

$$\begin{aligned}
& > f(R_A, N_A, N_B) := 0.45 - \frac{R_A}{N_A^2} - \frac{\left(\frac{1.5 \cdot \log(N_B)}{\log(2)} - 1 \right)}{N_B} - \left(0.1 + \frac{1}{(N_A + 1)} \right) \\
& \quad \cdot \left(\frac{1}{N_A} + \frac{1}{N_B} \right) \\
& f := (R_A, N_A, N_B) \mapsto 0.45 - \frac{R_A}{N_A^2} - \frac{\frac{1.5 \cdot \log(N_B)}{\log(2)} - 1}{N_B} - \left(0.1 + \frac{1}{N_A + 1} \right) \cdot \left(\frac{1}{N_A} \right. \\
& \quad \left. + \frac{1}{N_B} \right) \quad (1)
\end{aligned}$$

The 5 cases of a rooted tree T_A with 4 leaves; stem being a single vertex, K_2 (2 possible T_A), rooted P_3 or rooted S_3

Here for R_A, an upperbound equal to N_A*(ell/2+s) has been used, where s is the number of vertices in the stem, and ell=4 the number of leaves

$$\begin{aligned}
& > f(16 \cdot 3, 16, 2^7) \\
& \quad \quad \quad 0.1771139706 \quad (2)
\end{aligned}$$

$$\begin{aligned}
& > f(18 \cdot 4, 18, 2^7) \\
& \quad \quad \quad 0.1438870614 \quad (3)
\end{aligned}$$

$$\begin{aligned}
& > f(20 \cdot 4, 20, 2^7) \\
& \quad \quad \quad 0.1672470238 \quad (4)
\end{aligned}$$

$$\begin{aligned}
& > f(22 \cdot 5, 22, 2^7) \\
& \quad \quad \quad 0.1408658597 \quad (5)
\end{aligned}$$

$$\begin{aligned}
& > f(25 \cdot 5, 25, 2^7) \\
& \quad \quad \quad 0.1691610577 \quad (6)
\end{aligned}$$

The 2 cases where T_A has 3 leaves; stem with one vertex or 2 (S_4 or S_4 with one edge subdivided)

$$\begin{aligned}
& > f(8 \cdot 2.5, 8, 2^7) \\
& \quad \quad \quad 0.03524305556 \quad (7)
\end{aligned}$$

$$\begin{aligned}
& > f(31, 10, 2^7) \\
& \quad \quad \quad 0.04519886364 \quad (8)
\end{aligned}$$

Additionally, the computation for the case where N_A, N_B >= 32, with also verification that the ratio is decreasing (ln(N_B) > 1 + 1.5/ln(2))

$$\begin{aligned}
& > 0.45 - \frac{2 \cdot 6.5}{32} - \left(0.1 + \frac{1}{33} \right) \cdot \left(\frac{1}{32} \cdot 2 \right) \\
& \quad \quad \quad 0.03560606061 \quad (9)
\end{aligned}$$

$$\begin{aligned}
& > \text{diff} \left(\frac{\left(\frac{1.5 \cdot \log(N_B)}{\log(2)} - 1 \right)}{N_B}, N_B \right) \\
& \quad \quad \quad \frac{1.5}{N_B^2 \ln(2)} - \frac{\frac{1.5 \ln(N_B)}{\ln(2)} - 1}{N_B^2} \quad (10)
\end{aligned}$$

