Exercise 2 – Queueing models and Output analysis

2.1

Consider a single server queuing model. Customers arrive at random time intervals to a service station. One customer at a time is served while others form a first-in-first-out type of queue. Inter-arrival time of customers as well as service times follow the exponential distribution with mean values of λ and μ, respectively.

Implement the model to simulate the delay in queue for the first *n* customers arriving at the service station.

Notice, that a very simple recursion can be used: Let D_i be the delay in queue for the i^{th} customer, S_i the service time and T_i the inter-arrival time between $i+1^{st}$ and i^{th} customers. It holds:

$$D_{i+1}=(D_i+S_i-T_i)^+=max\{D_i+S_i-T_i,0\}$$

Use your model to estimate the average delay in queue when n=100, λ=1 and μ=0.9. Repeat the simulation to obtain multiple observations on the average delay. In addition, construct a confidence interval for the estimate based on Student's t-distribution as described in lecture 1.

2.2

Build a model of the following inventory system: A company uses a stationary (s,S)-policy, where s is the order point and S the order-up-to point. In other words, if the inventory level drops below s the company orders an amount of S-*inventory* new items.

Use the following assumptions in your simulation:

- The amount of items in the inventory is reduced according to demand that follows a discrete distribution and takes a value of 1 with probability of 1/6, 2 with 1/3, 3 with 1/3, and 4 with 1/6. The time between consecutive demands is exponentially distributed with a mean of 0.1 months.
- You may assume that inventory is either controlled continuously or at fixed time intervals.
- Order cost per item is \$3. Additionally, each order involves a fixed cost of \$32.
- Order delay is uniformly distributed between 0.5 and 1 month.
- Unsatisfied demand is backlogged.
- Holding cost (for items in the inventory) is \$1 per item per month.
- Shortage cost (for backlogged demand) is \$5 per item per month.
- Initial inventory is 60 items.

Simulate the system to define the 95% confidence interval for the difference in average monthly cost of operation during 120 months between policies (20,40) and (20,80).

2.3

A computer center is equipped with four identical mainframe computers. The number of users at any time is 25. Each user is capable of submitting a job through a terminal every 15 minutes, on the average, but the actual time between submissions is exponential. Arriving jobs will automatically go to the first available computer. The execution time per submission is exponential with mean 2 minutes. Build a simulation model of the system to determine the following measures of performance

a) The probability that a job is not executed immediately on submission.
b) The average time until the output of a job is returned to the user.
c) The average number of jobs awaiting execution.
d) The percentage of time the entire computer center is idle.
e) The average number of idle computers.

Note: in this exercise we estimate steady-state performance measures. Use Welch's procedure to first determine an appropriate warm-up period for the simulation. Then, estimate the performance measures using the replication/deletion approach.

2.4 (Demo)

Extend the model you built in 2.1. as follows:

- Observe the average queue length during the simulated time period
- Observe the utilization of the service station, i.e., the fraction of time that a customer is being served.
- Allow multiple servers in your model.

Observe the differences in average queuing delay, average queue length and server utilization between the following cases, *k* being the number of servers

1. k=1, n=100, λ=1 and μ=0.9 (the same as 2.1.)
2. k=2, n=200, λ=0.5 and μ=0.9 for both servers