In this document, we verify one of case of the essential inequality necessary to prove 2M_T>mu(T;v) for large j
We do the case where all mu_i^bullet are equal to Ni/2.

We start with the case where j>6.
We will write x=N1 and z=sum(N_i, 2 \le i \le j).
Note that product of (N_i, 2 \le i \le j) is at least 5 times the sum of them (z). Actually it is at least f=2^(j-2)/j > j-2 times z.
As such, we have that the left hand side is at least equal to

> $L := (x, z, k, f) \rightarrow (x \cdot f \cdot z + 1) \cdot \left(1 + k + \frac{(x+z)}{2}\right) + \frac{x}{2} \cdot \left(x + k \cdot x + \frac{x^2}{2}\right)$

$$L := (x, z, k, f) \rightarrow (x f z + 1) \left(1 + k + \frac{1}{2} x + \frac{1}{2} z\right) + \frac{1}{2} x \left(x + k x + \frac{1}{2} x^2\right) \quad \textbf{(1)}$$

For the right hand side, we **use** *that overline* $Ni = (k+1) Ni + \frac{Ni^2}{2}$ *en* $Ni^2 \backslash le\ N1\ Ni$ **for** *every* $i \backslash ge\ 2$.

> $R := (x, z, k) \rightarrow \left(1 + \frac{(x+z)}{2}\right) \cdot \left(\left(x + k \cdot x + \frac{x^2}{2}\right) + z \cdot (k+1) + \frac{x \cdot z}{2} + k\right)$

$$R := (x, z, k) \rightarrow \left(1 + \frac{1}{2} x + \frac{1}{2} z\right) \left(x + k x + \frac{1}{2} x^2 + z (k+1) + \frac{1}{2} x z + k\right) \quad \textbf{(2)}$$

Next we consider the difference of the two sides.

(3)

> $collect(simplify(L(x, z, k) - R(x, z, k)), k)$

$$\left(x f z - x z - \frac{3}{2} x - \frac{3}{2} z - \frac{1}{2} z^2\right) k + 1 + x f z + \frac{1}{2} f x^2 z + \frac{1}{2} f x z^2 - \frac{1}{4} x z^2 - \frac{3}{2} x z - \frac{1}{2} z^2 - \frac{1}{2} x^2 - \frac{1}{2} x - \frac{1}{2} x^2 z - \frac{1}{2} z \quad \textbf{(4)}$$

Using that z =< (j-1)x, j\ge 7, f> j-2 and 3/2(x+z) \le xz (since x >=2, z>=12), we note that $x f z - x z - \frac{3}{2} x - \frac{3}{2} z - \frac{1}{2} z^2$ > (j-7)/2 xz>=0,

The remaining is also positive, since f >5 and

$\frac{5}{2} x^2 z - \frac{1}{2} x^2 - \frac{1}{2} x - \frac{1}{2} x^2 z - \frac{1}{2} z$> 0

$\frac{5}{2} x z^2 - \frac{1}{4} x z^2 - \frac{3}{2} x z - \frac{1}{2} z^2 > 0$

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Finally, we consider the case j=6 separately. In this case, if z>=11, we have that f \ge 48/11>4,
We now also use that sum_{2 \le i\le 6}(Ni^2) \ge 1/5*z^2 (QM-AM)

> $L := (x, z, k) \rightarrow (x \cdot 4 \cdot z + 1) \cdot \left(1 + k + \frac{(x+z)}{2}\right) + \frac{x}{2} \cdot \left(x + k \cdot x + \frac{x^2}{2}\right) + \frac{1}{10} k \cdot z^2$

$$L := (x, z, k) \rightarrow (4\,x\,z + 1)\left(1 + k + \frac{1}{2}\,x + \frac{1}{2}\,z\right) + \frac{1}{2}\,x\left(x + k\,x + \frac{1}{2}\,x^2\right) + \frac{1}{10}\,k\,z^2 \quad \textbf{(5)}$$

> $R := (x, z, k) \rightarrow \left(1 + \frac{(x+z)}{2}\right) \cdot \left(\left(x + k \cdot x + \frac{x^2}{2}\right) + z \cdot (k+1) + \frac{x \cdot z}{2} + k - 6\right)$

$$R := (x, z, k) \rightarrow \left(1 + \frac{1}{2}\,x + \frac{1}{2}\,z\right)\left(x + k\,x + \frac{1}{2}\,x^2 + z\,(k+1) + \frac{1}{2}\,x\,z + k - 6\right) \quad \textbf{(6)}$$

> $collect(simplify(L(x, z, k) - R(x, z, k)), k)$

$$\left(-\frac{2}{5}\,z^2 + 3\,x\,z - \frac{3}{2}\,z - \frac{3}{2}\,x\right)k + 7 + \frac{7}{4}\,x\,z^2 + \frac{5}{2}\,x\,z - \frac{1}{2}\,z^2 - \frac{1}{2}\,x^2 + \frac{5}{2}\,x + \frac{3}{2}\,x^2\,z + \frac{5}{2}\,z \quad \textbf{(7)}$$

Since z<=5x, x>=3 and z>=11, we have that

$-\frac{2}{5}\,z^2 + 3\,x\,z - \frac{3}{2}\,z - \frac{3}{2}\,x \geq x\,z - \frac{3}{2}\,z - \frac{3}{2}\,x > 0$

and *so clearly the expression above is positive*

When z=10, we can turn back, and do the computations more precise.
Note that when Ni=2, we have mu_i^bullet=1/2 exactly.
We now verify one extreme, where also mu_1^bullet = (N1-1)/2.

> $j := 6$

$$j := 6 \quad \textbf{(8)}$$

> $LHS := (N1, N2, k, mu1, mu2) \rightarrow (N1 \cdot N2^5 + 1) \cdot (1 + mu1 + 5 \cdot mu2 + k) + k \cdot (1 + k - j) + mu1 \cdot (1 + k + mu1) \cdot N1 + 5 \cdot mu2 \cdot (1 + k + mu2) \cdot N2$

$$LHS := (N1, N2, k, \mu1, \mu2) \rightarrow (N1\,N2^5 + 1)\,(1 + \mu1 + 5\,\mu2 + k) + k\,(1 + k - j) + \mu1\,(1 + k + \mu1)\,N1 + 5\,\mu2\,(1 + k + \mu2)\,N2 \quad \textbf{(9)}$$

> $RHS := (N1, N2, k, mu1, mu2) \rightarrow (1 + mu1 + 5 \cdot mu2) \cdot ((1 + k + mu1) \cdot N1 + 5 \cdot (1 + k + mu2) \cdot N2 + k - j)$

$$RHS := (N1, N2, k, \mu1, \mu2) \rightarrow (1 + \mu1 + 5\,\mu2)\,((1 + k + \mu1)\,N1 + (5 + 5\,k + 5\,\mu2)\,N2 + k - j) \quad \textbf{(10)}$$

> $Dif := (N1, N2, k, mu1, mu2) \rightarrow LHS(N1, N2, k, mu1, mu2) - RHS(N1, N2, k, mu1, mu2)$

$$Dif := (N1, N2, k, \mu1, \mu2) \rightarrow LHS(N1, N2, k, \mu1, \mu2) - RHS(N1, N2, k, \mu1, \mu2) \quad \textbf{(11)}$$

> $collect\left(simplify\left(Dif\left(x, 2, k, \frac{x-1}{2}, \frac{1}{2}\right)\right), k\right)$

$$k^2 + (23\,x - 32)\,k + \frac{361}{4}\,x + \frac{57}{4}\,x^2 - \frac{33}{2} \quad \textbf{(12)}$$

The latter is clearly positive since x>=2,

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