In this document, we verify the essential inequality necessary to prove 2M_T>mu(T;v) by considering the critical values of mu^bullet, when there are (essentially) 4 sub-k-trees

We first write the formulas in general

> $k := 4$

$$k := 4 \tag{1}$$

>

> $LHS := (N1, N2, N3, N4, mu1, mu2, mu3, mu4) \to (N1 \cdot N2 \cdot N3 \cdot N4 + 1) \cdot (1 + mu1 + mu2 + mu3 + mu4 + k) + k + mu1 \cdot (1 + k + mu1) \cdot N1 + mu2 \cdot (1 + k + mu2) \cdot N2 + mu3 \cdot (1 + k + mu3) \cdot N3 + mu4 \cdot (1 + k + mu4) \cdot N4$

$$LHS := (N1, N2, N3, N4, \mu1, \mu2, \mu3, \mu4) \mapsto (N2 \cdot N1 \cdot N3 \cdot N4 + 1) \cdot (1 + \mu1 + \mu2 + \mu3 + \mu4 + k) + k + \mu1 \cdot (1 + k + \mu1) \cdot N1 + \mu2 \cdot (1 + k + \mu2) \cdot N2 + \mu3 \cdot (1 + k + \mu3) \cdot N3 + \mu4 \cdot (1 + k + \mu4) \cdot N4 \tag{2}$$

> $RHS := (N1, N2, N3, N4, mu1, mu2, mu3, mu4) \to (1 + mu1 + mu2 + mu3 + mu4) \cdot ((1 + k + mu1) \cdot N1 + (1 + k + mu2) \cdot N2 + (1 + k + mu3) \cdot N3 + (1 + k + mu4) \cdot N4)$

$$RHS := (N1, N2, N3, N4, \mu1, \mu2, \mu3, \mu4) \mapsto (1 + \mu1 + \mu2 + \mu3 + \mu4) \cdot ((1 + k + \mu1) \cdot N1 + (1 + k + \mu2) \cdot N2 + (1 + k + \mu3) \cdot N3 + (1 + k + \mu4) \cdot N4) \tag{3}$$

> $Dif := (N1, N2, N3, N4, mu1, mu2, mu3, mu4) \to LHS(N1, N2, N3, N4, mu1, mu2, mu3, mu4) - RHS(N1, N2, N3, N4, mu1, mu2, mu3, mu4)$

$$Dif := (N1, N2, N3, N4, \mu1, \mu2, \mu3, \mu4) \mapsto LHS(N1, N2, N3, N4, \mu1, \mu2, \mu3, \mu4) - RHS(N1, N2, N3, N4, \mu1, \mu2, \mu3, \mu4) \tag{4}$$

>

> $f1 := (N1, N2, N3, N4) \to Dif(N1, N2, N3, N4, 0, 0, 0, 0)$

$$f1 := (N1, N2, N3, N4) \mapsto Dif(N1, N2, N3, N4, 0, 0, 0, 0) \tag{5}$$

> $f2 := (N1, N2, N3, N4) \to Dif\left(N1, N2, N3, N4, \frac{N1 - 1}{2}, 0, 0, 0\right)$

$$f2 := (N1, N2, N3, N4) \mapsto Dif\left(N1, N2, N3, N4, \frac{N1}{2} - \frac{1}{2}, 0, 0, 0\right) \tag{6}$$

> $f3 := (N1, N2, N3, N4) \to Dif\left(N1, N2, N3, N4, \frac{N1 - 1}{2}, \frac{N2 - 1}{2}, 0, 0\right)$

$$f3 := (N1, N2, N3, N4) \mapsto Dif\left(N1, N2, N3, N4, \frac{N1}{2} - \frac{1}{2}, \frac{N2}{2} - \frac{1}{2}, 0, 0\right) \tag{7}$$

> $f4 := (N1, N2, N3, N4) \to Dif\left(N1, N2, N3, N4, \frac{N1 - 1}{2}, \frac{N2 - 1}{2}, \frac{(N3 - 1)}{2}, 0\right)$

$$f4 := (N1, N2, N3, N4) \mapsto Dif\left(N1, N2, N3, N4, \frac{N1}{2} - \frac{1}{2}, \frac{N2}{2} - \frac{1}{2}, \frac{N3}{2} - \frac{1}{2}, 0\right) \tag{8}$$

> $f5 := (N1, N2, N3, N4) \to Dif\left(N1, N2, N3, N4, \frac{N1 - 1}{2}, \frac{N2 - 1}{2}, \frac{(N3 - 1)}{2}, \frac{(N4 - 1)}{2}\right)$

$$f5 := (N1, N2, N3, N4) \mapsto Dif\left(N1, N2, N3, N4, \frac{N1}{2} - \frac{1}{2}, \frac{N2}{2} - \frac{1}{2}, \frac{N3}{2} - \frac{1}{2}, \frac{N4}{2} - \frac{1}{2}\right) \tag{9}$$

>

> $minimize(f1(N1, N2, N3, N4), N1 = 2 ..infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, N4 = 2 .. infinity)$

$$49 \tag{10}$$

> $minimize(f2(N1, N2, N3, N4), N1 = 2 ..infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, N4 = 2 .. infinity)$

$$\frac{83}{2} \tag{11}$$

> $minimize(f3(N1, N2, N3, N4), N1 = 2 ..infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, N4 = 2 .. infinity)$

$$33 \tag{12}$$

> $minimize(f4(N1, N2, N3, N4), N1 = 2 ..infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, N4 = 2 .. infinity)$

$$\frac{47}{2} \tag{13}$$

> $minimize(f5(N1, N2, N3, N4), N1 = 2 ..infinity, N2 = 2 .. infinity, N3 = 2 .. infinity, N4 = 2 .. infinity)$

$$13 \tag{14}$$

>