>

Here we denote \overline N_A as Na2 etc

> mu := $(Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \rightarrow \frac{(Ra \cdot Nb + Rb \cdot Na + Ra + Ra2 + Rb + Rb2)}{Na \cdot Nb + Na + Nb + Na2 + Nb2}$

$$\mu := (Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \mapsto \frac{Ra \cdot Nb + Rb \cdot Na + Ra + Ra2 + Rb + Rb2}{Na \cdot Nb + Na + Nb + Na2 + Nb2} \quad (1)$$

> $mue := (Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \rightarrow \frac{(Ra \cdot Nb + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2)}{Na \cdot Nb + Na2 + Nb2}$

$$mue := (Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \mapsto \frac{Ra \cdot Nb + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2}{Na \cdot Nb + Na2 + Nb2} \quad (2)$$

> $Pos := (Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \rightarrow simplify\Big(2 \cdot (Na \cdot Nb + Na2 + Nb2) \cdot (Na \cdot Nb + Na + Nb + Na2 + Nb2) \cdot \Big(\mu(Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) - mue(Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) - \frac{11}{20}\Big)\Big)$

$$Pos := (Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) \mapsto simplify\Big((2 \cdot Na \cdot Nb + 2 \cdot Na2 + 2 \cdot Nb2) \cdot (Na \cdot Nb + Na + Nb + Na2 + Nb2) \cdot \Big(\mu(Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) - mue(Ra, Na, Rb, Nb, Rb2, Ra2, Na2, Nb2) - \frac{11}{20}\Big)\Big) \quad (3)$$

We verify that the function is increasing in Rb.

> $collect(simplify(expand(Pos(8, 4, Rb, Nb, Rb2, 2, 2, Nb2))), Rb)$

$$(2\,Nb2 - 28)\,Rb + \frac{14}{5} - \frac{11\,Nb2^2}{10} + \frac{(-19\,Nb + 72)\,Nb2}{10} + 2\,Nb^2 + \frac{(33 - 10\,Rb2)\,Nb}{5} - 8\,Rb2 \quad (4)$$

It is thus sufficient to consider a lowerbound for Rb.
We use that R_b \ge N_b+N_b *Rb2/Nb2

> $G(Nb, Rb2, Nb2) := simplify\Big(expand\Big(Pos\Big(8, 4, \frac{Rb2 \cdot Nb}{Nb2} + Nb, Nb, Rb2, 2, 2, Nb2\Big)\Big)\Big)$

$$G := (Nb, Rb2, Nb2) \mapsto simplify\Big(expand\Big(Pos\Big(8, 4, \frac{Rb2 \cdot Nb}{Nb2} + Nb, Nb, Rb2, 2, 2, Nb2\Big)\Big)\Big) \quad (5)$$

> $diff(\ G(Nb, Rb2, Nb2), Rb2)$

$$-\frac{28\,Nb}{Nb2} - 8 \quad (6)$$

This function is decreasing in Rb2, so it is sufficient to consider an upper bound.
One upperbound is Rb2 <= Nb2*mu_b, where mu_B < 3/2*log_2(N_b)-1

> $F(Nb, Nb2) := G\Big(Nb, \Big(\frac{3}{2} \cdot log\_2(Nb) - 1\Big) \cdot Nb2, Nb2\Big)$

(7)

$$F := (Nb, Nb2) \mapsto G\left(Nb, \left(\frac{3 \cdot log\_2(Nb)}{2} - 1\right) \cdot Nb2, Nb2\right) \tag{7}$$

> $collect(F(Nb, Nb2), Nb2)$

$$-\frac{11\, Nb2^2}{10} + \left(-12\, log\_2(Nb) + \frac{Nb}{10} + \frac{76}{5}\right) Nb2 + \frac{14}{5} - 42\, Nb\, log\_2(Nb) + 2\, Nb^2 + \frac{33\, Nb}{5} \tag{8}$$

The extrema are attained when Nb2 is at one of its boundaries.
Now the conclusion follows as the following two expressions are always positive as Nb\ge 2^9.

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> $F(Nb, Nb)$

$$Nb^2 - 54\, Nb\, log\_2(Nb) + \frac{109\, Nb}{5} + \frac{14}{5} \tag{9}$$

> $F(Nb, 1)$

$$2\, Nb^2 - 42\, Nb\, log\_2(Nb) + \frac{67\, Nb}{10} - 12\, log\_2(Nb) + \frac{169}{10} \tag{10}$$

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