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In this file, we verify that mu(T)- mu(T\e) is at least 11/20 when e=AB is an edge in a series-reduced tree T of order at least 19, where T_A (the component of T\e containing A) is a P_3 whose center is A.

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Here we denote \overline N_A as Na2 etc

> $\mathrm{mu} := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \to \frac{(Ra \cdot Nb + Rb \cdot Na + Ra + Ra2 + Rb + Rb2)}{Na \cdot Nb + Na + Nb + Na2 + Nb2}$

$$\mu := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto \frac{Nb \cdot Ra + Rb \cdot Na + Ra + Ra2 + Rb + Rb2}{Na \cdot Nb + Na + Nb + Na2 + Nb2} \quad (1)$$

> $mue := ((Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2)) \to \frac{(Ra \cdot Nb + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2)}{Na \cdot Nb + Na2 + Nb2}$

$$mue := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto \frac{Nb \cdot Ra + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2}{Na \cdot Nb + Na2 + Nb2} \quad (2)$$

> $Pos := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \to simplify\left(2 \cdot (Na \cdot Nb + Na2 + Nb2) \cdot (Na \cdot Nb + Na + Nb + Na2 + Nb2) \cdot \left(\mu(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - mue(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - \frac{11}{20}\right)\right)$

$$Pos := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto simplify\left((2 \cdot Na \cdot Nb + 2 \cdot Na2 + 2 \cdot Nb2) \cdot (Na \cdot Nb + Na + Nb + Na2 + Nb2) \cdot \left(\mu(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - mue(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - \frac{11}{20}\right)\right) \quad (3)$$

We verify that the function is increasing in Rb.

> $collect(simplify(expand(Pos(8, 4, 2, 2, Rb, Nb, Rb2, Nb2))), Rb)$

$$(2\,Nb2 - 28)\,Rb + \frac{14}{5} - \frac{11\,Nb2^2}{10} + \frac{(-19\,Nb + 72)\,Nb2}{10} + 2\,Nb^2 + \frac{(33 - 10\,Rb2)\,Nb}{5} - 8\,Rb2 \quad (4)$$

It is thus sufficient to consider a lowerbound for Rb.
We use that R_b \ge N_b+N_b *Rb2/Nb2

> $G(Nb, Rb2, Nb2) := simplify\left(expand\left(Pos\left(8, 4, 2, 2, \frac{Rb2 \cdot Nb}{Nb2} + Nb, Nb, Rb2, Nb2\right)\right)\right)$

$$G := (Nb, Rb2, Nb2) \mapsto simplify\left(expand\left(Pos\left(8, 4, 2, 2, \frac{Rb2 \cdot Nb}{Nb2} + Nb, Nb, Rb2, Nb2\right)\right)\right) \quad (5)$$

> $diff(\ G(Nb, Rb2, Nb2), Rb2)$

$$-\frac{28\,Nb}{Nb2}-8 \quad (6)$$

The function G is thus decreasing in Rb2, so it is sufficient to consider an upper bound.
One upperbound is Rb2 <= Nb2*mu_b, where mu_B <= 3/2*log_2(N_b)-1

> $F(Nb, Nb2) := G\left(Nb, \left(\frac{3}{2}\cdot log\_2(Nb)-1\right)\cdot Nb2, Nb2\right)$

$$F := (Nb, Nb2) \mapsto G\left(Nb, \left(\frac{3\cdot log\_2(Nb)}{2}-1\right)\cdot Nb2, Nb2\right) \quad (7)$$

> $collect(F(Nb, Nb2), Nb2)$

$$-\frac{11\,Nb2^2}{10}+\left(-12\,log\_2(Nb)+\frac{Nb}{10}+\frac{76}{5}\right)Nb2+\frac{14}{5}-42\,Nb\,log\_2(Nb)+2\,Nb^2+\frac{33\,Nb}{5} \quad (8)$$

The minima of this parabola are attained when Nb2 is at one of its boundaries.
Now the conclusion follows as the following two expressions are always positive as Nb\ge 2^9 (the number of leaves of T_B is at least 9)

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> $F(Nb, Nb)$

$$Nb^2-54\,Nb\,log\_2(Nb)+\frac{109\,Nb}{5}+\frac{14}{5} \quad (9)$$

> $F(Nb, 1)$

$$2\,Nb^2-42\,Nb\,log\_2(Nb)+\frac{67\,Nb}{10}-12\,log\_2(Nb)+\frac{169}{10} \quad (10)$$

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