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In this file, we verify some of the computations related with the preliminary results for general trees.

We first prove the identity
$(N_v +1) \overline N_v \ge 3 \overline R_v$
for deg v=1 (with w being the neighbour of v) by induction.
Here we use Nvb for \overline N_v (Nv bar) and analogous.

> $Nv := 1 + Nw$

$$Nv := 1 + Nw \tag{1}$$

> $Nvb := Nw + Nwb$

$$Nvb := Nw + Nwb \tag{2}$$

> $Rv := Nv + Rw$

$$Rv := 1 + Nw + Rw \tag{3}$$

> $Rvb := Rw + Rwb$

$$Rvb := Rw + Rwb \tag{4}$$

> $simplify\left(Nv \cdot Nvb + Nvb - 3 \cdot Rvb - \left(Nw^2 + 2\,Nw + Nwb - 3 \cdot Rw\right) - (Nwb \cdot Nw + Nwb - 3\,Rwb)\right)$

$$0 \tag{5}$$

Next, we verify the identity $3 \cdot (Nvb \cdot Rv - Nv \cdot Rvb) \ge Nvb \cdot (Nv + Nvb)$ when deg v=1.

$$(Nw + Nwb)\,(1 + 2\,Nw + Nwb) \le -3\,(1 + Nw)\,(Rw + Rwb) + 3\,(Nw + Nwb)\,(1 + Nw + Rw) \tag{6}$$

> $simplify(3 \cdot (Nvb \cdot Rv - Nv \cdot Rvb) - Nvb \cdot (Nv + Nvb) - (Nv \cdot Nvb + Nvb - 3 \cdot Rvb) - (3 \cdot (Nwb \cdot Rw - Nw \cdot Rwb) - Nwb \cdot (Nw + Nwb)))$

$$0 \tag{7}$$

Finally, we verify $3 \cdot (Nvb \cdot Rv - Nv \cdot Rvb) \ge Nvb \cdot (Nv + Nvb)$ when deg v \ge 2.
We again use N1b and N2b for overline N_1, overline N_2.

$$(Nw + Nwb)\,(1 + 2\,Nw + Nwb) \le -3\,(1 + Nw)\,(Rw + Rwb) + 3\,(Nw + Nwb)\,(1 + Nw + Rw) \tag{8}$$

> $Nv := N1 \cdot N2$

$$Nv := N1\,N2 \tag{9}$$

> $Nvb := N1b + N2b$

$$Nvb := N1b + N2b \tag{10}$$

> $Rv := R1 \cdot N2 + R2 \cdot N1 - N1 \cdot N2$

$$Rv := -N1\,N2 + R2\,N1 + R1\,N2 \tag{11}$$

> $Rvb := R1b + R2b$

$$Rvb := R1b + R2b \tag{12}$$

> $F := (3 \cdot (N1b \cdot R1 - N1 \cdot R1b) - N1b \cdot (N1 + N1b)) \cdot N2 + (3 \cdot (N2b \cdot R2 - N2 \cdot R2b) - N2b \cdot (N2 + N2b)) \cdot N1 + \left((N2 - 1)\,N1b^2 + (N1 - 1)\,N2b^2\right)$

$$F := (-3\,N1\,R1b + 3\,N1b\,R1 - N1b\,(N1 + N1b))\,N2 + (-3\,N2\,R2b + 3\,N2b\,R2 - N2b\,(N2 \tag{13}$$

$+ N2b)) N1 + (N2 - 1) N1b^2 + (N1 - 1) N2b^2$

> $G := ((3\,R2 - 3\,N2) \cdot N1 - N2b)\,N1b + ((3\,R1 - 3\,N1) \cdot N2 - N1b)\,N2b$

$$G := ((3\,R2 - 3\,N2)\,N1 - N2b)\,N1b + ((3\,R1 - 3\,N1)\,N2 - N1b)\,N2b \qquad \textbf{(14)}$$

> $\mathit{simplify}(3 \cdot (Nvb \cdot Rv - Nv \cdot Rvb) - Nvb \cdot (Nv + Nvb) - F - G)$

$$0 \qquad \textbf{(15)}$$

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