \rightarrow Dif\left(N1, N2, N3, k, \frac{1}{2}\, N1 - \frac{1}{2}, \frac{1}{2}\, N2 - \frac{1}{2}, 0\right) \quad \textbf{(6)}$$

> $f4 := (N1, N2, N3, k) \rightarrow Dif\left(N1, N2, N3, k, \frac{N1 - 1}{2}, \frac{N2 - 1}{2}, \frac{(N3 - 1)}{2}\right)$

$$f4 := (N1, N2, N3, k) \rightarrow Dif\left(N1, N2, N3, k, \frac{1}{2}\, N1 - \frac{1}{2}, \frac{1}{2}\, N2 - \frac{1}{2}, \frac{1}{2}\, N3 - \frac{1}{2}\right) \quad \textbf{(7)}$$

> $minimize(f1(N1, N2, N3, k), N1 = 2 .. infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, k = 2 .. infinity)$

$$10 \quad \textbf{(8)}$$

> $minimize(f2(N1, N2, N3, k), N1 = 2 .. infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, k = 2 .. infinity)$

$$\frac{23}{4} \quad \textbf{(9)}$$

> $minimize(f3(N1, N2, N3, k), N1 = 2 .. infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, k = 2 .. infinity)$

$$\frac{19}{4} \tag{10}$$

> $minimize(f4(N1, N2, N3, k), N1 = 3..infinity, N2 = 2..infinity, N3 = 2..infinity, k = 2..infinity)$

$$5 \tag{11}$$

> $f4(2, 2, 2, k)$

$$12 - \frac{11}{2} k + k (k - 2) \tag{12}$$

There are only 3 cases for which the numbers do not work. Nevertheless, in the initial context, N=2, corresponds with T_i being a (k+1)-clique
For k >=4, the construction has no more than 2k vertices. For k=3, it is also immediately checked.

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