Simulation

Assignment 4.1 – Life sentence prisoners

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In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats.mstats import normaltest
from scipy.stats import rv_continuous
from scipy.stats import expon, norm, poisson, gamma
from scipy.stats import linregress
%matplotlib inline
```

In [33]:
```
sentences = [3,2,5,8,1,5,5,4,3,4,3,4,12,8,6,16,5,5,7,7]
avg_lengths = [16.5, 11.7, 10.7]
```

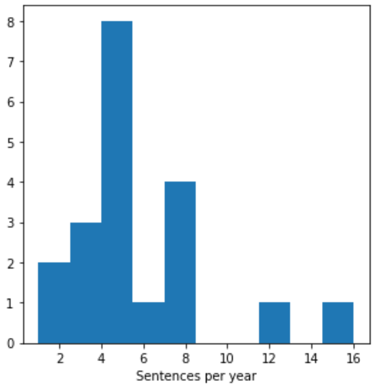
Let's take a shot in the dark and see if the sentences are normally distributed by doing a normality test.

In [41]:
```
normaltest(sentences)
```

Out[41]:
```
NormaltestResult(statistic=13.956503488528721, pvalue=0.00093193103363316959)
```

We ca see that the sentences are probably not normally distributed since the p-value is so small. Let's plot a histogram of the results and see what the distribution could be.

In [42]:
```
#plotting a histogram of the sentence lengths
plt.figure(1, (5,5))
plt.hist(sentences)
plt.xlabel("Sentences per year")
plt.show()
```

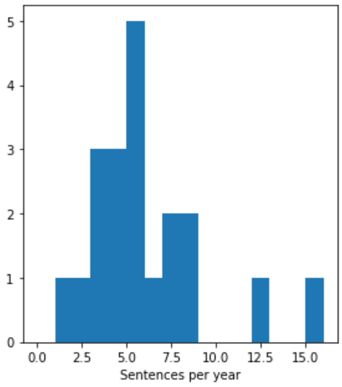


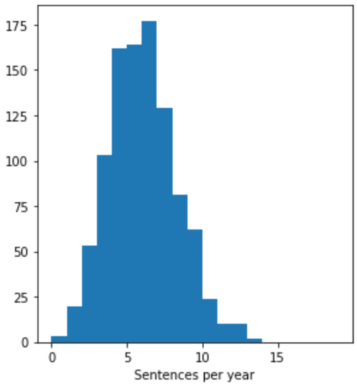
The distribution could be Poisson. Let's try fitting one.

In [43]:
```
print(np.mean(sentences))
```

```
5.65
```

In [44]:
```
#plotting the sentence lengths and the fitted poisson distribution
plt.figure(1, (10,5))
plt.subplot(121)
plt.hist(sentences, bins = range(0,17))
plt.xlabel("Sentences per year")

plt.subplot(122)
plt.hist((np.random.poisson(5.65, 1000)), bins = range(0, 20))
plt.xlabel("Sentences per year")
plt.show()
```





We can see that the amount of life sentences given each year is somewhat a poisson process with meadian value of 5 senteces per year. This means that the time between senteces is exponetially distributed with rate $\lambda = \frac{1}{5.65}$. Here we have a bit more low results than the originla sample like 1s and 2s, but not too many.

In [45]:
```
print(np.mean(avg_lengths))
print(np.std(avg_lengths))
```

```
12.9666666667
2.53157833947
```

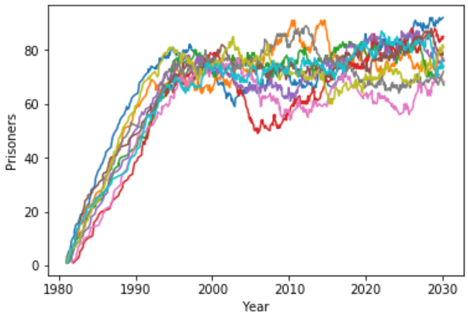
This is a pretty big assumption but let's say that the prison sentence lengths are normally distributed with mean of 13 years and standard deviation of 2.5 years. The duration is heavily influenced by the president and in the seventies it was Kekkonen and in the eighties it was Koivisto and most of the nineties it was Ahtisaari so the differences could be explained by just the different presidents.

Function for simulating prison system

In [46]:
```
def simulation(lambda_s, avg_l, std_l, stop):

    t = 1981
    next_customer = t + np.random.exponential(lambda_s)
    next_release = 10000000

    customers = []
    times = []
    history = []

    while t < stop:
        if next_customer < next_release:
            t = next_customer
            customers.append(t + np.random.normal(avg_l, std_l))
            customers = sorted(customers)
            times.append(t)
            history.append(len(customers))
            next_customer = t + np.random.exponential(lambda_s)
            next_release = customers[0]

        else:
            t = next_release
            customers.pop(0)
            times.append(t)
            history.append(len(customers))
            if len(customers) > 0:
                next_release = customers[0]
            else:
                next_release = 10000000
    return times, history
```

Driving the simulation with the deduced parameters until year 2030

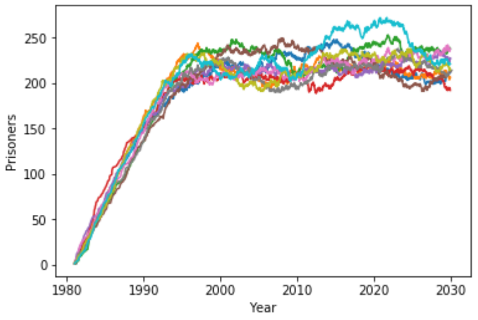
In [47]:
```
lambda_s = 1 / np.mean(sentences)
avg_l = 13
std_l = 2.5
stop = 2030

for i in range(0, 10):
    sim = simulation(lambda_s, avg_l, std_l, 2030)
    plt.plot(sim[0], sim[1])

plt.xlabel("Year")
plt.ylabel("Prisoners")
plt.show()
```



From the plot we can see that the prison system settles to around 60 to 80 prisoners serving the life sentence. This result seems reasonable considering the data at hand and assuming constat rate of life sentences but actually there were over 200 prisoners serving the life sentence in the year 2014 [1]. This is three times as many as the simulation would suggest.

Let's try increasing the ammount of people taken in yearly.

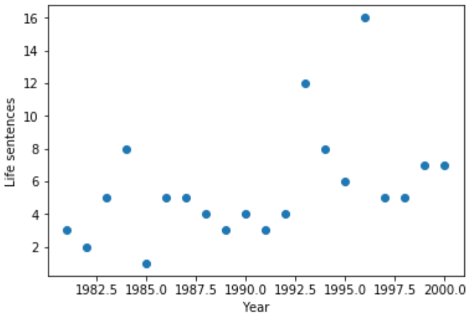
In [48]:
```
lambda_s = 0.06
avg_l = 13
std_l = 2.5
stop = 2030

for i in range(0, 10):
    sim = simulation(lambda_s, avg_l, std_l, 2030)
    plt.plot(sim[0], sim[1])

plt.xlabel("Year")
plt.ylabel("Prisoners")
plt.show()
```



If we increase the amount of people getting life sentence to about 20 per year we get naturally a much higher steady state. However this is still not correct since 20 prisoners per year is a gross overestimation for 80's and 90's and 200 prisoners is reached by mid 90's. Also there seems to be no steady state in the real data so this not reflect the too well.

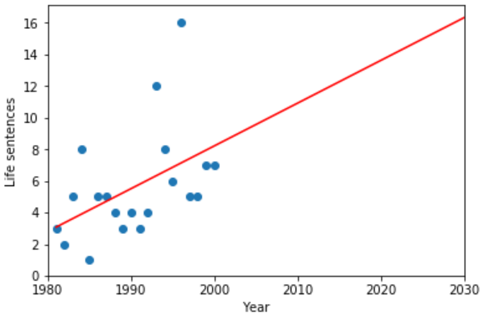
So far we have assumed that the average amount of yearly life sentences would remain constant. However, this might not be a valid assumption. Let's plot a scatter plot of the amount of people people who get the life sentence yearly.

In [49]:
```
time = list(range(1981,2001))
plt.scatter(list(range(1981,2001)), sentences)
plt.xlabel("Year")
plt.ylabel("Life sentences")
plt.show()
```



If we try really hard we can see that there is an increasing trend in life sentences. Let's try fitting a linear model to the data.

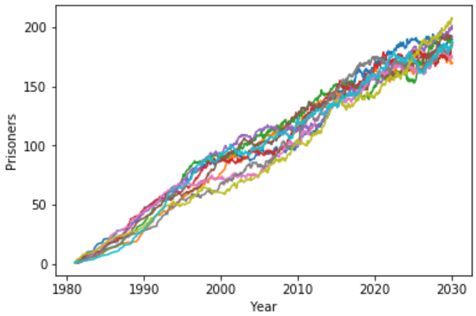
In [50]:
```
line = np.polyfit(time, sentences, 1)

pred_time = np.array(range(1981, 2031))
pred = pred_time * line[0] + line[1]

plt.plot(pred_time, pred, c = "red")
plt.scatter(time, sentences)
plt.xlim(1980, 2030)
plt.xlabel("Year")
plt.ylabel("Life sentences")
plt.show()
```



Let's create a new simulation function that will incorporate the increasing rate of life sentences. This can be done with just minor changes.

In [51]:
```
def simulation2(lambda_s, avg_l, std_l, stop):

    t = 1981
    next_customer = t + np.random.exponential(lambda_s[int(t) - 1981])
    next_release = 10000000

    customers = []
    times = []
    history = []

    while t < stop:
        if next_customer < next_release:
            t = next_customer
            customers.append(t + np.random.normal(avg_l, std_l))
            customers = sorted(customers)
            times.append(t)
            history.append(len(customers))
            next_customer = t + np.random.exponential(lambda_s[int(t) - 1981])
            next_release = customers[0]

        else:
            t = next_release
            customers.pop(0)
            times.append(t)
            history.append(len(customers))
            if len(customers) > 0:
                next_release = customers[0]
            else:
                next_release = 10000000
    return times, history
```

In [52]:
```
lambda_s = 1 / pred
avg_l = 13
std_l = 2.5
stop = 2030

for i in range(0, 10):
    sim = simulation2(lambda_s, avg_l, std_l, stop)
    plt.plot(sim[0], sim[1])
plt.xlabel("Year")
plt.ylabel("Prisoners")
plt.show()
```



Now we get a result that is much closer to the truth since the form is about right as there is a constant increase in the amount of prisoners. However the model is still a bit lacking since this model achieves the level of prisoners that were in prison 2014 by 2030. The change that has led to the current situation is not clearly visible in the dataset at hand so I don't think there are any more reasonable assumptions that could be made based on the data.

Sources:

[1] http://www.rikosseuraamus.fi/material/attachments/rise/julkaisut-monisteetjaraportit/c93POV0xz/2014-5_ELINKAUTISVANKIEN_UUSINTARIKOLLISUUDESTA.pdf (http://www.rikosseuraamus.fi/material/attachments/rise/julkaisut-monisteetjaraportit/c93POV0xz/2014-5_ELINKAUTISVANKIEN_UUSINTARIKOLLISUUDESTA.pdf)