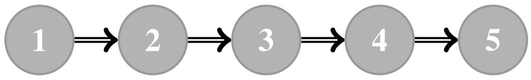
Non-cyclical timetabling: Problem description

You are the Infrastructure Manager of the following one-way single corridor railway network with five stations $S = \{1,2,3,4,5\}$.



You have collected the requests of different Train Operators for a total of four trains $T = \{A,B,C,D\}$. The first station f_j, the last station l_j, and the ordered set S_j of stations visited by train $j \in T$ are:

$$f_A = 1 \quad l_A = 5 \quad S_A = \{1,2,3,4,5\}$$
$$f_B = 1 \quad l_B = 5 \quad S_B = \{1,2,3,4,5\}$$
$$f_C = 1 \quad l_C = 3 \quad S_C = \{1,2,3\}$$
$$f_D = 3 \quad l_D = 5 \quad S_D = \{3,4,5\}$$

Table 1 reports, for each train $j \in T$, the ideal timetable along with the minimum dwell time at each station (a dash indicates that the train does not stop at that station).

Table 1: Ideal Timetables

Ideal Timetables

	Train A			Train B			Train C			Train D		
Station	**Arr.**	**Dep.**	**Min. Dwell**	**Arr.**	**Dep.**	**Min. Dwell**	**Arr.**	**Dep.**	**Min. Dwell**	**Arr.**	**Dep.**	**Min. Dwell**
1		8:02	-		8:00	-		8:02	-			-
2	8:14	8:16	2	8:08	8:08	-	8:12	8:14	2			-
3	8:26	8:29	3	8:14	8:18	4	8:20		-		8:18	-
4	8:41	8:43	2	8:26	8:26	-			-	8:28	8:30	2
5	8:48		-	8:30		-			-	8:35		-

For each train and each pair of consecutive visited stations, the ideal running times correspond to the running times of the ideal timetables. For the sake of simplicity, such running times are reported in Table 2.

Table 2: Ideal Running Times

	Train A	**Train B**	**Train C**	**Train D**
$1\ldots2$	12	8	10	-
$2\ldots3$	10	6	6	-
$3\ldots4$	12	8	-	10
$4\ldots5$	5	4	-	5

In order to solve this instance of the Train Timetabling Problem, you are provided with the arrival and departure headway times

$$\text{Arrival Headway Times} \qquad h_i^a = 3 \quad i \in S$$

$$\text{Departure Headway Times} \qquad h_i^d = 3 \quad i \in S$$

and with the profit pr_j of the ideal timetable, the penalty for each minute of shift π_j^{sh}, and the penalty for each minute of stretch π_j^{st} of each train $j \in T$ as described in Table 3.

Table 3: Ideal Profits and Shift/Stretch Penalties

	Train A	**Train B**	**Train C**	**Train D**
Ideal profit pr_j	100	500	5	5
Shift penalty π_j^{sh}	50	200	1	1
Stretch penalty π_j^{st}	30	200	2	2

Moreover, you are told that Train B, that passes through stations 2 and 4 without stopping (the minimum dwell time is 0), cannot stop at those two stations: so it can be stretched only at station 3.

Non-cyclical timetabling: Exercise

4. Provide a manual solution of the problem.
5. Comment on the profits and penalties assigned to the trains by the Train Operators.
6. Define the following components of the space-time graph:
   - the sets of arrival nodes R_i for each station $i \in S$;
   - the set of departure nodes D_i for each station $i \in S$;
   - the arc set A_j for train $j = 1$, also indicating the profit of each arc of the set A_j.