In this file,
we summarize the details of some computations that verify that there are no accumulation points **in** $[0.66, 0.67]$.
The excluded interval is slightly larger, but **for** *simplicity, we stick* **to** *crude* (*simpler*) *estimates.*

First, we observe that the contraction of an edge results **in** *a difference larger than* 0.7, **if** T_A **and** T_B *have both at least* 6 *leaves* •
For this, it is sufficient **to** *note that the following function is positive.*
Hereby we note that R_A \le $N\_A \left(\frac{3}{2} log\_2(N\_A) - 1\right)$ **and** *this divided* **by** N_A^2 *is a decreasing function.*

Finally, we also give more detailed estimates **to** *deduce that* mu(T) − mu$(T\backslash e)$ *is at least* $\frac{59}{99} - o(1)$ *when e is a non − pendent edge of a series − reduced tree.*

> $f(R\_A, N\_A) := 0.3 - \frac{2 \cdot R\_A}{N\_A^2} - \left(0.4 + \frac{1}{(N\_A + 1)}\right) \cdot \left(\frac{2}{N\_A}\right)$

$$f := (R\_A, N\_A) \mapsto 0.3 - \frac{2 \cdot R\_A}{N\_A^2} - \frac{2 \cdot \left(0.4 + \frac{1}{N\_A + 1}\right)}{N\_A} \qquad \textbf{(1)}$$

> $evalf(f(8 \cdot 64, 64))$

$$0.03701923077 \qquad \textbf{(2)}$$

>

In the remaining, we can assume that N_A << N_B, and the following is a lower bound for the difference of the mean,
which can be estimated for trees with N_A \ge 20, and the two remaining trees with 4 leaves.

> $Low(R\_A, N\_A) := 1 - \frac{2}{N\_A + 2} - \frac{(R\_A - N\_A)}{(N\_A + 1) \cdot N\_A}$

$$Low := (R\_A, N\_A) \mapsto 1 - \frac{2}{N\_A + 2} - \frac{R\_A - N\_A}{(N\_A + 1) \cdot N\_A} \qquad \textbf{(3)}$$

> $Low(6.5 \cdot 32, 32)$

$$0.7745098039 \qquad \textbf{(4)}$$

> $evalf\left(Low\left(\left(\frac{3}{2} \cdot \frac{\log(20)}{\log(2)} - 1\right) \cdot 20, 20\right)\right)$

$$0.6956198544 \qquad \textbf{(5)}$$

> $evalf(Low(16 \cdot 3, 16))$

$$0.7712418301 \qquad \textbf{(6)}$$

> $evalf(Low(18 \cdot 4, 18))$

$$0.7421052632 \qquad \textbf{(7)}$$

>

The above was a weaker lower bound for the lower bound below, which is complemented with an upper bound.

> $Low(R\_A, N\_A, c) := 1 - \frac{1+c}{N\_A+1+c} - \frac{\left(R\_A - N\_A - \frac{c\cdot(1+c)}{3}\right)}{(N\_A+1+c)\cdot(N\_A+c)};$

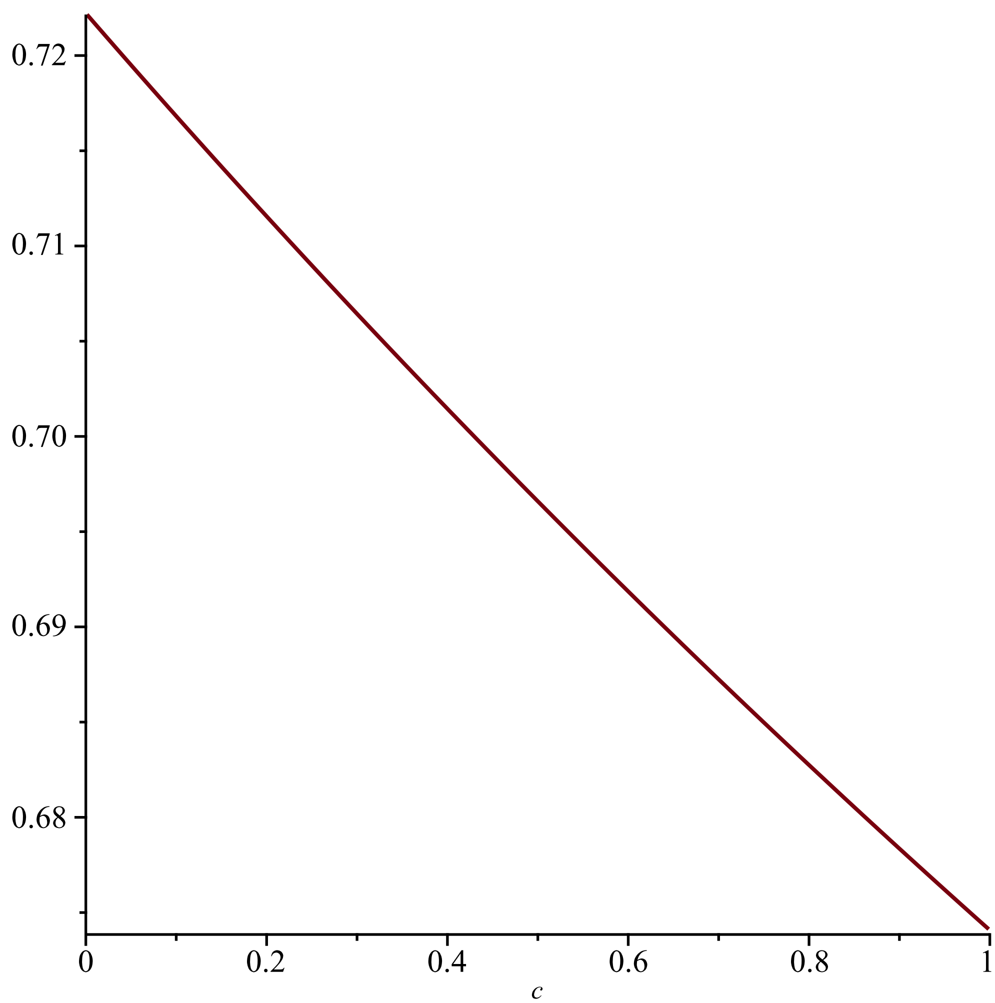
$$Low := (R\_A, N\_A, c) \mapsto 1 - \frac{c+1}{N\_A+1+c} - \frac{R\_A - N\_A - \frac{c\cdot(c+1)}{3}}{(N\_A+1+c)\cdot(N\_A+c)} \quad \textbf{(8)}$$

> $Upp(R\_A, N\_A, c) := 1 - \frac{1+c}{N\_A+1+c} - \frac{(R\_A - N\_A - (1+2\cdot c))}{(N\_A+1+c)\cdot(N\_A+c)}$

$$Upp := (R\_A, N\_A, c) \mapsto 1 - \frac{c+1}{N\_A+1+c} - \frac{R\_A - N\_A - 1 - 2\cdot c}{(N\_A+1+c)\cdot(N\_A+c)} \quad \textbf{(9)}$$

Finally, we estimate lower bounds for the 2 trees T_A with 3 leaves, and an upper bound for S_3.

> $plot(Low(8\cdot 2.5, 8, c), c = 0..1);$
> $Low(20, 8, 1); evalf(Low(20, 8, 1));$

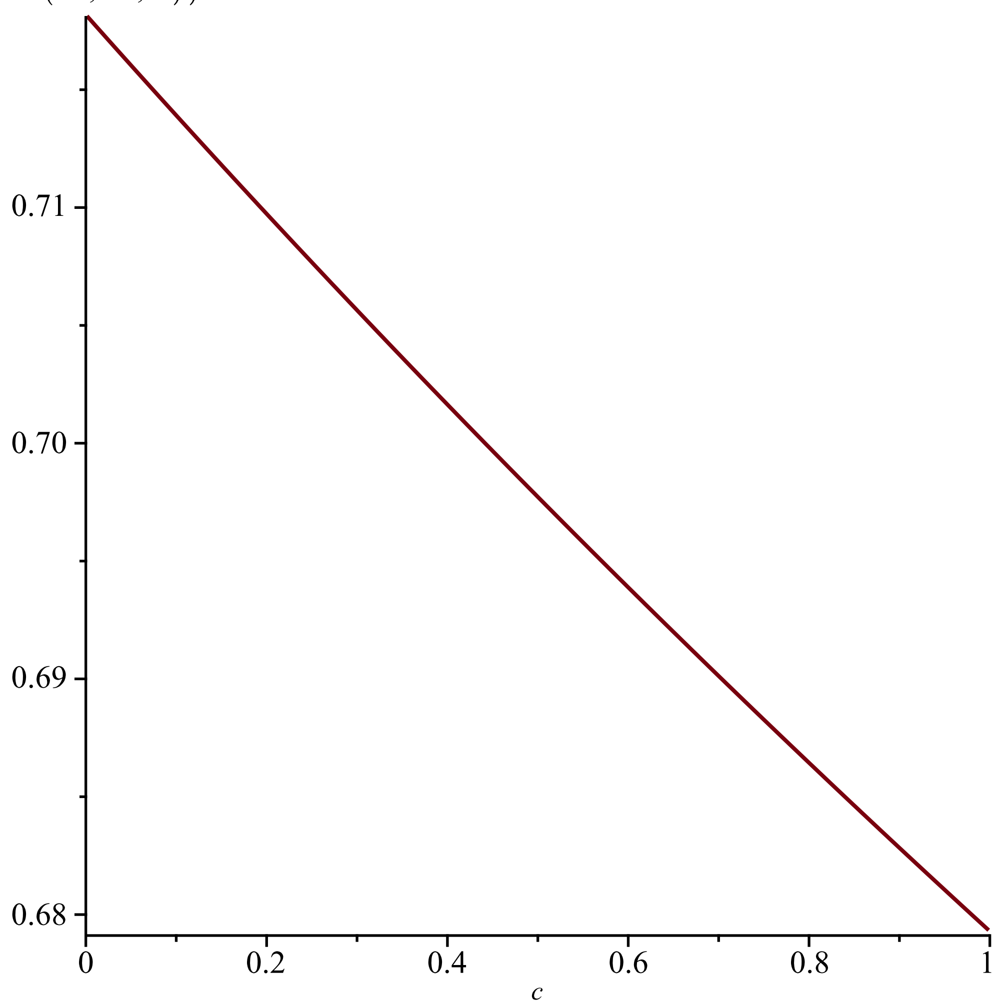


$$\frac{91}{135}$$

$$0.6740740741 \tag{10}$$

>

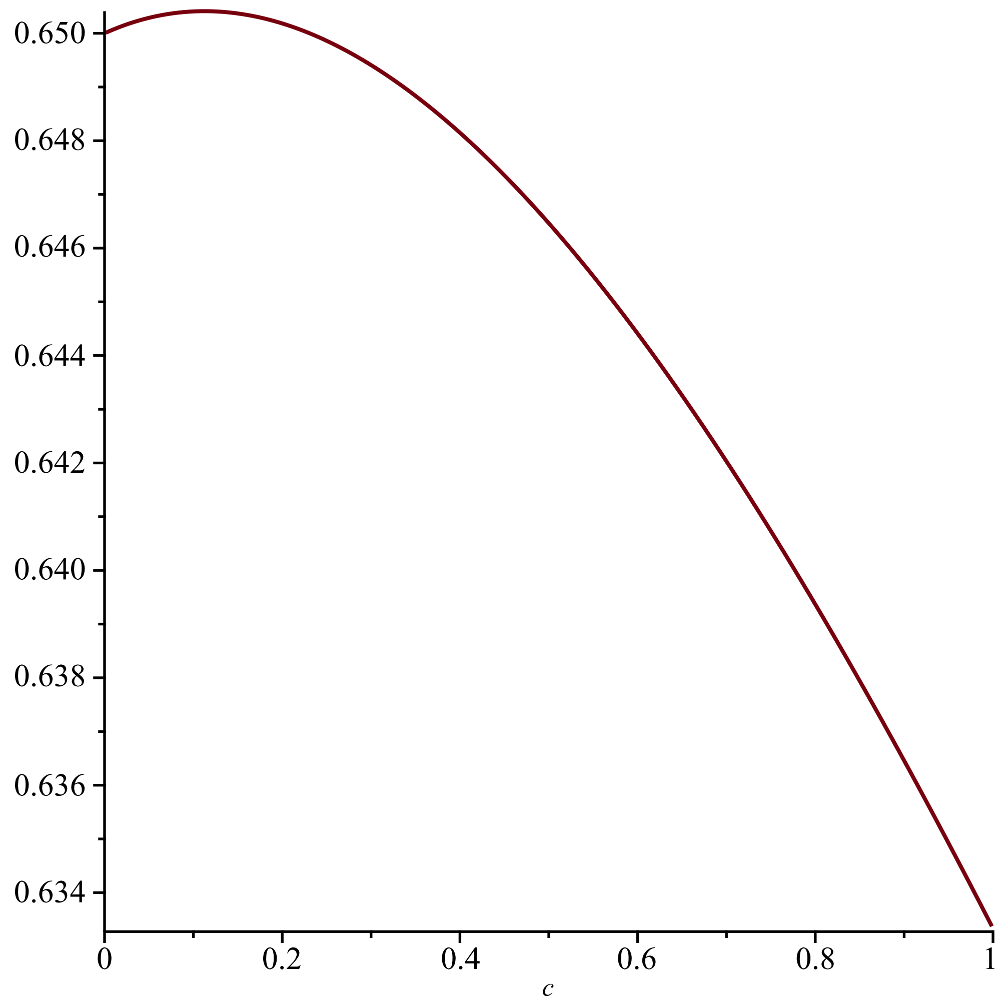
> $plot(Low(31, 10, c), c = 0..1); Low(31, 10, 1);$
$evalf(Low(31, 10, 1))$



$$\frac{269}{396}$$

$$0.6792929293 \tag{11}$$

> $plot(Upp(8, 4, c), c = 0..1)$



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>

The above was a weaker lower bound for the lower bound below, which is complemented with an upper bound.
Using that in T_B, deg(B) \ge 2 and thus mu_B \ge \overline\mu_B + 3/2-o(1), we can write more precise formulas in terms of c = \overline N_B / N_B.
If T_B \ B has more than 2 components, or its smallest component is not a singleton, then mu_B \ge \overline\mu_B + 2-o(1).

In the other case, T_B \backslash B is a partitioned in a singleton and a tree T_C, whose root C (neighbour of B) has degree at least 2.
Since C has degree at least 2, we derive that \mu_C \ge \overline \mu_C + 3/2-o(1).
Since N_B=2(1+N_C) \sim 2N_C and \overline N_B = \overline N_C + N_C+1 \sim \overline N_C + N_C,
we derive that c'=N_C/(\overline N_C + N_C) \sim 1/(2*c) when c>1/2.
In that case \overline \mu_B \le mu(T_C) \le c' \mu_C + (1-c')(\mu_C-3/2+o(1)).
Combined with mu_C \le \mu_B-3/2+o(1), this implies that
$\mu_B - \overline \mu_B \ge 3-3/(4*c)$ when $c \ge 1/2$.

Using the above observations, we consider the following 3 lower bounds.
As such, we can also conclude that 59/99-o(1) is only attained when c=1/2 and c' \sim 1.

$$> Low(R\_A, N\_A, c) := 1 - \frac{1+c}{N\_A+1+c} - \frac{\left(R\_A - N\_A - \frac{3}{2}\cdot c\right)}{(N\_A+1+c)\cdot(N\_A+c)};$$

$$Low := (R\_A, N\_A, c) \mapsto 1 - \frac{c+1}{N\_A+1+c} - \frac{R\_A - N\_A - \frac{3\cdot c}{2}}{(N\_A+1+c)\cdot(N\_A+c)} \tag{12}$$

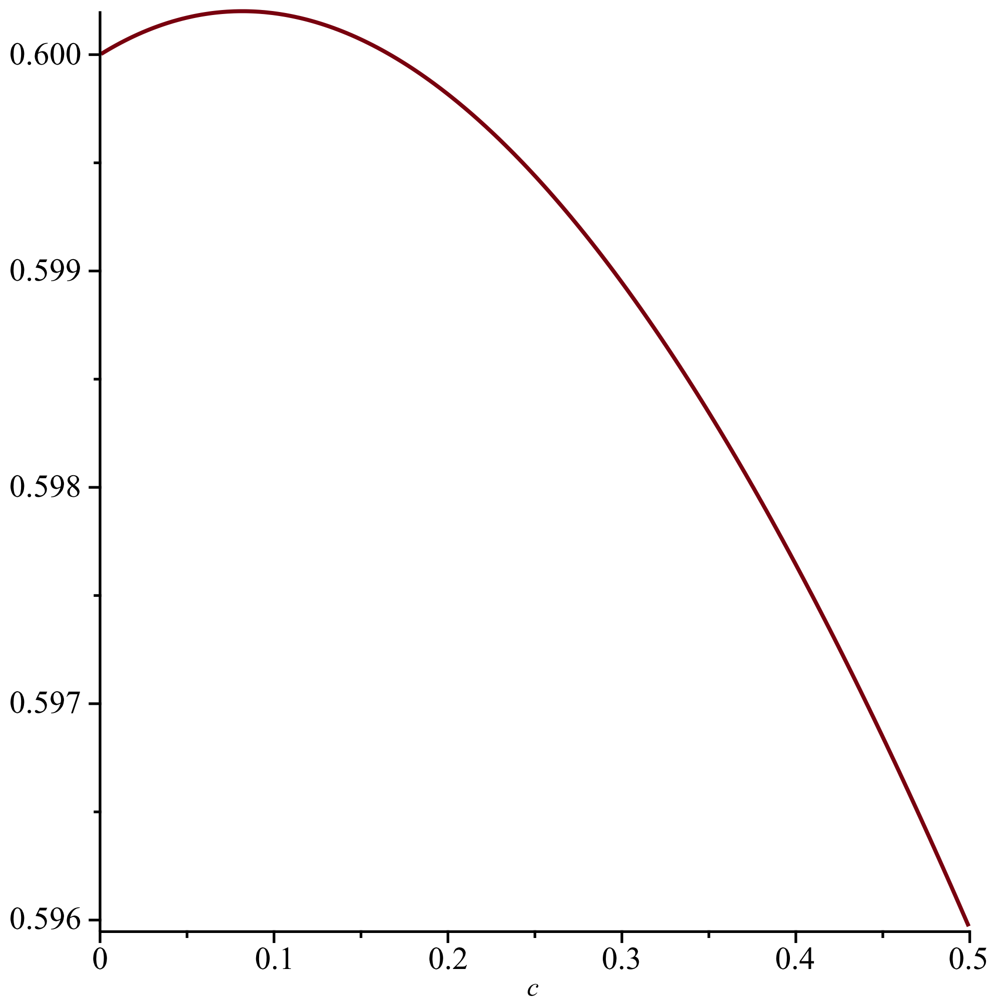
$$> Low1(R\_A, N\_A, c) := 1 - \frac{1+c}{N\_A+1+c} - \frac{(R\_A - N\_A - c\cdot 2)}{(N\_A+1+c)\cdot(N\_A+c)};$$

$$Low1 := (R\_A, N\_A, c) \mapsto 1 - \frac{c+1}{N\_A+1+c} - \frac{R\_A - N\_A - 2\cdot c}{(N\_A+1+c)\cdot(N\_A+c)} \tag{13}$$

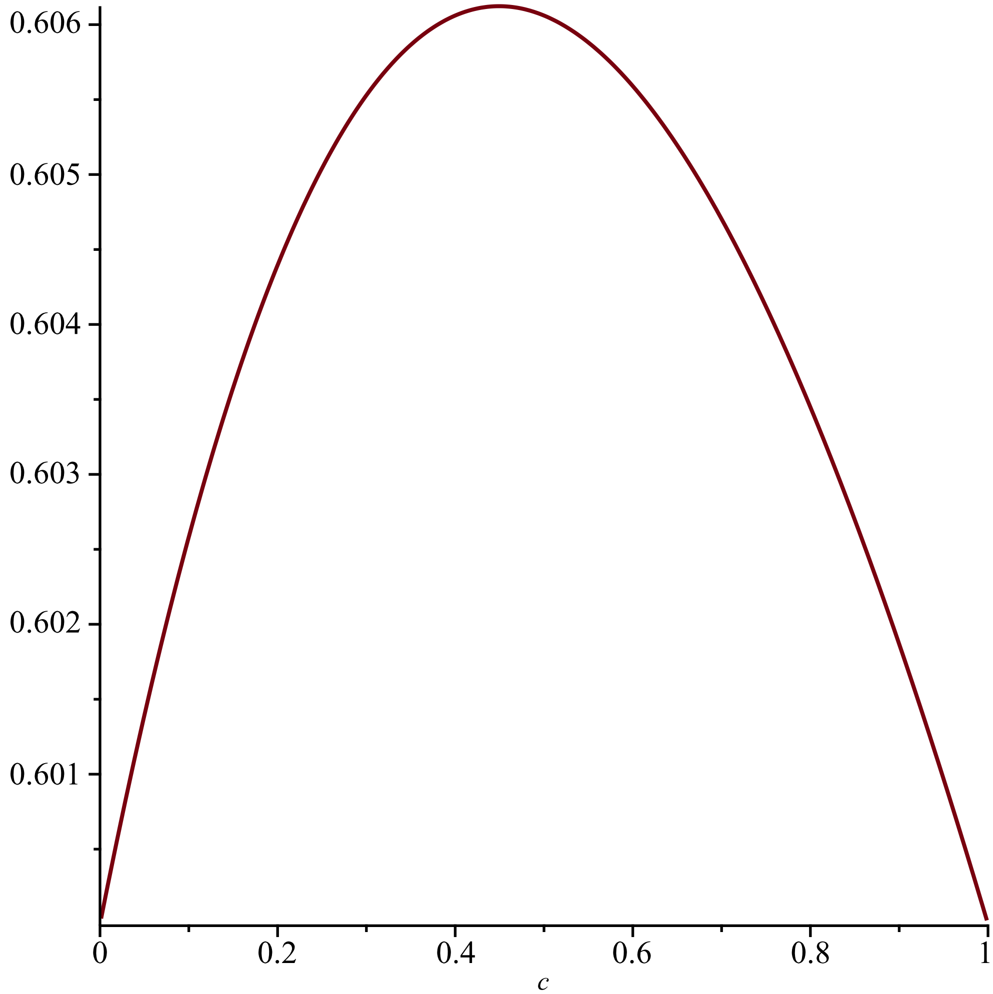
$$> Low2(R\_A, N\_A, c) := 1 - \frac{1+c}{N\_A+1+c} - \frac{\left(R\_A - N\_A - \left(3 - \frac{3}{4\cdot c}\right)\cdot c\right)}{(N\_A+1+c)\cdot(N\_A+c)};$$

$$Low2 := (R\_A, N\_A, c) \mapsto 1 - \frac{c+1}{N\_A+1+c} - \frac{R\_A - N\_A - \left(3 - \frac{3}{4\cdot c}\right)\cdot c}{(N\_A+1+c)\cdot(N\_A+c)} \tag{14}$$

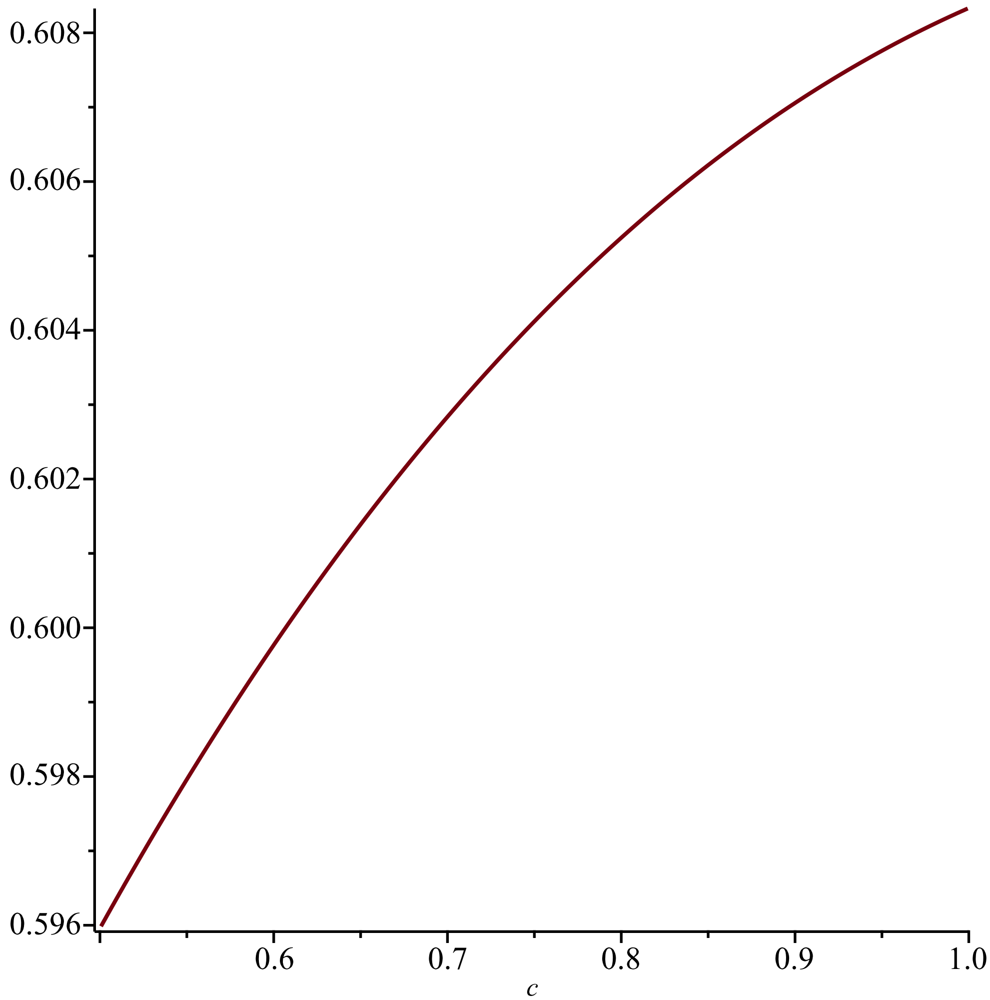
$$> plot(Low(8, 4, c), c = 0..0.5)$$



> $plot(Low1(8, 4, c), c = 0 ..1)$



> $plot(Low2(8, 4, c), c = 0.5 ..1)$



> $Low\left(8, 4, \frac{1}{2}\right);$
> $Low2\left(8, 4, \frac{1}{2}\right)$

$$\frac{59}{99}$$

$$\frac{59}{99} \tag{15}$$