- Report in maximum of 2 pages
- The total value of the assignment is 6 points
- You can write your answers either in Finnish, Swedish or English
- Deadline for this assignment is Thursday, March 22nd, 2018 at 16:00.
- Return your report via MyCourses

Assignment 3.2 – Importance sampling

Consider the simulation of the queue length process of a single server queueing system. Customers arrive at exponentially distributed inter-arrival times with rate λ. Service time is exponentially distributed with rate μ. The queue length X_i after ith state transition is

$$X_i = \begin{cases} X_{i-1}+1 & \text{with probability } p = \dfrac{\lambda}{\lambda+\mu} \\ X_{i-1}-1 & \text{with probability } 1-p = \dfrac{\mu}{\lambda+\mu} \end{cases}$$

Determine the probability γ that the queue length exceeds a given threshold c during a regenerative cycle. The cycle starts just after the first customer enters the queue and ends either when queue length returns to 0 or hits $c+1$:

$$\gamma = P(X_T > c),\ T = \inf_n \{ X_n > c \text{ or } X_n = 0 \},\ X_0 = 1$$

Use both standard Monte Carlo and Importance Sampling to estimate γ and compare the variance of the estimates. The Importance Sampling scheme is described as follows:

1. Using $p^* := \dfrac{\lambda^*}{\lambda^* + \mu^*}$ i.e.:

$$X_i = \begin{cases} X_{i-1}+1 & \text{with probability } p^* \\ X_{i-1}-1 & \text{with probability } 1-p^* \end{cases}$$

generate independent paths $(X_0^j, X_1^j, \dots, X_T^j)$, $j = 1, \dots, n$

- Let N_A^j be the number of arrivals in path j
- Let N_S^j be the number of service completions in path j

2. $\hat{\gamma}_{IS}(n) = \dfrac{1}{n} \sum_{j=1}^{n} I(X_T^j > c) \left(\dfrac{p}{p^*}\right)^{N_A^j} \left(\dfrac{1-p}{1-p^*}\right)^{N_S^j}$

, where $I(\cdot)$ is an indicator function.

Assume the original arrival and service rates are:

λ=1 and μ=5/4

For the Importance Sampling simulation:

λ^*=5/4 and μ^*=1

Consider c={3,5, and 7}. You may first verify your code by comparing your results to the analytic solution of c=3:

- Let π_i denote the probability of observing $X_T = c+1$ starting from state $X_0 = i$.
- Considering the underlying Markov chain of the queue length process, it is easily seen that:

$$\begin{cases} \pi_1 = p\pi_2 \\ \pi_2 = (1-p)\pi_1 + p\pi_3 \\ \pi_3 = (1-p)\pi_2 + p \end{cases}$$

- Solving for π_1 yields the desired probablity for $c = 3$,i.e., $\pi_1 = 0.173$.