Formula for Wiener index of path P_l (l vertices, tree in usual sense)

> $W(l) := \text{binomial}(l+1, 3)$

$$W := l \mapsto \binom{l+1}{3} \tag{1}$$

claimed optimum when n=ks+r and 0 \le r<k for total distance of connected k-uniform hypergraph of order n

> $f(s, k, r) := (s+1)\cdot \text{binomial}(r, 2) + s\cdot\text{binomial}(k-r, 2) + r^2\cdot(W(s+1)\cdot 2) + (k-r)^2 \cdot(W(s)\cdot 2) + r\cdot(k-r)\cdot(W(2\cdot s+1) - 2\cdot W(s) - 2\cdot W(s+1))$

$$f := (s, k, r) \mapsto (s+1)\cdot\binom{r}{2} + s\cdot\binom{k-r}{2} + 2\cdot r^2\cdot W(s+1) + 2\cdot(k-r)^2\cdot W(s) + r\cdot(k-r) \cdot(W(2\cdot s+1) - 2\cdot W(s) - 2\cdot W(s+1)) \tag{2}$$

> $simplify(expand(f(s, k, r)))$

$$\frac{k^2 s^3}{3} + r k s^2 + \frac{(k^2 + 6 r^2 - 3 k) s}{6} + \frac{r^2}{2} - \frac{r}{2} \tag{3}$$

maximum transmission of l additional vertices, when n=k*s+r other vertices are there
g1: if k-l < r
g2: if r \le k-l

> $g1(l, k, s, r) := k\cdot s^2 + l\cdot s + r\cdot(2\cdot s+1)$

$$g1 := (l, k, s, r) \mapsto k\cdot s^2 + l\cdot s + r\cdot(2\cdot s+1) \tag{4}$$

> $g2(l, k, s, r) := k\cdot s^2 + l\cdot s + r\cdot 2\cdot s$

$$g2 := (l, k, s, r) \mapsto k\cdot s^2 + l\cdot s + 2\cdot r\cdot s \tag{5}$$

> $simplify(\ expand(f(s+1, k, r+l-k) - (f(s, k, r) + l\cdot g1(l, k, s, r) + \text{binomial}(l, 2))))$

$$(-r-l+k)^2 \tag{6}$$

> $simplify(\ expand(f(s, k, r+l) - (f(s, k, r) + l\cdot g2(l, k, s, r) + \text{binomial}(l, 2))))$

$$r\,l \tag{7}$$