MS-E2122 - Nonlinear Optimization, Exercises Week 1

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Homework 1.1

a

The implementation of the problem in Julia can be found in file `homework1_1_a.jl`.

b

Using different initial values we can arrive in different local optima. One optimum can be found by setting initial property values in pooling nodes to zero. This way we achieve the objective function value of 4138.8921 and the flows and property values are as follows.

```
Flows:
x(s1,p1) = 1.0
x(s2,p1) = 0.0
x(s3,p1) = 3.0
x(s4,p1) = 0.0
x(s5,p1) = 2.3497
x(s6,p1) = 0.0
x(s7,p1) = 3.0
x(s8,p1) = 0.0
x(s9,p2) = 3.0
x(s10,p2) = 3.0
x(s11,p2) = 0.0
x(s12,p2) = 1.2899
x(s13,p2) = 3.0
x(s14,p2) = 3.0
x(s15,p2) = 0.0
x(s16,p2) = 0.0
x(p1,t1) = 5.1794
```

```
x(p1,t2) = 4.1704
x(p2,t1) = 4.8206
x(p2,t2) = 8.4693

Property values:
q1[p1] = 2.2553
q2[p1] = 24.8716
q1[p2] = 0.6885
q2[p2] = 35.5101
q1[t1] = 1.5
q2[t1] = 30.0
q1[t2] = 1.2055
q2[t2] = 32.0
```

If we set the initial property values to 50 we arrive at a solution with objective function cost of 4772.5457 and the following flows and property values.

```
Flows:
x(s1,p1) = 3.0
x(s2,p1) = 0.0
x(s3,p1) = 1.7083
x(s4,p1) = 0.0
x(s5,p1) = 3.0
x(s6,p1) = 2.2917
x(s7,p1) = 3.0
x(s8,p1) = 0.0
x(s9,p2) = 3.0
x(s10,p2) = 3.0
x(s11,p2) = 3.0
x(s12,p2) = 2.3258
x(s13,p2) = 3.0
x(s14,p2) = 0.6742
x(s15,p2) = 0.0
x(s16,p2) = 0.0
x(p1,t1) = 0.0
x(p1,t2) = 13.0
x(p2,t1) = 11.0
x(p2,t2) = 4.0

Property values:
q1[p1] = 3.7376
q2[p1] = 32.6154
q1[p2] = 0.4919
```

```
q2[p2] = 30.0
q1[t1] = 0.4919
q2[t1] = 30.0
q1[t2] = 2.9739
q2[t2] = 32.0
```

c

The implementation of this problem can be found in file `homework1_1_c.jl`. When we solve this problem we find an optimum with objective function cost of 4993.2634 and flows and property values

```
Flows:
x(s1,t1) = 1.0
x(s2,t1) = 0.0
x(s3,t1) = 0.0
x(s4,t1) = 0.0
x(s5,t1) = 1.0
x(s6,t1) = 0.0
x(s7,t1) = 0.0
x(s8,t1) = 0.0
x(s9,t1) = 1.0
x(s10,t1) = 2.6612
x(s11,t1) = 3.0
x(s12,t1) = 0.3772
x(s13,t1) = 1.0
x(s14,t1) = 0.0
x(s15,t1) = 0.9615
x(s16,t1) = 0.0
x(s1,t2) = 2.0
x(s2,t2) = 0.0
x(s3,t2) = 0.0
x(s4,t2) = 0.0
x(s5,t2) = 2.0
x(s6,t2) = 0.0385
x(s7,t2) = 0.0
x(s8,t2) = 0.0
x(s9,t2) = 2.0
x(s10,t2) = 0.3388
x(s11,t2) = 0.0
x(s12,t2) = 2.6228
x(s13,t2) = 2.0
x(s14,t2) = 3.0
```

```
x(s15,t2) = 0.0
x(s16,t2) = 3.0

Property values:
q1[t1] = 1.3562
q2[t1] = 30.0
q1[t2] = 1.801
q2[t2] = 32.0

Flow to t1 = 11.0
Flow to t2 = 17.0
```

We can see that the objective function increases if we remove the pooling nodes. This is probably because there are less restrictions and we can directly select the oil types that yield us the best results in the terminal nodes. In the problem with pooling nodes you had to distribute to two nodes from a single pool which required some kind of a compromise.

Homework 1.2.

a

The problem implementation can be found in file `homework1_2.jl`.

The objective function describes a sort of a balance between aversion of risk or the volatility of the portfolio and the expected return. The value λ describes the weight with which you want to balance these two traits. Naturally if you want to emphasize the return of the portfolio you must invest in stocks that have higher expected return but coincidentally this also means higher risk as higher return stocks are more usually more volatile. This also holds the other way. If you want a smaller risk portfolio, then you must invest in safer stocks. These stocks, however, usually come with a smaller expected return which also lowers the expected return of the portfolio.

b

The expected return of the portfolio and the volatility as a function of λ can be seen in figures 1 and 2. We can see that as lambda increases or we begin to emphasize return instead of risk aversion, the expected return increases but so does the volatility.

c

In figure 3 we can see the volatility of the portfolio as a function of the expected return. This tells a similar story as the ones before: the higher the expected return we want the more risky investmets we must make.

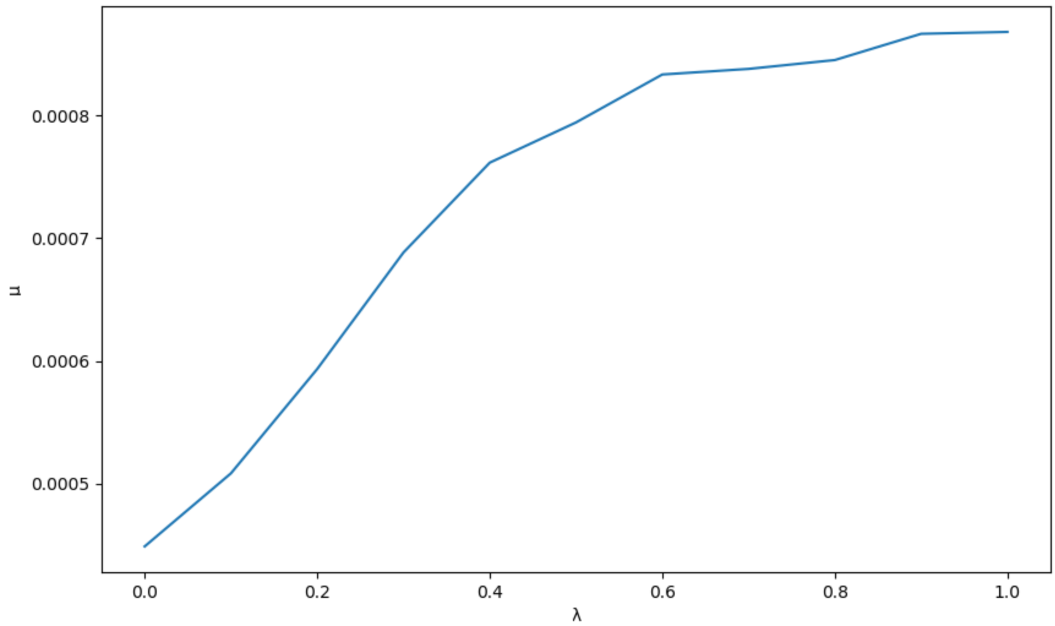

Figure 1: The expected return of the portfolio as a function of λ

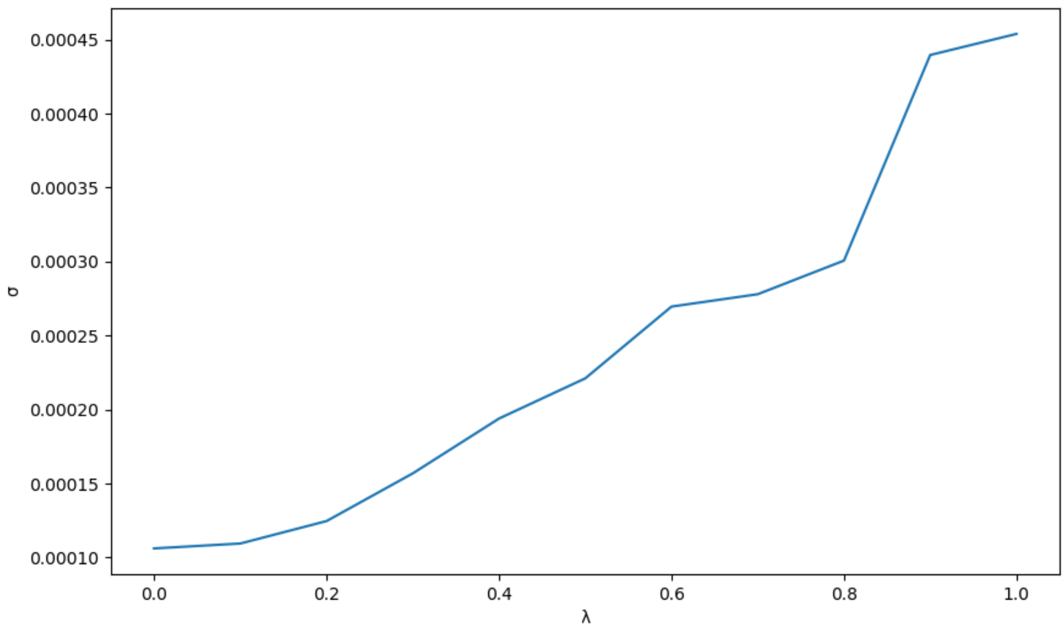

Figure 2: The volatility of the portfolio as a function of λ

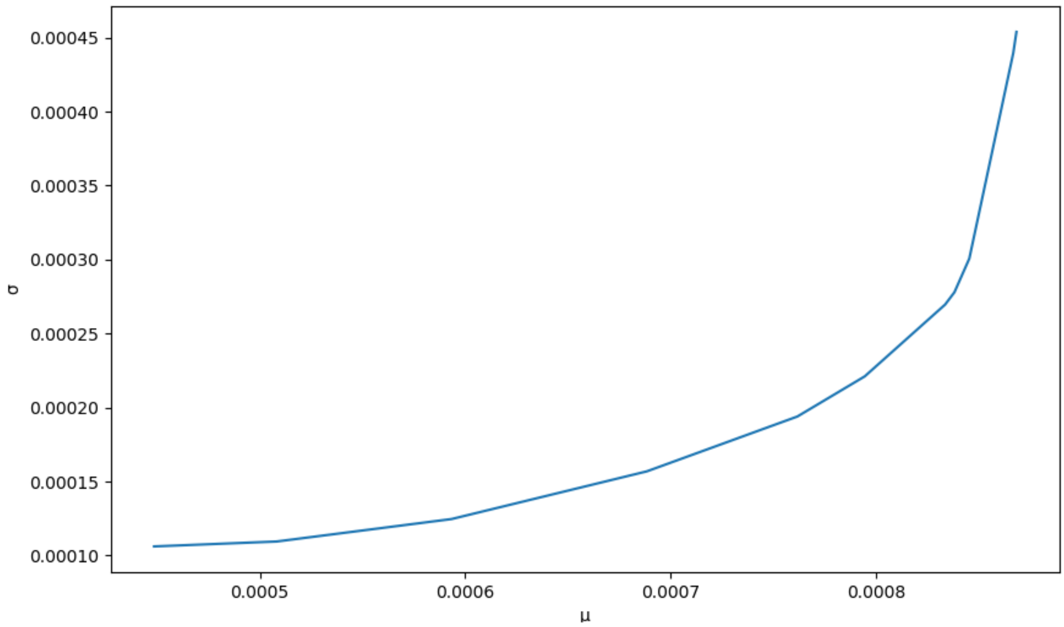

Figure 3: The volatility of the portfolio as a function of expected return