Danmarks
Tekniske
Universitet

DTU

42117 Transport Optimization
Project 2: On-Demand Multimodal Transit Systems with Rebalancing

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Part 0. Non-cyclical timetabling

Problem solution displayed in a space-time diagram

Figure 1 shows a manual solution to the problem where Train A is (blue), Train B is (red), Train C is (green) and Train D is (purple), dotted lines are used to represent where a train starts and ends, and full lines are used to represent the path of each train.

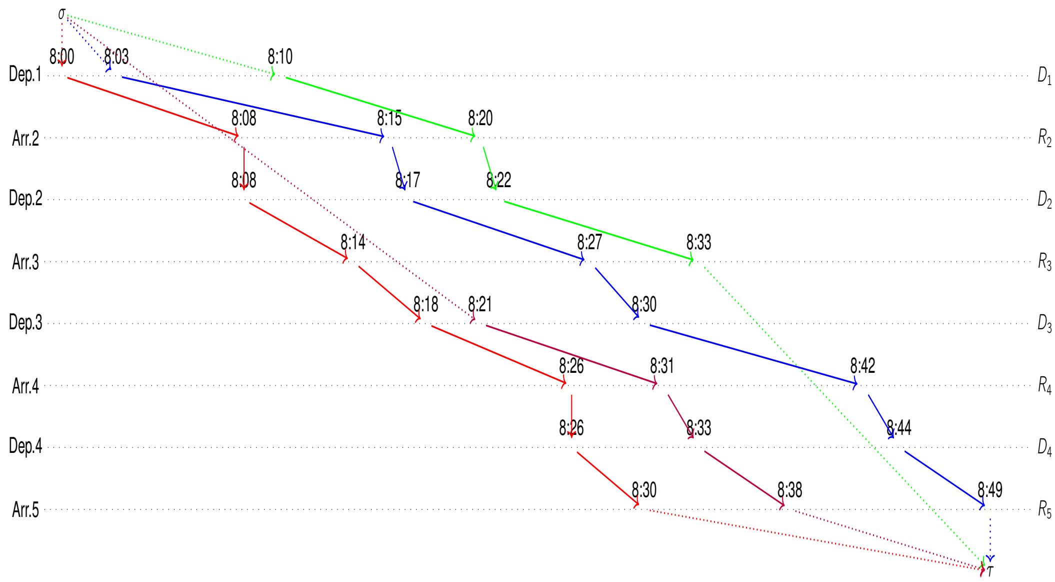


Figure 1: Manual solution to the problem where Train A is (blue), Train B is (red), Train C is (green) and Train D is (purple)

An event-activity graph of the same solution can be found in figure 2 where Train A is (blue), Train B is (red), Train C is (green) and Train D is (purple).

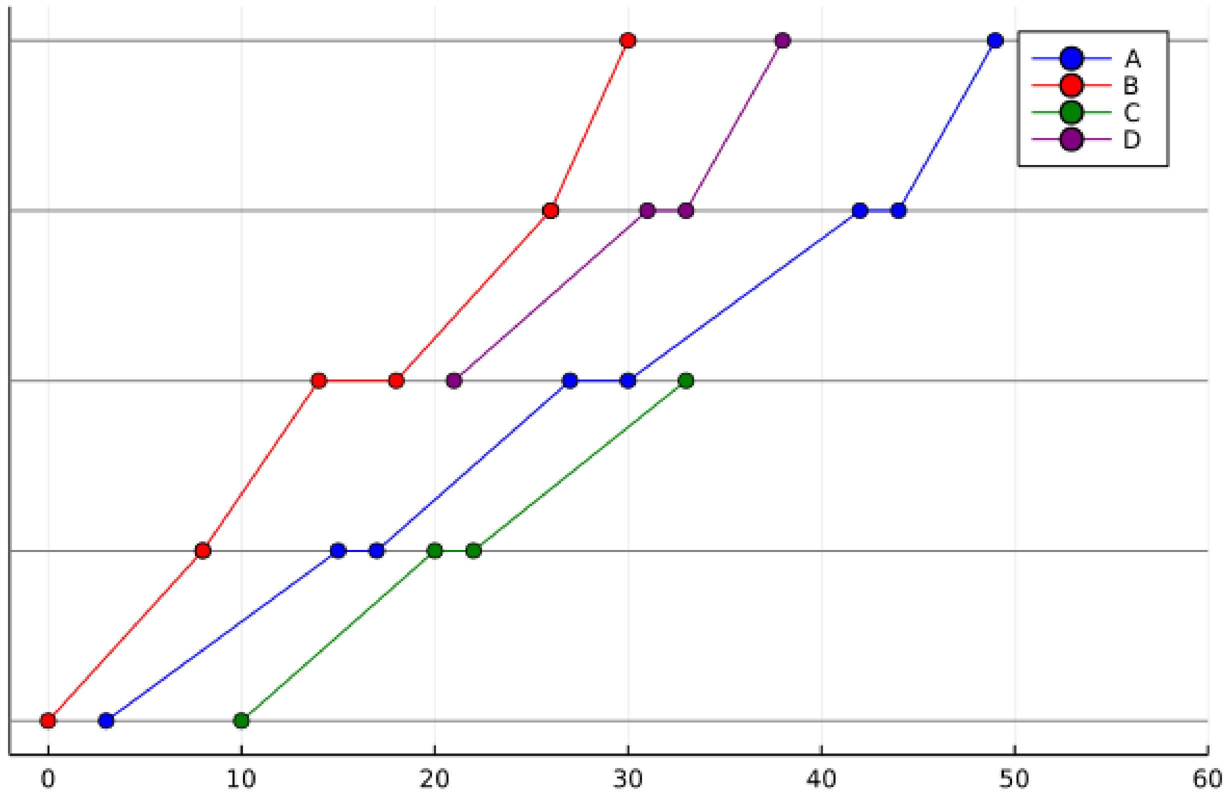


Figure 2: Problem solution in space-time diagram

Solution components for Train B

The set of arrival nodes R_i and the set of departure nodes D_i for each station $i \in S$ is displayed as a graph in Figure 3 and an overview of the arrival and depature times as well as each arcs profit can be found in Table 1.

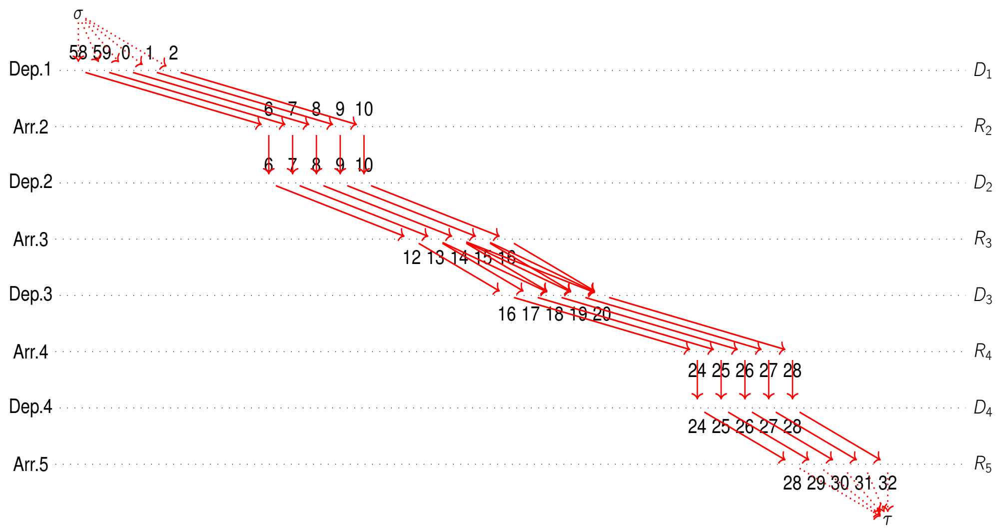


Figure 3: Components of the space-time graph for Train B

Table 1: Arrival and departure nodes for Train B

Train B	Station 1		Station 2		Station 3		Station 4		Station 5		Profit
	Arr.	Dep.	Arr.	Dep.	Arr.	Dep.	Arr.	Dep.	Arr.	Dep.	
Arc 1	-	7:58	8:06	8:06	8:12	8:16	8:24	8:24	8:28	-	100
Arc 2	-	7:59	8:07	8:07	8:13	8:17	8:25	8:25	8:29	-	300
Arc 3	-	7:59	8:07	8:07	8:13	8:18	8:26	8:26	8:30	-	200
Arc 4	**-**	**8:00**	**8:08**	**8:08**	**8:14**	**8:18**	**8:26**	**8:26**	**8:30**	**-**	**500**
Arc 5	-	8:00	8:08	8:08	8:14	8:19	8:27	8:27	8:31	-	300
Arc 6	-	8:00	8:08	8:08	8:14	8:20	8:28	8:28	8:32	-	100
Arc 7	-	8:01	8:09	8:09	8:15	8:19	8:27	8:27	8:31	-	300
Arc 8	-	8:01	8:09	8:09	8:15	8:20	8:28	8:28	8:32	-	100
Arc 9	-	8:02	8:10	8:10	8:16	8:20	8:28	8:28	8:32	-	100

Part 1. On-Demand Transit Network Design

A) Ridesharing and fleet sizing for On-Demand Multimodal Transit Systems

What is the problem studied in the paper?

The paper studies the design of On-Demand Multimodal Transit Systems (ODMTS), a concept for transportation systems where passengers travel from an origin to a destination using more than one mode of transportation.

What do the authors list as their main contributions?

The authors list their main contributions as:

- "it presents a framework to capture ridesharing in the design of an ODMTS, combining a" route-enumeration algorithm and a HALP;
- "it formulates the fleet-sizing optimization problem for the on-demand shuttles as a standard vehicle scheduling problem, and proposes an alternative flow formulation that is also totally unimodular but is constructed on a sparse underlying network, significantly enhancing its scalability";
- "it validates the proposed framework through a comprehensive set of experiments using real-world data from a local public transit system, including a sensitivity analysis of the most critical parameters and a comparison with the existing transit system";
- "it presents results that illustrate the potential benefits of ridesharing for ODMTS and the overall benefits in convenience and cost compared to the existing transit system."

Are the contributions practical/managerial, or advancing theory

Their contributions are an advancement of theory, by "capturing ridesharing in the design of an ODMTS". However, the theoretical advancement is applicable in a practical/managerial sense, meaning that the theoretical advancements can help plan out a multimodal transit system better than before.

Do the contributions have value outside of the application area?

The contributions have value outside the application area. Firstly it has value in related areas such as meal delivery, airplane scheduling and the multi-depot vehicle scheduling problem with this papers alternative arc elimination algorithm.

Do you think their case study is representative for the Danish setting (or your home country's setting)? Can their model be used in practice?

For the Danish setting there are some differences to the model, first being the capacity of busses, which in the model is uncapped. Secondly, if the model is used on a network of bus-hubs smaller than the total bus-network (eg. the network example for the project (DTU) problem), a lot of information is lost at the edge of the network. This information loss can lead to decision making that does not take all travelers into account. Even a network spanning the entirety of Sealand would not take into account the traffic coming from or leaving to Funen nor Sweden. The case study implementation relies on knowing demand a priori. Therefore implementation might encounter difficulties in assessing real world demand, which can fluctuate wildly due to a number of unknown reasons.

Do you think their case study results mostly represent an operator or a rider (user, passenger) perspective?

The case study results mostly represent an operator perspective, given that there is no consideration made to the price difference between a bus ride and shuttle ride for the individual. In the case study, regardless the assumably not insignificant price difference between each mode of transport, each rider will always need to travel from the same origin to the same destination. This is not representative of the real world and not the case from a riders perspective. On top of that, one of the goals is to optimize the number of shuttles in the fleet. In the results this leads to a slightly higher transit time, but lower costs for the operator. Therefore, we can conclude that the case study represents an operator, because that is only in the operator's interest.

List the differences between this paper and the project (DTU) problem.

The differences between the paper and the project we're studying for this assignment are that the paper is looking at a problem about an ODMTS that combines bus/train with a shuttle service. In the problem we're studying we are looking into an ODMTS that combines bus routes with cycling. In the paper they also develop a model to determine the number of shuttles needed, where we have the assumption that enough bikes will always be available. In the paper transfers have transfer times, which means that if I transfer is needed the cost will be higher. In the problem we're studying we ignore any transfer penalties.

From a sustainability perspective, in what way does the research contribute to increasing sustainability, and what could be added to increase the potential focus on sustainability?

The research contributed to increasing sustainability in a number of ways including; improving mobility for those who do not own a vehicle, increasing access to public transportation eg. for elderly people who otherwise would not be able to get to a bus stop.

When looking at the environmental impact aspects of sustainability, the research attempts to reduce the number of shuttles necessary during peak hours, by offering ridesharing opportunities resulting in shared shuttle rides and thereby achieving the reduction in demand for shuttle units.

On top of that, the transit times are reduced by 38%, which increases the probability of people choosing public transport over using their car. The rider aspect could be expanded by using incentives to use public transport, eg. by way of price reduction, would be an interesting parameter to see included in this case study. If it would be possible to cause modal swaps for riders from personal cars to public transport that would increase sustainability.

B) On-Demand Multimodal Transit System design around DTU

Mathematical model for the DTU ODTMS

The mathematical model uses the following data:

- N: Set of stations
- v_{bus}, v_{bike}: Respectively the speed of the bus or bike
- Connections: An NxN-matrix that has a 1 in it if a bus connection is allowed between and a 0 otherwise
- OD: An NxN-matrix that has the number of travellers per OD-pair in it
- Distances: An NxN-matrix that shows the Euclidean distance between each pair of stations
- Budget: The maximum budget to be spend on opening bus links
- BikeConnections: An NxN-matrix that is 1 if station j is a feasible hub for station i

The mathematical model uses the following decision variables:

- $x_{i,j}$: This binary variable is 1 if a bus link is opened between station $i \in N$ and station $j \in N$, otherwise 0.
- $y_{i,j,k,l}$: This binary variable is 1 if a bike connection is enabled between station $k \in N$ and station $l \in N$ as a last-leg link for travellers coming from origin station i going to destination j, otherwise 0.
- $q_{i,j}$: This binary variable is 1 if a direct bike connection is enabled between station $i \in N$ and station $j \in N$, otherwise 0.

- $z_{i,j,k,l}$: This binary variable is 1 if a traveller coming from origin station $i \in N$ going to destination $j \in N$ uses the link from $k \in N$ to $l \in N$, otherwise 0.

The mathematical model looks like this:

$$\text{Minimize} \sum_{k \in N} \sum_{k \in N} \Bigg(\frac{Distances_{k,l}}{v_{bus}} \cdot x_{k,l} + \frac{Distances_{k,l}}{v_{bike}} \cdot \sum_{i \in N} \sum_{j \in N} y_{i,j,k,l} + \frac{Distances_{k,l}}{v_{bike}} \cdot q_{k,l} \cdot OD_{k,l} \cdot \sum_{i \in N} \sum_{j \in N} z_{i,j,k,l}$$

$$\text{Subject to} \sum_{l \in N} z_{i,j,k,l} - 1|(i = k) + 1|(j = k) = \sum_{l \in N} z_{i,j,l,k} \qquad \forall i \in N, j \in N, k \in N$$

$$z_{i,j,k,l} \leq x_{k,l} + q_{k,l}|(i = k\&j = l)+ y_{i,j,k,l}|(BikeConnections_{i,l} = 1\&k = i\&l \neq j)+ y_{i,j,k,l}|(BikeConnections_{k,j} = 1\&j = l\&i \neq k) \qquad \forall i \in N, j \in N, k \in N, l \in N$$

$$\sum_{i \in N} \sum_{j \in N} x_{i,j} \cdot Distances_{i,j} \leq Budget$$

$$x_{i,j} \leq Connections_{i,j} \qquad \forall i \in N, j \in N$$

$$\sum_{k \in N} \sum_{l \in N} y_{i,j,k,l} \leq BikeConnections_{k,l} \qquad \forall i \in N, j \in N$$

$$x_{i,i} + q_{i,i} = 0 \qquad \forall i \in N$$

$$\sum_{k \in N} \sum_{l \in N} z_{i,i,k,l} + y_{i,i,k,l} = 0 \qquad \forall i \in N$$

$$y_{i,j,i,k} \leq \sum_{l \in N} x_{k,l}|(l \neq i) \qquad \forall i \in N, j \in N, k \in N$$

$$y_{i,j,k,j} \leq \sum_{l \in N} x_{l,k}|(l \neq i) \qquad \forall i \in N, j \in N, k \in N$$

$$q_{i,j} + \sum_{k \in N} y_{i,j,i,k} \leq 1 \qquad \forall i \in N, j \in N$$

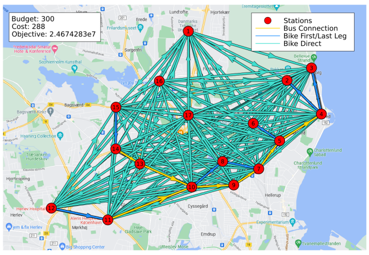
$$q_{i,j} + \sum_{k \in N} y_{i,j,k,j} \leq 1 \qquad \forall i \in N, j \in N$$

$$x_{i,j}, yi, j, k, l, q_{i,j}, z_{i,j,k,l} \in \{0, 1\} \qquad \forall i \in N, j \in N, k \in N, l \in N$$

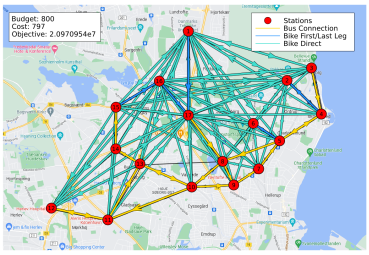
The first constraint ensures that what comes into a station, has to leave the station as well (except for the origin and destination). The second constraint limits the value of z, so it can only be 1 if either a bus link is available or if a traveller cycles directly from origin to destination or if the destination of the link is a feasible hub for this OD-pair. The third constraint limits the number of bus links being opened to the budget. The fourth constraint makes sure a bus link is only opened if this is allowed according to the connection matrix provided. The fifth constraint makes sure the value of y is only 1 if the bike route is feasible according to the feasible hubs in the bike connection matrix. The sixth constraint sets the variables $x_{i,j}$ and $q_{i,j}$ to 0 when $i = j$. The seventh constraint sets the variables $y_{i,j,k,l}$ and $y_{i,j,k,l}$ to 0 when $i = j$. The eighth and ninth constraint ensure that the first/last leg variable $y_{i,j,k,l}$ can only be 1 if there are respectively outgoing or incoming bus links to that node. The tenth and eleventh constraint ensure that for every OD-pair, maximum 1 of the variables $y_{i,j,k,l}$ (cycling for part of the journey) or $q_{i,j}$ (cycling directly from origin to destination) is 1 (or both are 0). The final constraint limits all decision variables to being binary variables.

Sensitivity analysis on the operating budget

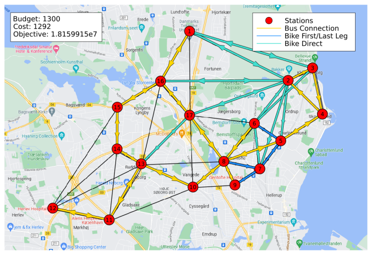
The model implementation can be found on Github.[1] The model is solved with a budget varying in the range $[300, 3300]$ and in increments of 500. The resulting optimal networks are shown in Figures 4(a)-4(g) and the quantitative results can be found in Table 2. The figures show cities as red numbered nodes, directional bus connections as yellow lines, directional directional first/last leg bike connections as dark blue lines and directional direct bike connections as turquoise lines. It is apparent, from both the figures and table, that the use of direct bike connection is very relevant while the budget is small. Namely, between $300 - 1300$ where the budget does not allows for enough bus connection to create a large enough busnetwork. Once the budget gets higher than 1800 the use for direct bike connections drop drastically and the amount of bus connections similarly increase, as can be seen in 2. Regardless of budget the first/last leg bike connections remain relatively steady. However, the number of these first/last leg connections does increase with the budget, due to the increased number of bus connections.



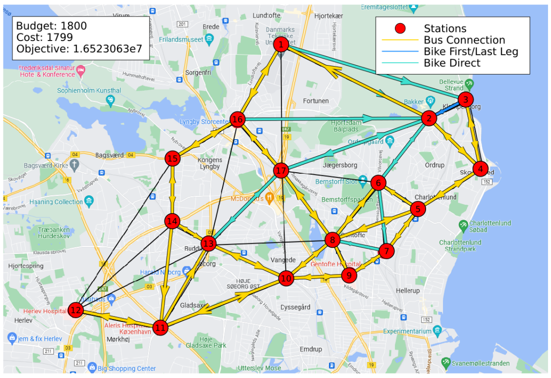
((a)) Busnetwork with a budget of 300



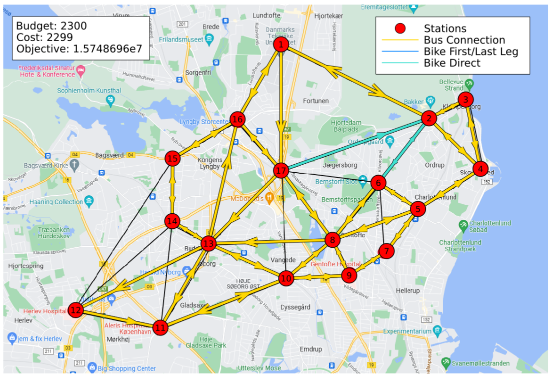
((b)) Busnetwork with a budget of 800



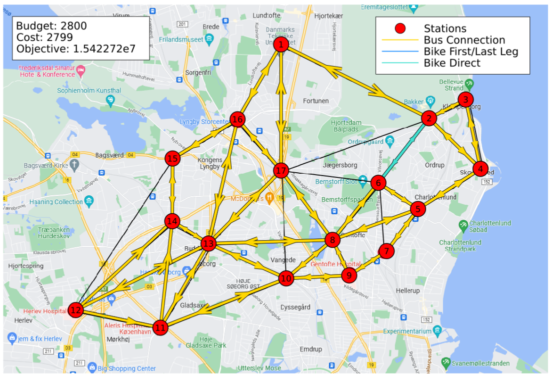
((c)) Busnetwork with a budget of 1300



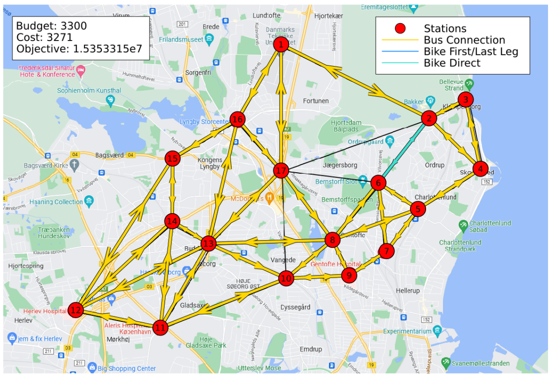
((d)) Busnetwork with a budget of 1800



((e)) Busnetwork with a budget of 2300



((f)) Busnetwork with a budget of 2800



((g)) Busnetwork with a budget of 3300

Figure 4: Busnetworks for all budgets

Table 2: Optimized busnetwork specificaitons at different budgets

			# Connections		
Budget	Cost	Objective value	Bus	Bike first/last leg	Bike direct
300	288	24674283	7	8	206
800	797	20970954	19	13	105
1300	1292	18159915	31	7	26
1800	1799	16523063	41	10	15
2300	2299	15748696	50	17	4
2800	2799	15422720	57	20	3
3300	3271	15353315	64	26	2

What would be the best busnetwork design, in your opinion?

There are a number of factors to consider when deciding on the best busnetwork design. One obvious factor is the cost of the network. A smaller network is obviously cheaper in terms of establishment and maintenance, but it is important to recognize the potential economic impact a smaller network could have on the customers it serves. Whether that be due to increased congestion on roads, increased commuter time or other similar factors that negatively impact commuters when reducing the size of the busnetwork.

Practicality is another aspect. The network seen in Figure 4(e) has bus connections that are not bi-directional, which is impractical for the commuter, due to potentially having drastically different commuter paths, depending on which direction in the network they travel. Additionally there are considerations to be made about the purpose of a busnetwork. An argument can be made that, the purpose of a busnetwork is to transport commuters from where they are, to where they want to be, effectively. With this assumption made, the idea of increasing the overall "time spent in the network", in order to reduce establishment and maintenance cost, is bad at best. For that reason it would be irresponsible to recommend a busnetwork design with a budget lower than 2300, as there is simply too much convenience to gain by increasing the cost of the busnetwork otherwise.

Taking a look at Table 2 all the networks with a budget higher than 2300 have a quite similar objective value, and thereby a similar total user inconvenience. There does however, seems to be diminishing return on investment, in terms of reducing in inconvenience with increased cost of busnetwork. This is to be expected, as the optimization will continually spend the budget in the most optimal place first. The Figures 4(e) and 4(f) show a significant difference in the number of uni-direcitonal bus connection between the network with a budget of 2300 and that with a budget of 2800. The increased practicality for the commuter accompanied by a not insignificant reduction in inconvenience makes the busnetwork with a budget of 2800 superior to the one with a lower budget. The decision between the largest two networks is however more complicated. Here the difference in inconvenience is not significant, yet the number of additional bus connections increase at the same rate as before. A case could be made for either of these two network, but given that a lot of the additional bus connections in the more expensive busnetwork, with a budget of 3300, occur along the edge of the network, the recommended optimal busnetwork design is that with a budget of 2800. However, we must acknowledge that there may exist a network with a budget between 2800 and 3300 that would be superior to the ones tested.

1 Part 2. Dynamic Bike Rebalancing

The problem researched in the paper by Contardo, Morency and Rousseau (2012) is a dynamic bike sharing balancing problem. In the paper they study a way to balance the number of bikes per station by relocating them with different vehicles to minimize the unmet demand (both shortage and excess of bikes at a station).

The main contributions of the paper are

- The introduction of a dynamic bike-sharing balancing problem based on the daily operations of a public bike-sharing system in its peak hours
- The mathematical formulations for that dynamic bike-sharing balancing problem
- The development of a scalable method that gives upper and lower bounds in a short time

The mathematical formulation uses the following data:

- K: Set of vehicles
- Q_k: Capacity of vehicle $k \in K$
- Q_k^0: Initial load of vehicle k
- u^k: Initial position of vehicle k
- V: Set of stations
- C_v: Capacity of station v
- C_v^0: Initial number of bikes at station v
- T: Set of time periods
- S: Set of states. Each state has three different components:
  - The initial positions of vehicles at time 0, $\{(u^k, 0) : k \in K\}$
  - Nodes for each station v at time t, $\{(v, t), v \in V, t \in T\}$
  - ϕ: A dummy node representing the end of a planned route in a schedule
- S_v: The subset of states made up of pairs (v, t), $v \in V, t \in T$.
- For each state $s \in S_v$, the predecessing and successing state can be calculated as

follows:

  - $pred(s) = (v(s), t(s) - 1)$ for $t(s) \geq 2$
  - $succ(s) = (v(s), t(s) + 1)$ for $t(s) \leq T - 1$

- f_s: Demand for bikes for state $s \in S_v$. $f_s \geq 0$ if s is a delivery point and $f_s \leq 0$ if s is a pickup point.

The paper uses the following decision variables:

- y_s^+, y_s^-: Variable that respectively show the shortage and excess of bikes at state $s \in S_v$
- $z_s \geq 0$: This is the number of bikes that is not used in state $s \in S_v$
- w_a^k: This is a binary variable that is 1 if a vehicle k uses arc a in its route
- $x_a^k \geq 0$: This is a continuous integer variable that shows the load of a vehicle k on arc a
- $\delta^+(s)$, $\delta^-(s)$: Respectively the set of arcs ending at s and the set of arcs starting at s for $s \in S$.

The data is used in the following mathematical model:

$$
\begin{aligned}
&\min \sum_{s \in \mathcal{S}_{\mathcal{V}}} (y_s^+ + y_s^-) && && (1)\\
&\text{s. t.} && &&\\
&\quad \sum_{k \in \mathcal{K}} \sum_{a \in \delta^+(s)} x_a^k - \sum_{k \in \mathcal{K}} \sum_{a \in \delta^-(s)} x_a^k - z_s + y_s^+ - y_s^- = f_s - C_{v(s)}^0 && s \in \mathcal{S}_{\mathcal{V}}, t(s) = 1 && (2)\\
&\quad \sum_{k \in \mathcal{K}} \sum_{a \in \delta^+(s)} x_a^k - \sum_{k \in \mathcal{K}} \sum_{a \in \delta^-(s)} x_a^k + z_{pred(s)} - z_s + y_s^+ - y_s^- = f_s && s \in \mathcal{S}_{\mathcal{V}}, t(s) \geq 2 && (3)\\
&\quad \sum_{k \in \mathcal{K}} \sum_{a \in \delta^+(s)} w_a^k \leq 1 && s \in \mathcal{S}_{\mathcal{V}} && (4)\\
&\quad y_s^+, y_s^- \geq 0 && s \in \mathcal{S}_{\mathcal{V}} && (5)\\
&\quad 0 \leq z_s \leq C_{v(s)} && s \in \mathcal{S}_{\mathcal{V}} && (6)\\
&\quad x_a^k \leq Q_k w_a^k && k \in \mathcal{K}, a \in \mathcal{A} && (7)\\
&\quad \sum_{a \in \delta^-(s)} w_a^k - \sum_{a \in \delta^+(s)} w_a^k = 0 && k \in \mathcal{K}, s \in \mathcal{S}_{\mathcal{V}} && (8)\\
&\quad \sum_{a \in \delta^-(u^k, 0)} w_a^k = 1 && k \in \mathcal{K} && (9)\\
&\quad \sum_{a \in \delta^-(u^k, 0)} x_a^k = Q_k^0 && k \in \mathcal{K} && (10)\\
&\quad x \geq 0 && && (11)\\
&\quad w \text{ binary.} && && (12)
\end{aligned}
$$

Figure 5: Mathematical model (Contardo, Morency and Rousseau, 2012)

The objective of the model is to minimize the total unmet demand (both the shortage and excess of bikes). The model has 11 constraints:

- Constraint 2 and 3: These constraint ensure that the demand is met for each state and make sure that there is flow conversation. In case perfect balance is not possible, variables y_s^+ and y_s^- get assigned a value here.
- Constraint 4: Each node can only be visited by 1 vehicle in a time period.
- Constraint 5: The variables y_s^+ and y_s^- are non-negative.
- Constraint 6: The number of unused bikes z_s is non-negative and limited to the capacity the station they are at.
- Constraint 7: The load of a vehicle k on arc a is limited to the capacity of that vehicle multiplied with w_a^k, which is 1 if the vehicle is travelling that arc.

- Constraint 8: This constraint ensures that a vehicle k does not visit previously visited stations.
- Constraint 9: Every vehicle k is only used once.
- Constraint 10: This equals the vehicles load to the vehicles initial load for the first time period.
- Constraint 11: This sets x to a non-negative value.
- Constraint 12: This sets w to a binary value.

To apply this model to the DTU problem, we would need to have more information about the demand. Currently, we only know the total demand between an origin-destination pair, but this is not enough for the model. We need to know at which time the bike trips are taking place.

After the model is solved, the Dantzig-Wolfe decomposition and a pricing procedure to generate columns with negative costs is applied to the arc-flow formulation, which leads to a significant reduction in the number of constraints. The benefit is that the problems are solved significantly faster than the original problem.

A polytope θ is introduced that satisfies the constraints 7-12 in the original MIP and the other constraints are rewritten to accommodate to the new polytope. The benefit of this is that the number of constraints is now linear instead of cubic. However, the number of variables θ is exponential, which is why a pricing algorithm is developed to dynamically generate add variables θ in the same way as column generation.

In the pricing subproblem, we can find the constraints 7-12 that were removed from the master problem. This can be rewritten as a shortest path problem, because of the similarities in constraints. Because the space-time network does not allow negative cycles, the shortest path problem can be solved by a label-correcting algorithm.

The upside of the sequential approach is that the problems can be solved in a shorter computation time. However, by using the sequential approach, the gap between upper and lower bound might be very big, which means the solution is not very good.

Part 3. Column Generation Exercise

A) General

1. Describe which two main mathematical programming models are at the core of the column generation procedure. Provide the formulation, and indicate the relevant input and output they provide to each other.

To solve the Capacitated Vehicle Routing Problem, it is formulated as a Set Partitioning Problem to which column generation is applied by using the Pricing Problem to generate columns. The goal of the CVRP is to find a set of routes that serve all customers' demand without exceeding the vehicle capacity. The Set Partitioning Problem is shown in figure 6 and the Pricing Problem in figure 7.

$$
\begin{aligned}
\text{(SP)} \quad & \min \sum_{r \in \Omega} c_r \theta_r \\
\text{s.t.} & \sum_{r \in \Omega} a_{ir} \theta_r = 1 \quad \forall i \in N \\
& \sum_{r \in \Omega} \theta_r \leq m \\
& \theta_r \in \{0, 1\} \qquad \forall r \in \Omega
\end{aligned}
$$

Figure 6: Set Partitioning Problem

With:

- N: Set of customers
- m: Number of vehicles
- Ω: Set of feasible CVRP routes
- c_r: Cost of route $r \in \Omega$
- a_{ir}: Binary variable that is 1 if customer $i \in N$ is served by route $r \in \Omega$
- $\theta_r \in \{0, 1\}$: Binary decision variable that is 1 if route $r \in \Omega$ is selected, 0 otherwise

$$
\begin{aligned}
\min \sum_{(i,j)\in A} & (c_{ij} - \lambda_i)x_{ij} \\
\text{s.t.} \sum_{(i,k)\in A} & x_{ik} = \sum_{(k,j)\in A} x_{kj} && \forall k \in N \\
\sum_{(0,j)\in A} & x_{0j} = 1 \\
\sum_{(i,0)\in A} & x_{i0} = 1 \\
& y_i + q_j + Mx_{ij} \le y_j + M && \forall (i,j) \in A : j \neq 0 \\
& 0 \le y_i \le Q && \forall i \in V \\
& x_{ij} \in \{0,1\} && \forall (i,j) \in A
\end{aligned}
$$

Figure 7: Pricing Problem

The linear relaxation of the Set Partitioning problem is the master problem. The reduced master problem is the master problem with a subset of routes. The dual problem of the RMP need to be solved to optimality and that is used as input for the Pricing Problem. When the Pricing Problem leads to negative reduced cost columns these variables are added to the RMP, after which the RMP and dual problem are solved again. If there are no negative reduced cost columns, the MP and DMP solutions that were found are optimal.

2. Do both problems need to be solved to optimality in each iteration? Why (not)? The DMP has to be solved to optimality each iteration, because this is used as input for the pricing problem. The pricing problem does not have to be solved to optimality in each iteration, because then columns of negative reduced cost can be produced by a heuristic algorithm.

3. In expectation the length of the column generation procedure can be reduced by adding multiple columns in each iteration. This can be done by adding a constraint to the appropriate problem. State to which of the two problems described in the first task this should be added. Formulate an example of a constraint that can be added to one of the problems in task 1.1 that allows finding multiple columns.

B) Subset row cuts

1. Can a subset row cut be added for the above solution? If yes, state which one. If not, explain why not.

We look at the subset of customers 3, 4 and 5. We add up the θ from the columns where at least 2 out of 3 customers are visited. So, $\theta_1 + \theta_{10} + \theta_{12} + \theta_{14} + \theta_{15} = \frac{2}{3} \le 1$. This means there is no violation, so the subset row cut can be added. Figure 8 shows this.

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15
c1	0	0	0	1	1	1	1	0	0	0	0	0	1	0	0
c2	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0
c3	1	0	0	0	1	0	0	0	0	1	0	0	0	1	1
c4	0	0	1	1	0	0	0	0	0	1	1	1	0	1	0
c5	1	0	0	0	0	1	0	0	1	1	0	1	1	0	1
c6	0	1	0	0	0	0	0	1	1	0	0	0	0	0	1
c7	1	0	0	0	0	0	1	1	0	0	0	0	1	0	0
c8	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
θ	0	0	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0

Figure 8: Subset Row Cuts

2. When adding a currently violated subset row cut to the column generation, could you in the next iteration find the same optimal solution for the linear relaxation of the set partitioning formulation of the VRPTW? What about a different solution with the same objective value of 29?

By adding a currently violated subset row cut to the column generation, it will lead to an infeasible solution, so you will not find the same solution. It would be possible to find a different solution with the same objective value.

References

[1] V. N. Malthe Jelstrup, “42117 transport optimization project 2.” https://github.com/Stinth/Transport-Optimization.