

$$> s := \frac{(n+k-3)}{2}$$

$$s := \frac{1}{2} n + \frac{1}{2} k - \frac{3}{2} \quad (1)$$

$$> l := s - k + 1$$

$$l := \frac{1}{2} n - \frac{1}{2} k - \frac{1}{2} \quad (2)$$

After having related s and l in terms of n and k,  
we compute N(T) and R(T) in terms of the coefficients.

The main terms in the simplified versions are the ones with  $2^l$

We count

all k-cliques that are not part of the stem (there are l of them)

all subtrees (sub-k-trees) containing the whole stem;  $2^{l+2}$  since we can add or not  
add any simplicial vertex independently

subtrees containing the stem, but not both endvertices

subtrees containing at least k vertices of the stem, and not both endvertices

$$> N := k \cdot (n-k) + 1 - l + 2^{l+2} + 2 \cdot \text{sum}(2^i, i=2..l) + \text{sum}((l-1-i) \cdot 2^i, i=1..l-2)$$

$$N := k(n-k) - \frac{11}{2} - \frac{3}{2} n + \frac{3}{2} k + 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{3}{2}} + 2 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} \quad (3)$$

$$+ \frac{1}{2} 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} n - \frac{1}{2} 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} k + \frac{1}{2} 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \\ - 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \left( \frac{1}{2} n - \frac{1}{2} k - \frac{3}{2} \right)$$

$$> \text{simplify}(N)$$

$$-k^2 + kn + 9 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{1}{2}} + \frac{3}{2} k - \frac{3}{2} n - \frac{11}{2} \quad (4)$$

This is approximately  $9 \cdot 2^l$

We multiply with the average order.

the k-cliques have order k

the stem has order s and the l+2 k-leaves are in exactly half of the subtrees

if we take (i-1)+(k-1) consecutive vertices of the stem, there are i additional leaves  
(appearing on average in half of the subtrees)

i+(k-1) vertices of the stem + i/2 k-leaves on average

$$> R := (k \cdot (n-k) + 1 - l) \cdot k + 2^{l+2} \cdot \left( s + \frac{(l+2)}{2} \right) + 2 \cdot \text{sum} \left( 2^i \cdot \left( \frac{i}{2} + i + k - 2 \right), i=2 \right. \\ \left. ..l \right) + \text{sum} \left( (l-1-i) \cdot 2^i \cdot \left( i + k - 1 + \frac{i}{2} \right), i=1..l-2 \right)$$

$$R := \left( k(n-k) + \frac{3}{2} - \frac{1}{2} n + \frac{1}{2} k \right) k + 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{3}{2}} \left( \frac{3}{4} n + \frac{1}{4} k - \frac{3}{4} \right) \quad (5)$$

$$\begin{aligned}
& + 2 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} k - 10 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} + 3 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} \left( \frac{1}{2}n - \frac{1}{2}k + \frac{1}{2} \right) \\
& - \frac{19}{2} k + \frac{39}{2} + \frac{1}{2} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} k n - 2 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} n - \frac{1}{2} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} k^2 \\
& + \frac{5}{2} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} k - 5 \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} + \frac{3}{4} n \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \left( \frac{1}{2}n - \frac{1}{2}k - \frac{3}{2} \right) \\
& - \frac{7}{4} k \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \left( \frac{1}{2}n - \frac{1}{2}k - \frac{3}{2} \right) + \frac{19}{4} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \left( \frac{1}{2}n - \frac{1}{2}k - \frac{3}{2} \right) \\
& - \frac{3}{2} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k - \frac{3}{2}} \left( \frac{1}{2}n - \frac{1}{2}k - \frac{3}{2} \right)^2 - k n + \frac{5}{2} n + k^2
\end{aligned}$$

> simplify(R)

$$\begin{aligned}
& -k^3 + k^2 n + \frac{9}{8} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} k + \frac{27}{8} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} n + \frac{3}{2} k^2 - \frac{3}{2} k n \\
& - \frac{111}{8} \cdot 2^{\frac{1}{2}n - \frac{1}{2}k + \frac{1}{2}} - 8 k + \frac{5}{2} n + \frac{39}{2}
\end{aligned} \tag{6}$$

Since the exponents are  $\lfloor l+1 \rfloor$ , we can see that R is dominated by  $(9/4k+27/4n-111/4) \cdot 2^{\lfloor l \rfloor}$

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