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In this document, we verify the essential inequality necessary to prove 2M_T>mu(T;v) by considering the critical values of mu^bullet, when there are 2 sub-k-trees

We first write the formulas in general

> $LHS := (N1, N2, k, mu1, mu2) \to (N1 \cdot N2 + 1) \cdot (1 + mu1 + mu2 + k) + k \cdot (k-1) + mu1 \cdot (1 + k + mu1) \cdot N1 + mu2 \cdot (1 + k + mu2) \cdot N2$

$$LHS := (N1, N2, k, \mu1, \mu2) \to (N1\,N2 + 1)\,(1 + \mu1 + \mu2 + k) + k\,(k-1) + \mu1\,(1 + k + \mu1)\,N1 + \mu2\,(1 + k + \mu2)\,N2 \quad (1)$$

> $RHS := (N1, N2, k, mu1, mu2) \to (1 + mu1 + mu2) \cdot ((1 + k + mu1) \cdot N1 + (1 + k + mu2) \cdot N2 + k - 2)$

$$RHS := (N1, N2, k, \mu1, \mu2) \to (1 + \mu1 + \mu2)\,((1 + k + \mu1)\,N1 + (1 + k + \mu2)\,N2 + k - 2) \quad (2)$$

> $Dif := (N1, N2, k, mu1, mu2) \to LHS(N1, N2, k, mu1, mu2) - RHS(N1, N2, k, mu1, mu2)$

$$Dif := (N1, N2, k, \mu1, \mu2) \to LHS(N1, N2, k, \mu1, \mu2) - RHS(N1, N2, k, \mu1, \mu2) \quad (3)$$

> $simplify(Dif(N1, N2, k, 0, 0))$

$$N1\,N2\,k + N1\,N2 - N1\,k - N2\,k + k^2 - N1 - N2 - k + 3 \quad (4)$$

> $collect\left(simplify\left(Dif\left(N1, N2, k, 0, \frac{(N2)}{2}\right)\right), k\right)$

$$k^2 + \left(\frac{1}{2}\,N1\,N2 - 1 - N1 - \frac{3}{2}\,N2\right)k + \frac{1}{2}\,N1\,N2 + 3 + \frac{1}{2}\,N1\,N2^2 + \frac{1}{2}\,N2 - \frac{1}{2}\,N2^2 - N1 \quad (5)$$

> $collect\left(simplify\left(Dif\left(N1, N2, k, \frac{(N1)}{2}, \frac{(N2)}{2}\right)\right), k\right)$

$$k^2 + \left(-\frac{3}{2}\,N1 - \frac{3}{2}\,N2 - 1\right)k + 3 + \frac{1}{4}\,N1\,N2^2 - \frac{1}{2}\,N2^2 + \frac{1}{2}\,N2 + \frac{1}{4}\,N1^2\,N2 - \frac{1}{2}\,N1^2 + \frac{1}{2}\,N1 \quad (6)$$

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> $f1(N1, N2, k) := Dif(N1, N2, k, 0, 0)$

$$f1 := (N1, N2, k) \to Dif(N1, N2, k, 0, 0) \quad (7)$$

> $f2(N1, N2, k) := Dif\left(N1, N2, k, \frac{N1}{2}, 0\right)$

$$f2 := (N1, N2, k) \to Dif\left(N1, N2, k, \frac{1}{2}\,N1, 0\right) \quad (8)$$

> $f3(N1, N2, k) := Dif\left(N1, N2, k, \frac{N1-1}{2}, \frac{N2-1}{2}\right)$

$$f3 := (N1, N2, k) \to Dif\left(N1, N2, k, \frac{1}{2}\,N1 - \frac{1}{2}, \frac{1}{2}\,N2 - \frac{1}{2}\right) \quad (9)$$

> $minimize(f1(N1, N2, k), N1 = 2..infinity, N2 = 2..infinity, k = 2..infinity)$

(10)

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> minimize(f2(N1, N2, k), N1 = 2 ..infinity, N2 = 2 .. infinity, k = 2 ..infinity)
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$$2 \tag{11}$$

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> minimize(f3(N1, N2, k), N1 = 2 ..infinity, N2 = 2 .. infinity, k = 2 ..infinity)
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$$1 \tag{12}$$

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