In this document, we give an example where studying the difference of contracting an edge results into the exclusion of some trees being optimal trees maximizing the average subtree order among all trees of order n.

First, we consider the formulas for mu(T) and mu(T\e), given the values on the two sides of the edge. Here we denote \overline N_A as Na2 etc

> mu := $(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \rightarrow \frac{(Ra \cdot Nb + Rb \cdot Na + Ra + Ra2 + Rb + Rb2)}{Na \cdot Nb + Na + Nb + Na2 + Nb2}$

$$\mu := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto \frac{Nb \cdot Ra + Rb \cdot Na + Ra + Ra2 + Rb + Rb2}{Na \cdot Nb + Na + Nb + Na2 + Nb2} \tag{1}$$

> $mue := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \rightarrow \frac{(Ra \cdot Nb + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2)}{Na \cdot Nb + Na2 + Nb2}$

$$mue := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto \frac{Nb \cdot Ra + Rb \cdot Na - Na \cdot Nb + Ra2 + Rb2}{Na \cdot Nb + Na2 + Nb2} \tag{2}$$

Differ denotes the difference in mean subtree order by contracting the edge AB, where A and B are the roots of trees T_A and T_B.

> $Differ := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \rightarrow \mu(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - mue(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2)$

$$Differ := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \mapsto \mu(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) - mue(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2) \tag{3}$$

D1 and D2 represent the differences for trees, which are built from three trees T_A, T_B and T_C, with T_A resp. T_C on one side of the edge, and the other two at the other side.

> $D1(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) := Differ(Ra, Na, Ra2, Na2, Rc + Nc \cdot Rb + Rc \cdot Nb, Nc + Nc \cdot Nb, Rb2 + Rb + Rc2, Nb + Nb2 + Nc2)$

$$D1 := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) \mapsto Differ(Ra, Na, Ra2, Na2, Rc + Nc \cdot Rb + Rc \cdot Nb, Nc + Nc \cdot Nb, Rb2 + Rb + Rc2, Nb + Nb2 + Nc2) \tag{4}$$

> $D2(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) := Differ(Rc, Nc, Rc2, Nc2, Ra + Na \cdot Rb + Ra \cdot Nb, Na + Na \cdot Nb, Rb2 + Rb + Ra2, Nb + Nb2 + Na2)$

$$D2 := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) \mapsto Differ(Rc, Nc, Rc2, Nc2, Ra + Rb \cdot Na + Nb \cdot Ra, Na + Na \cdot Nb, Rb2 + Rb + Ra2, Nb + Nb2 + Na2) \tag{5}$$

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Finally, we compare these two differences and conclude that the second difference is larger in the second case, when T_C is the larger one, T_A and T_B are of the order n^{1/2}, and these are broom-like.

> $D3(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) := simplify((D1(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) - D2(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2)) \cdot ((1 + (Nc + 1) Na) Nb + (Nc + 1) Na + Nc + Na2 + Nb2 + Nc2) ((Na Nc + Nc + 1) Nb + (Nc + 1) Na + Nc + Na2 + Nb2 + Nc2))$

$$D3 := (Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) \mapsto simplify((D1(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2) - D2(Ra, Na, Ra2, Na2, Rb, Nb, Rb2, Nb2, Rc, Nc, Rc2, Nc2)) \cdot ((1 + (Nc + 1) \cdot Na) \cdot Nb + (Nc + 1) \cdot Na + Nc + Na2 + Nb2 + Nc2) \cdot ((Na \cdot Nc + Nc + 1) \cdot Nb + (Nc + 1) \cdot Na + Nc + Na2 + Nb2 + Nc2)) \tag{6}$$

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> $collect\left(D3\left(Ra\cdot n^{1.5}, Na\cdot n, Ra2\cdot n^{2}, Na2\cdot n^{1.5}, Rb\cdot n^{1.5}, Nb\cdot n, Rb2\cdot n^{2}, Nb2\cdot n^{1.5}, Rc\cdot n^{3}, Nc\cdot n^{2}, Rc2\cdot n^{4}, Nc2\cdot n^{3}\right), n\right)$

$$-1.\, Nc^{2}\, n^{15/2}\, Nb^{2}\, Ra + \left(Na^{2}\, Nb^{2}\, Rc + (-1.\, Nc\, Rc2 + Nc2\, Rc)\, Nb\right) n^{7} + \left(-1.\, Nb\, Nc^{2}\, Ra + Nc\, Rb\, (Na\, Nc + Nc2)\right) n^{13/2} + \left(Na\, Nb^{2}\, Rc + Na\, (Na\, Rc + Rc2)\, Nb\right) n^{6} + \left(-1.\, Nc\, Nb^{2}\, Ra + (-1.\, Nc2\, Ra + Rc\, (Na2 + Nb2))\, Nb - 1.\, Na^{2}\, Nc\, Rb - 1.\, Na\, Nc2\, Rb + Nc^{2}\, Rb\right) n^{11/2} + \left(Nb^{2}\, Rc + (2.\, Na\, Rc + (-1.\, Ra2 - 1.\, Rb2)\, Nc)\, Nb + Nc\, Rb\, (Na2 + Nb2)\right) n^{5} - 2.\, Nb\, Nc\, n^{9/2}\, Ra + \left(((Ra2 + Rb2)\, Na + (-1.\, Na2 - 1.\, Nb2)\, Ra)\, Nb + (-1.\, Na2 - 1.\, Nb2)\, Rb\, Na\right) n^{4} + \left(-1.\, Na^{2}\, Rb - 1.\, Nb^{2}\, Ra\right) n^{7/2} \tag{7}$$

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